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REDUCED ORDER MODELS FOR NONLINEAR AERODYNAMICS

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ABSTRACT

Reduced order models are needed for reliable, accurate and efficient prediction of aerodynamic forces to analyze fluid-structure interaction problems in turbomachinery including prop fans. The phenomenological models, though efficient, require a large amount of experimental data for verification and are not always accurate. The models based on first principles of fluid mechanics, such as Navier-Stokes methods, are accurate but computationally expensive and difficult to integrate with structural mechanics models to obtain an interdisciplinary prediction capability. In the present work, a finite difference, time marching Navier-Stokes code is validated for unsteady airfoil motion by comparing with classical potential flow results. The Navier-Stokes code is then analyzed for calculation of primitive and exact estimates of eigenvalues and eigenvectors associated with fluid-airfoil interaction. A variational formulation for the Euler equations and Navier-Stokes equations will be the basis for reduction of order through an eigenvector transformation. This will help identify and exploit the relationships between the simpler phenomenological models and those based on first principles of fluid mechanics.

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Analysis of fluid-structure interaction problems in turbomachinery requires an accurate knowledge of fluid properties and forces (Dowell, 1978). Some unique features of propfans preclude the use of existing aeroelastic technology of conventional propellers, turbofans and helicopters. The accurate and efficient prediction of aerodynamic forces in nonlinear regimes, such as separated, transonic flows, is important for aeroelastic analysis of propfans (especially stall flutter, whirl flutter).

THE NEED FOR REDUCED ORDER MODELS

-Accurate prediction of aerodynamic forces is required for aeroelastic response and flutter analysis
- **-Low angles of attack - linear relationship**
- **-High angles of attack - nonlinear relationship**
  stall and dynamic stall
  separated flow

-Theoretical Approaches
- **-Methods - Navier-Stokes**
  discrete vortex
  zonal methods
- **-Limitations - computationally expensive**
  lack of generality
  difficult to use in routine aeroelastic analysis

-Reduced Order Models
- **-Methods - empirical / semi-empirical**
  new methods based on first principles
- **-Advantages - computationally fast**
  easily used in routine aeroelastic analysis
  allow for various airfoil motions and flow conditions

The nonlinear relationship between lift (also moment) and angle of attack is modelled in various ways. The easiest and most popular way is curve-fitting experimental data with algebraic or transcendental functions which represent qualitative approximations to physical behavior. Ordinary differential equations are also used to represent the lift-angle of attack relationship. These models are derived empirically or semi-empirically and have no direct relation to first principles of fluid mechanics.

The Navier-Stokes methods and Euler methods to calculate flow over airfoils are based on finite difference and/or finite elements techniques and involve a large number of degrees of freedom, depending on the grid/elements setup. The nonlinear equations of fluid mechanics are solved with linearized solution methods, such as approximate factorization, alternate direction implicit procedure, etc. Reduction in the number of degrees of freedom can be achieved by identifying the important or dominant modes and then writing the system of equations in terms of these modes.

AD HOC MODELS

Different ways to model nonlinearity in lift and moment.

- **-corrected angle of attack methods.**
- **-time-delay methods, synthesis procedures.**
- **-ordinary differential equation methods.**

FIRST PRINCIPLE MODELS

Flow over airfoils calculated using Navier-Stokes methods or Euler methods.

Identify/recognize the important or dominant modes. Write the system of equations in terms of dominant modes.

For development of a reduced order model, a Navier-Stokes code capable of calculating unsteady, transonic and separated flows for different airfoil motions, such as pitching and plunging, was required. A finite difference, time marching code developed by Sankar and Tang (1985) was selected. This code solves the unsteady, two-dimensional Navier-Stokes equations on a body-fitted moving coordinate system in a strong conservative form using the ADI procedure. The convective terms are treated implicitly and the viscous terms are treated explicitly. The code was modified to include step change response. The results from the code for inviscid flow were in reasonable agreement with classical potential flow results.

NAVIER-STOKES CODE MATHEMATICAL FORMULATION

$$(x,y,t) \rightarrow (\xi,\eta,\tau)$$

$$\xi = \xi(x,y,t), \quad \eta = \eta(x,y,t), \quad \tau = \tau(t)$$

Transformed Navier-Stokes Equations:

$$\tilde{q}_\tau + \tilde{F}_\xi + \tilde{G}_\eta = \tilde{R}_\xi + \tilde{S}_\eta$$

where $q = (\rho, \rho u, \rho v, e)$

$$F = (\rho u, \rho u^2+p, \rho uv, u(e+p))$$

$$G = (\rho v, \rho uv, \rho v^2+p, v(e+p))$$

$$R = (0, \tau_{xx}, \tau_{xy}, R_4)$$

$$S = (0, \tau_{xy}, \tau_{yy}, S_4)$$

$$R_4 = u\,\tau_{xx} + v\,\tau_{xy} + K\,(a^2)_x$$

$$S_4 = u\,\tau_{xy} + v\,\tau_{yy} + K\,(a^2)_y$$

Variational formulations not only concentrate all of the intrinsic features of the problem (governing equations, boundary conditions, initial conditions and constraints) in a single functional, but also provide a natural means for approximation. In solid mechanics, a variational formulation is easy to obtain. However, in fluid mechanics, use of an Eulerian reference frame and the nonlinearity in the expression for conservation of momentum make a variational formulation very difficult to obtain. Extremization of an energy functional has been formulated in the literature (Oden and Reddy, 1983, Girault and Raviart, 1986, Temam, 1984) for the cases listed below. The approximate variational formulations are used in methods of weighted residuals, collocation methods, Galerkin's method, least-squares methods and semi-discrete methods.

VARIATIONAL FORMULATIONS

-Inviscid potential flow

-incompressible flow (linear elliptic p.d.e.).

-compressible flow

-subsonic (small-disturbance equation).

-transonic (slender body assumption).

-Euler equations

-unsteady, compressible form.

-Navier-Stokes equations

-Stokes' problem.

-steady, incompressible N-S equations.

-unsteady, incompressible N-S equations.

-stream function-vorticity formulation.

To determine primitive modes, the airfoil is oscillated at different frequencies about several steady angles of attack with various amplitudes of oscillation. Primitive modes are amplitudes and phase differences for (ρ, ρu, ρv, e) over the entire grid. For the Euler Solution, the primitive modes are independent of oscillation amplitude, sufficiently far away from the airfoil.

To detrmine exact modes, an eigenvalue problem is formulated for the Navier-Stokes code. This eigenproblem is then solved to determine the exact eigenvalues and eigenmodes.

EIGENVALUE PROBLEM FORMULATION

$$\tilde{q}_\tau + \tilde{F}_\xi + \tilde{G}_\eta = \tilde{R}_\xi + \tilde{S}_\eta$$

$\tilde{F}, \tilde{G}, \tilde{R}, \tilde{S}$ **are functions of** $\tilde{q}$.

$$\tilde{q}_\tau = -\tilde{F}_\xi - \tilde{G}_\eta + \tilde{R}_\xi + \tilde{S}_\eta$$
$$= \tilde{Q}\,(\tilde{q})$$

Substitute $\tilde{q} = \bar{q} + \hat{q}$

$\bar{q}$ **: steady state value,** $\hat{q}$ **: small perturbation.**

$$\hat{q}_\tau = \tilde{Q}\,(\bar{q}) + \frac{d\tilde{Q}}{d\tilde{q}}_{\bar{q}}\,\hat{q}$$

$$\hat{q}_\tau = \frac{d\tilde{Q}}{d\tilde{q}}_{\bar{q}}\,\hat{q}$$

Substitute $\hat{q} = q\,e^{\lambda t}$

$$\lambda\, q\, e^{\lambda\tau} = \frac{d\tilde{Q}}{d\tilde{q}}_{\bar{q}}\; q\, e^{\lambda\tau}$$

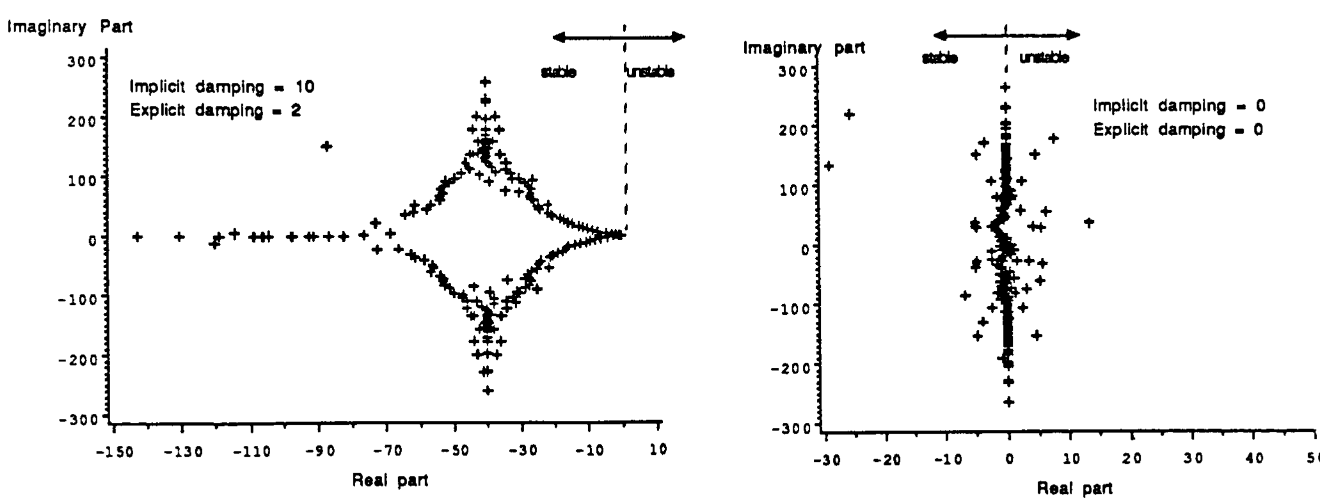
$$\lambda\,\{q\} = [A]\,\{q\}$$

A is a sparse, real, nonsymmetric matrix of order N, where
N = 4 x total number of grid points.
$\approx$ 24000.

The state of the art software available for an eigenvalue calculation is NOT capable of storing a 24000 x 24000 matrix or utilizing the sparsity and nonsymmetry of the present problem for obtaining a solution in a reasonable computer time. A procedure was developed to exploit the sparsity of the matrix for storage and calculation purposes. A modified Lanczos recursive procedure with no reorthogonalization is used to calculate eigenvalues (Cullum and Willoughby, 1986). These eigenvalues are found to be independent of the starting vectors used in the recursion.

Below are shown preliminary results from this eigenvalue calculation for different values of explicit and implicit damping in the Navier-Stokes code for NACA 0012 airfoil at M = 0.8, α = 0 using the Euler equations. The addition of artficial damping to the governing equations in the N-S code appears to change unstable eigenvalues into stable ones. Also the N-S code was able to calculate the time history of flow over the airfoil with artificial damping, but failed without this damping.

Eigenvalues for NACA 0012 at M = 0.8, α = 0. (Euler Solution)



Transformation of the system from physical coordinates to modal coordinates using the classical or approximate variational formulation for flow over an airfoil can be done as shown below. The dominant eigenmodes are used for this transformation. This transformation reduces the order of the fluid-airfoil system. For example, a single modal coordinate equation can then be compared to a simple phenomenological model represented by an ordinary differential equation.

CONSTRUCTION OF REDUCED ORDER MODEL USING EIGENVALUES AND VARIATIONAL PRINCIPLE

Variational principle in physical coordinates:

$$\int [L(q)] \{\delta q\}\, dt = 0$$

Eigenvector transformation:

$$\{q\} = [E]\ \{a\}$$

Variational principle in modal coordinates:

$$\int [L(a)]\,[E]\left\{\delta a\right\} dt = 0$$

(set all a_i = o for i > N)

From the preliminary results obtained in the present research effort, fluid-structure interaction problems can be analyzed using the modal behavior of the fluid. There is a strong relationship between the eigenvalues associated with the fluid and the damping present in the N-S code. Further study of eigenmodes will help understand the complex fluid-structure interaction on a modal basis and offers substantial potential for solving various other problems involving fluid forces.

SUMMARY OF RESULTS TO DATE

Survey of empirical and semi-empirical reduced order models.

Validation of N-S code for transient time responses.

Formulation of the eigenvalue problem using the finite difference code and calculation procedure for eigenvalues.

Assessment of potential for reduction of order using primitive modes and exact eigenmodes.

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