Bifurcations in Unsteady Aerodynamics–Implications for Testing

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SUMMARY

The various forms of bifurcations that can occur between steady and unsteady aerodynamic flows are reviewed. Examples are provided to illustrate the various ways in which bifurcations may intervene to influence the outcome of dynamics tests involving unsteady aerodynamics. The presence of bifurcation phenomena in such tests must be taken into consideration to ensure the proper interpretation of results, and some recommendations are made to that end.

INTRODUCTION

An understanding of unsteady aerodynamics is becoming increasingly important in the design of highly maneuverable aircraft. At present, however, unsteady aerodynamic flows are poorly understood, particularly when extensive changes (bifurcations) occur in the flow field. To develop an understanding of unsteady aerodynamics, it is important, first, to understand the various types of aerodynamics that can occur under conditions where the boundary conditions are steady as well as the types of bifurcations that can occur as the parameters representative of the steady boundary conditions, such as angle of attack, are changed. Second, the interaction of the bifurcations in the flow field with unsteady boundary conditions needs to be understood.

The present paper contains a review of the various types of aerodynamics that occur under steady-state boundary conditions and the various types of bifurcations that can occur under these conditions. Next, the roles that these bifurcations can play when the boundary conditions are unsteady are described. Then examples illustrating the impact of bifurcations on testing are introduced. Examples include cases of both forced and free oscillatory motions. Finally, some recommendations are made to assist the consideration of bifurcation phenomena in experiments involving unsteady aerodynamics.

AERODYNAMICS AND BIFURCATIONS WITH STEADY BOUNDARY CONDITIONS

Aerodynamics

Aerodynamic flows with steady boundary conditions can be categorized into five general types. They are: (1) steady-state single-valued, (2) steady-state multi-valued, (3) unsteady periodic, (4) unsteady quasi-periodic, and (5) chaotic. Figure 1(a) shows an example of the first type in which the response in lift coefficient C_L is steady (i.e., time-invariant) and a single-valued function of angle of attack α. An example of the second type is shown in figure 1(b). For a steady-state multi-valued response, there is a range of α within which the aerodynamic response, e.g., the lift coefficient C_L, can have two or more values for the same value of α. This is commonly referred to as static hysteresis. There are at least three different types of aerodynamic flows that are unsteady (i.e., time-varying) even though the boundary conditions are steady. The first of these is periodic unsteadiness. A common example is the periodic shedding of vortices in the wake behind a circular cylinder in cross flow. Vortex-shedding gives rise to a periodic variation of lift and a periodic component of drag. Another example of a periodic aerodynamic flow is illustrated in figure 1(c), showing the lift coefficient for an airfoil. At low α the lift coefficient is steady and is a single-valued function of α. Above some critical value of α, the flow separates from the upper surface of the airfoil. Depending on the Reynolds number, separation can give rise to a periodic lift about some nonzero mean value, as is indicated schematically in figure 1(c). Periodic flows normally first occur with a single frequency. However, in more complex situations, additional frequencies can occur, and if the additional frequencies are integer multiples of each other, we have what are referred to as multiply-periodic flows. On the other hand, flows in which the ratio of frequencies is nonintegral or irrational are referred to as quasi-periodic, because any finite sample of the time series of a coefficient will not contain any periodicity. However, a Fourier analysis or power spectrum will show the relevant frequencies. This case is not illustrated in figure 1.

The final case is that of chaos. Here, the flow or the aerodynamic response is aperiodic and its power spectrum will be very broad. The power spectrum may contain a peak at a frequency of a periodic state but the peak will be very broad and will not drop off to the background noise level indicative of experimental noise. The chaotic type of aerodynamic flow is illustrated in figure 1(d). Here again is shown the lift coefficient versus angle of attack for an airfoil. Again at low values of α, the lift is steady-state and single-valued. Separation will occur above a critical value of α, and, for a certain range of Reynolds numbers, the flow structure and hence the lift will be chaotic. According to the theory of nonlinear dynamical systems, the presence of what is called a strange attractor would be suggested by this type of flow. A more thorough description of chaotic flow properties and strange attractors is given in reference 1.

Bifurcations

In the above discussion, changes in flow have been described and illustrated in figure 1. These changes are often referred to as bifurcations. We have discussed the role of bifurcations in aerodynamics previously in references 1-4. The theoretical basis for our studies can be found in summary form in references 5-8. The following is a brief review.

Bifurcations can occur between any of the states illustrated in figure 1. In addition bifurcations can occur in two ways and hysteresis can occur in still another way. The two types of bifurcation are subcritical and supercritical. These are illustrated in figure 2. In the supercritical case (fig. 2(a)) a perturbation (labeled C) in a coefficient is zero until a critical value of a parameter representative of the steady boundary conditions, here α, is reached. At this point, solutions split into three branches. One branch remains the original solution (i.e., C = 0), but it is unstable and will not occur in practice. The other two branches, shown as symmetric around C = 0, represent stable perturbation solutions that depart from zero. The important point here is that departure from zero is continuous. That is, for a small change in the representative parameter (here, α) beyond the critical value, there is a corresponding small change in the perturbation solution.

In the case of subcritical bifurcation (fig. 2(b)), there is again a branching of solutions as the critical value of the parameter is crossed, but in this case all three solution branches are unstable at the critical point. However, the nonzero initially-unstable branches are multivalued, and they become stable beyond the turning points. As a result, after crossing the critical value of α, the flow field jumps to a new stable configuration, and hence there is a finite change in the perturbation solution for an infinitesimal change in α. This behavior is a source of static hysteresis. With further increases in α beyond the critical value, the perturbation solution continues to move along the stable branch. However, when α diminishes, the perturbation solution remains on the upper branch and returns to the lower branch only after α has diminished sufficiently below its critical value to pass below the value where the upper branch has a turning point.

There is another condition that leads to static hysteresis, namely, the fold. Note that in figures 2(a) and 2(b) both the subcritical and supercritical bifurcations have two branches beyond the critical value of α, reflecting a symmetry in the boundary conditions. Under these conditions, if we fix α at a value above the critical value and change another variable, e.g., the roll angle ϕ, the corresponding response (here the rolling-moment coefficient C_ℓ) will have the form shown in figure 2(c). Hence static hysteresis in the rolling-moment response will be present if the range of the roll-angle variation includes the turning points of the response.

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Importance of Bifurcations

The importance of bifurcations for changes in parameters such as the angle of attack is intuitively obvious. It is possible to make this intuition more mathematically precise by illustrating that bifurcation leads to a loss of Fréchet differentiability. Our use of indicial responses allows us to see this by inspection of figure 3. The top left part of this figure shows a plot of the angle of attack as a function of time ξ. At time τ and thereafter, the angle of attack is fixed at a constant value. In the bottom left part is shown the lift coefficient that results from this motion. The top right part of figure 3 again shows an angle-of-attack history. The history prior to time τ is the same as before, but now after time τ, the constant value of the angle of attack is increased by a small amount $\Delta\alpha$. The history of the lift response to this motion is shown at the bottom right of the figure. Now consider the ratio of the increment in lift to the increment in the angle of attack as the measuring time t goes to infinity for two cases. In the first case there is no bifurcation of the equilibrium flow at the given fixed value of $\alpha(\tau)$. In the second case a bifurcation occurs at $\alpha(\tau)$. In the first case we obtain the following relation.

$$\lim_{\Delta\alpha \to 0} \frac{\Delta C_L(t)}{\Delta\alpha} \quad \text{exists at} \quad t \to \infty$$

The limit in this case exists and is what is referred to as a Fréchet derivative. When a bifurcation occurs at $\alpha(\tau)$, the limit does not exist. This is easy to understand for subcritical bifurcation and for a fold, because in these cases the increment in C_L will be finite for an infinitesimal change in the angle of attack. For supercritical bifurcation, the loss of Fréchet differentiability is not as easy to see. Here we could have a bifurcation from a steady state to a periodic condition. In this case the equilibrium lift changes from a constant value to a mean value plus a time-periodic term. Then the limit may exist but it will be periodic with time, indicating that the form of the solution has to change.

BIFURCATIONS AND UNSTEADY BOUNDARY CONDITIONS

The role of bifurcations in cases where the boundary conditions are unsteady becomes exceedingly diverse and complex. We shall explore this role through examples of two distinct classes of flows having forced and free boundary conditions. In particular, the classes will be represented by examples taken from experiments involving forced and free oscillations.

Forced Oscillations

To fully understand the role of forced unsteady boundary conditions we must examine examples from forced oscillations under two sets of conditions: first, when

the oscillations occur in domains where no bifurcations exist under steady-state boundary conditions; second, when the oscillations occur over a domain within which a bifurcation exists under steady-state boundary conditions. In both of these cases, we will be concerned with low and high reduced frequency.

Forced oscillations in domains without bifurcations are examined for two reasons: first, to establish the traditional background of unsteady aerodynamics; second, to introduce the idea of bifurcations that are induced by the forced-oscillation boundary condition. The first case to be examined is an oscillation at low reduced frequency in a domain where the aerodynamic response to steady boundary conditions is steady and single-valued. This is the traditional case most often encountered in experiments involving unsteady aerodynamics. In this case the effect of the forced oscillation is to introduce a phase shift in the aerodynamic response. The out-of-phase term is the traditional damping term (e.g., $C_{m_q} + C_{m_{\dot{\alpha}}}$). Recent examples of this type of testing can be found in reference 9. For high reduced frequencies, it is possible for the oscillation itself to cause a bifurcation. This topic is being studied in depth for lower-order dynamical systems (e.g., ref. 10). An example in aerodynamics would be an airfoil undergoing pitching oscillations in which the peak angle of attack is just below the angle of attack where separation occurs. Here it is easy to see that if the frequency is high enough, the flow could be forced to separate on the downstroke.

If forced oscillations are performed in a domain where there are no bifurcations but the base flow is periodic, the aerodynamic behavior is analogous to that where the base flow is steady. Here again, for low reduced frequencies there is a phase shift (here, low is relative to the frequency of the periodic base flow). For high reduced frequencies near that of the base flow, the forcing can introduce a bifurcation. In this case, since the base flow is already periodic, the bifurcation will be either to a quasi-periodic or to a chaotic flow. These conditions have been well studied for lower-order dynamical systems (e.g., ref. 10).

The first case to be considered where oscillations occur in a domain that contains a bifurcation involves static hysteresis. The origin of the hysteresis could be either a subcritical bifurcation or a fold. If the oscillation has a low reduced frequency, the effect on the flow again is to cause a phase shift and hence the appearance of a classic damping term. However, care must be exercised in testing to ensure that the hysteresis loop is correctly taken into consideration. The usual method of determining the damping term is to measure the power required to drive the oscillation, in which case the hysteresis will contribute a fictitious damping. The form of this result can be illustrated by considering an example from the open literature (ref. 11). Here roll damping was measured for the forced rolling oscillation of a fighter-type model in a wind-tunnel test. The measured effective damping coefficient is shown plotted in figure 4(a) as a function of the amplitude of the oscillation for several reduced frequencies. Note that there is a very strong effect of the reduced frequency, particularly at the lower values. This is a very good indication that a static hysteresis loop may have been present. These data were reassessed by Schiff and Tobak (ref. 12). They found that if the results were analyzed under the assumption that static hysteresis was present, the results

could be plotted as shown in figure 4(b). When plotted in this manner, the results for the different amplitudes should fall on straight lines that intercept the zero reduced-frequency line at the same point. This intercept point is related to the area of the static hysteresis loop. With the exception of one data point, these data appear to be consistent with existence of the assumed hysteresis loop. Unfortunately, no static aerodynamic data were taken during this experiment which might have confirmed the presence of hysteresis. This omission shows the importance of taking data with steady boundary conditions over the entire domain where forced oscillation testing is to be carried out, and in such a way as to cover all of the possibilities for the existence of hysteresis.

The last forced oscillation case to be considered is that of oscillating over a domain in which there is a bifurcation from a steady state to a periodic state. This is the case in experiments involving dynamic stall. Here, the effect of the oscillation is extreme. Results from a typical set of tests are shown in figure 5 (ref. 13). We see that results even at low reduced frequencies are unexpected. At a low reduced frequency, k, we expect that oscillation might result in a loop surrounding the static data. We see, however, that even at the low reduced frequency of k = 0.004, the lift on the stroke with increasing angle of attack nearly matches the static results, whereas on the stroke with decreasing angle of attack, the lift is much less than static results. These results indicate that the separation which occurs at an angle of attack of 12° has an even more pronounced effect on the downstroke than it does on the static results. The same effect is further indicated at the high reduced frequency of k = 0.25. Here we see that the flow remains essentially attached on the upstroke, reflected by the resemblance of the variation of lift coefficient on the upstroke to that for an unstalled airfoil. Further evidence is provided by the water-tunnel flow visualization shown in figure 5 (ref. 14). Essentially-attached flow is observed on the upstroke, whereas on the downstroke, enhancement of the separation is indicated by both the much lower lift and the extensive region of separated flow shown by the flow visualization. Further discussion of the role of bifurcations in dynamic stall can be found in reference 2.

Free Oscillations

An additional factor must be considered in this case, which is that the body itself is a dynamical system that the aerodynamic system is now forcing and vice versa. If the free oscillations occur in a domain where the aerodynamic response is steady and no bifurcations are induced, there is nothing much of interest that can happen. If the entire system is statically and dynamically stable, a perturbed motion will simply decay to zero.

If the aerodynamic response for the stationary body is periodic, then the possibilities are many. If, for example, the body is supported in a spring-mounted system, the natural frequency of which is less than the frequency of the aerodynamic response, the motion of the body may be quasi-periodic or even chaotic. This in fact is the case for a spring-mounted circular cylinder in crossflow and also the case for a free-to-roll model in a wind-tunnel test.

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The case of the circular cylinder in crossflow is well illustrated by the results of Van Atta and Gharib (ref. 15) (an example is shown in fig. 6). The left portion of this figure shows measurements of the velocity in the wake and the motion of the wire (the circular cylinder in this case) for the case of a well-damped motion. Both the time series and the power spectra are shown. For this well-damped case, the wake velocity shows a very pronounced frequency at the Strouhal frequency. However, the wire motion is nearly nonexistent (lower-left-portion plots). The power spectrum is flat and at a level commensurate with experimental noise. The right portion of figure 6 shows the results for the same conditions except that there is no damping. Here the power spectrum for the motion of the wire (lower-right-portion plots) shows a broad peak at a frequency that is near the Strouhal frequency. In addition, and more importantly, the power level is 10 dB higher than the noise level of the experiment (left portion of figure) over a very broad portion of the spectrum. These results show a classic example of chaotic behavior. The peak frequency occurs at the harmonic of the natural frequency of the oscillating wire nearest to the Strouhal frequency. Note that under this condition, the velocity measurements in the wake still show a frequency near the Strouhal frequency but now the signal shows a very broad spectrum. Hence the wake flow may be classified as being even more chaotic than the wire motion itself. This problem is currently under study at NASA Ames. The major thrust of the research is to model the aerodynamics and hence to arrive at a simple predictive model of this type of motion. If successful, this research can lead to a better understanding of the modeling of motions induced by dynamic stall. A simple single-degree-of-freedom motion with a quasi-static modeling of the periodic aerodynamics is described in reference 4. Questions still remain whether modeling at the level of a single degree of freedom can capture the chaotic behavior. A two-degree-of-freedom motion is also under analysis and here it is already clear that chaotic motion occurs with periodic aerodynamics.

Another example of a free oscillation being forced into chaotic-like motion is that of a wind-tunnel model mounted so that it is free to roll. Some results are illustrated in figure 7 (ref. 16), which shows a typical example of the rolling motion. This motion appears to have a well-defined frequency, but the amplitude builds up and decays in bursts. These bursts of motion occur erratically in a manner which is similar to the intermittent bursts of motion that are known to occur in certain simple low-order dynamical systems. The phenomenon is often referred to as intermittent chaos. In this case, the motion is thought to be driven by an oscillatory rolling moment caused by the phasing between oscillatory shock-induced separations on each of the wing panels. The existence of such a periodic aerodynamic driving mechanism creates a dynamic situation in very close analogy to that of the spring-mounted cylinder in crossflow. An appropriate modeling of the aerodynamics for this motion would provide a good candidate for further study of the occurrence of chaotic-like motions in a flight-dynamics setting.

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CONCLUDING REMARKS

The different types of aerodynamics and bifurcations that can occur under steady-state boundary conditions have been reviewed. Bifurcations were found to involve a loss of Fréchet differentiability. The impact of bifurcations on testing that involves unsteady aerodynamics was reviewed by means of examples of both forced and free oscillatory motions. The following summarizes our main observations.

Body motion can be critical when passing through a bifurcation. Existence of a subcritical bifurcation or a fold can lead to the measurement of a spurious damping if a hysteresis loop is ignored. Body motion combined with bifurcation may cause major changes in the characteristics of the aerodynamics, such as those that occur in dynamic stall. Moreover, bifurcations can lead to changes in the characteristics of the motion itself. We saw that periodic vortex shedding from an elastically-mounted circular cylinder in crossflow may cause the cylinder to undergo a chaotic motion. These observations lead us to the following recommendations concerning the consideration of bifurcation phenomena in experiments involving unsteady aerodynamics. (1) There should be a complete base of testing under static boundary conditions that encompasses all possibilities for the presence of hysteresis. (2) Tests need to be conducted in each domain or type of aerodynamics under consideration and across all bifurcation points. (3) Data analysis must allow for the presence of bifurcations to ensure the proper interpretation of results.

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a) SINGLE VALUED

C_L

α

b) MULTI-VALUED

α

c) PERIODIC

C_L

$f_a = f_{a_0} + A \sin \omega t$

α

HOPF BIFURCATION

d) APERIODIC/CHAOTIC

$f_a = f_{a_0} + f(t)$

α

ONSET OF A STRANGE ATTRACTOR
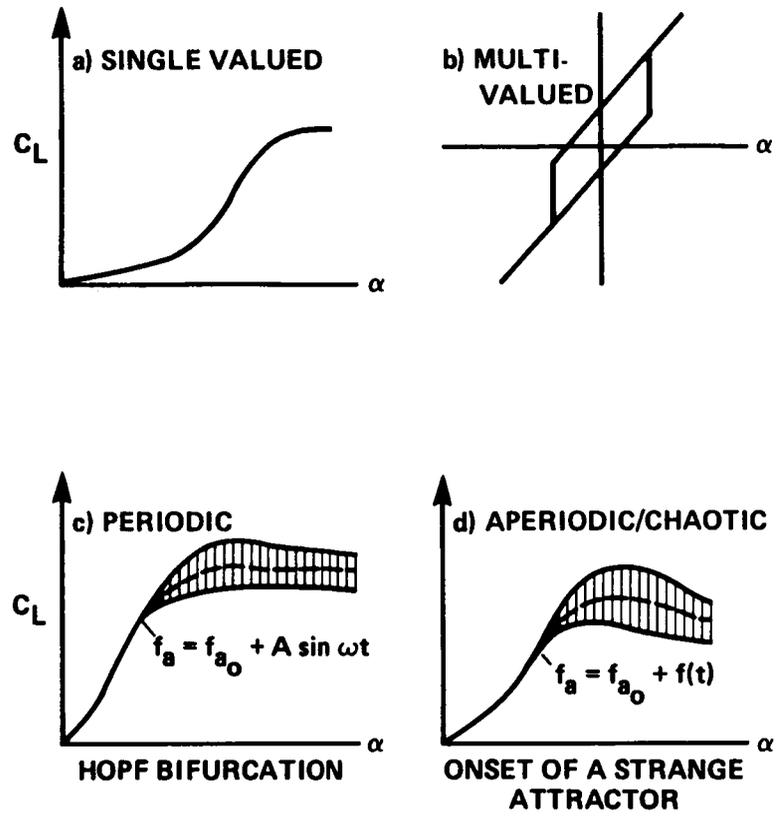
Figure 1.- Types of aerodynamic flows with steady boundary conditions.

a) SUPERCRITICAL   b) SUBCRITICAL

c) FOLD
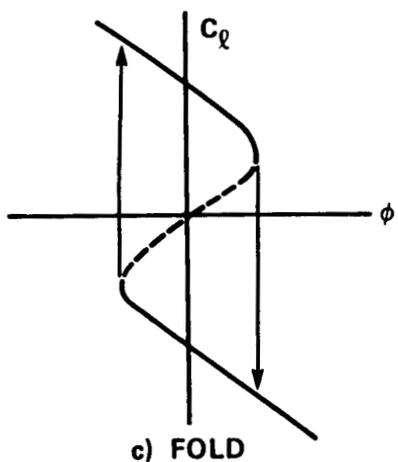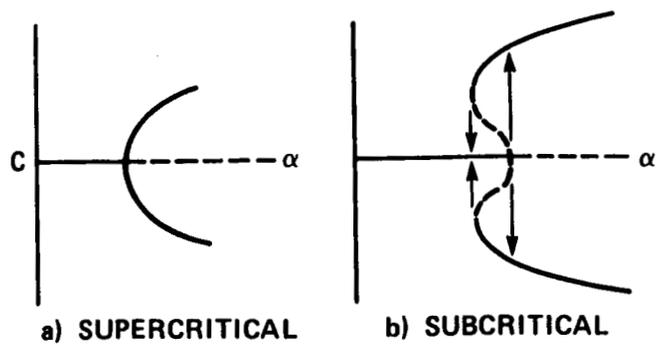
Figure 2.- Bifurcations and fold with steady boundary conditions.

$$\alpha_1(\xi) = \alpha(\xi); \ \xi < \tau$$
$$= \alpha(\tau); \ \xi \geqslant \tau$$

$$\alpha_2(\xi) = \alpha(\xi); \ \xi < \tau$$
$$= \alpha(\tau) + \Delta\alpha; \ \xi \geqslant \tau$$
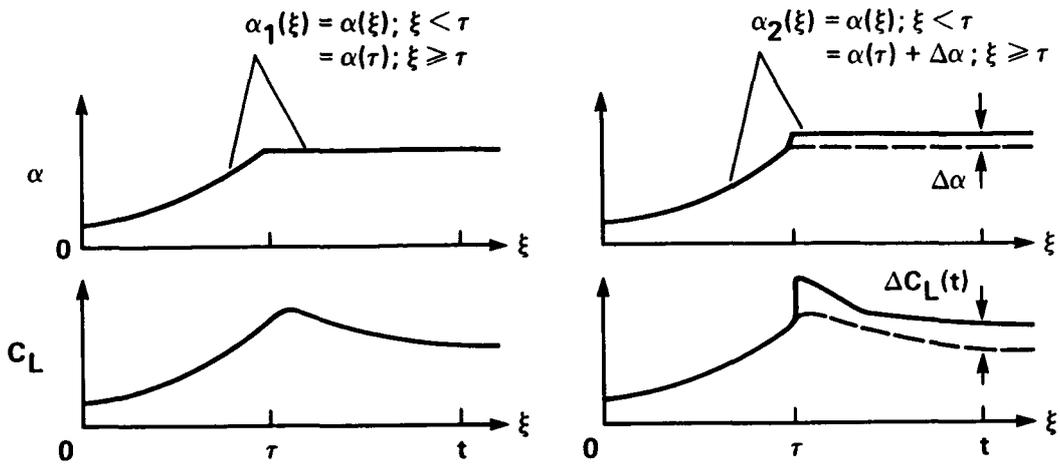
WITHOUT BIFURCATION AT $\alpha(\tau)$

$$\underset{\Delta\alpha \to 0}{\text{LIMIT}} \quad \frac{\Delta C_L(t)}{\Delta\alpha} \qquad \textbf{EXISTS AT } t \to \infty$$

WITH BIFURCATION AT $\alpha(\tau)$

$$\underset{\Delta\alpha \to 0}{\text{LIMIT}} \quad \frac{\Delta C_L(t)}{\Delta\alpha} \qquad \textbf{DOES NOT EXIST AT } t \to \infty$$

Figure 3.- Importance of bifurcations--indicial response approach.

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HYSTERESIS

$C_\ell(\infty; \psi, h, \sigma_0)$

$h = 0$ $h = 1$

ψ_c $-\psi_c$ ψ
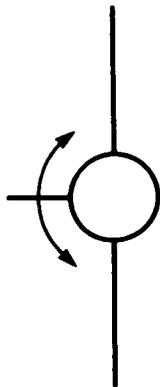
FORCED ROLL OSCILLATIONS

ψ_o, deg
○ 5.0
□ 10.5

$\psi_o^2 k (C_{\ell_\psi}^{\cdot})_{eff} \times 10^{-3}$

REDUCED FREQUENCY, k

b) DATA ANALYSIS WITH HYSTERESIS
(SCHIFF AND TOBAK)

k
○ 0.078
△ 0.109
□ 0.156 } REF. 18
◇ 0.203
△ 0.234

UNSTABLE

STABLE

$(C_{\ell_\psi}^{\cdot})_{eff}$

OSCILLATION AMPLITUDE, ψ_o, deg
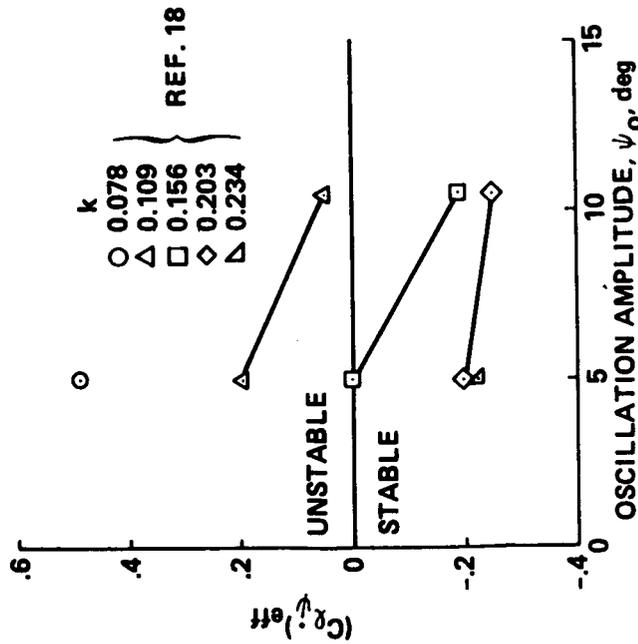
a) DATA ANALYSIS WITHOUT HYSTERESIS
(GRAFTON AND LIBBEY)
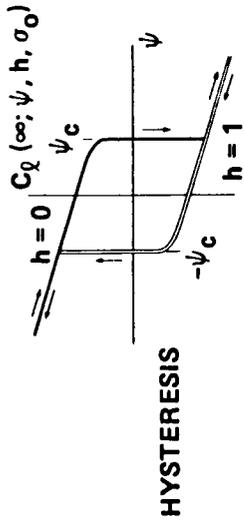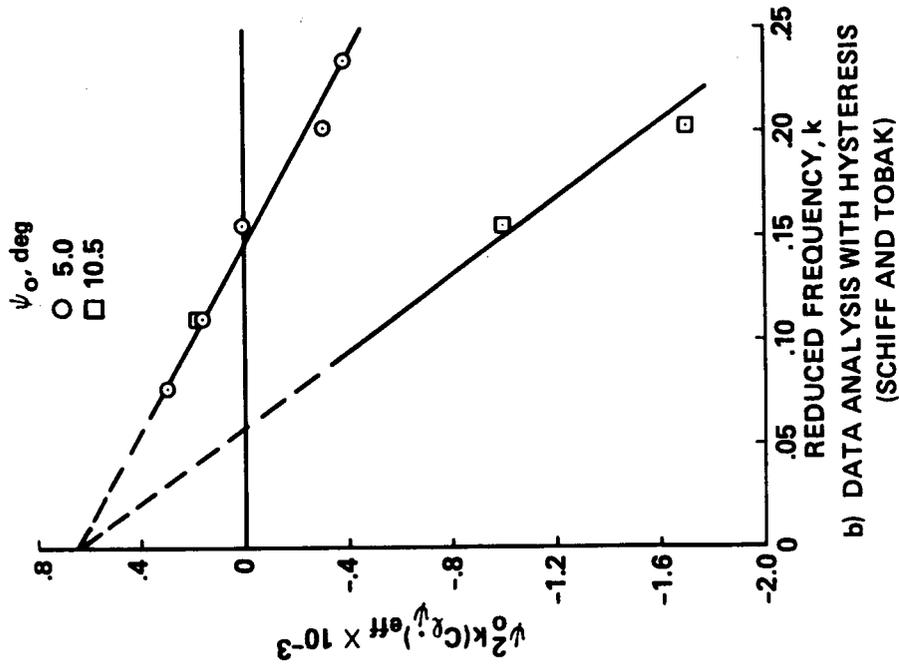
Figure 4.- Forced oscillations in the presence of hysteresis.
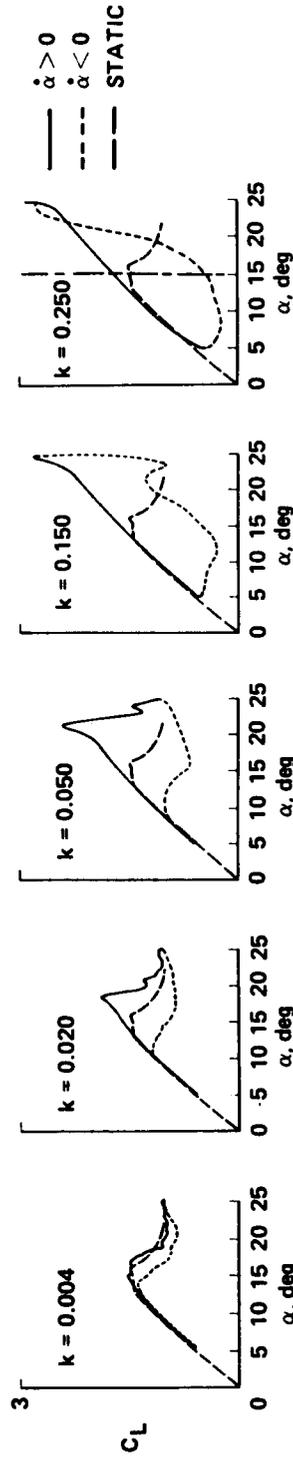
Figure 5.- Dynamic stall.

15
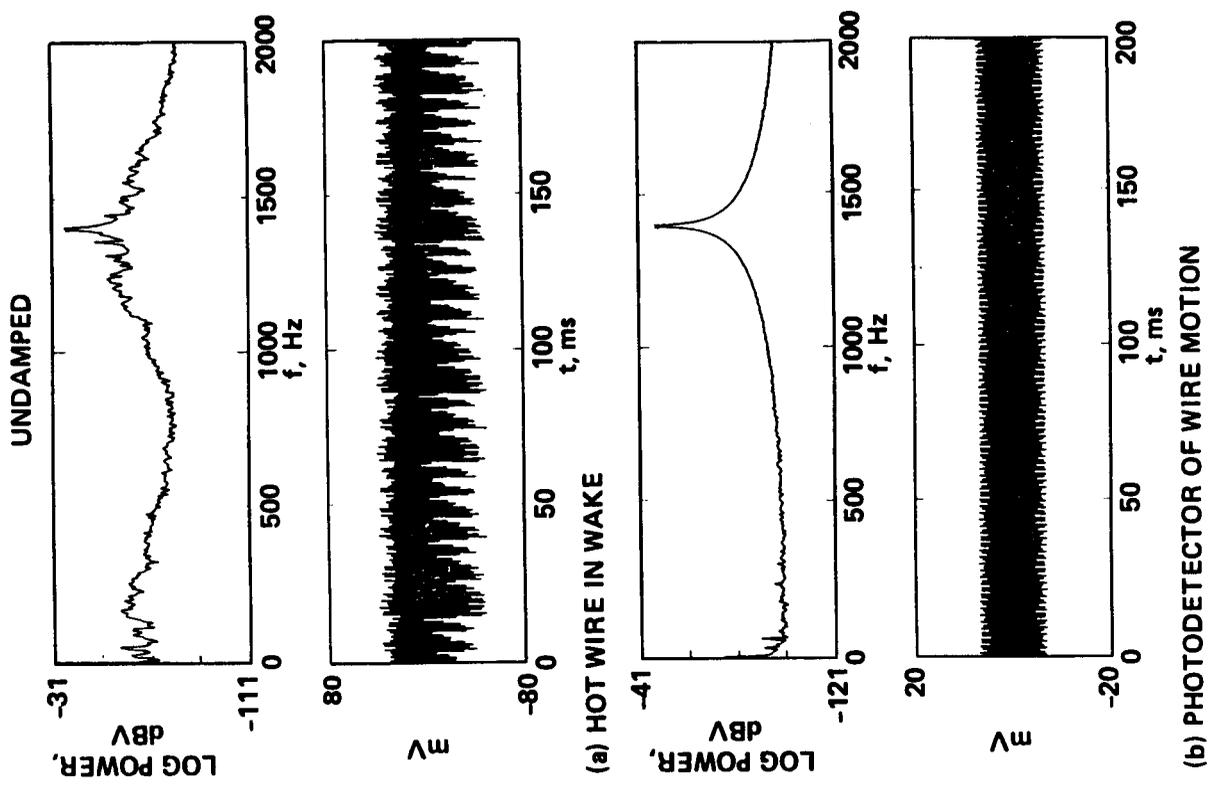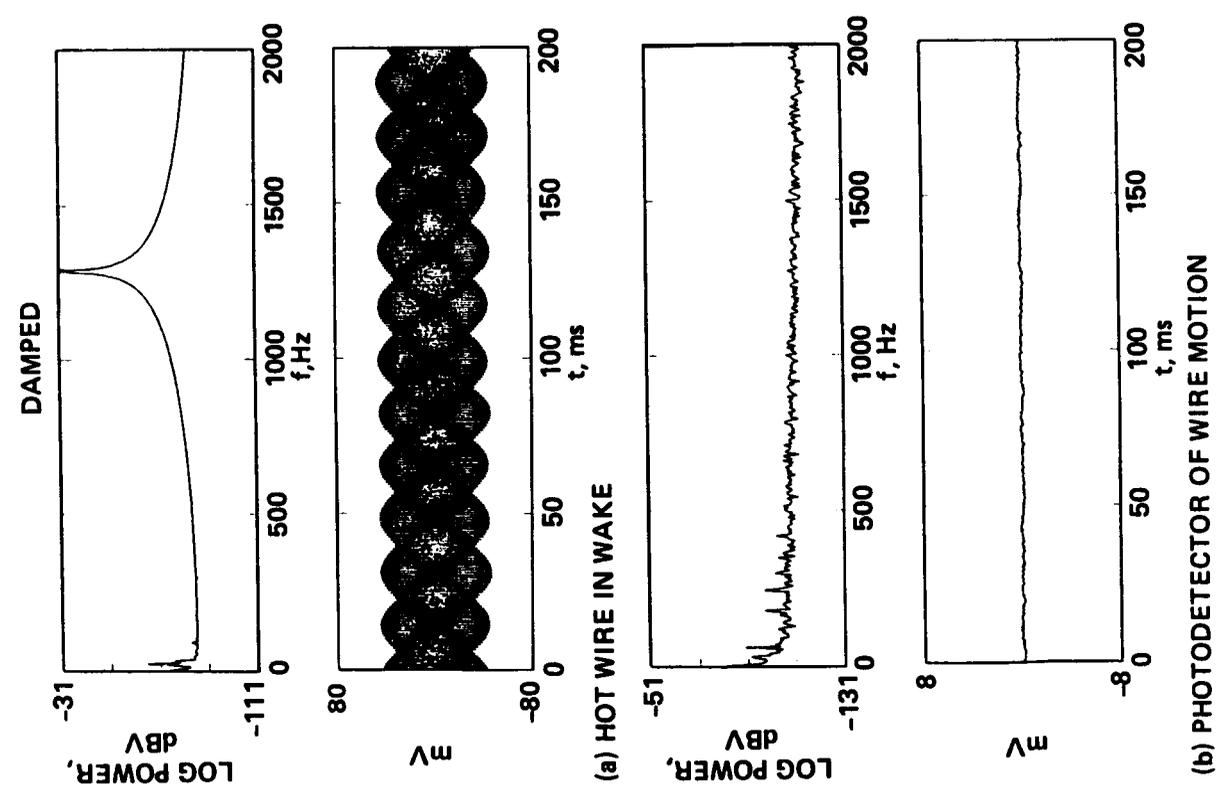
Figure 6.- Dynamics and aerodynamics of a circular cylinder in crossflow (ref. 15).

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REAR VIEW
OF
FREE OSCILLATION MODEL

SHOCK WAVE

SECTION A-A
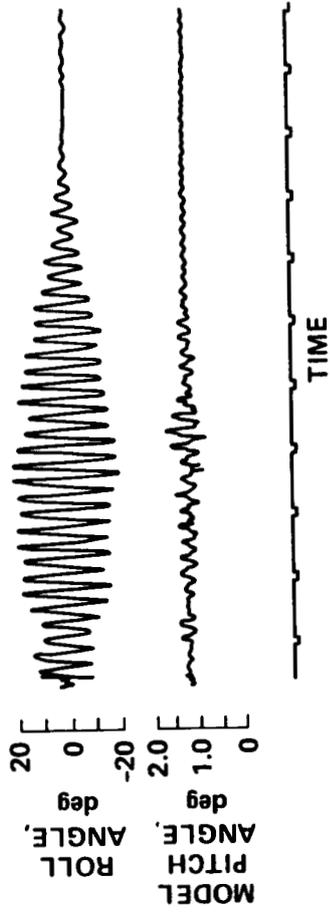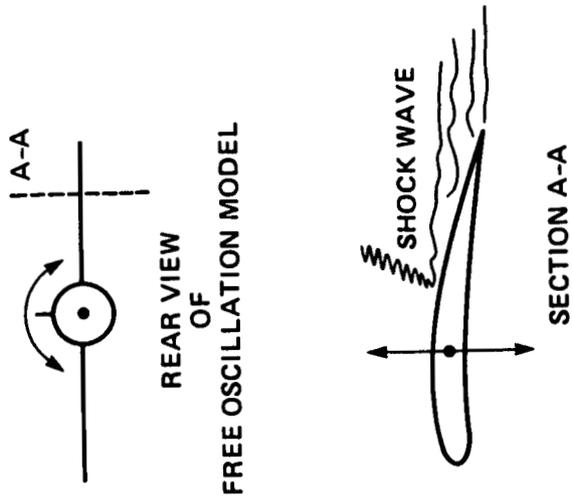
AERODYNAMIC COUPLING LEADS TO
INTERMITTENT BURSTS OF ROLL OSCILLATION
(A FORM OF CHAOTIC BEHAVIOR)

ROLL
ANGLE,
deg

PITCH
ANGLE,
deg
MODEL

TIME

Figure 7.- Wing-rock fluctuations caused by shock-induced separation (ref. 16).

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16. Abstract

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