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Computerized Life and Reliability Modelling for Turboprop Transmissions

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Abstract

A generalized life and reliability model is presented for parallel shaft geared prop-fan and turboprop aircraft transmissions. The transmission life and reliability model is a combination of the individual reliability models for all the bearings and gears in the main load paths. The bearing and gear reliability models are based on classical fatigue theory and the two parameter Weibull failure distribution. A computer program was developed to calculate the transmission life and reliability. The program is modular. In its present form, the program can analyze five different transmission arrangements. However, the program can be modified easily to include additional transmission arrangements. An example is included which compares the life of a compound two-stage transmission with the life of a split-torque, parallel compound two-stage transmission, as calculated by the computer program.

Nomenclature

C component dynamic capacity, N

D component or transmission dynamic capacity, N-m

F component load, N

L component or transmission life, million cycle

R component or transmission reliability

T transmission output torque, N-m

Subscripts:

a first component

b second component

c third component

i ith component

n number of components

t transmission

10 90 percent reliability

Superscripts:

e Weibull slope

p load-life factor

Introduction

One important measure of the performance of turboprop transmissions is the service time between scheduled maintenance overhauls. In the design of new prop-fan and turboprop aircraft propulsion systems, service life calculations are required. Tools that assist the transmission designer in comparing different configurations on a common life and reliability basis for a given mission spectrum loading before the transmissions are constructed are quite valuable. By determining transmission system life as well as component lives, weak links in a design can be identified and strengthened. This enables the designer to bring forward the best configuration and configuration embodiment possible to the final design stage of a transmission's development from a life and reliability standpoint.

Computer programs are available for life analysis of various bearings and bearing shaft arrangements based on the classical fatigue theory of Lundberg and Palmgren.[1,2] This theory has also been applied to the analysis of fatigue life for spur and helical gear sets.[3-7] In previous papers, the life and reliability of helicopter planetary and bevel transmissions have been developed as functions of the lives and reliabilities of the transmission component bearings and gears.[8-10]

The life models assume that the transmission is adequately lubricated and that the components are well designed. For gears, this means that sufficient rim thicknesses and proper materials are used to prevent premature tooth breakage failures. It is assumed also that the tooth form geometry and lubricant are adequate to avoid gear tip scoring. Both of these modes of failure are preventable with adequate design.[11] The life models assume steady power operation with minimal start-up or deceleration shock loading. The gas turbine power source of the turboprop engine provides this type of power input.

Surface pitting in the full load region of the gear tooth faces and in the bearing races is not preventable, however, due to the lack of a surface endurance limit for high strength steels.[1-7] Both elements will fail eventually in surface pitting regardless of the loads. Also, if similar components operate at the same conditions, they will exhibit a scatter in surface pitting fatigue life. Thus, statistical methods, such as the two parameter Weibull distribution are commonly used in predicting fatigue life.

In the present work, a computer program predicting life and reliability for parallel shaft gear transmissions of various configurations is presented. The transmission system life and reliability model is a combination of the component reliability models. The components included in the system model are the bearings and gears in the main load paths. The component life and reliability models are based on the two parameter Weibull distribution. The component life models are the same as was done in previous life modeling of helicopter transmissions.[8-10] The major contribution of the present work is the addition of parallel shaft transmission configurations and the modular structure of the computer program.

The computer model was written as a modular program so that many different transmission configurations could be treated with the same system analysis routine. In the present state of development, the program can treat a variety of parallel shaft transmission configurations. These include: (1) single pass gear transmissions, (2) compound gear transmissions, (3) parallel compound gear transmissions, (4) star gear transmissions, and (5) planetary gear transmissions.

These configurations were chosen to model the variety of turboprop transmissions in use today. Due to the modular form of the program, additional transmission configurations can be added easily to model transmission concepts not included in the original five basic configurations.

The program also incorporates a modular approach to the determination of the transmission system life and dynamic capacity. In the program, a common block property array is used to store transmission component information. The program then uses configuration independent subroutines to perform the system life and dynamic capacity analyses with the data in the common block. Configuration specific subroutines are used to prepare the common block property array for analysis and to display the results of the analysis. With this structure, the program can be easily expanded to include additional transmission configurations.

System Life Analysis

The life and reliability model of this analysis is Palmgren's,[12] originally developed for rolling element bearings:

$$L_{10} = \left(\frac{C}{F}\right)^p \tag{1}$$

where L_{10} is the life of the component for a 90 percent probability of survival, F is the equivalent applied load, C is the basic dynamic capacity of the component and p is the load-life factor. The basic dynamic capacity is the equivalent load at which 90 percent of the components will survive one million load cycles. This equation describes a load-life relationship for which there is no endurance limit.

The model for the component life as a function of load is normally combined with the two parameter Weibull distribution for probability of survival, R, as a function of life, L, at a given load. In terms of the 90 percent probability of survival life, L_{10}, and the Weibull slope, e, the two parameter Weibull distribution can be expressed as:

$$\text{Log}\left(\frac{1.0}{R}\right) = \text{Log}\left(\frac{1.0}{0.9}\right) \bullet \left(\frac{L}{L_{10}}\right)^e \tag{2}$$

The Weibull slope characterizes the shape or skewness of the distribution. For bearings, and to a lesser extent for gears, the life distribution is skewed high, with a larger percent of failures occurring before the mean life than after it.

The model for the system life of a transmission was chosen by comparing the transmission which is composed of load carrying gears and bearings to a chain of links. Just as a chain fails when any single link breaks, the transmission is considered to be failed when its first component has failed. This assumption yields conservative estimates of system failure events and is termed the "strict series probability model." In this model, the probability of survival of the system is equal to the product of the probabilities of survival of all the components.

$$R_t = R_a * R_b * R_c \bullet \ldots * R_n = \prod_{i=a}^{n} R_i \tag{3}$$

The strict series probability model is justified on the basis of the high speed of transmission components and the spray effect of loose debris. If any component fails, debris may be present in the transmission which could accelerate the fatigue damage in other components. So, if any single element in the transmission has failed, the transmission must be overhauled to return the transmission to its initial state of high reliability.

The log of the reciprocal of Eq. (3) is:

$$\text{Log}\left(\frac{1.0}{R_t}\right) = \sum_{i=1}^{n} \text{Log}\left(\frac{1.0}{R_i}\right) \tag{4}$$

Substitution of Eq. (2) into Eq. (4) for each component yields:

$$\text{Log}\left(\frac{1.0}{R_t}\right) = \text{Log}\left(\frac{1.0}{0.9}\right) \sum_{i=1}^{n} \left(\frac{L_t}{L_{i10}}\right)^{e_i} \tag{5}$$

In this equation, L_t is the life of the entire transmission for the system reliability, R_t. It is also the life of each component in the transmission at the same system reliability, R_t. To make this equation valid, all the component lives must be expressed in the same time base, such as million of transmission output shaft rotations.

This relation is not a simple two parameter Weibull relationship between system life and system reliability. The equation would be a true two-parameter Weibull distribution only in the case for which all the Weibull exponents, e_i, were equal. For the general case in which this is not true, the equation can be solved numerically for transmission reliability, R_t, as a function of transmission life, L_t, and plotted on Weibull coordinates.

This plot of percent probability of failure versus transmission life can be approximated by a straight line quite easily. As an example in this work, the straight line approximation is obtained with a linear regression in Weibull coordinates over the range $0.5 \leq R_t \leq 0.95$. The slope of this straight line approximation is taken as the transmission Weibull slope, e_t, and the life at which the transmission reliability, R_t, equals 90 percent for the straight line relationship is taken as the transmission 90 percent reliability life, L_{t10}. The equation for this fitted Weibull relation is:

$$\mathrm{Log}\left(\frac{1.0}{R_t}\right) = \mathrm{Log}\left(\frac{1.0}{0.9}\right) \bullet \left(\frac{L_t}{L_{t10}}\right)^{e_t} \qquad (6)$$

System Dynamic Capacity

The analysis for the transmission dynamic capacity proceeds in a similar manner. The basic dynamic capacity for the system, D_t, is the transmission output torque which will result in a 90 percent reliability transmission life of one million output shaft rotations. For these conditions, Eq. (5) becomes:

$$1.0 = \sum_{i=1}^{n} \left(\frac{1.0}{L_{i10}}\right)^{e_i} \qquad (7)$$

where the components lives ar expressed in terms of million of transmission output shaft rotations. By replacing the actual and basic dynamic component loads with the corresponding transmission output torques, Eq. (1) can be expressed as:

$$L_{i10} = \left(\frac{D_i}{T}\right)^{p_i} \qquad (8)$$

where D_i is the component dynamic capacity in units of output torque, L_{i10} is the component 90 percent reliability life and T is the transmission output torque which produces that component life. For a 90 percent reliability transmission life of one million output rotations, T becomes the transmission dynamic capacity, D_t. Substituting Eq. (8), with $T = D_t$ into Eq. (7) for each component, gives:

$$1.0 = \sum_{i=1}^{n} \left(\frac{D_t}{D_i}\right)^{e_i p_i} \qquad (9)$$

where D_t is the basic dynamic capacity of the transmission in units of output torque. This equation can be solved for D_t by iteration.

A series of 90 percent reliability lives for the transmission can be determined with Eq. (6) for a series of applied transmission loads. A log-log plot of output transmission torque versus transmission 90 percent reliability life can then be obtained over a range of output torques which varies from 10 to 100 percent of the dynamic capacity as found from Eq. (9). The slope of a least squares fit to this data is the negative of the reciprocal of the load-life exponent, p_t, for the transmission. The transmission dynamic capacity is taken as the output torque on the regression line which corresponds to a 90 percent reliability life of one million output shaft rotations.

Computer Life Analysis Program

The structure of the computer program which performs these analyses is described in the block diagram of Fig. 1. In this program, a common block array for properties is used to separate the component and transmission property values from the analysis subroutines. This array is two-dimensonal - its rows correspond to specific components with the first row containing the values for the entire transmission, and its column containing values for specific properties. Since the subroutines which determine the transmission life and dynamic capacity interface solely with this property array, they are thus separated from any specific transmission configuration. The system analysis subroutines work in an identical manner for all transmission configurations considered. Thus, other configurations can be added to the program by adding component property determination subroutines only.

As indicated in the figure, the program consists of a main program, a series of configuration specific subroutines, some generic component property analysis subroutines, the system analysis subroutines and the common block. The main program selects the routines to be used in the analysis and sequences their operation. The series of configuration specific subroutines input the configuration data, perform component force and life analyses with the help of the generic component property analysis subroutines, fill the property array, call up the system analysis routines, and finally print out the analysis results for the system and the components.

Transmission Configurations

The parallel axis transmissions treated by this program are: (1) single pass gear transmissions, (2) compound gear transmissions, (3) parallel compound gear transmissions, (4) star gear transmissions, and (5) planetary gear transmissions.

The single mesh gear transmission illustrated in Fig. 2 is composed of two gears and two shafts. The gears are in mesh, and both the input and output shafts are supported by two bearings each. The transmission life and dynamic capacity analyses are based on the lives and capacities of the two gears and the four bearings.

The compound gear transmission illustrated in Fig. 3 has two or more meshes and three or more gear shafts arranged in a single power path configuration. These transmissions contain a minimum of six bearings and four gears. For each additional mesh, two more gears and two more bearings are added. As with all the cases, the cited bearings and gears are the transmission components which influence the transmission life and capacity.

The parallel compound gear transmission shown in Fig. 4 has single input and output gears with split path power transfer through multiple intermediate gear shafts. These intermediate gears shafts need not be placed symmetrically around the input and output gears. This type of transmission includes an input and an output gear and multiple intermediate shafts with two gears each which mesh with the input and output gears. The four bearings on the input and output shafts and two additional bearings for each intermediate shaft are also included in the transmission.

The star gear transmission shown in Fig. 5 has concentric input and output external gears, with multiple intermediate gear shafts placed symmetrically around the common input and output shaft centerline. This type of transmission has an input and an output gear and multiple intermediate shafts with two gears each which mesh with the input and output gears. Due to the symmetry of gear loading, the loads on the four bearings on the input and output shafts cancel to zero and thus the bearings are not included in the life and dynamic capacity analyses for these transmissions. However, the lives and capacities of two bearings for each intermediate shaft are included in the transmission analysis.

The planetary gear transmission illustrated in Fig. 6 has a fixed ring gear, an input sun gear and output power taken from the planet carrier. Single bearings are placed in the centers of the planet gears. The life and capacity analyses of these transmissions include the lives and capacities of all the gears and planet bearings.

Examples

An in-line double mesh compound gear transmission is shown in Fig. 7. This transmission has an input speed of 14 000 rpm, an output speed of 2 000 rpm and a power rating of 300 kW. The first reduction is 7:3 and the second reduction is 3:1. Table 1 itemizes the component loads, component dynamic capacities in terms of transmission output torque, and 90 percent reliability lives of the components in million output rotations. The Weibull slopes for the bearings were given to be 1.2[1,2] while those for the gears were given to be 2.5.[3-7] The load-life exponents for the bearings were given to be 3.0 for the ball bearings and 3.33 for the roller bearings[1] while those for the gears were given to be 4.3.[3-7]

By program analysis, this transmission has a 90 percent reliability life of 213 million output cycles or 1779 hr, a dynamic capacity of 5.80 kN-m, a Weibull slope of 1.61 and a load-life factor of 3.75.

For comparison purposes, change this transmission to a parallel compound gear transmission with a second intermediate shaft identical to the first and located 95° from the first, as shown in Fig. 8. Table 2 itemizes the loads, dynamic capacities and 90 percent reliability lives of the components in this configuration.

By program analysis, this transmission has a 90 percent reliability life of 1026 million output cycles or 8550 hr, a dynamic capacity of 10.18 kN-m, a Weibull slope of 1.25 and a load-life factor of 3.60.

Adding the second power path reduced gear and bearing loads and thus increased the transmission life by almost four times and increased its dynamic capacity by 75 percent. Naturally, it increased the transmission weight as well. Figure 9 is a Weibull probability plot of the lives of the compound gear transmission and the parallel compound gear transmission. Other variations of this transmission could easily be evaluated for life and capacity.

Concluding Remarks

A generalized life and reliability model for parallel shaft geared prop-fan and turboprop aircraft transmissions was presented. The transmission life and reliability model is based on a combination of the component reliability and life models. The components included in the system model are the bearings and gears in the main load paths. The component lives and reliabilities are based on the two parameter Weibull failure distribution life-reliability model and the Palmgren classical fatigue load-life model. The theory of the reliability and dynamic capacity models was presented, including the approximations necessary to make the system models take on the same form as the component models.

A modular program was described which includes five separate transmission configurations. The intent of the program is to enable the designer to bring forward the best configuration and configuration embodiment possible to the final design stage of a transmission's development from a life and reliability standpoint.

The components included in each configuration system model are the bearings and gears in the main load path. The program uses a two-dimensional property array to separate the system analyses from configuration specific force and motion analyses. With this separation, it is easy to expand the program to include additional transmission configurations.

Finally, examples are included to compare two transmission configurations from a life and dynamic capacity standpoint. The parallel compound transmission with a second load path compared to a simple compound transmission with identical components had a life which was more than five and one-half times greater and a dynamic capacity which was 75 percent greater than that of its simpler and lighter counterpart.

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TABLE 1. - COMPOUND TRANSMISSION COMPONENT LOADS, CAPACITIES, AND LIVES

Component	Load F, kN	Weibull slope e	Load-life factor p	Capacity D, kN-m	Life L_{10}, million cycles
1st input bearing	1.25	1.2	3.0	36.34	16 317
2nd input bearing	4.15	1.2	3.3	11.25	898
Input gear	2.90	2.5	4.3	9.91	4 082
1st intermediate gear	2.90	2.5	4.3	11.15	6 787
1st intermediate bearing	2.52	1.2	3.3	28.66	19 682
2nd intermediate bearing	6.10	1.2	3.0	11.80	559
2nd intermediate gear	8.13	2.5	4.3	5.87	429
Output gear	8.13	2.5	4.3	6.84	829
1st output bearing	11.61	1.2	3.3	10.14	638
2nd output bearing	3.48	1.2	3.0	34.75	14 279
System	-----	1.61	3.75	5.80	213

TABLE II. - PARALLEL TRANSMISSION COMPONENT LOADS, CAPACITIES, AND LIVES

Component	Load F, kN	Weibull slope e	Load-life factor p	Capacity D, kN-m	Life L_{10}, million cycles
1st input bearing	0.84	1.2	3.0	53.78	52 919
2nd input bearing	2.80	1.2	3.3	16.65	3 275
Input gear	1.45	2.5	4.3	19.81	40 206
1st intermediate gear	1.45	2.5	4.3	22.30	133 694
1st intermediate bearing	.93	1.2	3.3	78.13	538 557
2nd intermediate bearing	3.05	1.2	3.0	23.60	4 472
2nd intermediate gear	4.07	2.5	4.3	11.73	8 452
Output gear	4.07	2.5	4.3	13.67	8 170
1st output bearing	7.85	1.2	3.3	15.01	2 327
2nd output bearing	2.35	1.2	3.0	51.44	46 304
System	----	1.25	3.60	10.18	1 026

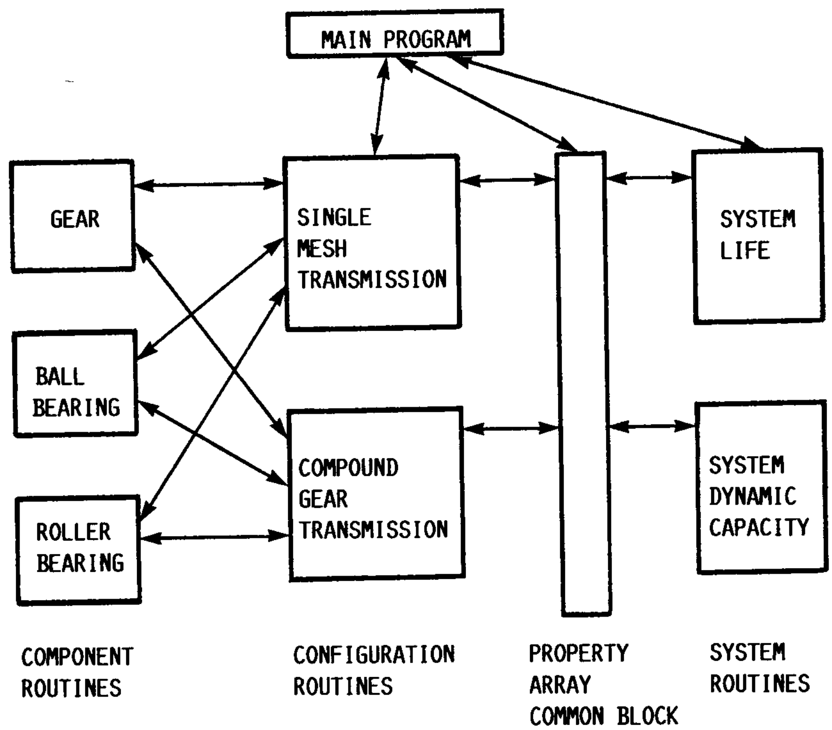

FIGURE 1. - PROGRAM STRUCTURE.

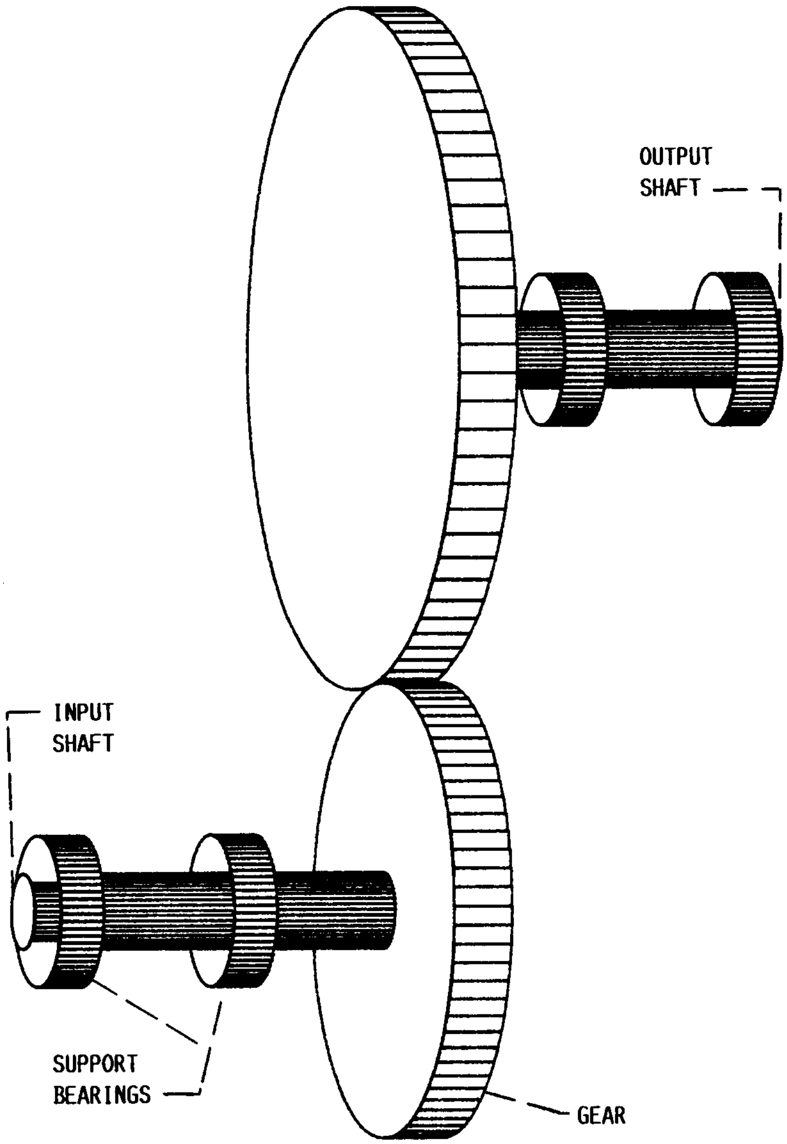

FIGURE 2. - SINGLE MESH GEAR TRANSMISSION.

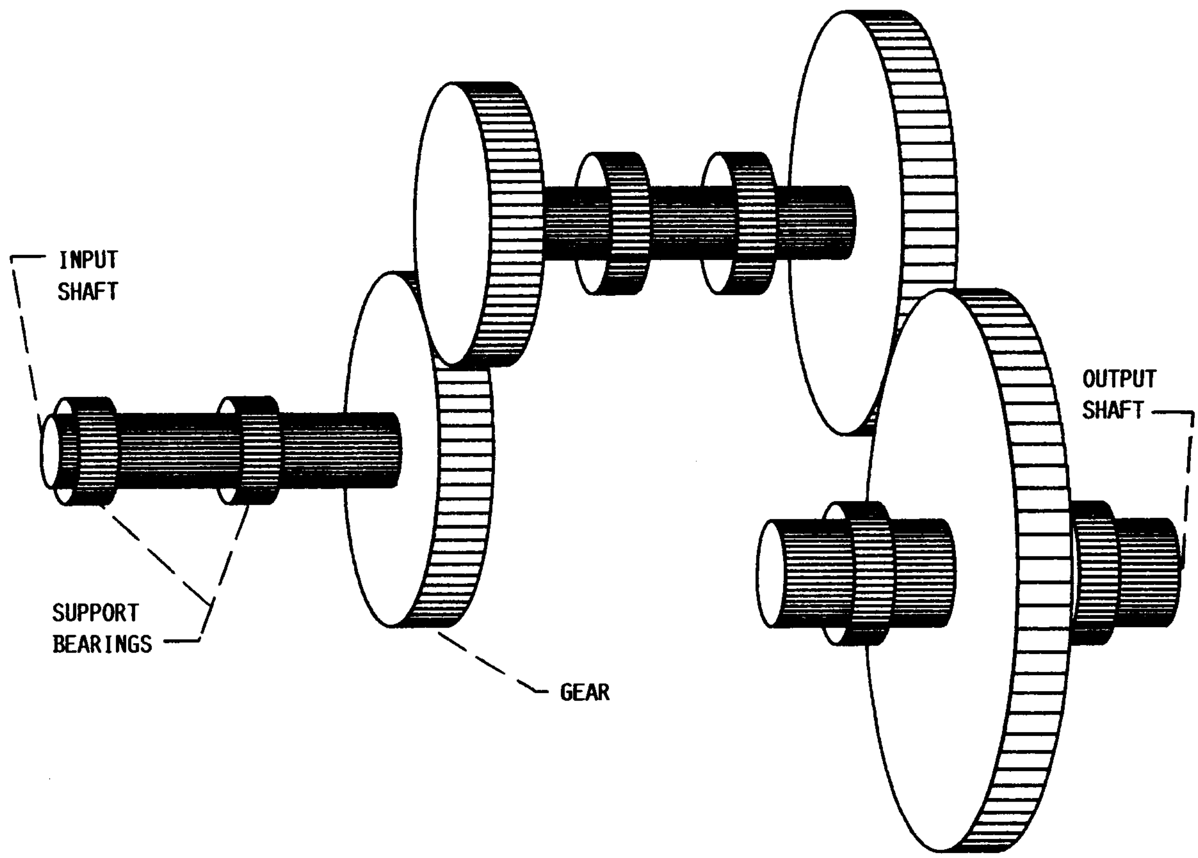

FIGURE 3. - COMPOUND GEAR TRANSMISSION.

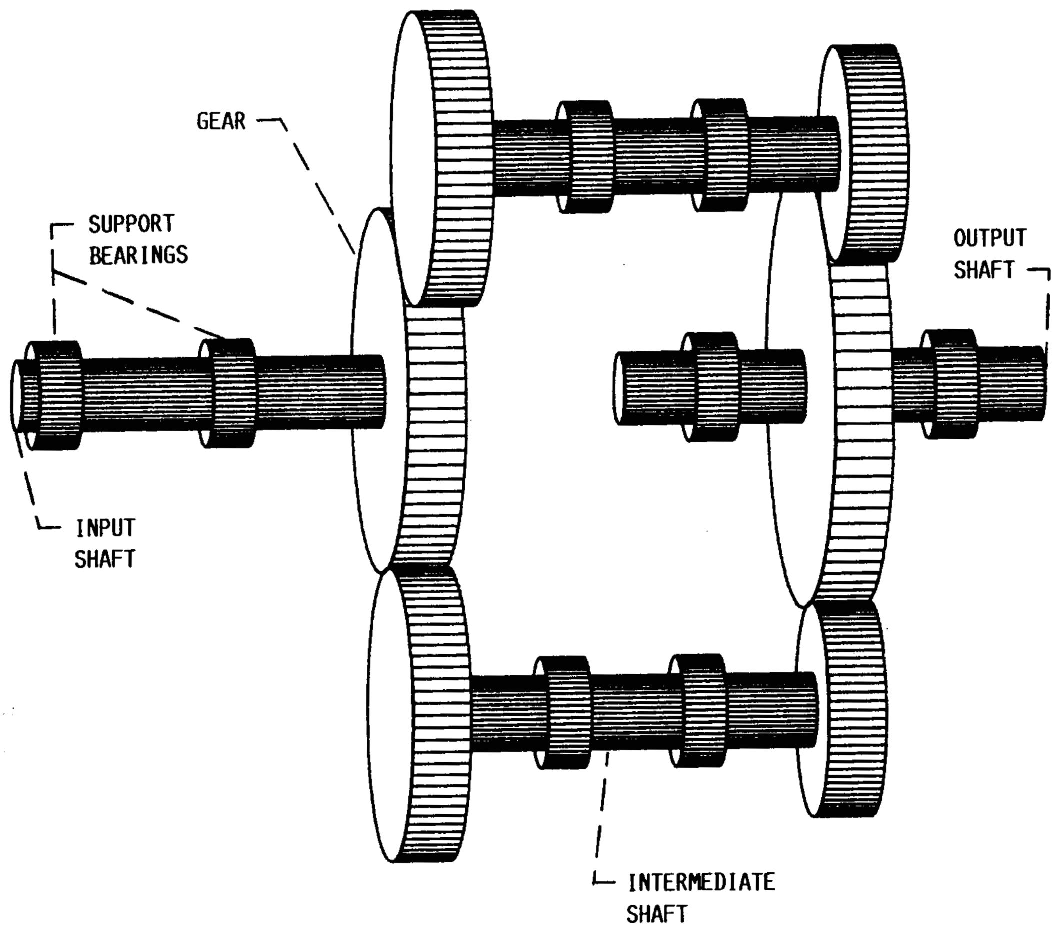

FIGURE 4. - PARALLEL COMPOUND GEAR TRANSMISSION.

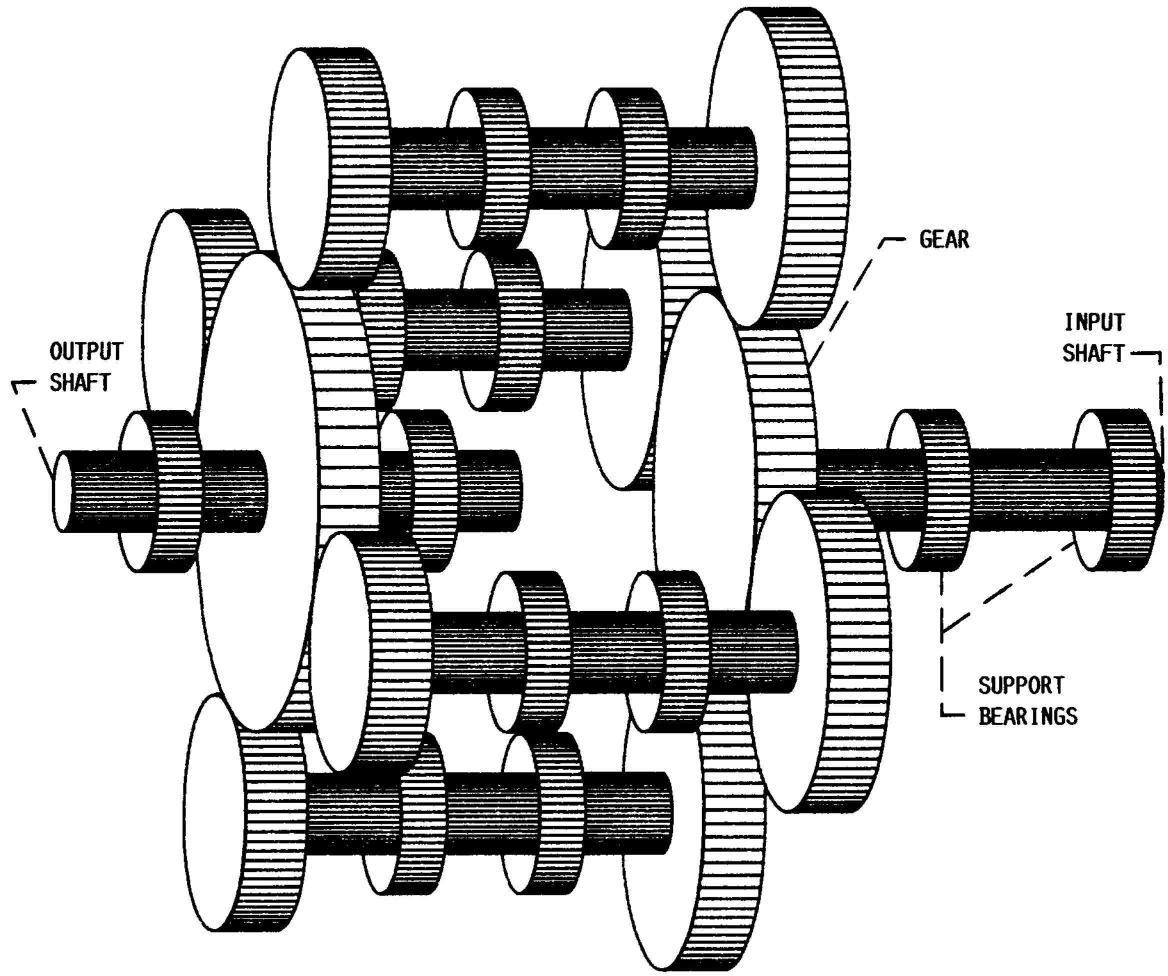

FIGURE 5. - STAR GEAR TRANSMISSION.

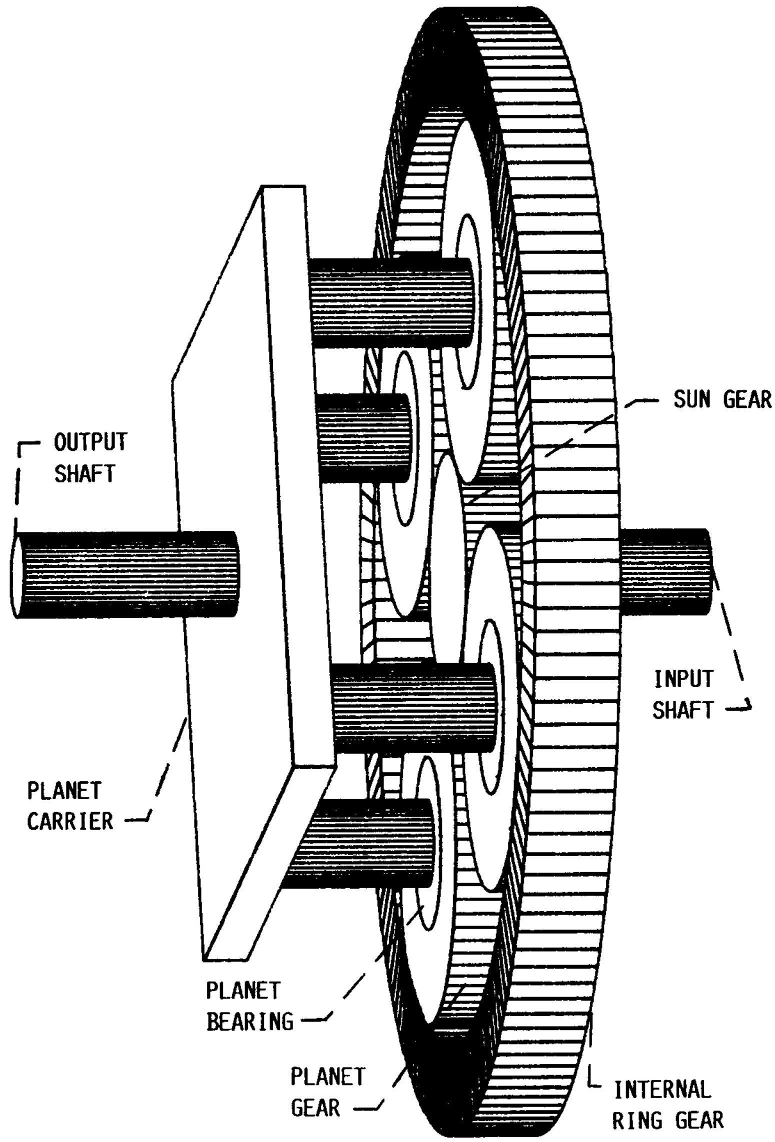

FIGURE 6. - PLANETARY GEAR TRANSMISSION.

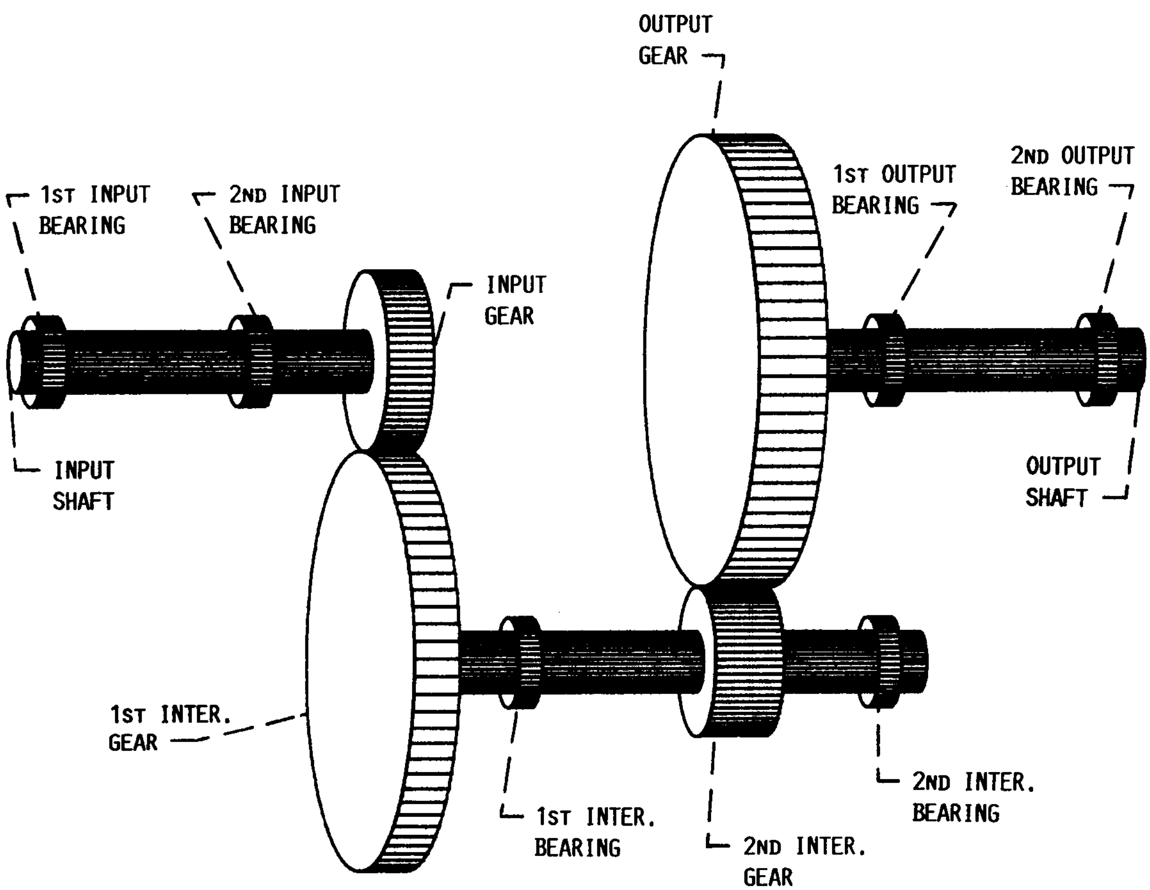

FIGURE 7. - COMPOUND TRANSMISSION EXAMPLE.

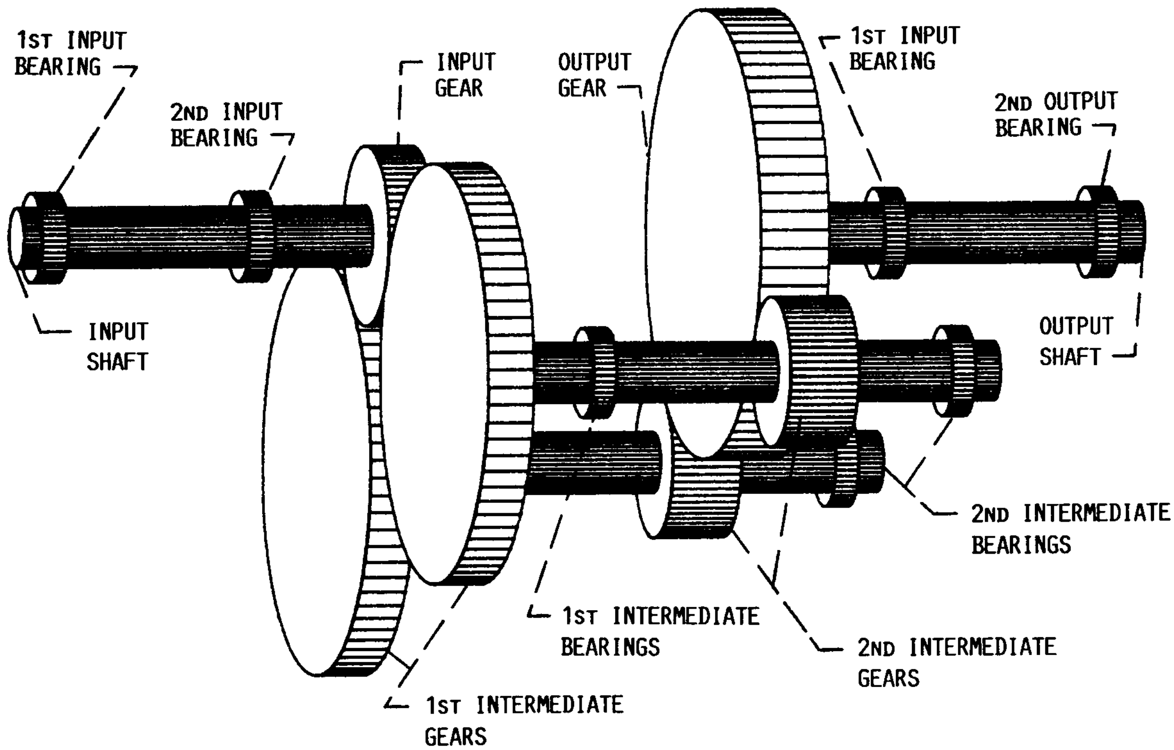

FIGURE 8. - PARALLEL COMPOUND TRANSMISSION EXAMPLE.

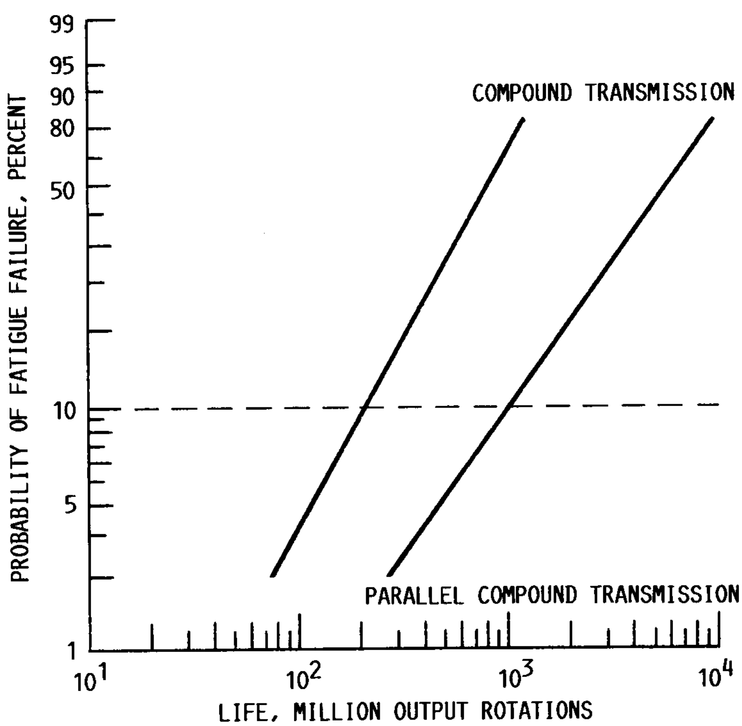

FIGURE 9. - EXAMPLE TRANSMISSION WEIBULL LIFE PLOTS.

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16. Abstract

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