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Coherent Structures and Dynamical Systems

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Any flow of a viscous fluid has a finite number of degrees of freedom, and can therefore be seen as a dynamical system. For a turbulent flow, an upper bound to this number was given by Landau & Lifshitz (1959) and scales as $Re^{9/4}$, which is usually a rather large number. Lower bounds have been computed for some particular turbulent flows, but also tend to be large. In this context, we can think of a coherent structure as a lower dimensional manifold in whose neighborhood the dynamical system spends a substantial fraction of its time. If such a manifold exists, and if its dimensionality is substantially lower than that of the full flow, it is conceivable that the flow could be described in terms of the reduced set of degrees of freedom, and that such a description would be simpler than one in which the existence of structure had not been recognized.

As a trivial example, consider a particular two-dimensional flow for which we can prove that, after some time, most of the vorticity concentrates in a few compact vortices. Such a flow could be described by a few differential equations, and, presumably, easily integrated. Homogeneous, two- dimensional decaying turbulence seems to follow roughly this model (Benzi et al. 1987).

Other examples of the same type are transitional Taylor- Couette flow and Rayleigh-Benard convection. After the initial loss of stability, both systems develop attractors, different from the initial equilibrium, and that can be described in terms of structures (rolls or cells). Although the initial bifurcate states of those flows can hardly be called turbulent, the secondary bifurcations that grow from them can be analyzed, at least for a while, as small perturbations of the initial structures, usually described in terms of their positions and intensities. Technically, we speak of a projection on a central manifold (Demay & Iooss 1984); physically we are talking about describing a turbulent flow in terms of a few degrees of freedom. Another recent example of the same situation is the appearance of disordered states in two-dimensional Poiseuille flow, starting from bifurcations of nonlinear trains of Tollmien Schlichting waves (Jimenez 1987).

The common feature of all these flows is the existence of stable attractors, whose dimensionality is much lower than that of the full flow, and towards which the flow tends after some time. Under those conditions, the flow can be described, up to some level, by the properties of the attractor. Most "attractors" found in nature, are, however, not stable, and cannot be really called attractors at all. The system will approach them for a while, only to be repelled once it gets near the central manifold.

The simplest example of this behavior is the linear differential equation $y_{tt} + sin y = 0$, which represents a circular pendulum. If the system is given proper initial conditions it will approach the position at which the pendulum is pointing "upwards," spend some time near it, and fall back to make a quick revolution across the lower part of its trajectory. Even in this case, the system expends most of its time in the neighborhood of its top (unstable) equilibrium point, and can be described approximately as a being in equilibrium at that position, together with some model for the fast motion in the lower part of its orbit. Perhaps the best example of this situation, in a flow, is the plane temporal mixing layer. Here the "attractor" is a uniform row of compact vortices, and the flow quickly tends towards it. But this solution is itself unstable (mainly through pairing), and is eventually abandoned by the flow, only to converge to a different solution of the same kind. Even so, a model of the flow as a uniform vortex row, with a suitable approximation to the "sudden" pairing process, has been shown to give rough approximations to quantities such as spreading rates (Jimenez 1980) and concentration distributions (Hernan & Jimenez 1982). A more careful perturbation analysis on the lines outlined above has not been attempted, but might be expected to give more accurate results.

Other turbulent flows have phase space structures which are presumably more complicated. The next best plausible candidate for eduction of the complexity of a turbulent flow are the sublayer ejections in wall-bounded turbulence. Recent observations (Jimenez et al. 1987) suggest that the basic structure in that flow is a self-reproducing ejection, that could perhaps be described as an unstable limit cycle. It is not clear, at present, how to treat a dynamical system in the

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neighborhood of such a manifold, but any identification of a low dimensional structure which describes a sizeable fraction of the time evolution of a flow, opens the possibility that a local analysis in its neighborhood might give results that capture the qualitative and perhaps even some of the quantitative features of the complete flow.

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