N88-23107

The Decay of Isotropic Turbulence in a Rapidly Rotating Frame

By C. G. SPEZIALE[1], N. N. MANSOUR[2], and R. S. ROGALLO[2]

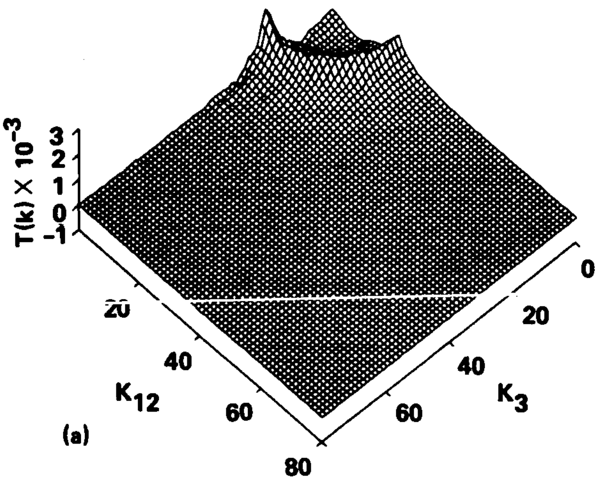
A direct numerical simulation of the decay of initially isotropic turbulence in a rapidly rotating frame was conducted. This 128x128x128 simulation was completed for a Reynolds number $Re_\lambda = 15.3$ and a Rossby number $Ro_\lambda = 0.07$ based on the initial turbulent kinetic energy and Taylor microscale. The numerical results indicate that the turbulence remains essentially isotropic during the major part of the decay (i.e., beyond the point where the turbulent kinetic energy has decayed to less than 10% of its initial value). The rapid rotation has the primary effect of shutting off the energy transfer so that the turbulence dissipation (and hence the rate of decay of the turbulent kinetic energy) is substantially reduced. Consequently, the anisotropy tensor remains essentially unchanged while the energy spectrum undergoes a nearly linear viscous decay — the same results that are predicted by Rapid Distortion Theory which is only formally valid for much shorter elapsed times. Surprisingly, no Taylor-Proudman reorganization of the flow to a two-dimensional state is observed. The implications that these results have on turbulence modeling are discussed briefly along with prospective future research.

1. Results

The research conducted this summer at the CTR concentrated on the development of improved Reynolds stress models for the description of rotating turbulent flows. It is envisioned that such models could also have important applications in the description of curved turbulent flows as a result of the analogy that can quite often be drawn between rotation and curvature.

In order to gain insight into the effects of rotation, a direct numerical simulation of decaying isotropic turbulence in a rapidly rotating frame was conducted. A Reynolds number of $Re_\lambda = 15.3$ based on Taylor microscale and a Rossby number of $Ro_\lambda = 0.07$ were considered (this Rossby number is more than an order of magnitude smaller than those which were considered previously). This direct simulation yielded some surprising results. As has been shown in previous numerical simulations and experiments (see Bardina, Ferziger and Rogallo 1985, and Wigeland and Nagib 1978), the turbulence remained isotropic after the rotation was imposed.

1 ICASE, NASA Langley Research Center
2 NASA Ames Research Center



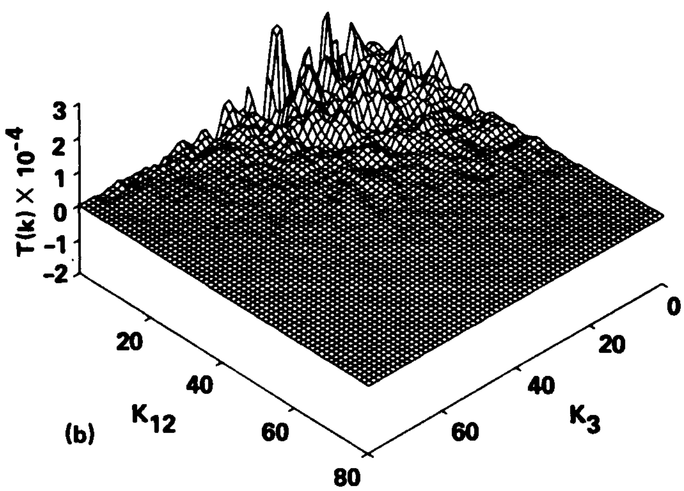


FIGURE 1. Transfer spectra as function of $k_{12} = \sqrt{k_1^2 + k_2^2}$ and k_3. a) Initial distribution (isotropic decay). b) Shortly after the rotation is started ($t\epsilon_0/q_0^2$).

The rotation killed the energy transfer (through the generation of inertial waves; see Fig. 1) and the turbulence underwent a pure viscous decay as would be predicted by Rapid Distortion Theory (RDT). More precisely, the energy spectrum, $E(k,t)$, decayed in time in good agreement with the formula,

$$E(k,t) = E(k,t_0)\exp\left[-2\nu k^2(t-t_0)\right] \tag{1}$$

where ν is the kinematic viscosity (see Fig. 2). Equation 1 is obtained from RDT for this problem. The surprising finding was that Eq. (1) remained an excellent approximation even after the turbulent kinetic energy had decayed to only 10% of its initial value. As a result of the energy transfer being suppressed, the turbulence

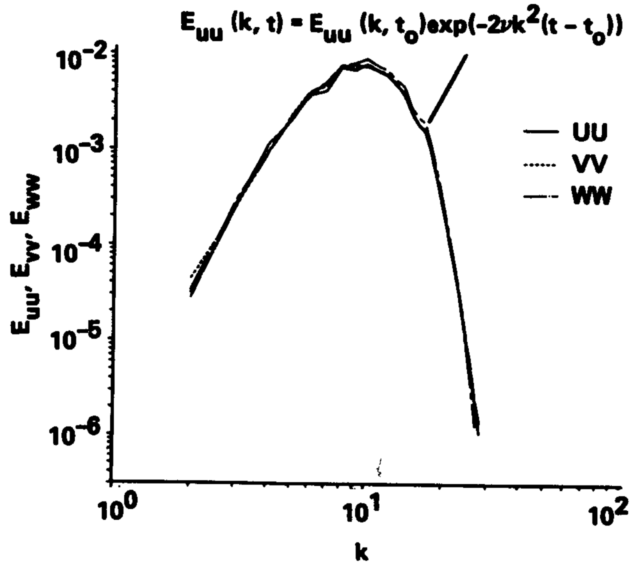


FIGURE 2. Energy spectra, E_{uu}, E_{vv} and E_{ww} $(q^2/q_0^2 = 0.03)$.

decayed slower in the rotating frame (see Fig. 3). Although the integral length scales showed the development of mild anisotropies, there was no discernable Taylor-Proudman reorganization to a two-dimensional flow. The tensor,

$$A_{ij} = \frac{\nu}{\epsilon}\overline{u'_{k,i}u'_{k,j}} \tag{2}$$

which is normalized by the dissipation rate ϵ, remained isotropic (under a complete Taylor-Proudman reorganization, the velocity gradient along the axis of rotation $u_{,3} \mapsto 0$ as the rotation rate $\Omega \mapsto \infty$ and, hence $A_{33} << A_{11}, A_{22}$). It can be shown that the RDT solution does not undergo a Taylor-Proudman reorganization since the Fourier transform of the velocity,

$$\hat{u}_i(k,t) \propto A(k)\exp(i\alpha(k)\Omega t) \tag{3}$$

and, hence, in the limit as $\Omega \mapsto \infty$,

$$\frac{1}{\Omega}\frac{\partial \omega_i}{\partial t} = O(1) \tag{4}$$

where ω_i is the vorticity vector.

In order for the Taylor-Proudman theorem to apply, $(1/\Omega)\partial\omega_i/\partial t$ must vanish as $\Omega \mapsto \infty$. Since RDT becomes a better approximation for longer instants of time as Ω gets larger (for a given turbulence level), it appears that no Taylor-Proudman

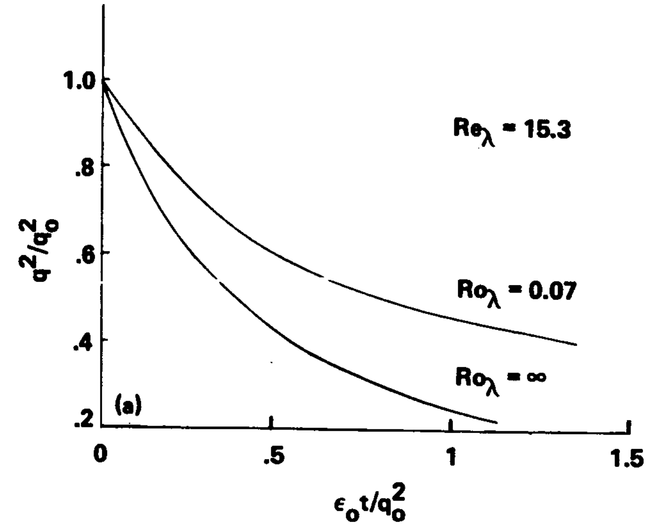


FIGURE 3. Effect of rotation on the decay of the turbulent kinetic energy.

reorganization will occur in a rapidly rotating isotropic turbulence. Previous investigators (*c.f.*, Bardina, Ferziger and Rogallo 1985) had speculated that such a reorganization to a two-dimensional flow could occur.

Since the results of these direct simulations on rotating isotropic turbulence clearly demonstrate (in support of Wigeland and Nagib 1978, and Bardina, Ferziger and Rogallo 1985) that there is a reduction in the dissipation rate with increasing rotation rate, it is clear that modifications need to be made in the dissipation rate equation. All of the commonly used dissipation rate equations (*c.f.*, Launder, Reece and Rodi 1975) predict that for a given mean flow, a system rotation has no effect on the evolution of the dissipation rate in contradiction of experimental and numerical simulation data. A recently proposed model by Bardina, Ferziger and Rogallo (1985) given by,

$$\dot{\epsilon} = -C_1 \frac{\epsilon^2}{q^2} - C_2 \Omega \epsilon \tag{5}$$

for an isotropic turbulence (where ϵ is the dissipation rate, q^2 is the trace of the Reynolds stress tensor, and $C_2 = 11/3$ and $C_1 = 0.15$ are empirical constants) was tested. It was found that this model, which compared favorably with the data of Wigeland and Nagib (1978) performed very poorly at the rapid rotation rates considered herein (see Fig. 4). Consequently, it appears that the dependence of the dissipation rate equation on Ω is unlike in Eq. (5). Furthermore, it is not clear at this time how such a modified dissipation rate equation could be generalized to anisotropic turbulent flows. Any smooth function of the invariants,

$$\Omega_{ij}\Omega_{ij}, \Omega^2_{ij}\tau_{ij}, \Omega^2_{ij}S_{ij}, \tag{6}$$

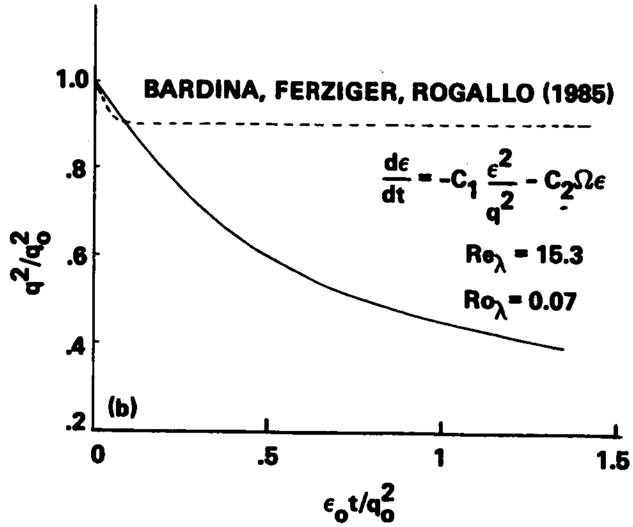


FIGURE 4. Comparison of the decay of the turbulent kinetic energy with the model of Bardina, Ferziger & Rogallo (1985).

where,

$$\Omega_{ij} = \frac{1}{2}\left(U_{i,j} - U_{j,i}\right), \quad S_{ij} = \frac{1}{2}\left(U_{i,j} + U_{j,i}\right) \tag{7}$$

U_i is the mean velocity, and τ_{ij} is the Reynolds stress tensor, reduce to functions of Ω in the limit of rotating isotropic turbulence (a model containg $\Omega_{ij}^2 S_{ij}$ was proposed by Pope 1978). Work on the development of a more general dissipation rate equation which can account for rotation in inhomogeneous turbulent flows was begun during the summer program and will continue in the future.

2. Future work

The time development of the energy spectrum for an isotropic turbulence is given by,

$$\dot{E}(k,t) = T(k,t) - 2\nu k^2 E(k,t) \tag{8}$$

where $T(k,t)$ is the energy transfer. The equation for the dissipation rate, ϵ, can be derived by multiplying Eq.(8) by $2\nu k^2 dk$ and integrating over all wave numbers,

$$\dot{\epsilon} = 2\nu \int_0^\infty k^2 T(k,t)dk - 4\nu^2 \int_0^\infty k^4 E(k,t)dk \tag{9}$$

The commonly used model for the right-hand-side of Eq. (9) is given as follows:

$$2\nu \int_0^\infty k^2 T(k,t)dk - 4\nu^2 \int_0^\infty k^4 E(k,t)dk = -C_1 \frac{\epsilon^2}{q^2} \tag{10}$$

where q^2 is twice the turbulent kinetic energy. Any modification to the above model that takes into account the effect of rotation, has to reflect the fact that the first term in Eq. (10) vanishes for high rotation rates.

Work was begun on the development of improved second-order closure models. In all of the currently popular second-order closure models it is assumed that the deviatoric part of the velocity pressure-gradient correlation ${}_D\Pi_{ij}$ and the deviatoric part of the dissipation rate corelation ${}_DD_{ij}$ are functions of τ_{ij}, S_{ij}, and Ω_{ij}. Hence, we considered the most general model of the form

$$ {}_D\Pi_{kl} + {}_DD_{kl} = f_{kl}(\tau_{ij}, S_{ij}, \Omega_{ij}) \tag{11} $$

Equation (8) should be subjected to the constraints:

(i) Form invariance under a change of coordinates (c.f., Smith 1971).

(ii) Material frame-indifference in the limit of two-dimensional turbulence (see Speziale 1981).

and the fact that ${}_D\Pi_{ij}$, and ${}_DD_{ij}$ are traceless. This led to the most general form,

$$ \begin{aligned} {}_D\Pi_{kl} + {}_DD_{kl} =& \beta_1 \frac{\epsilon}{q^2}(\tau_{kl} - \frac{2}{3}k\delta_{kl}) + \beta_2 k S_{kl} + \beta_3 \frac{\epsilon}{q^4}(\tau_{km}\tau_{ml} - \frac{1}{3}\tau_{mn}\tau_{mn}\delta_{kl}) \\ &+ \beta_4 \frac{q^4}{\epsilon}(S_{km}S_{ml} - \frac{1}{3}S_{mn}S_{mn}\delta_{kl}) \\ &+ \beta_5(\tau_{km}S_{ml} + \tau_{lm}S_{mk} - \frac{2}{3}\tau_{mn}S_{mn}\delta_{kl}) \\ &+ \frac{\beta_6}{q^2}(\tau_{km}\tau_{mn}S_{nl} + \tau_{lm}\tau_{mn}S_{nk} - \frac{2}{3}\tau_{mn}\tau_{np}S_{pm}\delta_{kl}) \\ &+ \beta_7 \frac{q^2}{\epsilon}(\tau_{km}S_{mn}S_{nl} + \tau_{lm}S_{mn}S_{nk} - \frac{2}{3}\tau_{mn}S_{np}S_{pm}\delta_{kl}) \\ &+ \beta_8 \frac{1}{\epsilon}(\tau_{km}\tau_{mn}S_{nr}S_{rl} + \tau_{lm}\tau_{mn}S_{nr}S_{rk} - \frac{2}{3}\tau_{mn}\tau_{np}S_{pr}S_{rm}\delta_{kl}) \\ &+ 2(1-\beta_9)(\tau_{km}\Omega_{lm} + \tau_{lm}\Omega_{km}) + \beta_9 \frac{1}{q^2}(\tau_{km}\tau_{mn}\Omega_{nl} + \tau_{lm}\tau_{mn}\Omega_{nk}) \end{aligned} \tag{12} $$

The coefficients β_i are functions of the invariants. This model is substantially simpler than previous attempts at general representations which contained several redundant terms (c.f., Reynolds 1987 for a summary of such previous representations). In the limit of a two component turbulence, Realizability (Shih 1987, private communication) requires that

$$ \begin{aligned} \beta_1 =& 2 - \frac{1}{2k^2}\tau_{kl}\tau_{lk}\beta_3 \\ \beta_2 =& 0 \\ \beta_4 =& 0 \\ \beta_5 =& -(\frac{1}{k}\tau_{kl}\tau_{lm}S_{mk}\beta_6 + \frac{k}{\epsilon}\tau_{kl}S_{lm}S_{mk}\beta_7 + \frac{1}{\epsilon}\tau_{kl}\tau_{lm}S_{mn}S_{nk}\beta_8)/\tau_{pq}S_{pq} \end{aligned} \tag{13} $$

and hence, in the first approximation this model has only five undetermined constants. This model will be investigated in the future in collaboration with Dr. T.-H. Shih. It is interesting to note that this model is consistent with the numerical results of this study which predict that an initially isotropic turbulence in a rotating frame decays isotropically.

REFERENCES

BARDINA, J., FERZIGER, J. H. & ROGALLO, R. S. 1985 Effect of rotation on isotropic turbulence: computation and modeling. *J. Fluid Mech.* **154**, 321 - 336.

LAUNDER, B. E., REECE, G. J. & RODI, W. 1975 Progress in the development of a Reynolds-stress turbulence closure. *J. Fluid Mech.* **68**, 537 - 566.

POPE, S. B. 1978 An explanation of the turbulent round jet / plane jet anomaly. *AIAA J.* **16**, 279 - 281.

REYNOLDS, W. C. 1987 Fundamentals of turbulence for turbulence modeling and simulation, Lecture Notes for Von Karman Institute, March 16-17.

SMITH, G. F. 1971 On isotropic functions of symmetric tensors, skew-symmetric tensors and vectors. *Int. J. Engng. Sci.* **9**, 899 - 916.

SPEZIALE, C. G. 1981 Some interesting properties of two-dimensional turbulence. *Phys. Fluids.* **24**, 1425 - 1427.

WIGELAND, R. A. & NAGIB, H. M. 1978 Grid-generated turbulence with and without rotation about the streamwise direction. *IIT Fluids and Heat Transfer Rep.* R78-1, Illinois Inst. of Tech., Chicago, Illinois.