N88-23228

Periodic Response of Nonlinear Systems

C. Nataraj, Ph.D
Trumpler Associates, Inc.
Westchester, PA 19380

H. D. Nelson, Ph.D
Mechanical and Aerospace Engineering
Arizona State University
Tempe, AZ 85287-6106

ABSTRACT

A procedure is developed to determine approximate periodic solutions of autonomous and non-autonomous systems. The trignometic collocation method (TCM) is formalized to allow for the analysis analysis of relatively small order systems directly in physical coordinates. The TCM is extended to large order systems by utilizing modal analysis in a component mode synthesis strategy. The procedure was coded and verified by several check cases. Numerical results for two small order mechanical systems and one large order rotor dynamic system are presented. The method allows for the possibility of approximating periodic responses for large order forced and self excited nonlinear systems.

NOMENCLATURE

$\mathbf{a}_0$	Fourier static coef. vector
$\mathbf{a}_i, \mathbf{b}_j$	Fourier cosine, sine coef. vector
$\underline{A}$, $\underline{B}$	state matrices
$\mathbf{c}$	vector of Fourier coef.
$\underline{C}$	matrix of collocation values
$\underline{D}$	tridiagonal matrix
$\mathbf{f}$	state function vector
$\mathbf{F}$	state force vector
$\underline{G}$	damping matrix
$\underline{K}$	stiffness matrix
$\underline{M}$	mass matrix
m	no. of harmonics
N	no. of collocation points
n	no. of modes
$\hat{n}$	truncated no. of modes
$\mathbf{q}$	physical coordinate vector
$\mathbf{q}^b$	nonlinear subvector of $\mathbf{q}$
$\mathbf{Q}^s$	linear system force vector
$\mathbf{Q}^b$	nonlinear force vector
R	norm
$\mathbf{r}, \mathbf{l}$	right, left displacement vectors
$\underline{r}$	right modal matrix
$\underline{S}$	connectivity matrix
t	time
T	fundamental period

$\mathbf{T}$	transformation matrix
$\mathbf{x}$	state vector
$\mathbf{y},\mathbf{z}$	state right, left vectors

GREEK

δ_{ij}	Kronecker delta
λ	eigenvalue
η	modal coordinate
ω	fundamental frequency

SUPERSCRIPT

o	collocation values
•	d/dt
T	transpose

INTRODUCTION

Nonlinear phenomena of many forms are clearly present in all complicated machinery. The future development and advancements in these machines depends strongly on our ability to identify, understand, model, analyze and design with these various nonlinear mechanisms present. Transient and steady state analysis capabilities are required with direct numerical integration, presently the most popular tool. This work presents a method based on trignometric collocation for approximating periodic solutions of forced systems and for locating limit cycles of self excited systems. The use of modal analysis allows the method to be extended to large order systems.

Previous work on the steady state response of systems which include nonlinear components is limited except by direct numerical integration. This can be very time consuming, especially for large order systems, and is not particularly economical in parametric design applications. It is really the only option available for transient analysis, however, and also serves as a useful means for verifying final designs.

Some quantitative methods for steady state analysis of nonlinear systems include perturbation techniques, describing function procedures, harmonic balance procedures, and methods of weighted residuals. Perturbation techniques (Nayfeh, 1981) have a limited range of applicability due primarily to high algebraic complexity for large order systems. They also require the introduction of a small parameter, thus restricting the solution validity to systems with weak nonlinearities. Describing function methods (Atherton, 1982) are a good choice for many problems since they can accommodate non-analytic nonlinearities. They can be used, however, only when higher harmonics are small compared to the fundamental component.

The harmonic balance method, (Hagedorn, 1982), has been recently applied to the analysis of engineering systems (Yamauchi, 1983; Saito, 1985) and the preliminary results indicate that the method may be quite effective. An alternate approach is the use of methods of weighted residuals which have been used quite extensively in the past to solve nonlinear boundary value problems. Some of these methods, which have been extended to the problem of determining periodic response, include Galerkin's method (Urabe, 1965; Urabe and Reiter, 1966; Stokes, 1972) and the Trignometric Collocation method (Samoilenko and Ronto, 1979).

The primary objective of this work is to formulate the mathematical procedures for the analysis of periodic motion in nonlinear systems. The proposed procedure involves a coupling of the Trignometric Collocation method (TCM) with modal analysis techniques, thereby effecting a substantial reduction in the number of unknown quantities in the iterative part of the solution process.

MATHEMATICAL DEVELOPMENT

The focus of this research is on the TCM that was developed and formalized by Ronto (Samoilenko and Ronto, 1979) with applicability to small order systems. Described below are the essential features of the procedure for both non-autonomous and autonomous systems.

Trignometric Collocation Method

Many engineering systems can be modelled by a set of n nonlinear ordinary differential equations of the non-autonomous form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) \tag{1}$$

where the RHS is continuous and periodic with a period T. It is required to determine a periodic solution $\mathbf{x}(t)$ of Eq. (1). It is assumed that the required solution can be approximated by a finite trigonometric series:

$$\mathbf{x} = \mathbf{a}_0 + \sum_{j=1}^{m} \left[\mathbf{a}_j \cos(j\omega t) + \mathbf{b}_j \sin(j\omega t) \right] \tag{2}$$

where ω is is the fundamental frequency. The unknown coefficients of the above series can be ordered into a vector,

$$c_i = (a_0, a_1, b_1, a_2, b_2, \ldots . a_m, b_m)^T \tag{3}$$

corresponding to each variable x_i.

The collocation method essentially consists of substituting the assumed solution form, Eq. (2), into the system state, Eq. (1), and requiring that the equations be identically satisfied at a specified number of points, N. This gives rise to N∗n nonlinear algebraic equations which must be solved to obtain the unknown coefficients. For a unique solution, the following inequality must be satisfied

$$N \geq (2m + 1) \tag{4}$$

Rigorous investigations of the applicability and foundation of the TCM have been carried out by Ronto (Samoilenko and Ronto, 1979), and only the formalism of the procedure is presented here.

The state variables can be evaluated at the collocation points in terms of the unknown coefficients leading to the form:

$$\overset{\circ\,\bullet}{\mathbf{x}}_i = \underline{T}\, \mathbf{c}_i \tag{5}$$

where,

$$\overset{0\,\bullet}{\mathbf{x}}_j = \left(x_i(t_o), x_i(t_1), \ldots\ldots, x_i(t_N) \right)^T \tag{6}$$

is a vector of values of the trignometric polynomial at the collocation points. The array $\underline{T}$ is an N * (2m + 1) transformation matrix whose elements are defined as:

$$T_{ij} \begin{cases} 1 & j = 1 \\ \cos[(i-1)j\pi/N] & j = 2, 4, \ldots \\ \sin[(i-1)(j-1)\pi/N] & j = 3, 5, \ldots \end{cases} \qquad (7)$$

The derivative of each state variable can be expressed in a trignometric series and is obtained by differentiation of Eq. (2). Hence, the following relation, which is similar to Eq. (5), is obtained:

$$\overset{\circ\,\bullet}{\mathbf{x}}_i = \omega \underline{T} \; \underline{D} \; \mathbf{c}_i \qquad (8)$$

The array D is a (2m + 1) square tridiagonal matrix of the form:

$$\begin{bmatrix} 0 & & & & & & & \\ & 0 & +1 & & & & & \\ & -1 & 0 & & & & & \\ & & & 0 & +2 & & & \\ & & & -2 & 0 & & & \\ & & & & & \ddots & & \\ & & & & & & 0 & +m \\ & & & & & & -m & 0 \end{bmatrix} \qquad (9)$$

and the elements are given by,

$$D_{i,i+1} = -D_{i+1,i} = i/2, \qquad i = 2, 3, \ldots\ldots, (2m)$$

The requirement that the set of system state equations, Eq. (1), be satisfied exactly at N collocation points leads to N algebraic equations of the form:

$$\omega \underline{T} \; \underline{D} \; \mathbf{c}_i = \mathbf{f}_i(\underline{T} \; \mathbf{c}_i, t_k) \qquad (10)$$

where $\mathbf{f}_i$ is the vector of the ith function evaluated at the N collocation time points. Hence, the collocation process yields N*n nonlinear algebraic equations in the (2m + 1) * n unknown coefficients. These equations are then solved using a secant method from standard subroutine packages of IMSL.

If the system state equations are <u>autonomous</u>, then Eq. (1) may be written as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \qquad (11)$$

The frequency or frequencies of self-oscillation for such systems are unknown *a priori*. The analysis procedure is essentially the same as in the non-autonomous systems and leads to a set of nonlinear algebraic equations,

$$\omega \underline{T}\ \underline{D}\ \mathbf{c}_i = \mathbf{f}_i(\underline{T}\ \mathbf{c}_i) \tag{12}$$

The number of unknown quantities is increased by one, the unknown frequency, to [(2m+1)*n + 1], and the number of collocation points must satisfy the following inequality to assure a unique solution

$$N \geq (2m+1) + 1 \tag{13}$$

Hence, this situation is a case of non-linear least squares and cannot be solved by the secant method used for non-autonomous systems. An IMSL developed procedure, however, based on the Levinberg-Marquardt algorithm for nonliunear curve-fitting can be applied to the autonomous problem and has succesfully yielded satisfactory results, for several problems.

Rotor System Equations

The equations of motion for a typical multi-shaft flexible rotor system can be written in the second order form

$$\underline{M}\ddot{\mathbf{q}} + \underline{G}\dot{\mathbf{q}} + \underline{K}\mathbf{q} = \mathbf{Q}^s + \mathbf{Q}^b(\dot{\mathbf{q}}^b, \mathbf{q}^b) \tag{13}$$

or equivalently in the first order form

$$\underline{A}\dot{\mathbf{x}} + \underline{B}\mathbf{x} = \mathbf{F} \tag{14}$$

where

$$\mathbf{x} = \begin{Bmatrix} \dot{\mathbf{q}} \\ \mathbf{q} \end{Bmatrix} , \quad \mathbf{F} = \begin{Bmatrix} \mathbf{Q}^s \\ \mathbf{Q}^b \end{Bmatrix}$$

and

$$\underline{A} = \begin{bmatrix} \underline{O} & \underline{M} \\ \underline{M} & \underline{G} \end{bmatrix} , \quad \underline{B} = \begin{bmatrix} -\underline{M} & \underline{O} \\ \underline{O} & \underline{K} \end{bmatrix} \tag{15}$$

The linear forces of the system are included in the vector **Q** and the nonlinear component forces are included in the sparse vector $\mathbf{Q}^b$. A direct application of the TCM to a large order system such as Eq. (14) would almost always be computationally untenable. Thus, to obtain a mathematical model that is sufficiently small for the TCM to be effective, it is necessary to reduce the order of the original model.

It is propsosed here to develop a procedure to analyze the periodic motion of large order structural systems with nonlinear supports or pseudo supports by using the TCM in conjunction with modal analysis. This algorithm will reduce the original problem to a set of nonlinear algebraic equations involving only the physical coordinates which are associated with the nonlinear supports. This would normally result in a susbstantial reduction in order hopefully rendering the TCM computationally tenable.

The connectivity between the nonlinear coordinates and the system displacement is specified by the connecticity matrix $\underline{S}$ and can be written as

$$\mathbf{q}^b(t) = \underline{S}\ \mathbf{q}(t) \tag{16}$$

where $\mathbf{q}^b$ is the displacement vector associated with the coordinates of the nonlinear supports. The connectivity matrix S is a sparse matrix consisting mostly of zero elements and a few unity elements that ensure that the displacements in the nonlinear supports are identical with the displacements at the corresponding connection points of the linear system. Hence, Eq. (16) is a statement of geometric displacement compatibility for the system.

Modal Analysis

The $\underline{A}$ and $\underline{B}$ arrays of Eq. (14) are not generally symmetric, thus both the right eigenvectors $\mathbf{y}_i$ and adjoint left eigenvectors $\mathbf{z}_i$ must be evaluated for use in a modal expansion. These two sets of vectors satisfy the biorthoganality conditions

$$\begin{aligned} \mathbf{z}_j^T \underline{A}\, \mathbf{y}_i &= R_i\, \delta_{ij} && \text{a)} \\ \mathbf{z}_j^T \underline{B}\, \mathbf{y}_i &= -\lambda_i R_i\, \delta_{ij} && \text{b)} \end{aligned} \tag{17}$$

where R_i is the system norm associated with the eigenvalue λ_i.

With the state vector defined in Eqs. (15), the system eigenvectors are of the form

$$\mathbf{y}_i = \begin{Bmatrix} \lambda_i \mathbf{r}_i \\ \mathbf{r}_i \end{Bmatrix}, \quad \mathbf{z}_i = \begin{Bmatrix} \lambda_i \mathbf{l}_i \\ \mathbf{l}_i \end{Bmatrix} \tag{18}$$

where $\mathbf{r}_i$ and $\mathbf{l}_i$ are the right and left displacement eigenvectors associated with the physical coordinate vector $\mathbf{q}$. The state response of Eq. (14) is represented by the modal expansion

$$\mathbf{x} = \sum_{i=1}^{2n} \mathbf{y}_i\, \eta_i \tag{19}$$

where η_i is the ith modal coordinate. Substitution of Eq. (19) into Eq. (14) and premultiplication by $\mathbf{z}_i^T$, using the biorthogonality conditions of Eq. (17), yield the 2n equations

$$\dot{\eta}_i - \lambda_i \eta_i = \frac{1}{R_i} \mathbf{l}_i^T (\mathbf{Q}^s + \mathbf{Q}^b) \qquad i = 1, 2, \ldots., 2n \tag{20}$$

These equations are still coupled due to the nonlinear force vector $\mathbf{Q}^b$. For a large order system, it is not normally necessary nor is it feasible to retain all the modal information when determining the system steady state response. Usually only n lower modes are retained in the modal expansion of Eq. (19), thus there are correspondingly $\hat{n}$ equations in Eq. (20).

SOLUTION PROCEDURE

Following the TCM procedure, periodic solution forms are assumed for the system physical coordinates, the nonlinear subset of the system physical coordinates, and the generalized coordinates in the modal expansion. i.e.,

$$\mathbf{q} = \mathbf{a}_0 + \sum_{j=1}^{m} \left[\mathbf{a}_j \cos(\omega_j t) + \mathbf{b}_j \sin(\omega_j t) \right] \qquad \text{a)}$$

$$\mathbf{q}^b = \mathbf{a}_0^b + \sum_{j=1}^{m} \left[\mathbf{a}_j^b \cos(\omega_j t) + \mathbf{b}_j^b \sin(\omega_j t) \right] \qquad \text{b)} \qquad (21)$$

$$\eta = \mathbf{a}_0^\eta + \sum_{j=1}^{m} \left[\mathbf{a}_j^\eta \cos(\omega_j t) + \mathbf{b}_j^\eta \sin(\omega_j t) \right] \qquad \text{c)}$$

By choosing N equally spaced collocation points and evaluating the variables of Eq. (21) at these time points, the following set of relations is obtained

$$^o\underline{q} = \underline{T}\,\underline{C} \qquad \text{a)}$$

$$^o\underline{q}^b = \underline{T}\,\underline{C}^b \qquad \text{b)} \qquad (22)$$

$$^o\underline{\eta} = \underline{T}\,\underline{C}^\eta \qquad \text{c)}$$

where the ith column of $\underline{C}$ corresponds to the variable q_i (t) evaluated at each of the N collocation points. The i^{th} typical column of $\underline{C}$ is defined by Eq. (3). Similar definitions apply for the arrays of Eqs. (22 b,c).

The unknown coefficient arrays ($\underline{C}$,$\underline{C}^b$,$\underline{C}^\eta$) are dependent and are related through the geometric displacement relation of Eq. (16) and the modal expansion of Eq. (19). From Eqs. (22 a,b) and (16),

$$\underline{T}\,\underline{C}^b = \underline{T}\,\underline{C}\,\underline{S}^T \qquad (23)$$

and by utilizing the form of the system right vectors, Eq. (18), the modal expansion for the system physical coordinates may be written as

$$\mathbf{q} = \sum_{i=1}^{\hat{n}} \mathbf{r}_i\,\eta_i = \mathbf{r}\,\eta \qquad (24)$$

Thus, from Eq. (22a) and (24) the following relation between physical coordinate and normal coordinate Fourier coefficients is obtained:

$$\underline{T}\,\underline{C} = \underline{T}\,\underline{C}^\eta\,\underline{r}^T \qquad (25)$$

The substitution of this constraint relation into Eq. (23) gives

$$\underline{C}^b = \underline{C}^\eta \, \underline{r}^T \, \underline{S}^T \tag{26}$$

The next step in establishing the solution is to apply the TCM procedure to the set of pseudo modal equations, Eq. (20). Utilizing the equivalent form of Eqs. (5) and (8), Eq. (20) may be written as

$$\omega \, \underline{T} \, \underline{D} \, c_i^\eta \; - \lambda_i \, \underline{T} \, c_i^\eta \; = \; {}^o\mathbf{f}_i \tag{27}$$

where:

$$ {}^o\mathbf{f}_i = \frac{1}{R_i} \begin{bmatrix} \mathbf{l}_i^T \left(\mathbf{Q}^s(t_1) + \mathbf{Q}^b(t_1) \right) \\ \bullet \\ \bullet \\ \bullet \\ \mathbf{l}_i^T \left(\mathbf{Q}(t_N) + \mathbf{Q}^b(t_N) \right) \end{bmatrix} \tag{28}$$

The elements of the ${}^o\mathbf{f}_i$ vector are the RHS values of Eq. (20) evaluated at the collocation points, and are functions of the nonlinear displacements $\mathbf{q}^b$ and velocities $\mathbf{q}^b$.

Equation (27) can be rearranged to the form

$$c_i^\eta = \left[\, (\omega \underline{D} - \lambda_i \underline{I})^{-1} \, (\underline{T}^T \underline{T})^{-1} \, \right] \underline{T}^T \, {}^o\mathbf{f}_i \tag{29}$$

$$i = 1, 2, ..., n$$

which is a typical column of the array of modal coordinate Fourier coefficients, $\underline{C}^\eta$. The combination of relations (29) with Eq. (26) results in a set of nonlinear algebraic equations in terms of Fourier coefficients for the physical coordinate subset $\mathbf{q}^b$.Thus, the size of the problem has been substantitally reduced and the location of a solution is computationally more feasible.

The iterative procedure for estimating the Fourier coefficients $\underline{C}^b$ can be summarized in the following steps:

1. Choose a starting value for $\underline{C}^b$.
2. Compute ${}^o\mathbf{q}^b$ using Eq. (22 b).
3. Compute $\underline{C}^\eta$ using Eqs. (19).
4. Evaluate and update the value for $\underline{C}^b$ using Eq. (26).
5. Check convergence between steps 1. and 4.

The procedure involves the solution of a set of nonlinear equations and its success depends upon the effectiveness of the numerical routine utilized. The optimization routine based on the secant method from the IMSL subroutine library proved to be very effective with convergence being achieved for a wide variety of starting points. The error norm in the algebraic equations appears to be a reasonable measure of the solution accuracy.

NUMERICAL EXAMPLES

The results of analyzing three example systems are presented below. The first two are relatively small order systems and are analyzed directly in terms of physical coordinates. The third example is a larger order dual shaft rotor system that utilizes modal analysis in conjunction with the TCM.

Journal - Hydrodynamic Bearing System

Consider a single rotating journal on a hydrodynamic bearing as illustrated in Fig. 1. With reference to this figure, O is the bearing center, C is the geometric center of the journal, and c represents the radial bearing clearance. The mass center of the journal is assumed to be displaced from the geometric center by the cg offset e. During rotation this offset gives rise to a rotating unbalance force which is synchronous with journal spin frequency. The converging wedge that arises due to the eccentricity of the journal gives rise to a pressure field in the fluid film that supports the load.

The nondimensionalized equations of motion of the journal assume the form:

$$\begin{aligned} \ddot{v} &= F_r \cos\phi + F_t \sin\phi + u \cos(t+\beta) + g \\ \ddot{w} &= F_r \sin\phi - F_t \cos\phi + u \sin(t+\beta) \end{aligned} \tag{30}$$

In these equations, v and w represent the nondimensional displacement coordinates of the journal center with respect to a fixed reference frame. The quantities g and u represent the gravity and unbalance parameters, and F_r and F_t represent the radial and tangential fluid film force components acting on the journal.

Using short bearing theory, Reynolds equation can be integrated to obtain closed form expressions for the plain journal bearing force components, e.g. (Holmes, 1960). Thus,

$$\begin{aligned} F_r &= -B\left[\frac{\pi(1+2\varepsilon^2)\dot{\varepsilon}}{(1-\varepsilon^2)^{5/2}} + \frac{2\varepsilon^2(1-2\dot{\phi})}{(1-\varepsilon^2)^2}\right] \\ F_t &= +B\left[\frac{4\varepsilon\dot{\varepsilon}}{(1-\varepsilon^2)^2} + \frac{\pi\varepsilon(1-2\dot{\phi})}{2(1-\varepsilon^2)^{5/2}}\right] \end{aligned} \tag{31}$$

where, $\varepsilon = (v^2 + w^2)^{1/2}$, $\phi = \arctan(w/v)$
and B is a bearing parameter that is dependent on the fluid viscosity, and geometry of the bearing.

Clearly F_r and F_t are highly nonlinear functions of the response variables. Typically, the journal equations, Eqs. (30) are linearized about the static equilibrium position. The resulting linear response corresponds to an elliptical orbit centered at the equilibrium position. Application of the TCM to this problem can yield an orbit which is quite different the linearized response as displayed in Fig. 2.

An interesting fact revealed by the TCM is that the higher harmonics in the response of the journal may not be negligible as contrasted with many nonlinear problems. In the results presented, at least 8 harmonics were required to obtain close agreement between the TCM and numerical integration. In this case, many analytical procedures, such as the Describing Function Method, which neglect harmonics above the fundamental would not adequately describe the dynamics of the journal.

Flow Induced Vibration

Consider the problem of quasi-steady analysis of the transverse galloping of a long prism of square cross-section (Blevins, 1977; Parkinson and Smith, 1964).The equation of motion for the single degree-of-freedom oscillator is

$$y'' + 2\zeta y' + y = n U^2 c_f \tag{32}$$

where U is the non-dimensional velocity of wind and y is the non-dimensional vibration displacement. The non-dimensional aerodynamic force coefficient c_f is obtained by experimental measurements in a wind tunnel and can be approximated by a polynomial in α, the angle of attack, or equivalently (y'/U).

$$c_f = n\left[A U y' - \frac{B}{U}(y')^3 + \frac{C}{U^3}(y')^5 - \frac{D}{U^5}(y')^7\right] \tag{33}$$

From a curve-fit to experimental values, A = 2.69, B = 168, C = 6270, D = 59,900 (for a Reynolds number = 22,300) ; n is a mass parameter, which is a function of the prism dimensions and the density of air; ζ is the linear viscous damping coefficient.

The second-order nonlinear autonomous equation (32) has been shown to exhibit self-excited oscillations and an analysis by the method of averaging was carried out by (Parkinson, 1964). It was found that the amplitude (A_1) vs wind velocity curves for various values of the damping coefficient collapse into a single curve if normalized by $nA_1 / 2\zeta$.

The first harmonic amplitudes obtained by the application of TCM are shown in Fig. 3. As is evident from the figure, the response exhibits a hysteresis loop. A choice of different initial guesses helped the procedure converge to the multiple solution points. It is identical to the figure in Parkinson and Smith (1964) and indicates that the collocation procedure developed here is valid for problems with multiple solution points.

Dual Shaft Rotor System

The dual-shaft rotor system with configuration shown in Fig. 4 includes a nonlinear bearing at station 6 and excited by rotating unbalance in shaft 1 and a static side load at station 6. Rotor (1,2) is modelled as a (6,4) station, (24,16) degree - of - freedom, (5,3) element assembly with stations as indicated in Fig. 4. Detailed rotor configuration data is provided in (Nelson and Alam, 1983). The rotating assemblies are connected to a rigid foundation by linear bearings at stations 1 and 7 and are interconnected by a linear bearing between stations 4 and 10. A nonlinear bearing with cubic stiffness variation and linear viscous damping connects shaft 1 to the rigid foundation at station 6. The nonlinear bearing force components are given by the relations:

$$F_Y = -(k_1 r + k_3 r^3)\frac{v}{r} - c_v \dot{v}$$

$$F_Z = -(k_1 r + k_3 r^3)\frac{w}{r} - c_w \dot{w} \qquad (34)$$

where $r = (v^2 + w^2)^{1/2}$, and k_1 = 50,000 lbf/in, $k_3 = 50 * 10^9$ lbf / in^3, and $c_v = c_w$ = 20 lbf-s/in.

The unbalance distribution of the rotating assembly consists of a single concentrated unbalance at station 2 with a cg eccentricity of 0.95 mils. In addition, a static side load acts on the system at station 6. Shaft 1 spins at 80,000 Rpm and shaft 2 co-rotates at 120,000 Rpm.

It should be noted that the linear subsystem is not totally constrained. Thus, an "artificial support" is added at station 6 to eliminate a singularity. This influence is then subsequently removed from the model by subtracting its influence in the nonlinear forces. A value of 10,000 lbf / in at station 6 was arbitrarily selected for this system. The nonlinear radial force versus displacement is shown in Fig. 5. The linear bearing stiffnesses are 150,000, 50,000, and 100,000 lbf / in at station 1, 4-10, and 7 respectively.

Displacement orbits, as determined using the TCM procedure, for this system are plotted in Fig. 6 for two stations and a side load of 100 lbf acting in the negative z direction. The orbit distortion clearly indicates the presence of higher harmonics in the response.

CONCLUSIONS

A numeric-analytic procedure based on the trignometric collocation method has been developed and implemented for estimating the periodic response of engineering systems. The procedure allows for estimating periodic forced response and for locating limit cycles of self-excited systems. A component mode synthesis strategy coupled with the TCM extends the method to large order system application.

Three example analyses are presented. Two of small order in physical coordinates and the third on larger order using the modal strategy. Preliminary indications are that this method may be very effective in estimating the periodic response of both small and large order systems. Additional work is required to further test its generality, to handle systems with subharmonic response and, to ascertain the stability of the located periodic responses. Study on the speed and accuracy of the necessary computational work should also lead to improvement in the utility of the approach.

REFERENCES

Atherton, D. F., 1982, *Nonlinear Control Engineering*, Van Nostrand Reinhold: London.

Blevins, R. D., 1977, "Flow Induced Vibration," Van Nostrand Reinhold, New York.

Hagedorn P., 1982, *Nonlinear Oscillations*, Clarendon Press: Oxford.

Holmes, R., 1960, "Vibration of a Rigid Shaft on Short Sleeve Bearings," J. Mech. Engr. Sciences, Vol. 2, No. 4m pp. 337-341

Nataraj,C., Nelson, H. D. and Arakere, N., 1985, "Effect of Coulomb Spline on Rotor Dynamic Response," *Instability in Rotating Machinery*, NASA CP 2409, pp. 225-233.

Nayfeh, A. H., 1981, *Introduction to Perturbation Techniques*, John Wiley & Sons: New York.

Nelson, H. D., and Alam, M., 1983, "Transient Response of Rotor Bearing Systems using Component Mode Synthesis: Part VI Blade Loss Response Spectrum," NASA Grant NAG3-6 Report.

Parkinson, G. V. and Smith, J. D., 1964, "The Square Prism as an Aeroelastic Nonlinear Oscillator," Quart. J. Mech. & App. Math., Vol. 17, pt. 2, pp. 225-239.

Saito, S., 1985, "Calculation of Nonlinear Unbalance Response of Horizontal Jeffcott Rotors Supported by Ball Bearings with Radial Clearances," *ASME J. of Vibration, Accoustics, Stress and Reliability in Design*, Vol. 107, pp. 416-420.

Samoilenko, A, M, and Ronto, N. I., 1979, *Numerical-Analytical Methods of Investigating Periodic Solutions*, Mir. Publishers: Moscow.

Stokes, A., 1972, "On the Approximation of Nonlinear Oscillations," *J. Differential Equations*, Vol. 12, pp. 535-558.

Urabe, M., 1965, "Galerkin's Procedure for Nonlinear Periodic Systems," Archives of Rational Analysis, Vol. 20, pp. 120-152.

Urabe, M. and Reiter, N., 1966, "Numerical Computation of Nonlinear Forced Oscillations by Galerkin's Procedure," J. Mathematical Analysis Applications, Vol. 14, pp. 107-140.

Yamauchi, S. 1983, "The Nonlinear Vibration of Flexible Rotors: First Report, Development of a New Analysis Technique," J. ASME, Vol. 49, No. 446, Series C, pp. 1862-1868.

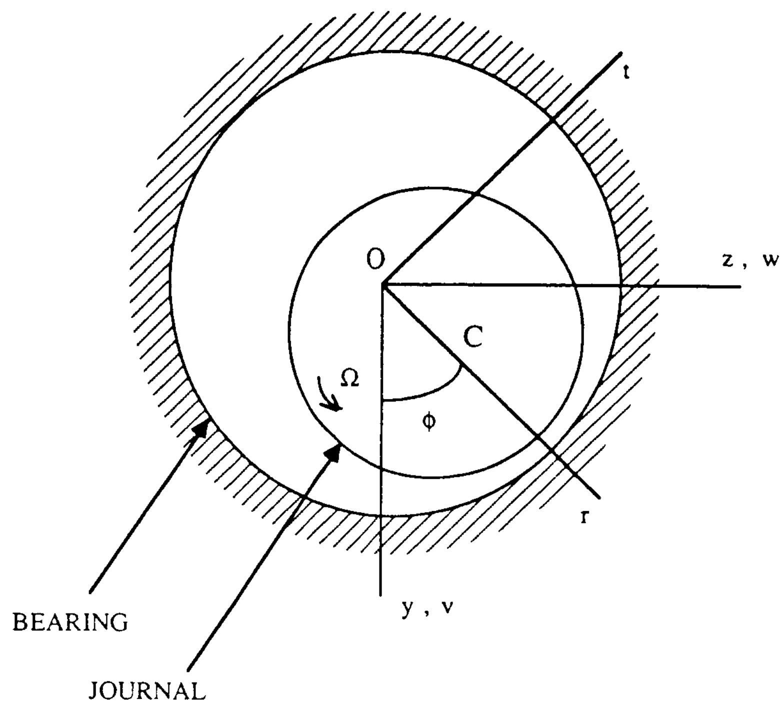


Fig. 1 Journal Bearing Configuration

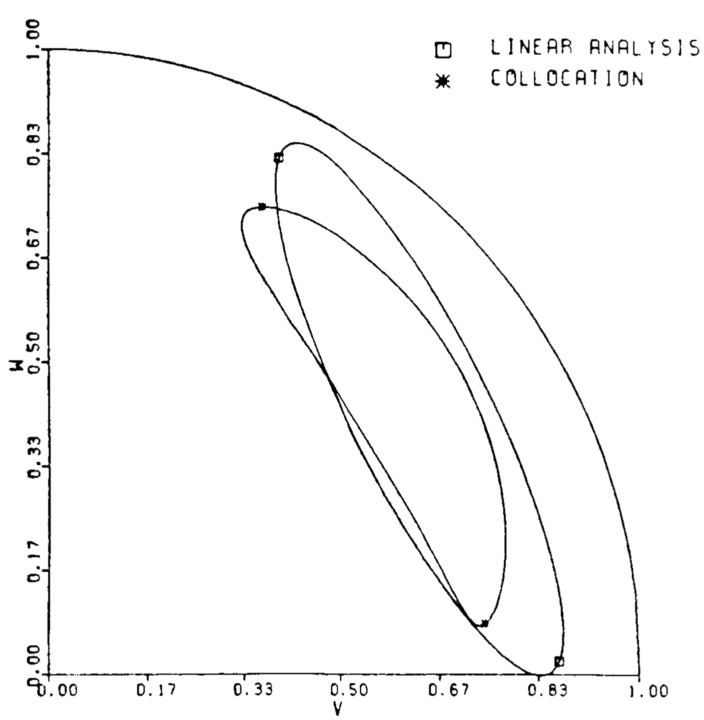


Fig. 2 Linearized and Collocation Solutions

ORIGINAL PAGE IS
OF POOR QUALITY

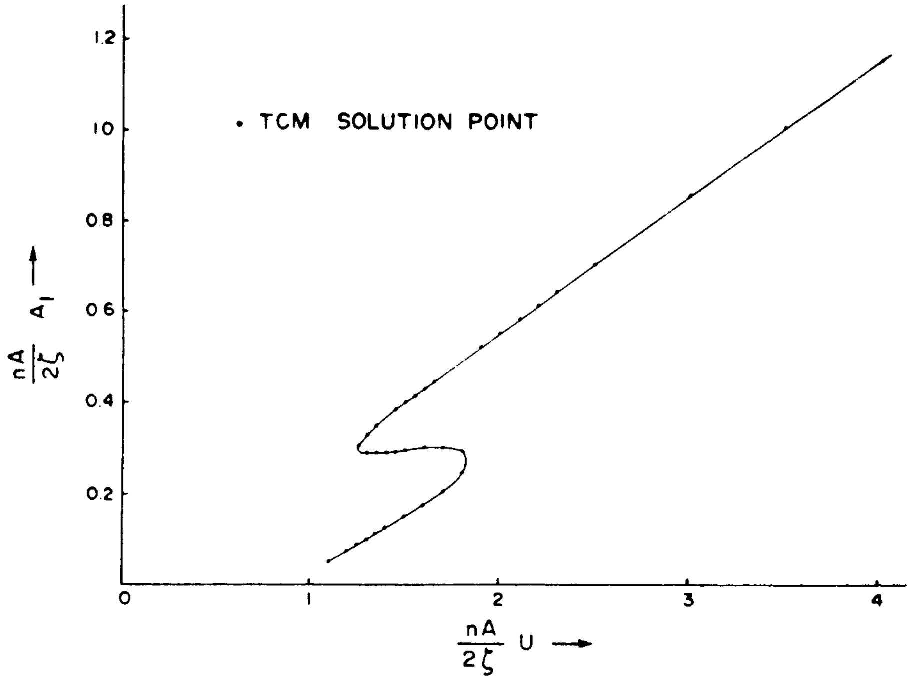


Fig. 3 First Harmonic Amplitude vs. Wind Velocity

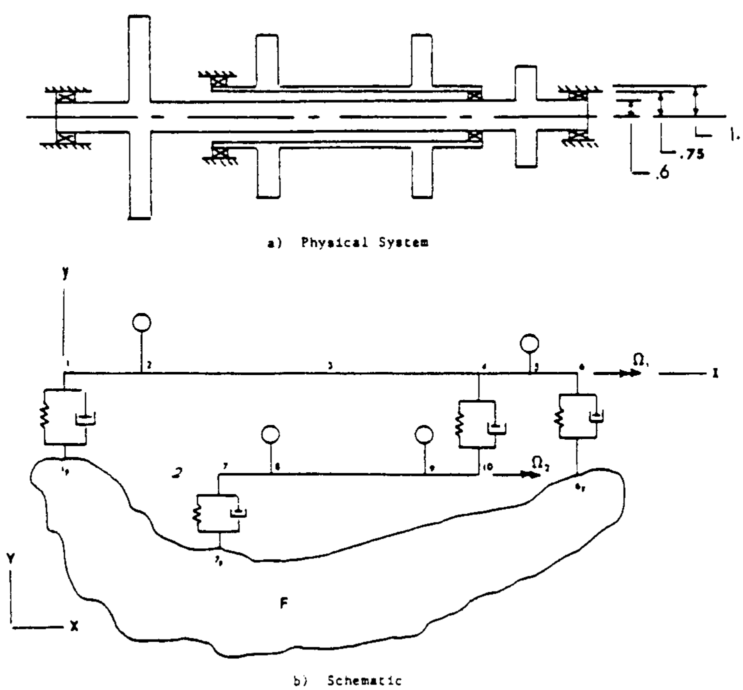


Fig. 4 Dual Rotor Schematic and Model

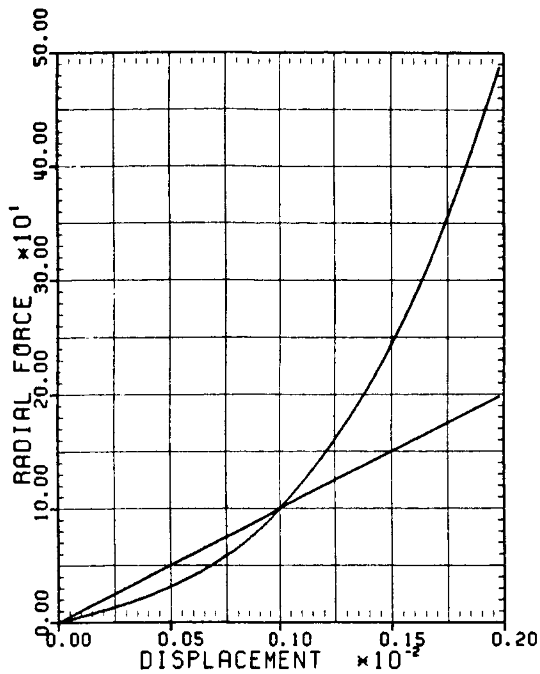
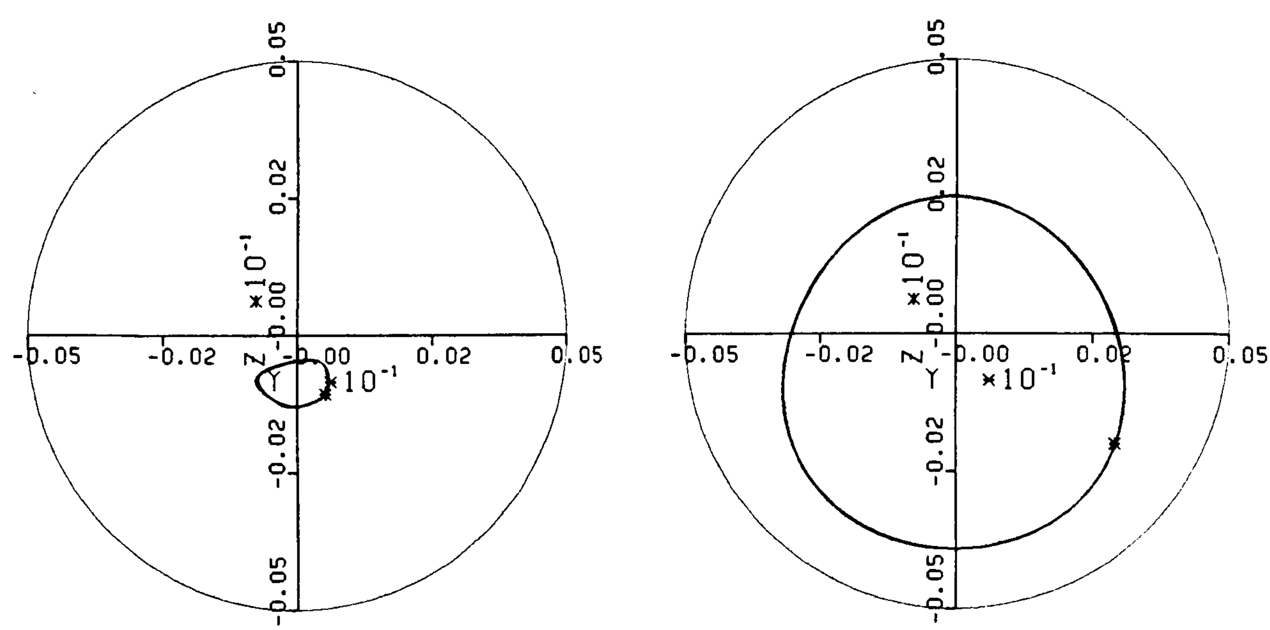


Fig. 5 Nonlinear Force-Displacement Relation



(a) Station 6

(b) Station 10

Fig. 6 Rotor Station Orbits