BIAXIAL EXPERIMENTS SUPPORTING THE DEVELOPMENT OF

CONSTITUTIVE THEORIES FOR ADVANCED HIGH-TEMPERATURE MATERIALS

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ABSTRACT

In most engineering applications, complex states of stress and strain are introduced into components during service. It follows that analysis of such components requires material descriptions, or constitutive theories, which reflect the tensorial nature of stress and strain. In most applications involving low homologous temperatures and stress levels below yield, material response is linear and elastic. These material characteristics lead to simple constitutive relationships involving two material constants.

For applications involving stress levels above yield, the situation is more complex in that material response is both nonlinear and history dependent. Plasticity theories have been developed to model these features by using flow laws and evolutionary laws expressed in differential form. At elevated temperatures, the situation is more complicated still in that material response is time dependent. This has led to the development of viscoplastic constitutive theories which introduce time by expressing the flow and evolutionary equation in the form of time derivatives. Most recently, attempts have been made to extend viscoplastic theories to transversely isotropic materials. The intent here is to develop models which can be used to analyze high-temperature components manufactured from advanced composite materials.

In parallel with the theoretical studies, considerable effort has been directed at developing multiaxial testing techniques to verify the various theories. This work faces two technical challenges. The first is identifying the type of experiment best suited to verifying particular theories. In the case of plasticity, yield surface testing techniques were developed and used successfully to support the theoretical effort. A similar approach has recently been proposed to support development of viscoplastic theories. In this case, the time-dependent characteristics of elevated temperature response are captured by using inelastic strain rate as a measure of inelastic state and by determining flow surfaces. It is interesting to note that this approach can be extended without difficulty to experimental studies of composite materials.

The second challenge is being able to make stress and strain measurements to high levels of precision under biaxial loading conditions. Presently,

tension-torsion load cells are fully developed, and so stress measurements do not present much difficulty. In contrast, the systems available for measuring biaxial strains need further development. At room and intermediate temperatures, foil strain gage rosettes have been used with some success in yield surface studies. Attempts to extend this approach to elevated temperatures have proved less successful because of the unreliability of high-temperature strain gages when used in biaxial stress fields. This has led to the development of a number of mechanical extensometers designed specifically for use in flow surface determinations at elevated temperature. Again, this instrumentation can be used to characterize advanced composite materials provided these materials can be fabricated in the form of thin-walled tubes.

The primary aim of this paper is to outline recent progress in the development of constitutive theories both from the theoretical and experimental viewpoints. One important aspect of this work is that it is leading to verified material descriptions for advanced composite materials which can be implemented in general purpose finite element codes and used for practical design.

BIAXIAL EXPERIMENTS SUPPORTING DEVELOPMENT OF PLASTICITY THEORIES

Yield surface experiments have been used extensively to investigate multiaxial deformation behavior at room and intermediate temperatures. In these experiments, tubular specimens are loaded under computer control at fixed ratios of torsional stress and axial stress. The specimens are instrumented with strain gage rosettes which provide decoupled measures of axial strain and tensorial shear strain. During the initial stages of loading, straight line relationships are established between stress and strain for both components of loading. The stress levels are increased proportionately until the stress-strain response deviates a predetermined amount from linear. These deviations, or small offset definitions of yield, are kept small, 25 $\mu\varepsilon$ or less, to avoid changing the material's state significantly. After achieving the target offset, the specimen is unloaded, and the procedure is repeated for stress ratios in all quadrants of tension-torsion stress space. The combinations of torsional stress and axial stress giving the required yield condition are noted at the termination of each probe and used to construct yield surfaces.

INDIVIDUAL LOADING PROBES ARE TERMINATED WHEN A TARGET VALUE OF EQUIVALENT INELASTIC STRAIN $\bar{\varepsilon}^p$ IS REACHED
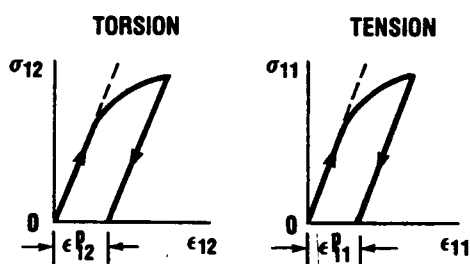
PROBING SEQUENCE USED TO DETERMINE YIELD SURFACES IN TENSION/TORSION STRESS SPACE

$$\bar{\varepsilon}^p = \left[(\varepsilon^p_{12})^2 + 3/4\,(\varepsilon^p_{11})^2\right]^{1/2}$$

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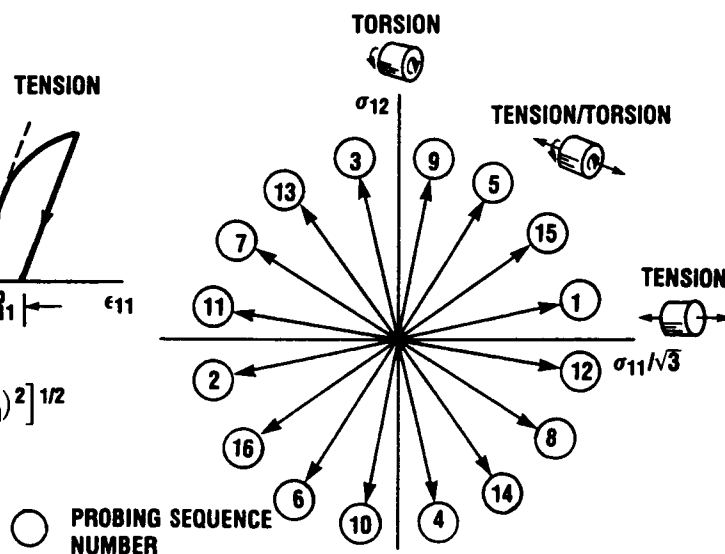
PLASTICITY THEORIES FOR INITIALLY ISOTROPIC MATERIALS

The aim of plasticity theories is to provide mathematical descriptions of material response at stress levels above yield. Here, material response is both nonlinear and history dependent. In the case of initially isotropic materials, material descriptions should not be directional. This requirement can be met by using as many as three invariants to introduce stress into the theory. Classically, the single invariant used for this purpose is the J_2, or Von Mises, form of equivalent stress. One approach adopted in treating history-dependent material response is to use an internal state variable α_{ij} which provides a measure of inelastic state. Evolutionary equations are used to relate changes in α_{ij} to incremental changes of inelastic strain (Prager) or effective deviatoric stress (Ziegler). It is important to note that these theories can only be verified through careful experimentation.

FLOW LAW: $\qquad d\epsilon_{ij}^{p} = \lambda \dfrac{\partial f}{\partial \sigma_{ij}} \dfrac{\partial f}{\partial \sigma_{kl}} d\sigma_{kl}$ $\qquad$ **(DRUCKER)**

POSSIBLE EVOLUTIONARY EQUATIONS: $\qquad d\alpha_{ij} = C\, d\epsilon_{ij}^{p}$ $\qquad\qquad$ **(PRAGER)**

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad d\alpha_{ij} = (S_{ij} - \alpha_{ij})\, d\mu$ $\qquad$ **(ZIEGLER)**

POSSIBLE YIELD FUNCTIONS: $\qquad f(J_2) = J_2 - K$ $\qquad\qquad$ **(VON MISES)**

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad f(J_2, J_3) = J_2 + \beta\, J_3^{2/3} - K$ $\qquad$ **(DRUCKER)**

WHERE

$\qquad J_2 = 1/2\, S_{ij}S_{ij};$ $\qquad\qquad\qquad J_3 = 1/3\, S_{ij}S_{jk}S_{ki}$

$\qquad S_{ij} = \sigma_{ij} - 1/3\, \sigma_{kk}\, \delta_{ij};$ $\qquad \alpha_{ij}$ **– INTERNAL STATE VARIABLE**

AND c, λ, **AND** μ **ARE MATERIAL CONSTANTS.**

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COMPARISON OF EXPERIMENTAL YIELD SURFACES WITH THEORETICAL PREDICTIONS

The yield surface concept described earlier provides a convenient means of interpreting plasticity theories. Adopting this approach, the yield function describes the shape of the yield surface, and the evolutionary equation describes the translation of the yield surface in stress space. Interpretation of the data can be simplified further by using modified stress space σ_{12} versus $\sigma_{11}/\sqrt{3}$. This is because the Von Mises yield function plots as a circle in this stress space. Theoretical and experimental yield surfaces for a series of nonproportional loadings beyond initial yield are shown here. The experiments were conducted at 20 °C on 9Cr-1Mo steel and used a 25-$\mu\epsilon$-offset definition of yield (Ellis, 1985). As is typical for most structural alloys, the Von Mises yield function gives a close representation of initial yield behavior and a poor representation of subsequent yield behavior. Also, Prager and Ziegler forms of evolutionary equation provide reasonable approximations of yield surface translation in stress space.

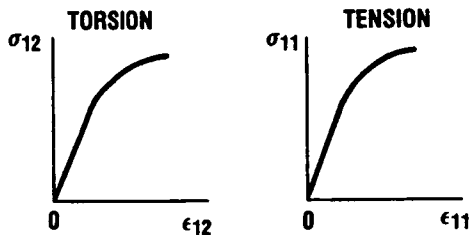## TRANSLATION OF YIELD SURFACES RESULTING FROM NONPROPORTIONAL LOADINGS BEYOND INITIAL YIELD

IDEALIZED   EXPERIMENTAL

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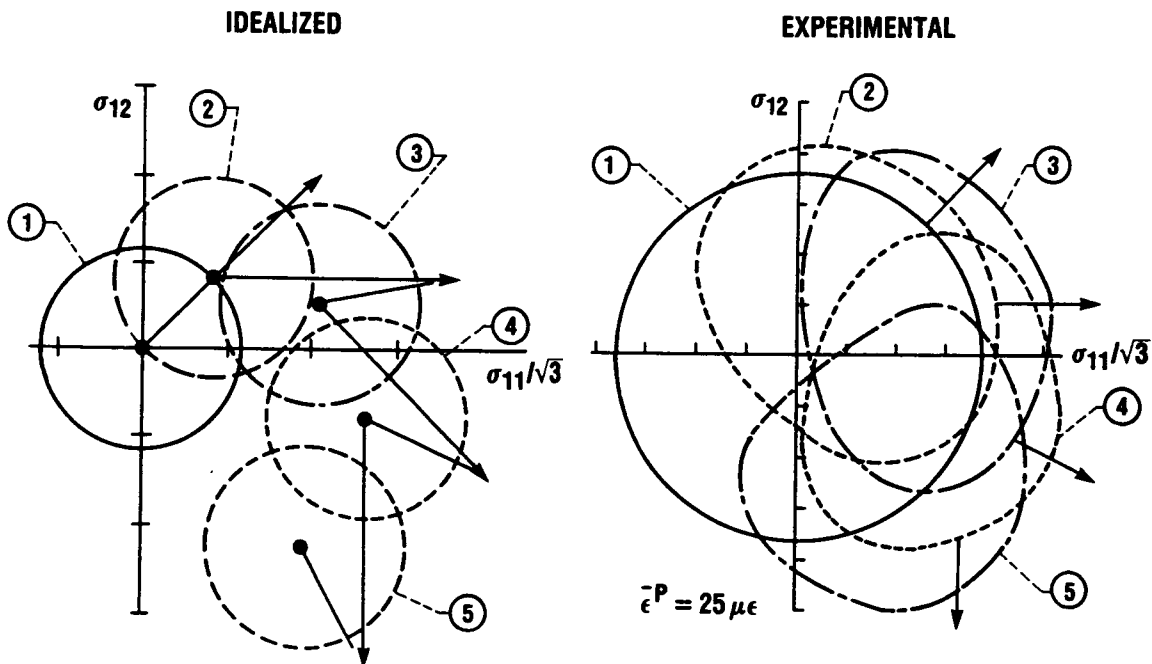
ELEVATED-TEMPERATURE BIAXIAL TESTING SUPPORTING THE DEVELOPMENT OF VISCOPLASTIC CONSTITUTIVE THEORIES

Attempts to extend yield surface testing techniques to elevated temperatures have proved less than successful. One major difficulty has been the unreliability of high-temperature strain gages when used in biaxial stress fields. This led to the development of a number of biaxial extensometers which have recently shown promise in probing-type experiments (Ellis, 1983). Another difficulty is that the traditional concept of yield breaks down at elevated temperatures when material response becomes time dependent. One experimental approach developed to resolve this difficulty is to use inelastic strain rate $\dot{\varepsilon}^P$ as a measure of change in inelastic state. By adopting this approach, flow surfaces can be determined for particular values of $\dot{\varepsilon}^P$ and used to guide the development of viscoplastic constitutive theories.
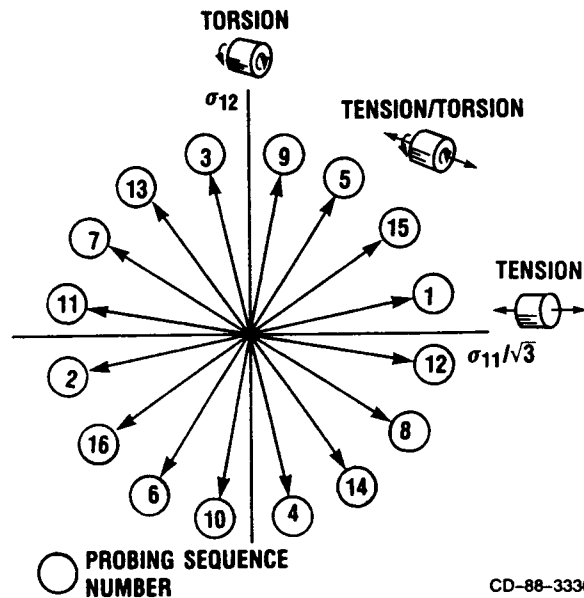
INDIVIDUAL LOADING PROBES ARE TERMINATED WHEN A TARGET VALUE OF EQUIVALENT INELASTIC STRAIN RATE $\dot{\bar{\varepsilon}}^P$ IS REACHED

$$\dot{\varepsilon}^{p}_{12} = \dot{\varepsilon}_{12} - \dot{\sigma}_{12}/2G \qquad \dot{\varepsilon}^{p}_{11} = \dot{\varepsilon}_{11} - \dot{\sigma}_{11}/E$$

$$\text{AND} \quad \dot{\bar{\varepsilon}}^{p} = \left[(\dot{\varepsilon}^{p}_{12})^2 + 3/4\,(\dot{\varepsilon}^{p}_{11})^2 \right]^{1/2}$$

PROBING SEQUENCE USED TO DETERMINE FLOW SURFACES IN TENSION/TORSION STRESS SPACE

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A VISCOPLASTIC CONSTITUTIVE THEORY FOR INITIALLY ISOTROPIC MATERIALS

In addition to treating the complexities of plasticity, viscoplastic constitutive theories attempt to model the time-dependent features of material response at elevated temperature. These features include creep, relaxation, recovery, and rate dependence. A convenient method of introducing time into the material description is to express the flow and evolutionary equations in the form of time derivatives. The majority of viscoplastic theories use two internal state variables, α_{ij} and K, to treat the complexities of elevated temperature behavior (Freed, 1988). As in the case of plasticity, stress is introduced into the material description by means of invariants. Again, the Von Mises form of equivalent stress is used in the majority of these models for initially isotropic materials. The validity of this approach remains to be verified experimentally.

FLOW LAW:
$$\dot{\epsilon}_{ij}^{p} = \theta(T)\, Z\left(\frac{\Sigma_2}{K}\right) \frac{\Sigma_{ij}}{\Sigma_2}$$

EVOLUTIONARY LAWS:
$$\frac{\dot{\alpha}_{ij}}{H} = \dot{\epsilon}_{ij}^{p} - \frac{\alpha_{ij}}{K}\dot{\epsilon}_2^{p}$$

$$\frac{\dot{K}}{h} = \frac{1}{K}\Sigma_{ij}\dot{\epsilon}_{ij}^{p} - \theta(T)\, r(K)$$

WHERE $Z(\Sigma_2)$, $r(K)$, **AND** $\theta(T)$ **ARE MATERIAL FUNCTIONS; H AND h ARE MATERIAL PARAMETERS; AND**

$$\Sigma_{ij} = S_{ij} - \alpha_{ij} \qquad S_{ij} = \frac{3}{2}\sigma_{ij} - \frac{1}{2}\sigma_{kk}\delta_{ij}$$

$$\Sigma_2 = \left[\frac{2}{3}\Sigma_{ij}\Sigma_{ij}\right]^{1/2} \qquad \dot{\epsilon}_2^{p} = \left[\frac{2}{3}\dot{\epsilon}_{ij}^{p}\dot{\epsilon}_{ij}^{p}\right]^{1/2}$$

## DETERMINATION OF FLOW SURFACES AT ELEVATED TEMPERATURES

The feasibility of using flow surfaces to investigate multiaxial response at elevated temperatures has been demonstrated in a series of experiments conducted on type-316 stainless steel at 20 and 650 °C (Battiste and Ball, 1986). As indicated earlier, the main challenge in conducting these experiments is strain measurement. Obvious requirements in probing-type experiments are linearity and high resolution. A requirement for minimum crosstalk is unique to biaxial testing. This problem arises when loading in the axial sense produces apparent torsional strains and vice versa. Further complications arise in conducting these experiments at elevated temperatures. The problem here is that the electrical noise produced by most heating systems causes problems with test system control and resolution. As demonstrated by the results shown here, mechanical extensometers have been developed which meet most of the requirements discussed previously. Further improvements are necessary, however, before some of the more fundamental theoretical assumptions can be tested with a high degree of confidence.

FLOW SURFACES FOR SMALL VALUES OF EQUIVALENT INELASTIC STRAIN RATE CAN BE DETERMINED IN POST-TEST ANALYSIS OF THE EXPERIMENTAL DATA
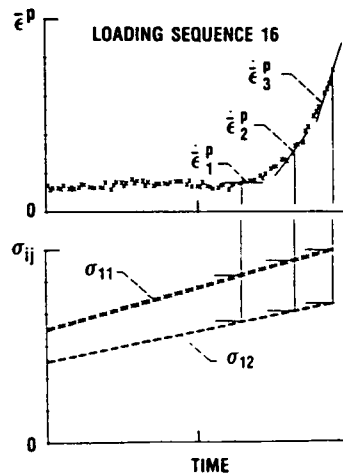
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**PRELIMINARY FLOW SURFACES DETERMINED FOR TYPE-316 STAINLESS STEEL AT 20 °C**

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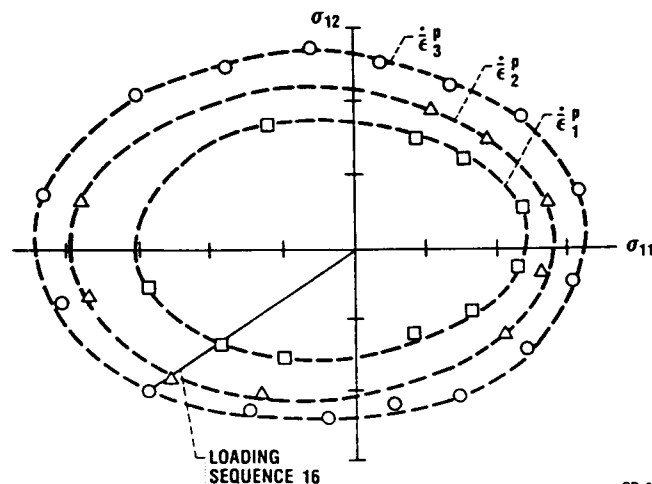
EXTENSION OF BIAXIAL TESTING AND CONTINUUM THEORIES TO ADVANCED COMPOSITE MATERIALS
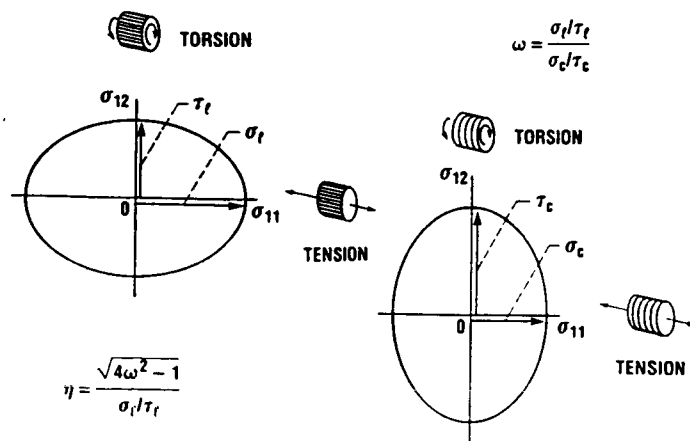
It has been recognized for some time that the thin-walled tube and tension-torsion loading provide an ideal means of investigating deformation behavior in composite materials. One advantage of this approach is that properties can be determined for compressive loadings without too much difficulty. Further, shear properties can be obtained with minimum ambiguity by using the torsional component of loading. Thus, it appears that the high-temperature instrumentation and experimental techniques developed for isotropic materials can be used to advantage in developing deformation theories for advanced composite materials.

APPLICATION OF CONTINUUM THEORIES TO STRUCTURED MATERIALS REQUIRES IDENTIFICATION OF A CONTINUUM ELEMENT (D) THAT IS SMALL COMPARED TO CHARACTERISTIC STRUCTURAL DIMENSIONS BUT LARGE COMPARED TO CELL SIZE DIMENSIONS

$$\frac{D}{L} << 1$$

$$\frac{C}{D} << 1$$
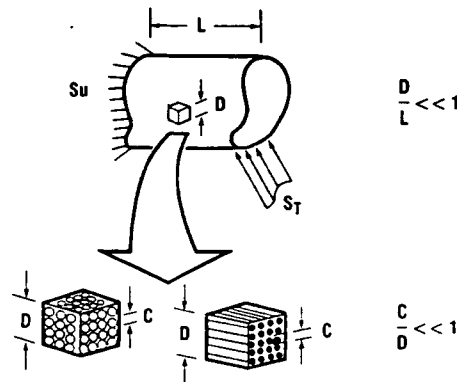
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MEASURES OF ANISOTROPY, ω AND η, CAN BE OBTAINED BY DETERMINING FLOW SURFACES FOR TUBES REINFORCED IN THE LONGITUDINAL AND TRANSVERSE SENSES

TORSION

$$\omega = \frac{\sigma_\ell / \tau_\ell}{\sigma_t / \tau_t}$$

TORSION

TENSION

TENSION

$$\eta = \frac{\sqrt{4\omega^2 - 1}}{\sigma_\ell / \tau_\ell}$$

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VISCOPLASTIC CONSTITUTIVE THEORY FOR TRANSVERSELY ISOTROPIC MATERIALS

Procedures are well established for extending continuum mechanics concepts to composite materials. Here, the challenge is to incorporate directionality into the material description without violating the laws of mechanics. A viscoplastic constitutive theory developed by using this approach is shown below for the case of transversely isotropic materials (Robinson et al., 1987). This form of directionality was incorporated into the theory by specifying a strong direction, di. This increased the number of invariants initially involved in the theoretical development to seven. This number subsequently, was reduced to three by tailoring the theory to match assumed material response. The strength of the continuum mechanics approach is that it makes use of more than three decades of progress in the field of viscoplasticity. This almost assures that the theory will prove successful in predicting time dependence and history dependence in composite structures involving a single strong direction. Whether the theory can be extended to predict the response of complex composite structures is open to question.

FLOW LAW: $\dot{\epsilon}^p_{ij} = f(F)\Gamma_{ij}$

EVOLUTIONARY LAW: $\dot{\alpha}_{ij} = h\,(G)\,\dot{\epsilon}^p_{ij} - \gamma(G)\,\pi_{ij}$

WHERE

$f(F)$, $h(G)$, AND $\gamma(G)$ ARE MATERIAL FUNCTIONS, AND

$$F = \frac{1}{K_T^2}\left[I_1 + \frac{I_2}{\eta^2} + \frac{9}{4(4\omega^2 - 1)}I_3\right] - 1$$

$$\Gamma_{ij} = \Sigma_{ij} - \xi\,[d_k d_i \Sigma_{jk} + d_j d_k \Sigma_{ki} - 2I_0 d_i d_j] - \frac{1}{2}\,\zeta I_0(3d_i d_j - \delta_{ij})$$

$$G = \frac{1}{K_T^2}\left[I_1' + \frac{I_2'}{\eta^2} + \frac{9}{4(4\omega^2 - 1)}I_3'\right]$$

$$\pi_{ij} = a_{ij} - \xi\,[d_k d_i a_{jk} + d_j d_k a_{ki} - 2I_0' d_i d_j] - \frac{1}{2}\,\zeta\,I_0'(3d_i d_j - \delta_{ij})$$

$$\xi = \frac{\eta^2 - 1}{\eta^2} \qquad\qquad \zeta = \frac{4(\omega^2 - 1)}{4\omega^2 - 1}$$

$$\eta = K_L/K_T \qquad\qquad \omega = Y_L/Y_T$$

$$I_1 = J_2 - I + \frac{1}{4}I_3 \qquad\qquad I_2 = I - I_3$$

$$I_3 = (I_0)^2 \qquad\qquad J_2 = \frac{1}{2}\Sigma_{ij}\Sigma_{ij}$$

$$I = d_i d_j \Sigma_{jk}\Sigma_{ki} \qquad\qquad I_0 = d_i d_j \Sigma_{ji}$$

NOTE THAT THE PRIMED INVARIANTS IN G AND π_{ij} ARE OBTAINED BY REPLACING Σ_{ij} BY a_{ij} IN THE ABOVE EXPRESSIONS.
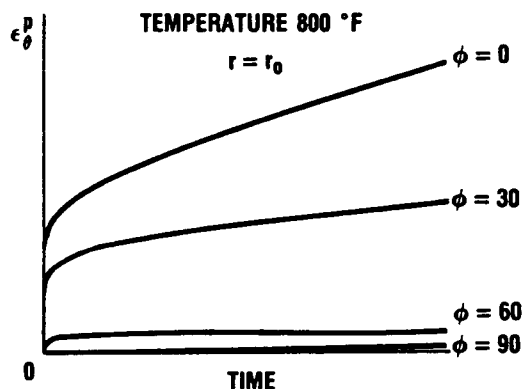
IMPLEMENTATION OF THE VISCOPLASTIC CONSTITUTIVE THEORY FOR TRANSVERSELY ISOTROPIC MATERIALS IN THE MARC FINITE ELEMENT CODE

The subject viscoplastic model has been implemented in the MARC finite element code and exercised in a number of trial calculations (Arya, 1987). The material properties used in these calculations were similar to those of a metal matrix composite material, tungsten-copper, at about 800 °F. Initially, the analyses were limited to uniaxial stress states. Under these conditions, it was shown that the model can treat cyclic plasticity, rate dependence, and creep for a range of fiber orientations. More recently, a thick-walled cylinder was analyzed to determine performance in calculations involving multiaxial stress states. As indicated below, the theory was successful in predicting creep-like behavior which was highly dependent on fiber orientation. For example, creep in the hoop direction can be seen to be negligible for the case of $\phi = 90$.

THE SUBJECT CONSTITUTIVE THEORY HAS BEEN IMPLEMENTED IN THE MARC FINITE ELEMENT CODE AND USED TO ANALYZE A THICK-WALLED CYLINDER WITH A RANGE OF FIBER ORIENTATIONS (ϕ)
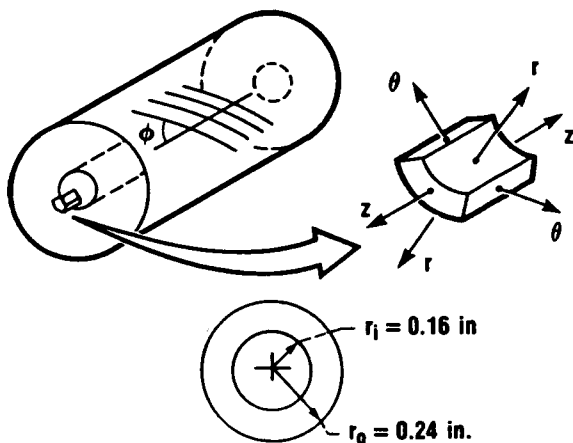
WHEN LOADED UNDER INTERNAL PRESSURE, $\approx$ 4000 psi, THE MATERIAL EXHIBITED CREEP-LIKE BEHAVIOR WHICH WAS HIGHLY DEPENDENT ON FIBER ORIENTATION

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2-47

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