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A COMPUTATIONAL PROCEDURE FOR AUTOMATED FLUTTER ANALYSIS

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ABSTRACT

A direct solution procedure for computing the flutter Mach number and the flutter frequency is applied to the aeroelastic analysis of propfans. The procedure uses a finite element structural model and an unsteady aerodynamic model based on a three-dimensional, subsonic, compressible lifting-surface theory. An approximation to the Jacobian matrix that improves the efficiency of the iterative process is presented. The Jacobian matrix is indirectly approximated from approximate derivatives of the flutter matrix. Examples are used to illustrate the convergence properties. The direct solution procedure facilitates the automated flutter analysis and contributes to the efficient use of computer time as well as the analyst's time. Further details of the numerical procedure are given by Murthy and Kaza (1987).

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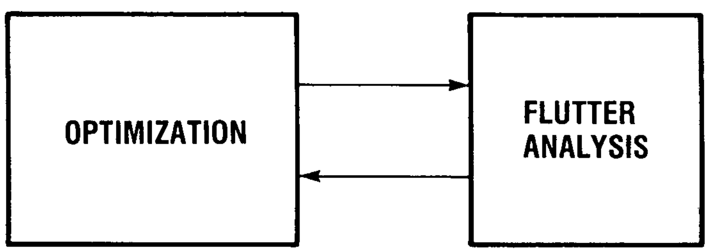
MOTIVATION

Flutter of propfans and other types of turbomachinery blading is an important phenomenon that has generated considerable interest. Flutter prevention has been a significant factor in the design of propfan blades. Flutter prevention is also significant for turbomachinery, particularly for unshrouded blades. With the recent advances in computer technology, automated design of propfan and turbomachinery blades by using optimization techniques has become practical. Design optimization employing flutter constraints requires repeated solution of the aeroelastic equations of motion to obtain the flutter parameters as the design is updated. For the optimization to be performed in a realistic period of time an automated flutter analysis capability is essential. It is also desirable for the analysis to be computationally efficient in order to keep the central processing unit (CPU) time and the turnaround time within reasonable limits. Automated flutter analysis can also shorten the nonautomated design process by reducing the analyst's time.

WHY AUTOMATED FLUTTER ANALYSIS?

- ESSENTIAL FOR REPEATED EXECUTION OF FLUTTER ANALYSIS CODE



- ALSO USEFUL IN NONAUTOMATED DESIGN PROCESS

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FORMULATION OF FLUTTER ANALYSIS PROBLEM

The computational procedure to be presented is applied to the analytical formulation described in detail by Kaza, et al. (1987a, 1987b). This formulation is applicable to the flutter analysis of a single-rotor propfan containing an arbitrary number of blades rotating at a fixed speed in an axial flow. The structure is modeled by finite elements. The aerodynamic model (Williams and Hwang, 1986) is based on a three-dimensional, subsonic, compressible lifting-surface theory.

For simplicity, the effect of steady deformations due to aerodynamic loads on the flutter boundary is neglected. The error introduced by ignoring the steady aerodynamic deformations is shown in Kaza, et al. (1987a) to be small enough to produce an approximate flutter point. In particular, the effect of the steady aerodynamic deformations on the flutter Mach number is not very significant. Thus the approximate flutter analysis neglecting steady aerodynamic deformations is suitable for use in design optimization procedures that require repeated efficient execution of the flutter analysis. The optimal design can be easily checked for the flutter condition by using the refined flutter analysis with steady deformations and the conventional procedure.

The propfan is assumed to have identical groups of blades symmetrically distributed about a rigid disk. The linearized aeroelastic equations of motion are then uncoupled for different intergroup phase angle modes σ_r. The flutter Mach number for the propfan is then the lowest Mach number at which one of the intergroup phase angle modes becomes unstable.

$$[M_g]\{\ddot{q}\} + [K_g]\{q\} = [A(M,\omega)]\{q\}$$

$[M_g]$	**GENERALIZED MASS MATRIX**
$[K_g]$	**GENERALIZED STIFFNESS MATRIX**
$\{q\}$	**GENERALIZED COORDINATE VECTOR**
$[A(M,\omega)]$	**GENERALIZED AERODYNAMIC MATRIX**

$[A(M,\omega)]$ IS USUALLY VALID ONLY FOR SIMPLE HARMONIC MOTION

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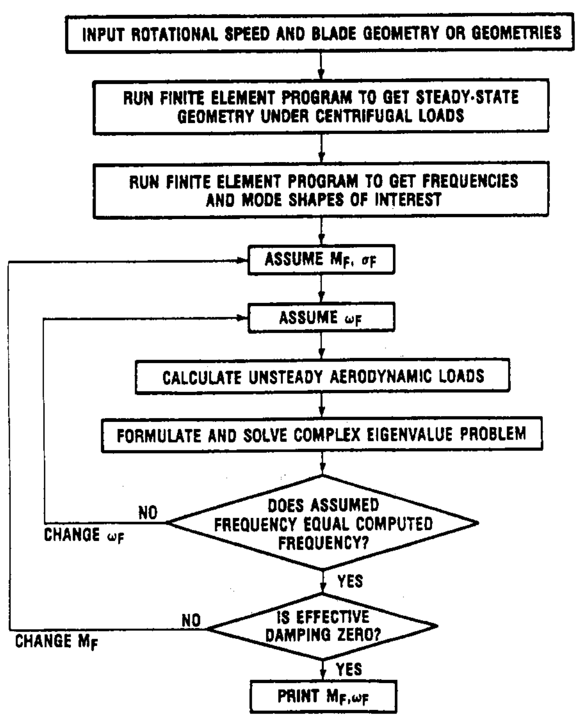
CONVENTIONAL PROPFAN FLUTTER ANALYSIS

The conventional procedure for obtaining the flutter Mach number M_F and frequency ω_F is as follows. The aerodynamic matrix is evaluated at an assumed Mach number M_F, an assumed frequency ω_F, and an assumed interblade phase angle σ_F, and then the flutter equation is solved for all the eigenvalues ω^2. This procedure has been implemented for propfans in a program called ASTROP3. In general, these eigenvalues are complex. The real and imaginary parts of $i\omega$ represent the effective damping and frequency, respectively. The assumed frequency is varied until it is equal to the frequency corresponding to the eigenvalue with the least effective damping. This frequency matching forms the inner iteration. When this iteration reaches convergence, the Mach number is varied until the effective damping of the eigenvalue corresponding to the matched frequency is equal to zero. This forms the outer iteration. The flutter Mach number and the flutter frequency are obtained at the convergence of the outer iteration.

The conventional procedure cannot be reliably automated because it requires that the identity or ordering of the eigenvalues be preserved over a wide range of assumed frequencies and Mach numbers. Most eigensolution routines do not compute the eigenvalues in any particular order, and the sorting of eigenvalues by frequency or magnitude does not usually preserve the continuity. Loss of continuity necessitates user intervention and complicates the automated analysis. A direct solution of the flutter equation that alleviates these problems is proposed and described. It views the flutter equation as an inplicit double-eigenvalue problem.



- NEEDS EIGENVALUE TRACKING AND IS THUS DIFFICULT TO AUTOMATE
- REQUIRES DOUBLE ITERATION (INNER-OUTER LOOPS)
- NEEDS AS MANY EIGENVALUES AS THERE ARE MODES

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DIRECT SOLUTION OF FLUTTER EQUATION

When the dependence of the aerodynamic matrix on the assumed Mach number and frequency is considered explicitly, the flutter equation can be viewed as an implicit double-eigenvalue problem. In general, the aerodynamic matrix $[A(M_F, \omega_F]$ is a transcendental function of the assumed frequency and Mach number. Only real values of M_F and ω_F are of interest. The two eigenvalues M_F and ω_F are coupled since the aerodynamic matrix is complex.

We now have two equations in two unknowns, M_F and ω_F. These equations can be solved by any of the methods for solving a system of nonlinear equations. When they are solved for M_F and ω_F, no further iterations are required for the purpose of matching assumed and computed quantities. This procedure is illustrated below. Once M_F and ω_F are found, inverse iteration can be used to find the flutter mode.

IF M_F IS THE FLUTTER MACH NUMBER AND ω_F THE FLUTTER FREQUENCY,

$$[B]\{q_0\} = \{0\}$$

WHERE

$$[B] = -\omega_F^2[M_g] + [K_g] - [A(M_F,\omega_F)]$$

FOR A NONTRIVIAL FLUTTER MODE, WE HAVE

$$\det\{-\omega_F^2[M_g] + [K_g] - [A(M_F,\omega_F)]\} = 0$$

LET

$$\begin{aligned} D &= \det\{-\omega_F^2[M_g] + [K_g] - [A(M_F,\omega_F)]\} \\ &= D_R(M_F,\omega_F) + iD_I(M_F,\omega_F) \end{aligned}$$

WHERE D_R AND D_I ARE THE REAL AND IMAGINARY PARTS OF THE CHARACTERISTIC DETERMINANT D, RESPECTIVELY. THEN AT FLUTTER CONDITION

$$D_R(M_F,\omega_F) = 0$$

$$D_I(M_F,\omega_F) = 0$$

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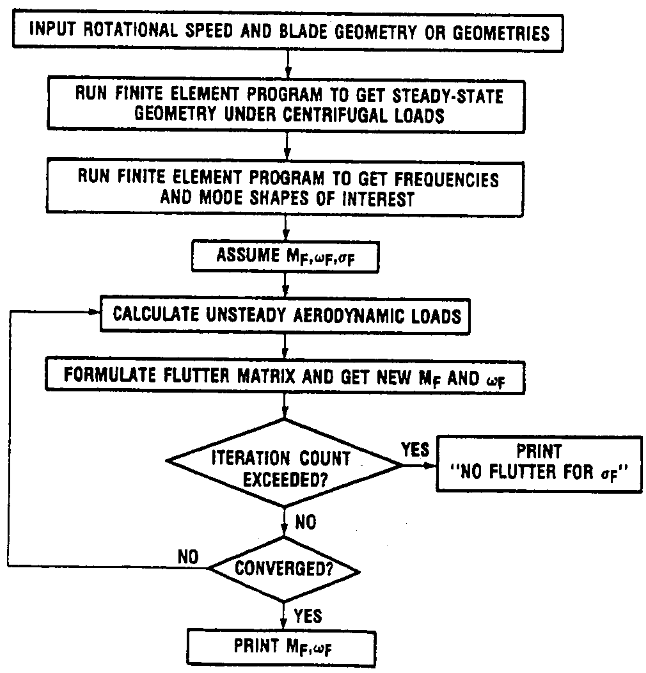
PROPFAN FLUTTER ANALYSIS BY DIRECT SOLUTION

In contrast to the conventional procedure the direct solution outlined here eliminates the need to track eigenvalues to determine the flutter point. In addition to this important benefit the double iteration on the complex eigensolution is replaced by a single solution of a system of two real nonlinear equations. The Mach number and frequency are varied simultaneously in this procedure rather than one at a time as in the conventional procedure. Thus the flutter Mach number and the flutter frequency are determined simultaneously.

The formulation of a transcendental double-eigenvalue problem in preference to a linear single-eigenvalue problem may seem to defeat the objective of increased efficiency, even if it is more suitable for automation. However, the price to be paid is not as great as it may seem. The transcendental eigenvalue problem needs to be solved for only one set of eigenvalues in most cases, whereas the linear eigenvalue problem has to be repeatedly solved for all the eigenvalues, which are equal in number to the number of assumed mode shapes.

The direct solution may not find the lowest flutter Mach number if more than one structural mode were to flutter in the Mach number and frequency range of interest for the selected intergroup phase angle mode. Under these circumstances one will be forced to search the entire range of interest for the roots M_F and ω_F, starting with different initial guesses. This is not amenable to an efficient automated procedure. However, it is expected that these circumstances will rarely occur for tuned or alternately mistuned propfans. This is not a major limitation for two other reasons: (1) the frequency interval in which flutter occurs is usually determined early in the design phase and (2) the search domain can be considerably reduced after a few orienting runs.



• ELIMINATES EIGENVALUE TRACKING
• REQUIRES ONLY SINGLE ITERATION
• REQUIRES SOLUTION OF ONLY ONE SET OF M AND ω

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NUMERICAL METHODS FOR DIRECT SOLUTION

The flutter Mach number and flutter frequency M_F and ω_F may be solved for by Newton's method. The iterative scheme for Newton's method is

$$\begin{Bmatrix} M_F \\ \omega_F \end{Bmatrix}_{(k+1)} = \begin{Bmatrix} M_F \\ \omega_F \end{Bmatrix}_{(k)} - [J_k]^{-1} \begin{Bmatrix} D_R \\ D_I \end{Bmatrix}_{(k)}$$

where k is the iteration number and $[J_K]$ is the Jacobian matrix given by

$$[J_k] = \begin{bmatrix} D_{kR,M_F} & D_{kR,\omega_F} \\ D_{kI,M_F} & D_{kI,\omega_F} \end{bmatrix}$$

The Jacobian $[J_k]$ is expensive to compute because the evaluation of the aerodynamic matrix $[A_k]$ is computationally intensive. Several quasi-Newton algorithms that approximate the Jacobian in various ways are available.

NEWTON'S METHOD:

$$\begin{Bmatrix} M_F \\ \omega_F \end{Bmatrix}_{(k+1)} = \begin{Bmatrix} M_F \\ \omega_F \end{Bmatrix}_{(k)} - [J_k]^{-1} \begin{Bmatrix} D_R \\ D_I \end{Bmatrix}_{(k)}$$

$[J_k]$ EXPENSIVE TO COMPUTE

QUASI-NEWTON METHODS:

$[J_k]$ APPROXIMATED IN VARIOUS WAYS

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A BETTER QUASI-NEWTON METHOD

A quasi-Newton algorithm is proposed that is more efficient than the algorithms currently available for determining the flutter Mach number and the flutter frequency. The numerical scheme is based on the hypothesis that approximating the derivatives of the flutter matrix $[B_k]$ provides a more accurate approximation to the Jacobian matrix $[J_k]$ than directly approximating the derivatives of the characteristic determinant. The numerical scheme based on this hypothesis approaches Newton's method in its superior convergence characteristics with the same cost per iteration as the secant method.

The derivatives $[B_k]_{,M_F}$ and $[B_k]_{,\omega_F}$ of the flutter matrix are approximated by following a reasoning similar to that employed in deriving Broyden's method (Johnston, 1982). Let $\Delta M_k = M_{F(k-1)} - M_{F(k)}$ and $\Delta\omega_k = \omega_{F(k-1)} - \omega_{F(k)}$. The derivatives are approximated in the direction of the last move to satisfy

$$[B_{k-1}] = [B_k] + [B_k]_{,M_F} \cdot \Delta M_k + [B_k]_{,\omega_F} \cdot \Delta\omega_k$$

and are assumed to be unchanged in the direction orthogonal to the last move.

APPROACH

- APPROXIMATE THE JACOBIAN INDIRECTLY BY APPROXIMATING THE DERIVATIVES OF THE FLUTTER MATRIX
- UPDATE THE DERIVATIVES OF THE FLUTTER MATRIX ONLY IN THE DIRECTION OF THE LAST MOVE

RESULT

- A QUASI-NEWTON METHOD MORE LIKE NEWTON'S METHOD THAN OTHERS
- THE FLUTTER MODE ALMOST A BYPRODUCT

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EFFICIENCY OF NUMERICAL PROCEDURE

The direct solution procedure was demonstrated by performing flutter boundary calculations at various rotational speeds for two propfan rotor configurations. The first configuration consisted of eight identical blades. The second configuration was an alternately mistuned rotor with eight blades.

The typical progress of iteration, for initial guesses for flutter Mach number and flutter frequency of 0.5 and 310 Hz, respectively, with the direct solution procedure and the conventional procedure, is shown in the first table. Recall that the conventional procedure relies on user interaction and judgment. The progress of iteration shown for the conventional procedure is typical. The direct solution procedure, in addition to being suitable for automation, is also more efficient as evidenced by the considerably smaller number of analysis steps. Thus both the CPU time and the analyst's time are considerably reduced by using the direct solution procedure.

The results show that a fair initial guess would converge to the "exact" flutter point after about 5 to 10 flutter matrix evaluations. The second table shows the CPU times on the Cray-XMP required to obtain the flutter boundary for good initial guesses and poor initial guesses. The CPU times for one flutter eigenvalue analysis at a given set of Mach number and assumed frequency are also shown for comparison. With a good initial guess the flutter Mach number and the flutter frequency can be obtained for two or three times the cost of a single eigenanalysis. The direct solution procedure is much less expensive in terms of CPU time as well as analyst's time than the conventional procedure, although precise comparisons have not been made.

PROGRESS OF ITERATION
(5280 rpm; BLADE SETTING ANGLE AT 0.75 RADIUS, 61.6°; σ_r=225°)

COUNT	CONVENTIONAL PROCEDURE		DIRECT SOLUTION PROCEDURE	
	FLUTTER MACH NUMBER, M	FLUTTER FREQUENCY, ω, Hz	FLUTTER MACH NUMBER, M	FLUTTER FREQUENCY, ω, Hz
1	0.500	310.0	0.500	310.0
2	↓	267.5	.499	310.0
3		268.9	.500	313.0
4		268.9	.701	289.7
5	.700	268.9	.590	287.7
6	↓	299.8	.641	293.6
7		298.9	.642	294.1
8		298.9	.641	294.1
9	.616	286.3	[a].641	294.1
10	↓	290.1		
11		290.5		
12		290.4		
13	.640	292.9		
14	.640	293.9		
15	.640	294.0		
16	.641	293.9		
17	.641	294.1		
18	[a].641	294.1		

[a]Converged.

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CPU TIME FOR AUTOMATED FLUTTER ANALYSIS

	EIGENSOLUTION AT A SINGLE SET OF MACH NUMBER M AND FREQUENCY ω	DIRECT SOLUTION TO FIND M_F AND ω_F	
		GOOD INITIAL GUESS[a]	POOR INITIAL GUESS[a]
	CPU TIME, SEC		
TUNED ROTOR[b]—8 BLADES; 6 MODES/BLADE; 5280 rpm; $\sigma_r=225°$	4.332	10.356 ($M=0.70$; $\omega=310$ Hz)	22.146 ($M=0.45$; $\omega=340$ Hz)
MISTUNED ROTOR[c]—8 BLADES; 4 GROUPS; TWO MODES/BLADE; 5190 rpm; $\sigma_r=90°$	10.020	22.084 ($M=0.65$; $\omega=310$ Hz)	31.970 ($M=0.5$; $\omega=340$ Hz)

[a]INITIAL GUESSES ARE GIVEN IN PARENTHESES.
[b]"EXACT" $M_F=0.641$ AND "EXACT" $\omega_F=294$ Hz.
[c]"EXACT" $M_F=0.718$ AND "EXACT" $\omega_F=285$ Hz.

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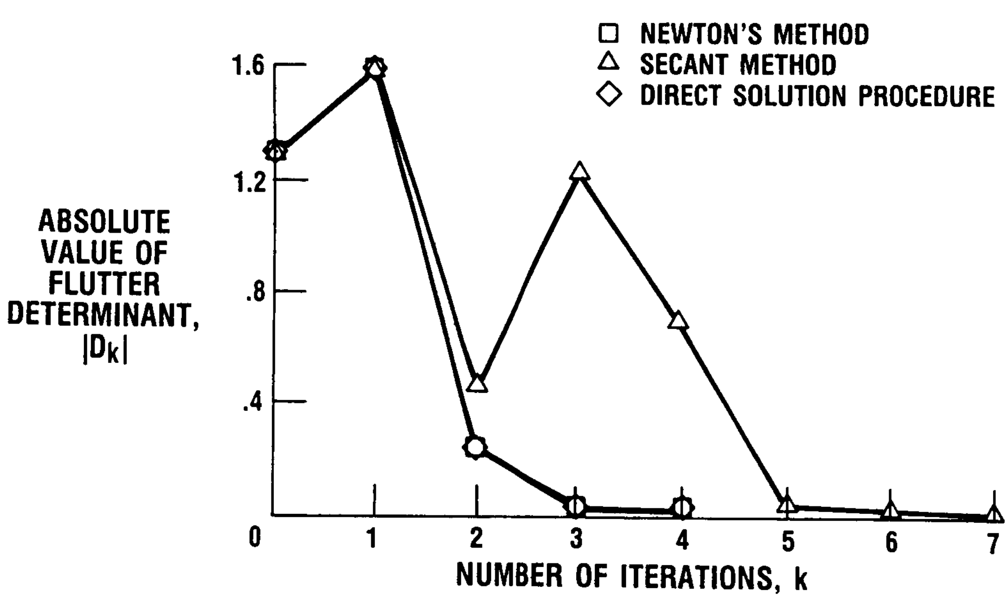
ACCURACY OF JACOBIAN

So that the accuracy of the approximate Jacobian could be judged, the nonlinear equations were solved by Newton's method, by the present numerical procedure, and by alternative quasi-Newton methods such as the multipoint secant method (implemented in IMSL routine ZSCNT), the modified Powell algorithm (implemented in IMSL routine ZSPOW), and Broyden's method (Johnston, 1982). For the cases tested, the present procedure outperformed all three alternative methods in terms of efficiency. Even though the characteristic determinant D_k is never calculated in the present procedure, the variation of the absolute value of D_k with each iteration is shown in the graph so that the procedure can be compared with Newton's method and the multipoint secant method. For these cases, the "exact" flutter Mach number was 0.641, the "exact" flutter frequency was 294 Hz, and the initial values for M_F and ω_F were 0.65 and 330 Hz, respectively. The determinant value has been scaled so that $1.0 \leq D_0 \leq 10.0$, where D_0 is the characteristic determinant at the beginning of iteration. The iteration history for the current numerical procedure closely resembles that for Newton's method, indicating the accuracy of the approximation proposed here for the Jacobian matrix. In contrast, the secant approximation for the Jacobian matrix requires almost double the number of iterations.

5280 rpm; BLADE SETTING ANGLE AT 0.75 RADIUS, 61.6°; σ_r=225°



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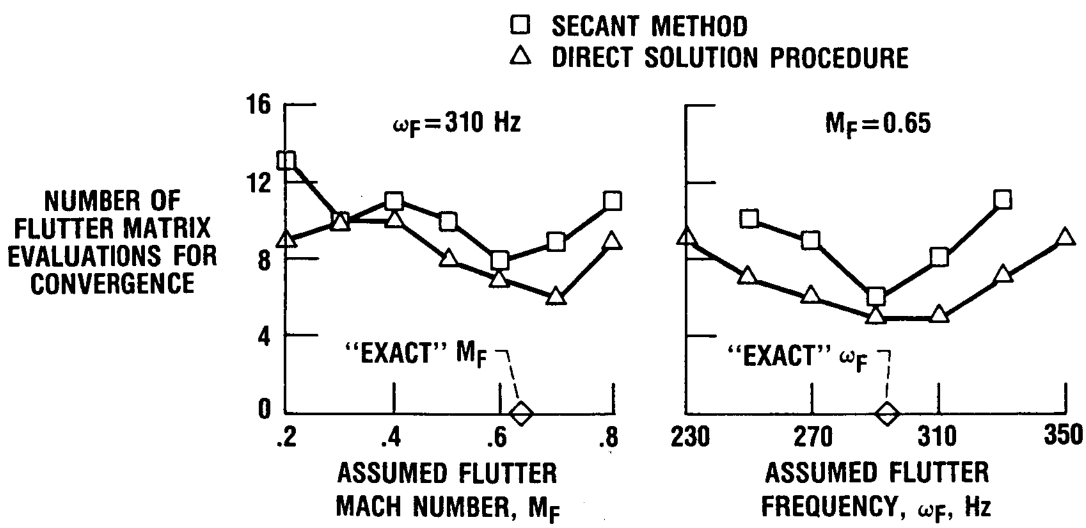
RANGE OF CONVERGENCE

The range of convergence is an important factor in any iterative procedure since it has an important effect on how closely the initial solution must approximate the final solution. The graph on the left shows the number of flutter matrix evaluations required for convergence; the initial guesses for M_F are varied and the initial guess for ω_F is fixed at 310 Hz. The range of Mach number convergence is from 0.2 to 0.8. The graph on the right similarly shows the number of flutter matrix evaluations required for convergence; the initial guess for M_F is fixed at 0.65 and the initial guesses for ω_F are varied. The frequency range of convergence with the direct solution procedure, 230 to 350 Hz, is slightly larger than that with the secant method. From these graphs it can be stated that the present procedure has a large range of convergence.



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SUMMARY

A direct solution of the equations of motion is demonstrated to be a reliable automated flutter analysis procedure if steady aerodynamic deformations are ignored. The direct solution procedure replaces the inner-outer iteration loop of the conventional procedure by a single iteration loop. A numerical procedure, based on an accurate and efficient approximation to the Jacobian matrix, is presented. The procedure is straightforward in concept, and results for test cases show good convergence properties. Since the procedure is iterative, it is particularly suitable for design optimization. As the optimal design is evolved, the flutter solution is expected to change incrementally from design to design, so that the previous solution provides good estimates for the current solution.

- **DEVELOPED A QUASI-NEWTON METHOD FOR DETERMINANT ITERATION**
- **AUTOMATED THE PROPFAN FLUTTER ANALYSIS BY DIRECT SOLUTION**
- **DEMONSTRATED GOOD CONVERGENCE AND EFFICIENCY OF DIRECT SOLUTION METHOD WITH ADVANCED AERODYNAMIC MODEL**

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