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MHOST: AN EFFICIENT FINITE ELEMENT PROGRAM FOR INELASTIC ANALYSIS OF SOLIDS AND STRUCTURES

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ABSTRACT

The main objective of this development is to construct and validate an efficient finite element program for 3-D inelastic analysis of gas tubine hot section components. A novel mixed-iterative solution strategy is derived from the *augmented* Hu-Washizu variational principle in order to nodally interpolate coordinates, displacements, deformations, strains, stresses and material properties. A series of increasingly sophisticated material models incorporated in MHOST include elasticity, secant plasticity, infinitesimal and finite deformation plasticity, creep and unified viscoplastic constitutive model porposed by Walker. To detect the effects of embedded discontinuities, such as cooling channels in turbine blades, the local-global analysis procedure called *subelement iteration* is developed in the framework of mixed-iterative formulation. The performance of this numerical procedure is demonstrated in the elastic and elastic-plastic computations.

A library of high performance elements is built into this computer program utilizing the concepts of selective reduced integrations and independent strain interpolations. A strain filtering scheme based on the polar decomposition of isoparametric coordinate trainsformation is devised to improve the accuracy of highly distorted element shapes. The MHOST linear isoparametric elements exhibit high coarse mesh accuracy even when the element bending mode is predominant. Also included are 6 dof/node linear beam and shell elements. For Shells, a simple and efficient hourglass control mechanism is built in.

A family of efficient solution algorithms is implemented in MHOST for lenear and nonlinear equation solution including the classical Newton-Raphson, modified-, quasi- and secant Newton methods with optional line search and the conjugate gradient method. Also automatic load increment control in the spirit of arc length is incorporated in the mixed-iterative solution framework. The eigenvalue extraction for the vibration mode and buckling analysis utilizes the subspace iteration method driven by the profile solution for the matrix factorizations. In addition to the conventional feature, MHOST allow users to dig into the analysis of deformation modes for the finite element model looking at the eigenstructure of stiffness matrix itself. The transient dynamic calculations are driven by the Newmark method recast in the mixed-iterative form.

The compact computer program consists of about 47,000 *Fortran 77* statements including extensive comments. To provide a comfortable user interface, a free format data reader and an industry standard formatted post processing data writer are included in the package. The portability of this program has been demonstrated on various computer systems from small engineering workstations to supercomputers.

The robustness of numerical and computational technology built in the MHOST program has served as an ideal software platform for further research and development in finte elements such as the probablistic structural analysis methods.

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MIXED ITERATIVE FINITE ELEMENT METHOD

The new mixed finite element formulation and its iterative solution algorithms are developed in this project. This strategy enabled us to boost the performance of elements and computer programs by taking advantages of nodal stress and strain interpolations. The conventinal displacement solution is used as the *preconditioner* and independent nodal interpolations for stresses and strains are introduced *iteratively* to improve the quality of numerical solution inexpensively.

The structural model is fed into the numerical computation in a usual way by constructing the stiffness matrix. The nodal strain is then calculated by a projection process which eliminates the numerical instability in strain/stress approximations such as spurious oscillations. The inelastic material models are incorporated in the nodal stress integration in a modular fashion. The resulting nodally interpolated stress field is more accurate in comparison with the conventional displacement method. This improvement is brought back into the displacement field through the equilibrium iteration loop.

The resulting control structure for linear and nonlinear computations as schematically presented in Figure 1 is similar to what is used for the nonlinear finite element analysis derived from the displacement formulation. Indeed the mixed iterative solution concept is used as an framework to derive and analyze the solution strategies for nonlinear finite elements.

Additional computer cost of mixed solution for linear problems is insignificant performing only a few re-solution for the same stiffness matrix with modified residual vector. The nodal strain projection is carried out utilizing the diagonalized projection operator not requiring extra matrix manipulations. No additional computation is needed when the method is used for nonlinear computations. The modern iterative solution technology based on the quasi-Newton update is incorporated to further improve the performance of this solution method.

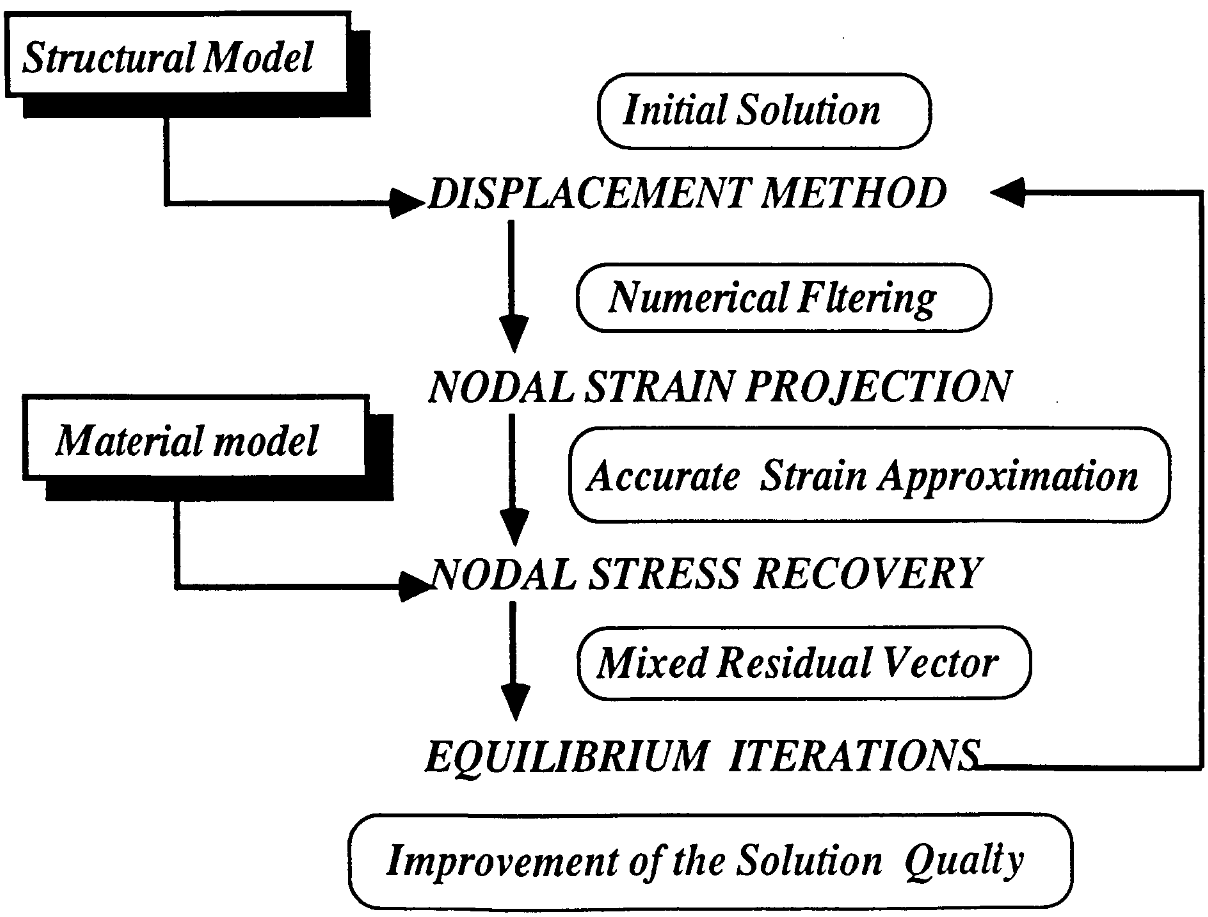


Figure 1 The Concept of Mixed-Iterative Finite Element Solution

THE ITERATIVE SOLUTION ALGORITHMS

The system of finite element equations solved in the MHOST program is symbolically shown in Figure 2 with the equivalent mixed stiffness equations derived by direct elimination. Computationally the construction and inversion of mixed finite element equation is prohibitively expensive in terms of both computing time and the memory requirement. Hence the iterative solution algorithms are used to overcome this difficulty and make the mixed solution feasible.

The linear convergence of basic mixed iterative method is illustrated in Figure 3a. The difference of displacement and mixed stiffness matrices generates the driving residual vector for the iterations even for linear elastic problems. The quasi-Newton type update algorithms, an example illustrated in Figure 3b, improve the convergence rate of the iterative solution siginificantly.

In the MHOST program, other than the constabt metric iteration scheme, the optional line search, the conjugate gradient, the secant implementation of Davidon rank-one quasi-Newton, BFGS rank-two quasi-Newton update algorithms are implemented. It is observed that the displacement stiffness gives steeper gradient in the load-deflection relation that the exact one whereas the mixed stiffness lies in the flexible side. Therefore the iterative solution algorithm starting from the displacement method gradually recovering the mixed solution is almost always stable and convergent.

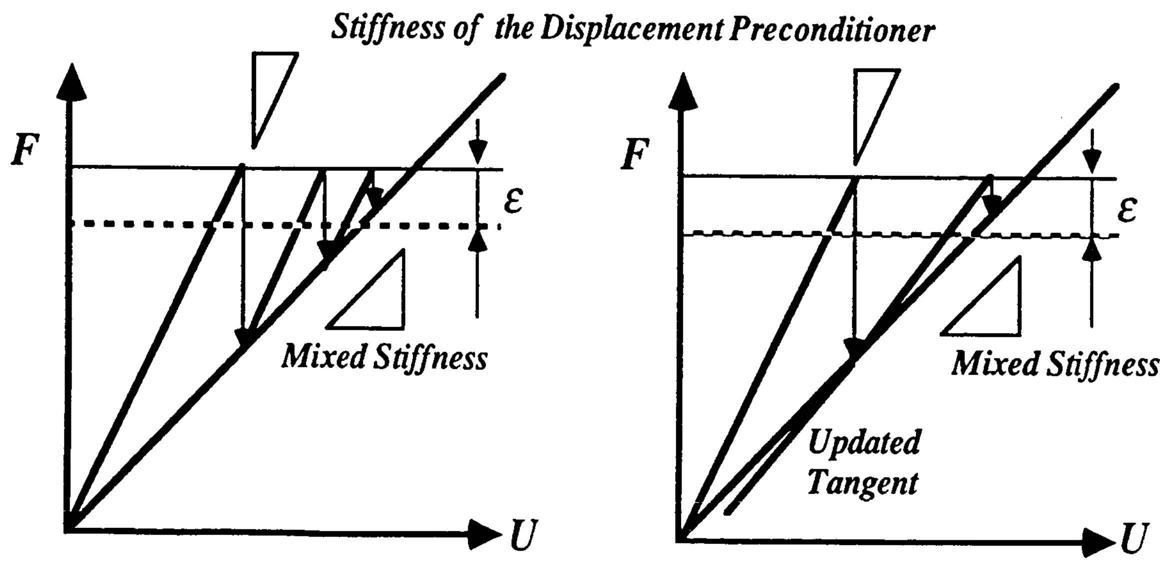
The Augmented Hu-Washizu Finite Elment Equations

$$\begin{vmatrix} K & 0 & B^t \\ 0 & D & -C \\ B & -C^t & 0 \end{vmatrix} \begin{vmatrix} U^{n+1} \\ E \\ S \end{vmatrix} = \begin{vmatrix} F\text{-}K\,U^n \\ E_0 \\ 0 \end{vmatrix}$$

Equivalent Mixed Stiffness Equations

$$(B^t C^{-1} D\, C^{-t} B)\, U = F - D\, E_0$$

Figure 2 Mixed Finite Element Equations and Equivalent Stifness Matrix



a.) Constant Metric Iteration

b.) Variable Metric Iteration

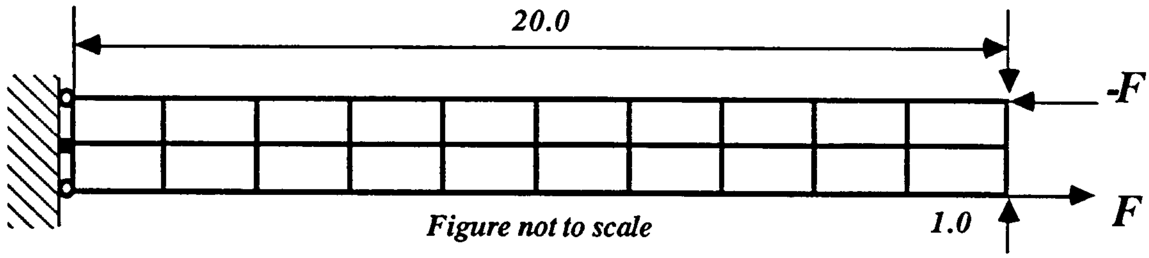
Figure 3 Convergence of Mixed Iterative Solution for a Linear Problem

HIGHLY ACCURATE LINEAR FINITE ELEMENTS

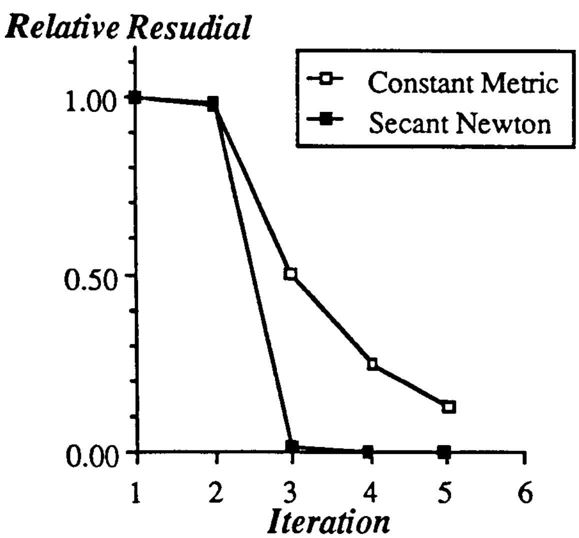
The MHOST program consists of a library of highly accurate linear quadrilateral and hexahedral elements. The selective reduced integration and assumed stress interpolation are used to improve the element responses under bending load. To take the full advantage of these formulations, a rational local coordinate transformation derived from the polar decomposition of the Jacobian matrix is developed and implemented in the code. This highly sophisticated algorithm makes the response of element insensitive to the isoparametric distortion.

For the sampling and nodal projection of element strain components, the trapezoidal integration rule is utilized for the maximum accuracy and stability. The oscillation of strain/stress fields often observed at the element integration points are filtered out by this procedure. The resulting mixed iterative finite element method is capable of producing very accurate displacement, strain and stress simultaneously.

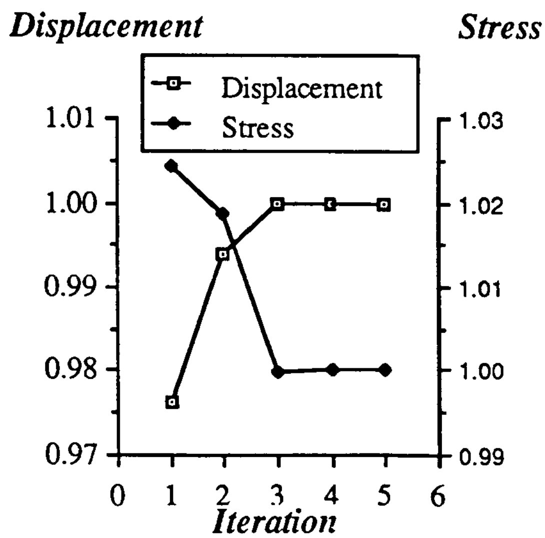
A cantilever beam modelled by the MHOST plane stress elements is used as an example to demonstrate the performance of both the methodology and the element formulation. For this simple problem, the nodally exact values are obtained for the displacement, strain and stress after a few iterations as shown in Figure 4 c). The optional acceleration algorithms such as the Davidon rank-one secant update procedure improve the convergence rate significantly over the standard constant metric scheme as typical result shown in Figure 4 b) in terms of the reduction of normalized mixed residual vector.



a) A Cantilever Beam. Plane Stress Finite Element Model.



b) Convergence of iterative Solution



c) Convergence of Nodal Solution

Figure 4 High Accuracy of MHOST Linear Quadrilateral Element

GLOBAL-LOCAL SOLUTION BY SUBELEMENT ITERATIONS

In order to include the effects of the embedded singularities such as cooling holes and cracks without excessive mesh refinement, the subelement concept is developed in the framework of mixed-iterative solution strategy and implemented in the MHOST program. The basic idea is to set up a coarse global mesh without considering embedded singularities, but to use locally refined mesh to recover these effects in the mixed residual force calculation process. An example shown in Figure 5 illustrates the concept of this methodology. Instead of using the global mesh with the local effect (top left), a simple mesh without the local singularity is built (top right). The local effect is embedded in the regular grid (shaded area in the top right mesh) and used for the calculation of the residual force. Note that the mixed-iterative process is used in the subelement region for the given initial stress field obtained as the global solution. Under unform traction loading, the stress at the bottom edge of the circular hole is expected to be three times larger than the unifom uniaxial stress value. In the subelement regions, the linear and quadratic elements are made available in MHOST and, in this particular example, the quadratic subelements produce significantly more accurate stress results (2.708) than the linear subelement solution (2.504). The stress calculated from the linear subelement model is slightly more accurate than the global finite element solution with a hole (2.349). In these analyses, the iterations are carried out until the relative residual value becomes below 10%.

The solution of each subelement can be performed independently and, for a multiple number of subelements, the operation can be fully parallelizable. The subelement meshes for individual global elements can be introduced based on *a posteriori* error estimate and incorporated with the adaptive refinement strategies.

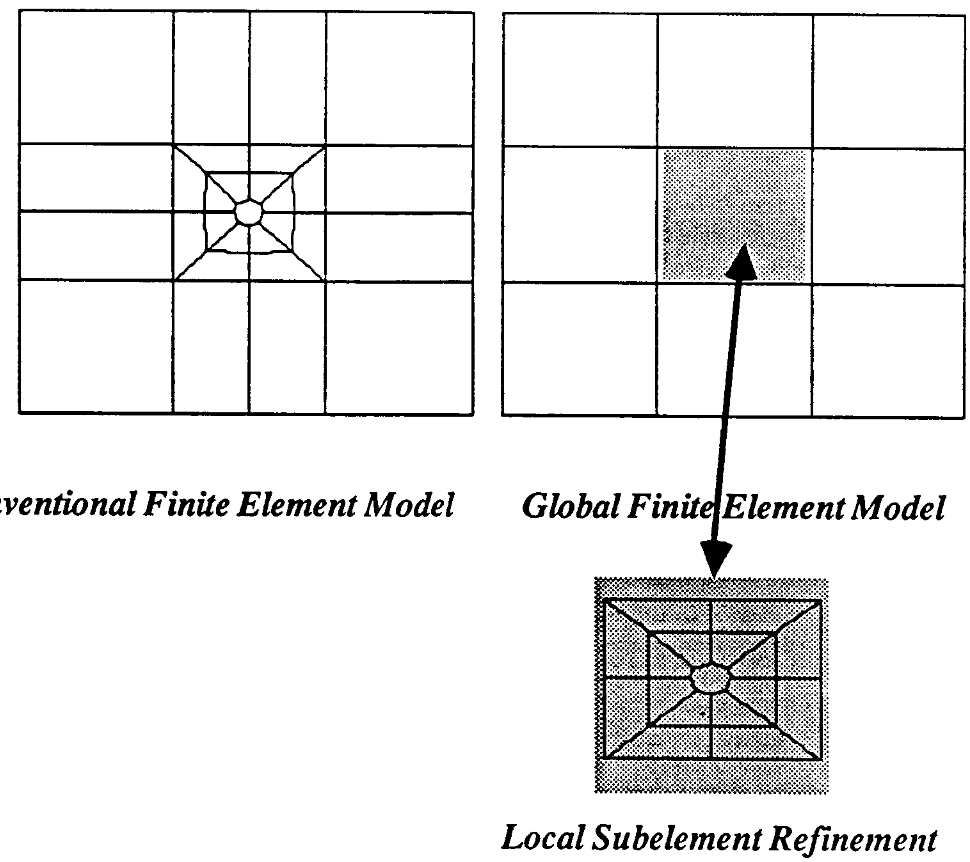


Figure 5 A Square Plate with a Circular Hole.

Global-Local Analysis by Subelement Iteration.

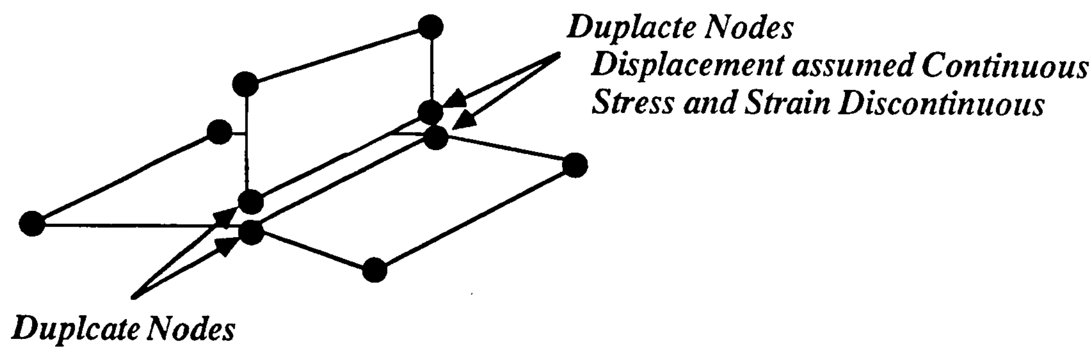
THE THREE DIMENSINAL SHELL ELEMENT

The nodally continuous stress assumption used in the beams and shells improves the accuracy of MHOST solution cosiderably in comparison with the conventinal displacement strategies. For example, the nodally exact displacement and moment solution is obtained for a cantilever beam problem modelled by the MHOST shell element subjected to a point load in the transverse direction at the tip. This shows the capability of the mixed iterative shell formulation to capture the linear moment field exactly by using only the linear finite element basis functions.

To avoid numerical instabilities commonly observed for the four node shell elements such as the *numerical locking* and *hourglass* displacement modes, the stiffness matrix for the shell element uses the selective integration for the transverse shear terms and a simple and efficient hourglass control scheme proposed by Belytschko, Tsay and Liu (1981).

Difficulties are encountered when the intersecting shell problems aremodelled by the mixed iterative finite element method, in which the continuity of stress at the intersection is no longer a valid assumption. The MHOST program provides a user with the DUPLICATE NODE option which allow the stress to be discontinuous at the specified interelement boundaries whereas the displacement field is kept continuous as the concept illustrated in Figure 6 (top).

An hollow shell structure, the MHOST finite element model for the wind turbine blade with an internal stiffener, is shown at the bottom of Figure 6. In this example, the COMPOSITE material option is also invoked, which allows the users to directly input the constitutive resultant array (D matrix defined in terms of curveture and bending moment) for anisotropic material responses.



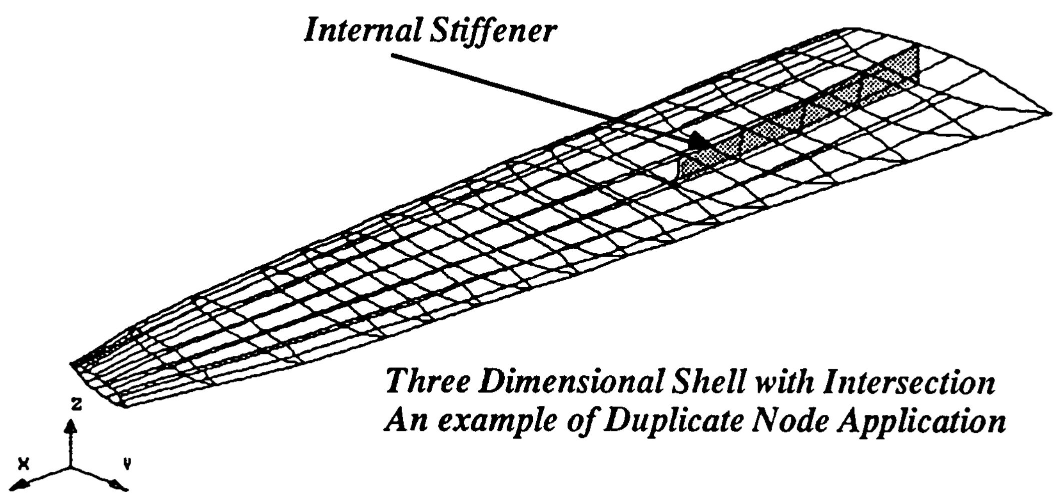


Figure 6 Treatment of Intersecting Shells in the MHOST Mixed Formulation

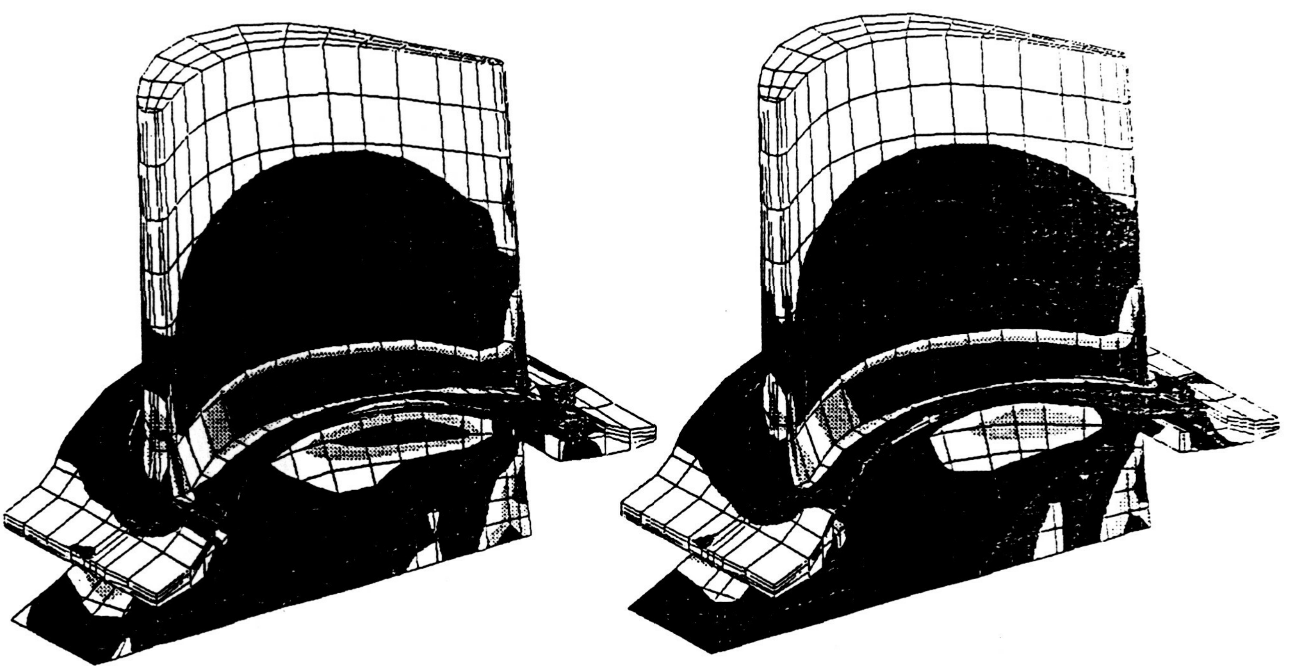
AN EXAMPLE OF THREE DIMENSIONAL ANALYSIS

The sophistcated three dimensional element implemented in the MHOST program exhibits superior accuracy in comparison with the conventional isoparamatric 8 node brick element derived directly from the displacement method formulation. The selective shear integration/assumed stress formulation in conjunction with the local coordinate trasformation based on the isoparametric mapping results in a highly accurate displacement field even at the initial displacement preconditioning phase without iterative improvement. The iterative process drives the solution further toward the direction of high accuracy for the given mesh.

The three dimensional brick element implemented in the MHOST program controls the effects of parasitic shear strains when it is subjected to the bending load. When this element is used for the solution to probelms in which local bending behavior is predominant (such as the burner blister specimen - a thick circular plate subjected to a intensive thermal loading from one side at the center - a typical three dimensional model uses two layers of elements through the thickness), the conventinal displacement element results in a displacement 25% of *the exact solution* (obtained from extremely fine axisymmetric model) whereas the MHOST element manages to repoduce more than 90% of displacement using the same mesh.

An example shown below is a three dimensional model of the SSME HPFTP blade subjected to the thermal and centrifugal loading. This example demonstrates the improvement of the stress field by the mixed iterative process in practical applications. The stress concentrations occuring below the platform and at the leading edge of the blade are captured by the mixed-terative solution (left), which is not so clearly visible in the displacement solution included in the same figure (right). The additional computational effort by the mixed iterations is insignificant (40% of full displacement solution for 5 extra iterations for this linear elastic problem) involving a few re-solution with the updated mixed residual vectors.

An additonal advantage of the present mixed iterative formulation is that the stress is calculated, stored and reported at nodes which makes the interpretation of results and graphic post processing simple and straightforward. The MHOST program produces an industry standard post processing file which can be dealt with by most of the commercially avilable softwares.



a) Mixed Finite Element Solution. b) Displacement Solution By MHOST Element

Figure 7 Three Dimensinal Solid Finite Element Model of the SSME HPFTP Blade. Equivalent Mises Stress Under Thermal and Centrifugal Loading.

MHOST ANALYSIS CAPABILITIES

As suumarized in the table below, MHOST has become a versatile tool for the linear and nonlinear solid and structural analysis. Similar to the commercially available nonlinear finite element packages such as MARC and ABAQUS, the code consists of the solution algorithm library, the element library (both of which has been discussed extensively in this paper) and the constitutive equation library. The MHOST constitutive equation library consists of: (i) linear elasticity with optional temperature dependency and anisotropy definitions; (ii) simplified secant plasticity; (iii) the infinitesimal and finite deformation versions of J2 plasticity with isotropic and anisotropic yield conditions, and isotropic and kinematic hardening rules with optional temperature dependency and creep; and (iv) the unified viscoplasticity law poposed by Walker. For experienced users to be able to modify/enhance the constitutive equation library, the open ended architecure concept is implemented via user definable subrotines.

The MHOST program is written in the standard *Fortran 77* and highly portable. Currently versions are available on CRAY (COS/CFT), PRIME (PRIMOS/F77), VAX (VMS/FOR), IBM (MVS/VS- FORTRAN) and Alliant (CONCENTRIX/fortran).

Table 1 MHOST Analysis Capability

			ELEMENT LIBRARY		
Analysis Option	**Beam**	**Plane Stress**	**Plane Strain**	**3-D Solid**	**3-D Shell**
Quasi-Static Analysis*1	*X*	*X*	*X*	*X*	*X*
Buckling Analysis*1	*X*	*X*	*X*	*X*	*X*
Deformation Mode Extraction*1,*2	*X*	*X*	*X*	*X*	*X*
Modal Analysis*1	*X*	*X*	*X*	*X*	*X*
Linear Dynamics	*X*	*X*	*X*	*X*	*X*
Transient Dynamics	*X*	*X*	*X*	*X*	*X*

*1The effects of the stress stiffening and the centrifugal mass may optionally included in these calculations.

*2This option extracts the eigenvalues and eigenvectors of the stiffness matrix. The facility is particularly useful to test the element formulations and implementations. Often excessive kinematic modes are detected by this option. It may be considered as a tool complementary to the *patch test*.

CONCLUDING REMARKS

As briefly discussed in this paper, the finite element package MHOST developed under the HOST project consists of an innovative numerical method which has turned out to be highly accurate and efficient. The implementation of the state-of-the-art numerical processes for the advanced element formulations and the constitutive integrations in this program package extends its functionality from a mere research code to an attractive alternative to the commercially available finite element programs for the serious engineering development. The heavy usage of the code indeed indicates its capability as a versatile numerical tool in the research and development environment. Also the fact that the extension of this methodology and computer program to the probablistic structural analysis demonstrates its potential for a wide range of applications not limited to the straightforward structural analyses but as a reliable engine to extend the usage of finite elements in design environments where the nonlinear responses of the objects have started to play important roles in assessing the reliability.

The advantages of the new method has not yet been fully exploited and the drawbacks need to be identified and circumvented. The mathematics of this method has not yet fully understood. The establishment of the formal error estimates and stability criteria would help increasing the level of confidence in this class of methodology. Further utilization and users' feedback of MHOST would motivate further research and development to make this technology really fly as a robust computational mechanics method.

The iterative solution strategy implemented for the solution of the mixed finite element method has a control structure suitable for the further efficiency improvement in the vector and parallel processing environments. In the initial development of the MHOST program, the emphasis was placed on the demonstration of the concept rather than the refinement in the program development. It is now clear that the method is more suitable structured for the modern computing machineries than the traditional finite element procedures. The use of displacement stiffness matrix as a preconditioner to the mixed solution in the current implementation is the most time and memory consuming part in the entire computations, and needs critically reviewed. Modern preconditioning techniques such as the element-by-element algorithm would increase the efficiency of the mixed solution processes considerably. Also the use of relaxation techniques in the framework of mixed finite element method could potentially economize the overall solution time and memory requirement.

The coding strategy used in the MHOST program is rather traditional consisting of multiply nested loops and numerous subroutine calls. Recent programming experiments demonstrate that the unrolled short loops and the inline expansion of subroutines cuts down the redundant computing time, which had not been visible until the new high performance floating point processors emerged. To take advantages of the simplicity of the mixed iterative solution, in particular the constitutive integration operations looped over the nodes, the stress recovery subroutines would be the first one to be rewritten for the performance gain on those machines.

Also the memory allocation schemes to minimize the data transfer between cache, main memory and disk drives need to be reviewed and streamlined. The compactness of the code with full inline documentation makes the MHOST code as an ideal starting point to explore this avenue. The internal nodal database unique to the mixed-iterative code makes the memory management operations far simpler and more compact than the element database used in the displacement type codes. Indeed this was one of the major motivation behind the extension of mixed strategy to the probablistic analysis, in which multiple number of the perturbed finte element solutions with respect to the random variables are calculated and stored in an accessible manner from the interactive probalility analysis packages.

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