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A CONSTITUTIVE MATERIAL MODEL FOR NONLINEAR FINITE ELEMENT STRUCTURAL ANALYSIS USING AN ITERATIVE MATRIX APPROACH

Herbert A. Koenig
University of Connecticut
Storrs, Connecticut 06268

Kwai S. Chan
Southwest Research Institute
San Antonio, Texas

Brice N. Cassenti
United Technologies Research Center
East Hartford, Connecticut 06108

Richard Weber
Pratt & Whitney
East Hartford, Connecticut 06108

A unified numerical method for the integration of stiff time-dependent constitutive equations is presented. The solution process is directly applied to a constitutive model proposed by Bodner. The theory confronts time-dependent anelastic behavior coupled with both isotropic hardening and directional hardening behaviors. Predicted stress-strain responses from this model are compared to experimental data from cyclic tests on uniaxial specimens. An algorithm is developed for the efficient integration of the Bodner flow equation. A comparison is made with the Euler method for integrating these relations. Additional comparisons are made with the model developed by Walker using the Euler integration method. An analysis of computational time is presented for the three algorithms.

I. Introduction

The development of constitutive models for the use in the structural analysis of aircraft gas turbine engine components has been an on-going process for many years.

Recent investigators have chosen to combine the physical aspects of the various non-linear effects into a unified theory in which all phenomena are coupled. One such theory was proposed by Bodner and Partom (1). This theory forms the basis for the investigations in this paper.

The usual approach for the integration of this type of model is either an Euler integration technique of direct "marching" or the second-order Adams-Moulton predictor-corrector technique (2).

II. Method

A new approach which uses a matrix integration technique has been presented by Tanaka (3). This approach solves for all the variables at the same time, thereby proposing to shorten the computational process and result in a stable integration scheme. The process should allow for larger time steps to be chosen than the Euler

technique to achieve the same accuracy. The process is also stable. Therefore short computational times are the goal to be achieved for a given problem. Unified computative models such as

$$\dot{\underset{\sim}{\sigma}} = \underline{D} \{ \dot{\underset{\sim}{\varepsilon}} - \dot{\underset{\sim}{\varepsilon}}^{N} \} \tag{1}$$

$$\dot{\underset{\sim}{\varepsilon}}^{N} = \underline{f} \{ \sigma, \underline{x}, T \} \tag{2}$$

$$\dot{\underset{\sim}{x}} = \underline{g} \{ \dot{\underset{\sim}{\varepsilon}}^{N}, \underline{x}, T \} \tag{3}$$

may be treated by this technique.

Equation (3) may be enhanced by including work hardening and recovery formats, i.e.,

$$\dot{\underset{\sim}{x}} = \underline{h} \{ \sigma, \underline{x}, T \} \dot{\underset{\sim}{\varepsilon}}^{N} - \underline{r} \{ \sigma, \underline{x}, T \} \tag{4}$$

Algorithm:

A numerical algorithm has been developed to numerically solve for the state variables represented in equations (2) - (4) subjected to a time - varying total strain function.

Equations (2) - (4) are formed into a matrix of the form

$$M\ x = b$$

Where M is the system Jacobian matrix, x is the vector of unknown variables, and b is the solution and error correction matrix.

This matrix is successively reformed and inverted to affect the solution of the state variables. No attempt is made to iterate on the matrix construction during a particular strain incremental step. Thus, the process constructed herein is a matrix extension of an Euler method in which all variables are calculated simultaneously based upon a driver of the state variables from the previous strain state.

Results:

The basic disadvantage in using Tanaka's NONSS (Noniterative Self-Correcting Solution) technique is the solution time which is excessive. The execution times for the load cases studied are summarized in Table I. The times shown are for a single load history at 2000 F, a strain rate of 4 x 10 -5 per second, a strain range of .6%, and a R-ratio of minus infinity. All computation times shown in Table I represent actual numerical computation time, the computer time spent for I/O and for accessing the clock were subtracted since the three routines differed in their respective I/O burdens. The convergence criterion used in each of the three routines was identical, namely the calculated out-of-plane stress for the uniaxial specimen was forced to be within a small tolerance of zero.

The first 5 cases shown in Table I summarize the experience with the NONSS integration method. It can be seen that the method is in general slower than the other two routines studied (Bodner's model using Euler integration shown as cases 6 through 10 and Walker's model using Euler integration shown as cases 11 through 15). In addition it can be seen that there is a tendency to fail to converge for coarser time steps. One difference between the NONSS routine and the other two studied is the lack of a self adaptive time step. This is evident in the comparison of the computing times in cases 3 and 4 where the number of integration time subincrements was doubled resulting in an approximately doubled computing gime. For the other two routines the number of subincrements was set initially for each increment, but underwent an automatic readjustment internally (self-adaptive time steps). This self adaptive feature often results in the same overall computation time even though the initial subincrement value was high. Inclusion of a self adaptive time step with the NONSS method would likely have prevented the convergence failures which occurred in cases 1 and 2 without significant loss of efficiency.

The present study supports the conclusion reached by Kumar et. al. (7) and by Imbrie, Haisler, and Allen (6); namely that the Euler forward difference or a minor modification of it is the most efficent method. Evidence of this for the NONSS and Euler methods is seen by comparing cases 5 and 9 in Table II. For case number 10, the self adaptive time stepping was suppressed in order to get a direct comparison of the two methods.

The cases 6 through 9 and 12 through 14 compare the response of Bodner's and Walker's models to various initializations of the subincrement parameter. For Walker's model the self adaptive time step quickly finds the lowest level and consequently completes the integration in the same amount of computing time regardless of the initial value of the subincrement. For Bodner's model this is also true to some extent. However an examination of case number 7 shows that initializing the subincrement parameter at too small a value can actually increase the total integration time. Thus there appears to be an inherent instability for Bodner's model which detracts from the overall efficiency. Additional study is required to identify the source of this instability and to determine if strategies exist which might prevent it.

A detailed analysis of the portions of each code where the computing time was actually spent showed that nearly 33% of computing time consumed by the NONSS method was spent in matrix inversion. This result suggests that considerable efficiency for the NONSS method could be gained if the inverted matrix were assembled directly. It was also noted that the algorithm used for Walker's model contained some redundant calculations which, if removed, would save approximately 20% of computing time.

References

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Table I

A Comparison of Computing Times For 3 Constitutive Model Routines

CASE #	THEORY	METHOD	CYCLES	INCREMENTS	SUB-INCREMENTS	TIME
1.	Bodner -Partom	NONSS.	2	80	2	failed
2.			2	80	3	failed
3.			2	80	4	3.50
4.			2	80	8	6.34
5.			2	320	1	3.53*
6.	Bodner -Partom	Euler (self-adaptive)	2	80	1	1.28
7.			2	80	3	1.62
8.			2	80	4	1.26
9.			2	80	8	1.26
10.			2	320	1	1.76
11.	Bodner -Partom	Euler self-adaptive, without directional hard-ening terms	2	320	1	1.43
12.	Walker	Euler	2	80	1	0.45
13.		(self-adaptive)	2	80	2	0.45
14.			2	80	8	0.45
15.			2	320	1	1.03**

* 0.78 to assemble matrix, 1.15 to invert matrix

** 0.81 when adjusted for excess material properties calculations