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AN APPROXIMATION THEORY FOR THE IDENTIFICATION OF NONLINEAR DISTRIBUTED PARAMETER SYSTEMS[1]

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ABSTRACT

An abstract approximation framework for the identification of nonlinear distributed parameter systems is developed. Inverse problems for nonlinear systems governed by strongly maximal monotone operators (satisfying a mild continuous dependence condition with respect to the unknown parameters to be identified) are treated. Convergence of Galerkin approximations and the corresponding solutions of finite dimensional approximating identification problems to a solution of the original infinite dimensional identification problem is demonstrated using the theory of nonlinear evolution systems and a nonlinear analog of the Trotter-Kato approximation result for semigroups of bounded linear operators. The nonlinear theory developed here is shown to subsume an existing linear theory as a special case. It is also shown to be applicable to a broad class of nonlinear elliptic operators and the corresponding nonlinear parabolic partial differential equations to which they lead. An application of the theory to a quasilinear model for heat conduction or mass transfer is discussed.

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1. Introduction

In this paper we develop a general abstract approximation framework for the identification of nonlinear distributed parameter evolution systems. Our intent is to define relatively straightforward and easily verified criteria that are applicable to broad classes of nonlinear systems; these criteria will guarantee the convergence of solutions to a sequence of finite dimensional Galerkin approximation based parameter estimation problems to a solution of the original, underlying, infinite dimensional identification problem. The results that we present below generalize and extend the theory recently developed by Banks and Ito in [2] and [3] for regularly dissipative or abstract parabolic, linear systems. It is, to the best of our knowledge, the first such general approximation theory for inverse problems involving nonlinear distributed systems.

The sufficient conditions set down in our framework include a relatively mild continuity assumption with respect to the unknown parameters to be identified, an equi-boundedness and an equi-strong monotonicity assumption on the nonlinear operator describing the system dynamics. In addition our theory requires a standard approximation assumption on the Galerkin subspaces used to effect the finite dimensional, or finite element, approximations. We demonstrate that solutions to the finite dimensional identification problems approximate a solution to the infinite dimensional identification problem via a convergence result for solutions to the forward problems. This result is obtained using the theory of nonlinear evolution systems and a nonlinear analog of the well-known Trotter- Kato approximation result for linear semigroups.

In the present paper, we are concerned only with theory; implementation questions and conclusions drawn from our numerical or computational studies will be reported on elsewhere. Also, while we have tried to make our framework as versitile as possible, the treatment below does have limitations. For example, our theory can handle quasi-autonomous systems but it is not applicable in the fully nonautonomous case. The development of a general theory which can handle nonlinear systems involving time dependent operators requires additional effort and is currently the focus of our ongoing investigations. The particular difficulties inherent in the time dependent case will be described in greater detail in our discussions below.

We provide a brief outline of the remainder of the paper. In Section 2 we state a fundamental existence and uniqueness result for infinite dimensional nonlinear systems and prove a general approximation result which is especially well suited for application in the context of the inverse problems which are the central focus of our study. In Section 3 we define a class of nonlinear distributed systems and the associated parameter identification problems. We define the Galerkin approximations and prove the general convergence result. Section 4 contains some examples. We show that our nonlinear theory subsumes the linear theory presented in [2] and [3] as a special case; we also consider the application of our framework to a class of nonlinear elliptic operators and the corresponding nonlinear parabolic partial differential equations to which they lead. In particular, we look at the application of our results to a well known quasilinear model for heat conduction or mass transfer. In Section 5 we summarize our findings and provide some concluding remarks.

2. An Approximation Result for Nonlinear Evolution Systems

Let X_0 be a Banach space with norm $|\cdot|_0$. We consider the nonlinear, quasiautonomous initial value problem in X_0 given by

(2.1) $\dot{x}_0(t) + A_0 x_0(t) \ni f_0(t), \quad 0 < t \leq T,$

(2.2) $x_0(0) = x_0^0$

where $x_0^0 \in X_0$, f_0: $[0,T] \to X_0$ and the nonlinear operator A_0: $X_0 \to 2^{X_0}$ is in general multivalued, not everywhere defined, and not continuous. The existence of solutions to the initial value problem (2.1), (2.2) and the subsequent approximation result to follow, are both consequences of Theorem 2.1 to be given below.

We shall require the following definitions. Let X be a Banach space with norm $|\cdot|_X$. For A: $X \to 2^X$, a nonlinear, multivalued operator, the domain and range of A are defined by $\mathrm{Dom}(A) = \{x \in X: Ax \neq \emptyset\}$ and $\mathcal{R}(A) = \bigcup_{x \in \mathrm{dom}(A)} Ax$ respectively. We say that the operator A is accretive if for every $\lambda > 0$, $x_1, x_2 \in \mathrm{Dom}(A)$ and $y_1 \in Ax_1$, $y_2 \in Ax_2$ we have

$$|x_1 - x_2|_X \leq |x_1 - x_2 + \lambda(y_1 - y_2)|_X.$$

We say that A is m-accretive if A is accretive and $\mathcal{R}(I + \lambda A) = X$ for some $\lambda > 0$. We note that if A is m-accretive then $\mathcal{R}(I + \lambda A) = X$ for every $\lambda > 0$ and for each $\lambda > 0$ the resolvent of A at λ, $J(\lambda;A)$: $X \to X$, a single valued, everywhere defined, nonlinear operator on X can be defined as $J(\lambda;A) = (I + \lambda A)^{-1}$.

A two parameter family of nonlinear operators $\{U(t,s): 0 \leqslant s \leqslant t \leqslant T\}$ defined on a subset $\Omega \subset X$ is called a nonlinear evolution system on Ω if for each $x \in \Omega$ we have $U(t,s)x \in \Omega$, $U(s,s)x = x$ and $U(t,r)U(r,s)x = U(t,s)x$ for $0 \leqslant s \leqslant r \leqslant t \leqslant T$ and $U(t,s)x$ is continuous from the triangle $\Delta = \{[s,t]: 0 \leqslant s \leqslant t \leqslant T\}$ into X.

A strongly continuous function $x: [0,t] \to X$ is called a strong solution to the quasiautonomous initial value problem

(2.3) $\dot{x}(t) + Ax(t) \ni f(t), \quad 0 < t \leqslant T$

(2.4) $x(0) = x^0$

where $f: [0,T] \to X$ and $x^0 \in X$ if x is absolutely continuous on compact subintervals of (0,T), differentiable almost everywhere and satisfies $f(t) - \dot{x}(t) \in Ax(t)$ for almost every $t \in [0,T]$ and $x(0) = x^0$.

Theorem 2.1. *Let* X *be a Banach space with norm* $|\cdot|_X$ *and suppose that* $A: X \to 2^X$ *and* $f: [0,T] \to X$ *appearing in* (2.3) *satisfy*

(1) *there exists an* $\omega \in R$ *for which the operator* $A + \omega I$ *is* m-*accretive,*

(2) $f \in L_1(0,T;X)$.

Then a unique, nonlinear evolution system $\{U(t,s): 0 \leqslant s \leqslant t \leqslant T\}$ *on* $\overline{Dom(A)}$ *can be constructed which satisfies*

(i) $|U(t,s)\phi - U(t,s)\psi|_X \leqslant e^{\omega(t-s)}|\phi - \psi|_X$, *for* $\phi,\psi \in \overline{Dom(A)}$ *and* $0 \leqslant s \leqslant t \leqslant T$,

(ii) $|U(s+t,s)\phi - U(r+t,r)\phi|_X \leqslant 2\int_0^t e^{\omega(t-\tau)}|f(\tau+s) - f(\tau+r)|_X d\tau$, *for all* $\phi \in \overline{Dom(A)}$ *and all* $t > 0$ *such that* $s+t, r+t \leqslant T$.

(iii) *if* $x^0 \in \overline{\mathrm{Dom}(A)}$ *and the initial value problem* (2.3), (2.4) *has a strong solution* x, *then*

$$x(t) = U(t,s)x(s), \quad \textit{for } 0 \leqslant s \leqslant t \leqslant T.$$

When $x^0 \in \overline{\mathrm{Dom}(A)}$, *the strongly continuous function* $x\colon [0,T] \to X$ *given by* x(t) $U(t,0)x^0$ *is referred to as a mild or generalized solution to* (2.3), (2.4).

Theorem 2.1 is a direct consequence of results given by Crandall and Evans and Evans in [7] and [9]. Henceforth, we shall assume that $A_0\colon X_0 \to 2^{X_0}$ and $f_0\colon [0,T] \to X$, satisfy (1) and (2) in the statement of Theorem 2.1 and that $x_0 \in \overline{\mathrm{Dom}(A_0)}$. We then let $\{U_0(t,s)\colon 0 \leqslant s \leqslant t \leqslant T\}$ denote the corresponding nonlinear evolution system on $\overline{\mathrm{Dom}(A_0)}$ and consider the approximation of mild solutions to the initial value problem (2.1), (2.2).

Our approximation result is in the spirit of those given for nonlinear semigroups and evolution systems by Crandall and Pazy in [8] and Goldstein in [10]. However, our theorem differs from these earlier treatments in two ways. First, we require that the time dependent perturbation f_0 be only L_1 as opposed to it being continuous as in [8] and it satisfying a Lipschitz-like condition in [10]. This distinction is especially relevant in the case of control systems where discontinuous input is common. The second difference is that we give our result in a form that is most appropriate for application to the development of a general approximation theory or framework and computational schemes for the parameter identification problems to be discussed in the next section.

We shall require some set theoretic notation. For sets H_n, n=0,1,2,..., by $\lim H_n \supset H_0$ we shall mean: Given $x_0 \in H_0$, there exist $x_n \in H_n$ such that $x_n \to x_0$ as $n \to \infty$.

Theorem 2.2. *For each* $n \in Z^+ = \{1,2,3, ...\}$ *let* X_n *be a closed linear subspace of* X_0. *For* $n = 0,1,...,$ *let* $A_n: X_n \to 2^{X_n}$ *be a possibly multivalued nonlinear operator on* X_n, *and let* $f_n: [0,T] \to X_n$ *be an* X_n*-valued measurable function defined on* $[0,T]$. *Suppose that there exists an* $\omega_0 \in R$, *independent of* n, *for which the operators* $A_n + \omega_0 I$ *are* m*-accretive, that there exists a function* $g \in L_1(0,T;X_0)$ *for which* $|f_n(t)| \leq g(t)$, a.e. $t \in [0,T]$, *and that* $\lim \bar{D}_n \supset \bar{D}_0$ *where* $D_n = \text{Dom}(A_n)$ and $D_0 = \text{Dom}(A_0)$. *Suppose further that for some* $\lambda_0 > 0$ *we have*

$$\lim_{n\to\infty} J(\lambda_0; A_n + \omega_0 I)\phi_n = J(\lambda_0; A_0 + \omega_0 I)\phi_0 \tag{2.5}$$

whenever $\phi_n \in X_n$ *with* $\lim_{n\to\infty} \phi_n = \phi_0 \in X_0$,

and that

$$\lim_{n\to\infty} f_n(t) = f_0(t) \text{ for a.e. } t \in [0,T].$$

Then for each $n \in Z^+$ *there exists a unique nonlinear evolution system* $\{U_n(t,s): 0 \leq s \leq t \leq T\}$ *on* $\bar{D}_n$ *corresponding (in the sense of Theorem* 2.1) *to* A_n *and* f_n *and for* $\phi_n \in \bar{D}_n$ *with* $\lim_{n\to\infty} \phi_n = \phi_0 \in \bar{D}_0$ *we have*

$$\lim_{n\to\infty} U_n(t,s)\phi_n = U_0(t,s)\phi_0, \quad 0 \leq s \leq t \leq T, \tag{2.6}$$

with the limit being uniform in t *for* $t \in [s,T]$.

Proof. We follow Goldstein (see [10], [11]) and use an approach first suggested by Kisynski [13] for demonstrating the convergence of approximations to linear semigroups, to prove the theorem via an application of our existence result, Theorem 2.1.

Let $\chi = \{\hat{x} = \{x_n\}_{n=0}^{\infty}: x_n \in X_n,\ n = 0,1,2, \ldots,$ and $\lim_{n\to\infty} x_n = x_0\}$ and for $\hat{x} \in \chi$ set $\|\hat{x}\| = \sup_n |x_n|_0$. Then $\|\cdot\|$ defines a norm on the linear vector space χ, and the space χ together with the norm $\|\cdot\|$ is a Banach space. Define the operator $\mathcal{A}: \chi \to 2^{\chi}$ by

$$\begin{aligned}\mathrm{dom}(\mathcal{A}) = \{\hat{x} = \{x_n\}_{n=0}^{\infty} \in \chi:\ & x_n \in \mathrm{Dom}(A_n), \text{ and for each } n = 1,2, \ldots \\ & \text{there exists a } y_n \in A_n x_n \text{ such that } \lim_{n\to\infty} y_n = y_0 \in A_0 x_0\},\end{aligned}$$

$$\begin{aligned}&\text{for } \hat{x} \in \mathrm{Dom}(\mathcal{A}),\ \hat{y} = \{y_n\}_{n=0}^{\infty} \in \mathcal{A}\hat{x} \text{ if and only if } y_n \in A_n x_n, \\ &n = 0,1,2, \ldots \text{ and } \lim_{n\to\infty} y_n = y_0.\end{aligned}$$

Define an essentially χ-valued function f on the interval [0,T] by $f(t) = \{f_n(t)\}_{n=0}^{\infty}$. The assumptions on the f_n are such that $f_n(t) \to f_0(t)$ for almost every $t \in [0,T]$. However, by appropriately redefining on a set of measure zero, we may infer from the assumptions on the functions f_n that $f:[0,T] \to \chi$ with $f \in L_1(0,T;\chi)$.

It is readily seen that the operator $\mathcal{A} + \omega_0 I$ is m-accretive. Let $\hat{x}^1 = \{x_n^1\}_{n=0}^{\infty}$, $\hat{x}^2 = \{x_n^2\}_{n=0}^{\infty} \in \mathrm{Dom}(\mathcal{A})$ and let $\hat{y}^1 = \{y_n^1\}_{n=0}^{\infty} \in \mathcal{A}\hat{x}^1$ and $\hat{y}^2 = \{y_n^2\}_{n=0}^{\infty} \in \mathcal{A}\hat{x}^2$. Since for each $n = 0,1,2, \ldots, A_n + \omega_0 I$ is assumed to be m-accretive, for $\lambda > 0$ we have

$$\begin{aligned}\|\hat{x}^1 - \hat{x}^2\| &= \sup_n |x_n^1 - x_n^2|_0 \leq \sup_n |x_n^1 - x_n^2 + \lambda(y_n^1 + \omega_0 x_n^1 - (y_n^2 + \omega_0 x_n^2))|_0 \\ &= \|\hat{x}^1 - \hat{x}^2 + \lambda(\hat{y}^1 + \omega_0 \hat{x}^1 - (\hat{y}^2 + \omega_0 \hat{x}^2))\|,\end{aligned}$$

and therefore that $\mathcal{A} + \omega_0 I$ is accretive. Now let $\hat{y} = \{y_n\}_{n=0}^{\infty} \in \chi$ and set $\hat{x} = \{x_n\}_{n=0}^{\infty}$ with $x_n = J(\lambda_0; A_n + \omega_0 I)y_n$, $n = 0,1,2, \ldots$ where λ_0 is chosen as in (2.5). It is immediately clear that for each $n = 0,1,2, \ldots, x_n \in \mathrm{Dom}(A_n) \subset X_n$. Since

$\hat{y} \in \mathcal{X}$ we have $\lim_{n\to\infty} y_n = y_0$ and therefore, by assumption (2.5), that $\lim_{n\to\infty} x_n = x_0$ or $\hat{x} \in \mathcal{X}$. Setting $z_n = (y_n - (1 + \lambda_0\omega_0)x_n)/\lambda_0$, $n = 0,1,2, \ldots$, it follows that $z_n \in A_n x_n$ and $\lim_{n\to\infty} z_n = z_0 \in A_0 x_0$. We conclude that $\hat{x} \in \mathrm{Dom}(\mathcal{A})$, $(I + \lambda_0(\mathcal{A}+\omega_0 I))\hat{x} \ni \hat{y}$, and that $\mathcal{R}(I + \lambda_0(\mathcal{A} + \omega_0 I)) = \mathcal{X}$.

We have shown that the operator $\mathcal{A}$ and the function f satisfy conditions (1) and (2) given in the statement of Theorem 2.1. Therefore, a unique nonlinear evolution system $\{\mathcal{U}(t,s): 0 \leq s \leq t \leq T\}$ on $\overline{\mathrm{Dom}(\mathcal{A})}$ corresponding to $\mathcal{A}$ and f can be constructed with $\mathcal{U}(t,s) = \{\mathcal{U}_n(t,s)\}_{n=0}^{\infty}$. Using assumption (2.5) it can be shown that $\overline{\mathrm{Dom}(\mathcal{A})} = \{\hat{x} = \{x_n\}_{n=0}^{\infty} \in \mathcal{X}: x_n \in \bar{D}_n,\ n = 0,1,2, \ldots$ and $\lim_{n\to\infty} x_n = x_0\}$. Since $\mathcal{R}(\mathcal{U}(t,s)) \subset \mathcal{X}$, it follows that

$$\lim_{n\to\infty} \mathcal{U}_n(t,s)\phi_n = \mathcal{U}_0(t,s)\phi_0, \quad 0 \leq s \leq t \leq T \tag{2.7}$$

whenever $\phi_n \in \bar{D}_n$ and $\lim_{n\to\infty} \phi_n = \phi_0 \in \bar{D}_0$. Since each of the operators A_n and the functions f_n satisfy conditions (1) and (2) of Theorem 2.1, unique nonlinear evolution systems $\{U_n(t,s): 0 \leq s \leq t \leq T\}$ on $\bar{D}_n$ corresponding to A_n and f_n can be constructed. Recalling that $\overline{\mathrm{Dom}(\mathcal{A})} \subset \mathop{\times}_{n=0}^{\infty} \bar{D}_n$, we may define the family of operators $\{V(t,s): 0 \leq s \leq t \leq T\}$ on $\overline{\mathrm{Dom}(\mathcal{A})}$ by

$$V(t,s)\hat{x} = \{V_n(t,s)x_n\}_{n=0}^{\infty} \equiv \{U_n(t,s)x_n\}_{n=0}^{\infty} \tag{2.8}$$

for $\hat{x} = \{x_n\}_{n=0}^{\infty} \in \overline{\mathrm{Dom}(\mathcal{A})}$. Uniqueness (see [9]) dictates that for each $n = 0,1,2, \ldots$, $\mathcal{U}_n(t,s)x_n = V_n(t,s)x_n$ whenever $\{x_n\}_{n=0}^{\infty} \in \overline{\mathrm{Dom}(\mathcal{A})}$. This together with (2.7) and (2.8) establish (2.6). The fact that the convergence in (2.6) is uniform in t for $t \in [s,T]$ is argued exactly as it was for the convergence of approximations to nonlinear semigroups in the proof of Theorem 3.2 in [10].

We note that (2.5) is also a necessary condition for the conclusion to hold (see, for example, Theorem 1 in [14]).

3. An Approximation Theory for Identification Problems

Let H be a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and corresponding norm $|\cdot|$. Let V be a reflexive real Banach space with norm $\|\cdot\|$ and let V^* be its dual. (All of our theory can be developed in complex spaces if necessary; see [6].) We denote the usual dual norm on V^* by $\|\cdot\|_*$ and assume that V is densely and continuously embedded in H with $|v| \leq \mu\|v\|$, $v \in V$, for some positive constant μ. Identifying H with its dual, we obtain $V \subset H = H^* \subset V^*$. For $\phi \in V^*$ and $v \in V$ the duality pairing between ϕ and v is denoted by $\langle\phi,v\rangle$. When $\phi \in H$, its pairing with $v \in V$ agrees with the inner product of ϕ with v. It follows for $u \in H$ and $v \in V$ that $\|u\|_* \leq \mu|u|$ and $\|v\|_* \leq \mu^2\|v\|$. Let $\mathcal{Q}$ and Z be metric spaces and let Q be a nonempty, sequentially compact, subset of $\mathcal{Q}$. The spaces $\mathcal{Q}$ and Z, and the set Q are referred to as the parameter space, the observation space, and the admissible parameter set respectively.

We recall that a single valued operator $A:V\to V^*$ is hemicontinuous if $\lim_{t\to 0} A(u+tv) = Au$ for all $u,v\in V$ where the limit is taken in the weak sense.

For each $q \in Q$ let $A(q)\colon V \to V^*$ be a single valued, hemicontinuous, (in general, nonlinear) operator satisfying:

(A) (Continuity): For each $v \in V$, the map $q \to A(q)v$ is continuous from $Q \subset \mathcal{Q}$ into V^*.

(B) (Equi V- monotonicity): There exist an $\omega \in R$ and an $\alpha > 0$, both independent of $q \in Q$, such that

$$\langle A(q)u - A(q)v, u-v\rangle + \omega|u-v|^2 \geq \alpha\|u-v\|^2,$$

for every $u,v \in V$.

(C) (Equi-boundedness): There exist a constant $\beta > 0$, independent of $q \in Q$ such that

$$\|A(q)v\|_* \leq \beta(\|v\| + 1),$$

for every $v \in V$.

For each $q \in Q$, let $f(\cdot;q) \in L_1(0,T;H)$ and $u^0(q) \in H$ and assume that the mapping $q \to u^0(q)$ is continuous from $Q \subset \mathcal{Q}$ into H and that the mapping $q \to f(t;q)$ is continuous from $Q \subset \mathcal{Q}$ into H for almost every $t \in [0,T]$. Also, for every $z \in Z$, let $u \to \Phi(u;z)$ be a continuous map from $C(0,T;H)$ into R^+.

We consider parameter identification or inverse problems of the form:

(ID) Given observations $z \in Z$, determine parameters $\bar{q} \in Q$ which minimize

$$\phi(q) = \Phi(u_0(q);z)$$

where $u_0(q) = u_0(\cdot;q)$ is a mild solution to the initial value problem

$$\dot{u}(t) + A(q)u(t) = f(t;q), \quad 0 < t \leq T, \tag{3.1}$$

$$u(0) = u^0(q) \tag{3.2}$$

corresponding to $q \in Q$.

By a mild solution to (3.1), (3.2) we mean a solution in the sense of Theorem 2.1. To be more precise, for each $q \in Q$ we define the operator $A_0(q)$: $\mathrm{Dom}(A_0(q)) \subset H \to H$ to be the restriction of the operator $A(q)$ to the subset of V given by $\mathrm{Dom}(A_0(q)) = \{v \in V: \ A(q)v \in H\}$, and prove the following theorem.

Theorem 3.1. *For each* $q \in Q$ *the operator* $A_0(q)$: $\mathrm{Dom}(A_0(q)) \subset H \to H$ *is densely defined and the operator* $A_0(q) + \omega I$ *is* m-*accretive.*

Proof. We first show that for each $q \in Q$ the operator $A(q) + \omega I$: $V \to V^*$ is coercive. If $\{v_n\} \subset V$ with $\lim_{n\to\infty} \|v_n\| = \infty$, then from assumptions (B) and (C) we obtain

$$\lim_{n\to\infty} \langle (A(q) + \omega I)v_n, v_n \rangle / \|v_n\|$$

$$= \lim_{n\to\infty} \{(\langle A(q)v_n - A(q)\theta, v_n\rangle + \omega|v_n|^2)/\|v_n\| + \langle A(q)\theta, v_n\rangle/\|v_n\|\}$$

$$\geq \lim_{n\to\infty} \{\alpha\|v_n\|^2/\|v_n\| - |\langle A(q)\theta, v_n\rangle|/\|v_n\|\}$$

$$\geq \lim_{n\to\infty} \{\alpha\|v_n\| - \|A(q)\theta\|_*\} \geq \lim_{n\to\infty} \alpha\|v_n\| - \beta = \infty$$

where θ denotes the zero vector in V. It follows that for each $\lambda > 0$, the operator $I + \lambda(A(q) + \omega I)$: $V \to V^*$ is monotone, everywhere defined on V, hemicontinuous, and coercive. Consequently $\mathcal{R}(I + \lambda(A(q) + \omega I)) = V^*$ (see Barbu [6], Theorem II.1.3) and therefore $\mathcal{R}(I + \lambda(A_0(q) + \omega I)) = H$. Also, for $u,v \in \mathrm{Dom}(A_0(q))$, we may use assumption (B) to conclude

$$\left(1 + \frac{\lambda\alpha}{\mu^2}\right)|u-v|^2 \leq |u-v|^2 + \lambda\alpha\|u-v\|^2$$

$$\leq |u-v|^2 + \lambda\langle (A(q) + \omega I)u - (A(q) + \omega I)v, u-v\rangle$$

$$= \langle (I + \lambda(A(q) + \omega I))u - (I + \lambda(A(q) + \omega I))v, u-v\rangle$$

$$\leq |(I + \lambda(A_0(q) + \omega I)u - (I + \lambda(A_0(q) + \omega I)v|\,|u-v|$$

or

$$|u-v| \leq |u - v + \lambda((A_0(q) + \omega I)u - (A_0(q) + \omega I)v)|$$

which proves that $A_0(q) + \omega I$ is m-accretive on $\mathrm{Dom}(A_0(q)) \subset H$.

To show $\overline{\mathrm{Dom}(A_0(q))} = H$, we let $u \in H$ and for each $n = 1,2,\ldots$ we set

$u_n = J(1/n;A_0(q)+\omega I)u \in \mathrm{Dom}(A_0(q))$. Then, arguing as we have above, we find

$$|u_n|^2 + (1/n)\alpha\|u_n\|^2 \leq \langle u - (1/n)A(q)\theta, u_n\rangle$$
$$\leq |u|\,|u_n| + (1/n)\|A(q)\theta\|_*\|u_n\|$$

where θ is again the zero vector in V. But then

$$(3.3)\qquad (1/2)|u_n|^2 + (1/n)(\alpha/2)\|u_n\|^2 \leq (1/2)|u|^2 + (1/n)(1/2\alpha)\|A(q)\theta\|_*^2$$
$$\leq (1/2)|u|^2 + (1/n)(\beta^2/2\alpha),$$

from which it immediately follows that the u_n are uniformly bounded in H. Indeed, from (3.3) we see that $(1/n)\|u_n\|^2$ and, hence $\|u_n\|/\sqrt{n}$, is bounded so that $\|u_n\|/n \to 0$ as $n\to\infty$.

Also, assumption (C) yields

$$\|u_n - u\|_* = (1/n)\|(A_0(q) + \omega I)u_n\|_* \leq (1/n)\{(\beta + \omega\mu^2)\|u_n\| + \beta\}.$$

Since the last term in the estimate above tends to zero as $n \to \infty$, we find $u_n \to u$ in V* as $n \to \infty$. This, together with the fact that V is dense in H imply that $u_n \to u$ weakly in H as $n \to \infty$ from which $\overline{\mathrm{Dom}(A_0(q))} = H$ immediately follows.

In light of Theorem 3.1, we may apply Theorem 2.1 with X = H, A = $A_0(q)$ and f = $f(\cdot;q)$. We conclude that there exist a unique nonlinear evolution system $\{U_0(t,s;q)\colon 0 \leq s \leq t \leq T\}$ on H satisfying (i), (ii) and (iii).

The mild solution $u_0(\cdot;q)$: $[0,T] \to H$ to the initial value problem (3.1), (3.2) is given by $u_0(t;q) = U_0(t,0;q)u^0(q)$ for $t \in [0,T]$.

Remark. Under additional hypotheses on $f(\cdot;q)$ and $u^0(q)$ other existence results can be applied to obtain somewhat different notions of a solution to the initial value problem (3.1), (3.2). For example (see [6, p.140-144]) if $f(\cdot;q) \in W^{1,1}(0,T;H)$ and $u^0(q) \in Dom(A_0(q))$, then there exists a unique $u(\cdot,q)$: $[0,T] \to V$ satisfying $u(\cdot;q) \in W^{1,\infty}(0,T;H)$, $A(q)u(\cdot;q) \in L_\infty(0,T;H)$ and $\dot{u}(t;q) + A(q)u(t;q) = f(t;q)$ a.e. $t \in [0,T]$. Or, if $u^0(q) \in H$ and $f(\cdot;q) \in L_2(0,T;V^*)$ then there exists a unique $u(\cdot;q)$ which is V^*-valued absolutely continuous almost everywhere on $[0,T]$, $u(\cdot;q) \in C(0,T;H) \cap L_2(0,T;V)$, $\dot{u}(\cdot;q) \in L_2(0,T;V^*)$ and $\dot{u}(t;q) + A(q)u(t;q) = f(t;q)$, a.e. $t \in [0,T]$. If, in addition, the mapping $t \to t^\gamma f'(t;q)$ is an element in $L_2(0,T;V^*)$ for some $\gamma \geq 1$, then the mapping $t \to t^\gamma \dot{u}(t;q)$ is in $L_2(0,T;V) \cap L_\infty(0,T;H)$. In particular, when $f(\cdot;q) = 0$, the nonlinear semigroup $\{S_0(t;q): 0 \leq t \leq T\}$ on H defined by $S_0(t;q) = U_0(t;0;q)$, $t \in [0,T]$, with generator $-A_0(q)$ behaves like a holomorphic linear semigroup in that it smooths. That is, $S_0(t;q)u^0(q) \in Dom(A_0(q))$, $t \in (0,T]$, and the mapping $t \to t \frac{d}{dt} S(t;q)u^0(q)$ is an element in $L_\infty(0,T;H)$ for every $u^0(q) \in H$. Also, some generalizations are possible. For example, in assumption (B), the term $\alpha\|u - v\|^2$ can be replaced by a term of the form $\alpha(\|u-v\|)\|u - v\|$ where $\alpha(\cdot)$ is a continuous, strictly increasing function on $[0,\infty)$ satisfying $\alpha(0) = 0$ and $\lim_{x\to\infty} \alpha(x) = \infty$. Or, the terms $\|u - v\|^2$ in (B) and $\|v\|$ in (C) can be replaced by $\|u - v\|^p$ and $\|v\|^{p-1}$, respectively, for any $p \geq 2$.

The development of computational methods for the solution of the infinite dimensional optimization problem (ID) requires the finite dimensional approximation of the abstract initial value problem (3.1), (3.2). The general framework that we are proposing is based upon a classical Galerkin approach. For each $n = 1,2, \ldots$ let H_n denote a finite dimensional subspace of H which is a subset of V. Let $P_n: H \to H_n$ denote the orthogonal projection of H onto H_n with respect to the $\langle \cdot,\cdot \rangle$ inner product. We assume that the approximating subspaces H_n, and the projections P_n satisfy

(D) For each $v \in V$, $\lim_{n\to\infty} \| P_n v - v \| = 0$.

Note that assumption (D) and V densely and continuously embedded in H imply that $\lim_{n\to\infty} |P_n u - u| = 0$ for each $u \in H$.

For each $q \in Q$ and $n = 1,2, \ldots$ we define the single-valued operator $A_n(q)$: $H_n \to H_n$ by $A_n(q)u_n = v_n$ for $u_n \in H_n$ where v_n satisfies

$$\langle A(q)u_n, w_n \rangle = \langle v_n, w_n \rangle, \quad w_n \in H_n.$$

That $A_n(q)$ is a well defined operator from H_n into H_n follows from the Riesz Representation Theorem applied to the Hilbert space H_n and the bounded linear functional $\langle A(q)u_n, \cdot \rangle$ on H_n. Also, define $f_n(\cdot;q)$: $[0,T] \to H_n$ and $u_n^0(q) \in H_n$ by $f_n(t;q) = P_n f(t;q)$, $0 \leq t \leq T$, and $u_n^0(q) = P_n u^0(q)$, respectively. Note that $f_n(\cdot;q) \in L_1(0,T;H_n) \subset L_1(0,T;H)$ and that $|f_n(t;q)| \leq |f(t;q)|$ for $q \in Q$ and almost every $t \in [0,T]$.

We consider the sequence of approximating identification problems given by:

(ID_n) Given observations $z \in Z$, determine parameters $\bar{q}_n \in Q$ which minimize

$$\phi_n(q) = \Phi(u_n(q);z)$$

where $u_n(q) = u_n(\cdot;q)$ is a mild solution to the initial value problem in H_n

$$\dot{u}_n(t) + A_n(q)u_n(t) = f_n(t;q), \quad 0 < t \leq T \tag{3.4}$$

$$u_n(0) = u_n^0(q) \tag{3.5}$$

corresponding to $q \in Q$.

From the definition of the $A_n(q)$ and the assumptions (B) and (C) on $A(q)$, using arguments analogous to those used to prove Theorem 3.1, it can be shown that the operators $A_n(q) + \omega I$ are m-accretive on H_n. It then follows from Theorem 2.1 that for each $n = 1,2, \ldots$ there exists a unique nonlinear evolution system $\{U_n(t,s;q): 0 \leq s \leq t \leq T\}$ on H_n satisfying (i) - (iii) in the statement of that theorem with $X = H_n$, $f(t) = f_n(t;q)$, and $x^0 = u_n^0(q)$. The mild solution to the initial value problem (3.4), (3.5) is given by $u_n(t;q) = U_n(t,0;q)u_n^0(q)$, $t \in [0,T]$.

If we assume for the moment that the approximating identification problems (ID_n) have solutions $\bar{q}_n \in Q$, then it is desirable that they in some sense approximate a solution $\bar{q}$ to the original identification problem (ID). This is in fact the case. For suppose that it can be shown that for any sequence $\{q_n\} \subset Q$ with $\lim_{n\to\infty} q_n = q \in Q$ we have

$$\lim_{n\to\infty} u_n(q_n) = u_0(q_0) \text{ in } C(0,T;H). \tag{3.6}$$

Then $\{\bar{q}_n\} \subset Q$ and Q a compact subset of the metric space $\mathcal{Q}$ imply that there exist a subsequence $\{\bar{q}_{n_j}\} \subset \{\bar{q}_n\}$ and a $\bar{q} \in Q$ such that $\lim_{j\to\infty} \bar{q}_{n_j} = \bar{q}$. For any $q \in Q$ the continuity of Φ implies

$$\begin{aligned}
\phi(\bar{q}) &= \Phi(u_0(\bar{q});z) = \Phi\,(\lim_{j\to\infty} u_{n_j}(\bar{q}_{n_j});z) \\
&= \lim_{j\to\infty} \Phi(u_{n_j}(\bar{q}_{n_j});z) = \lim_{j\to\infty} \phi_{n_j}(\bar{q}_{n_j}) \\
&\leq \lim_{j\to\infty} \phi_{n_j}(q) = \lim_{j\to\infty} \Phi(u_{n_j}(q);z) \\
&= \Phi\,(\lim_{j\to\infty} u_{n_j}(q);z) = \Phi(u_0(q);z) \\
&= \phi(q).
\end{aligned}$$

Note that in the discussion above we did not assume that a solution to problem (ID) exists. But rather we have shown that the existence of solutions $\bar{q}_n$ to the approximating problems (ID_n) and (3.6) imply the existence of a solution $\bar{q}$ to problem (ID). When the solution to problem (ID) is unique, the sequence $\{\bar{q}_n\}$ itself converges to $\bar{q}$.

The existence of a solution $\bar{q}_n$ to problem (ID_n) for each $n = 1,2, \ldots$ will follow from the compactness of Q and the continuity of Φ once the continuous dependence result: $\lim_{m\to\infty} u_n(q_m) = u_n(q_0)$ in $C(0,T;H_n)$ whenever $\{q_m\} \subset Q$ with $\lim_{m\to\infty} q_m = q_0$, has been established. Although continuous dependence for the finite dimensional systems (3.4), (3.5) could be demonstrated via a modification to any one of a number of familiar continuous dependence results for ordinary differential equations (see, for example, Hale [12], Theorem I.3.4), it is also easily handled with the approximation theory developed in the previous section. This and the convergence in (3.6) are addressed in the following theorem.

Theorem 3.2. *If assumptions* (A) - (D) *hold, then*

(a) *If* $\{q_n\} \subset Q$ *with* $\lim_{n\to\infty} q_n = q_0$ *then* $\lim_{n\to\infty} u_n(q_n) = u_0(q_0)$ *in* $C(0,T;H)$, *and*

(b) *If* $\{q_m\} \subset Q$ *with* $\lim_{m\to\infty} q_m = q_0$ *then* $\lim_{m\to\infty} u_n(q_m) = u_n(q_0)$ *for each* $n \in Z^+$.

Proof. Assumption (D) and the continuity of the map $q \to u^0(q)$ from Q into H imply $\lim_{n\to\infty} u_n^0(q_n) = u^0(q_0)$ in H. Hence, we will have verified (a) if we can show that $\lim_{n\to\infty} U_n(t,s;q_n)w_n = U_0(t,s;q_0)w_0$, $0 \leq s \leq t \leq T$, uniformly in t for $t \in [s,T]$ whenever $w_n \in H_n$ with $\lim_{n\to\infty} w_n = w_0 \in H$. We argue this using Theorem 2.2. Note that assumption (D) implies $\lim_{n\to\infty} H_n \supset H$ and assumption (D) together with the assumed continuity of the map $q \to f(t;q)$ from $Q \subset \mathcal{Q}$ into H for almost every $t \in [0,T]$ imply $\lim_{n\to\infty} f_n(t;q_n) = f(t;q_0)$ in H for almost every $t \in [0,T]$ with the $f_n(\cdot;q_n)$ dominated by a function $g \in L_1(0,T;H)$ which is independent of n. Thus, we need only to demonstrate that for some $\lambda_0 > 0$ we have

$$\lim_{n\to\infty} J(\lambda_0;A_n(q_n) + \omega I)w_n = J(\lambda_0;A_0(q_0) + \omega I)w_0 \tag{3.7}$$

in H whenever $w_n \in H_n$, $n \in Z^+$ with $\lim_{n\to\infty} w_n = w_0$.

Let $\lambda_0 > 0$ and set $v_n = J(\lambda_0;A_n(q_n) + \omega I)w_n$ and $v_0 = J(\lambda_0;A_0(q) + \omega I)w_0$. We first show that $\|v_n\|$ is uniformly bounded in n. From assumption (B) we obtain

$$\begin{aligned}
\lambda_0\alpha\|v_n\|^2 &\leq \lambda_0\omega|v_n|^2 + \lambda_0\langle A(q_n)v_n - A(q_n)\theta, v_n\rangle \\
&= \langle (I + \lambda_0(A_n(q_n) + \omega I))v_n, v_n\rangle - |v_n|^2 \\
&\quad + \lambda_0\langle A(q_0)\theta - A(q_n)\theta, v_n\rangle - \lambda_0\langle A(q_0)\theta, v_n\rangle \\
&= \langle w_n, v_n\rangle - |v_n|^2 + \lambda_0\langle A(q_0)\theta - A(q_n)\theta, v_n\rangle - \lambda_0\langle A(q_0)\theta, v_n\rangle \\
&\leq \|w_n\|_*\|v_n\| + \lambda_0\|A(q_0)\theta - A(q_n)\theta\|_*\|v_n\| + \lambda_0\|A(q_0)\theta\|_*\|v_n\|
\end{aligned}$$

where θ denotes the zero vector in V. This estimate together with assumption (C) yields

$$\|v_n\| \leq (\lambda_0\alpha)^{-1}\mu|w_n| + \alpha^{-1}\|A(q_n)\theta - A(q_0)\theta\|_* + \alpha^{-1}\beta.$$

Recalling assumption (A) and that $\lim_{n\to\infty} w_n = w_0$ in H, we find that the desired uniform bound on $\|v_n\|$ has been established.

Once again, from assumption (B), we find

$$\begin{aligned}
\lambda_0\alpha\|v_n-v_0\|^2 &\leq \lambda_0\omega|v_n-v_0|^2 + \lambda_0\langle A(q_n)v_n - A(q_n)v_0, v_n - v_0\rangle \\
&= \lambda_0\omega|v_n-v_0|^2 + \lambda_0\langle A(q_n)v_n - A(q_0)v_0, v_n - P_nv_0\rangle \\
&\quad + \lambda_0\langle A(q_n)v_n - A(q_0)v_0, P_nv_0 - v_0\rangle \\
&\quad + \lambda_0\langle A(q_0)v_0 - A(q_n)v_0, v_n - v_0\rangle \\
&= \lambda_0\omega\langle P_nv_0 - v_0, v_n - v_0\rangle + \langle(I + \lambda_0(A_n(q_n) + \omega I))v_n \\
&\quad - (I + \lambda_0(A_0(q_0) + \omega I))v_0, v_n - P_nv_0\rangle + \langle v_0 - v_n, v_n - P_nv_0\rangle \\
&\quad + \lambda_0\langle A(q_n)v_n - A(q_0)v_0, P_nv_0 - v_0\rangle \\
&\quad + \lambda_0\langle A(q_0)v_0 - A(q_n)v_0, v_n - v_0\rangle \\
&= \lambda_0\omega\langle P_nv_0 - v_0, v_n - v_0\rangle + \langle w_n - w_0, v_n - P_nv_0\rangle - |v_n - P_nv_0|^2 \\
&\quad + \lambda_0\langle A(q_n)v_n - A(q_0)v_0, P_nv_0 - v_0\rangle \\
&\quad + \lambda_0\langle A(q_0)v_0 - A(q_n)v_0, v_n - v_0\rangle \\
&\leq \lambda_0\omega\|P_nv_0-v_0\|_*\|v_n-v_0\| + \|w_n-w_0\|_*\|v_n-v_0\| \\
&\quad + \|w_n-w_0\|_*\|P_nv_0-v_0\| + \lambda_0\|A(q_n)v_n - A(q_0)v_0\|_*\|P_nv_0 - v_0\| \\
&\quad + \lambda_0\|A(q_0)v_0 - A(q_n)v_0\|_*\|v_n - v_0\|.
\end{aligned}$$

The estimate $ab \leq \frac{1}{2\eta}a^2 + \frac{\eta}{2}b^2$ for any $\eta > 0$ and assumption (C) allow us to argue

$$\frac{\lambda_0\alpha}{2}\|v_n-v_0\|^2 \leq \frac{3\omega^2\lambda_0}{2\omega}\|P_nv_0-v_0\|_*^2 + \frac{3}{2\lambda_0\alpha}\|w_n-w_0\|_*^2 + \|w_n-w_0\|_*\|P_nv_0-v_0\|$$

$$+ \lambda_0 \|A(q_n)v_n - A(q_0)v_0\|_* \|P_n v_0 - v_0\| + \frac{3\lambda_0}{2\alpha}\|A(q_0)v_0 - A(q_n)v_0\|_*^2$$

$$\leq \left\{\frac{3\omega^2\lambda_0\mu^4}{2\alpha} + \frac{(\lambda_0+1)}{2}\right\}\|P_n v_0 - v_0\|^2 + \left\{\frac{3\mu^2}{2\lambda_0\alpha} + \frac{\mu^2}{2}\right\}|w_n - w_0|^2$$

$$+ \lambda_0\{\beta(\|v_n\| + \|v_0\|) + 2\}\|P_n v_0 - v_0\| + \frac{3\lambda_0}{2\alpha}\|A(q_0)v_0 - A(q_n)v_0\|_*^2 .$$

From this, the uniform bound on $\|v_n\|$, $\lim_{n\to\infty} w_n = w_0$ in H and assumptions (A) and (D) allow us to conclude $\lim_{n\to\infty} v_n = v_0$ in V and that (3.7) holds.

An analogous, but somewhat simpler argument can be used to verify (b). We use Theorem 2.2 to show that for $n \in Z^+$ fixed, $\lim_{m\to\infty} U_n(t,s;q_m)w_m = U_n(t,s;q_0)w_0$, $0 \leq s \leq t \leq T$, uniformly in t for $t \in [s,T]$ whenever $w_m, w_0 \in H_n$ with $\lim_{m\to\infty} w_m = w_0$ in H. Clearly $\lim_{m\to\infty} f_n(\cdot;q_m) = f_n(\cdot;q_0)$ in $L_1(0,T;H_n)$ so that we need only to show that for some $\lambda_0 > 0$,

$$\lim_{m\to\infty} J(\lambda_0;A_n(q_m) + \omega I)w_m = J(\lambda_0;A_n(q_0) + \omega I)w_0$$

in H whenever $\lim_{m\to\infty} w_m = w_0$ in H. Let $v_m = J(\lambda_0;A_n(q_m) + \omega I)w_m$ and $v_0 = J(\lambda_0;A_n(q_0) + \omega I)w_0$. Then from assumption (B)

$$\lambda_0\alpha\|v_m - v_0\|^2 \leq \lambda_0\omega|v_m - v_0|^2 + \lambda_0\langle A(q_m)v_m - A(q_m)v_0, v_m - v_0\rangle$$

$$= \langle (I + \lambda_0(A_n(q_m) + \omega I))v_m - (I + \lambda_0(A_n(q_0) + \omega I))v_0, v_m - v_0\rangle$$

$$- |v_m - v_0|^2 + \lambda_0\langle A(q_0)v_0 - A(q_m)v_0, v_m - v_0\rangle$$

$$= \langle w_m - w_0, v_m - v_0\rangle - |v_m - v_0|^2 + \lambda_0\langle A(q_0)v_0 - A(q_m)v_0, v_m - v_0\rangle$$

$$\leq \|w_m - w_0\|_* \|v_m - v_0\| + \lambda_0\|A(q_0)v_0 - A(q_m)v_0\|_* \|v_m - v_0\|$$

or

$$\|v_m - v_0\| \leq \frac{\mu}{\lambda_0\alpha}|w_m - w_0| + \frac{1}{\alpha}\|A(q_0)v_0 - A(q_m)v_0\|_* .$$

Assumption (A) and $\lim_{m\to\infty} w_m = w_0$ in H yield the desired result and the theorem is proved.

Remark. In practice, the approximating identification problems (ID_n) are solved using standard iterative search techniques (for example, steepest descent, Newton's method, etc.) requiring the evaluation of $\phi_n(q)$ for $q \in Q$ at each step. This in turn requires the integration of the finite dimensional initial value problem (3.4), (3.5). Once a basis for H_n has been chosen, the solution to (3.4), (3.5) can be computed using any standard numerical integrator for ordinary differential systems. Also, the parameter space $\mathcal{Q}$ and the admissible parameter set Q are frequently functional in nature and infinite dimensional. When this is the case the set Q must also be discretized. Suppose that for each m = 1,2, ..., $I^m: Q \subset \mathcal{Q} \to \mathcal{Q}$ is a continuous map with finite dimensional range and that $\lim_{m\to\infty} I^m(q) = q$ with the convergence uniform in q for $q \in Q$. Set $Q^m = I^m(Q)$ (note that Q^m is a compact subset of $\mathcal{Q}$) and consider the identification problems (ID_n^m) defined to be the problems (ID_n) with Q replaced by Q^m. It is clear that each of these problems admit a solution $\bar{q}_n^m$ and it is not difficult to argue that there exists a subsequence $\{\bar{q}_{n_k}^{m_j}\} \subset \{\bar{q}_n^m\}$ with $\lim_{j,k\to\infty} \bar{q}_{n_k}^{m_j} = \bar{q}$, $\bar{q}$ a solution to problem (ID) (see, for example, [4]). Once bases for H_n and the range of I^m have been chosen, problem (ID_n^m) involves the minimization of a functional over a compact subset of Euclidean space subject to finite dimensional constraints.

Remark (Nonautonomous systems). Theorems 2.1 and 2.2 remain valid for certain classes of temporally inhomogeneous or time dependent operators A = A(t). To be more precise, the family of operators $A(t): X \to 2^X$ must be m-accretive on X for almost every $t \in [0,T]$ and must satisfy

(3.8) $|J(\lambda;A(t))x - J(\lambda;A(s))x|_X \leq \lambda|h(t) - h(s)|_X L(|x|_X)$

for each $x \in X$, every λ satisfying $0 < \lambda \leq \lambda_0$ for some $\lambda_0 > 0$, some $h \in L_1(0,T;X)$, some continuous, non decreasing function $L: [0,\infty) \to [0,\infty)$ and almost every $t,s \in [0,T]$ (see [8], [9]). (Note that for simplicity we have taken $\omega = 0$; however, the discussion to follow remains valid for any $\omega \in R$.) The primary motivation for developing the framework outlined above was to define readily verifiable conditions on the operators $A(q): V \to V^*$ that if satisfied would (i) also automatically be satisfied by the Galerkin approximation $A_n(q)$ and (ii) lead to the desired convergence of solutions to the approximating identification problems to a solution to problem (ID). The natural assumption to add to (A) - (C) that certainly satisfies criterion (i) and that could conceivably lead to an estimate of the form (3.8) in H is that

(3.9) $\|A(t;q)v - A(s;q)v\|_* \leq |h(t) - h(s)| \check{L}(|v|)$

for each $v \in V$, almost every $s,t \in [0,T]$ and some $h \in L_1(0,T;H)$ and some continuous nondecreasing $\check{L}: [0,\infty) \to [0,\infty)$, both of which do not depend upon $q \in Q$. Unfortunately, however, we can only show that (3.9) leads to an estimate of the form

(3.10) $|J(\lambda;A_0(t;q))u - J(\lambda;A_0(s;q))u| \leq \sqrt{\lambda}\, |h(t) - h(s)|\, L(|u|)$

for each $u \in H$. Moreover, it is not clear to us how, or if, the proof of the fundamental Theorem 2.1 given in [9] could be modified so that (3.10) would suffice. We have explored alternative approaches and developed other techniques for treating the nonautonomous case (for example, in the linear case, based upon some ideas in Tanabe [18], and in the strongly monotone case, via a variational formulation which can be found in Barbu [6]). These results will appear soon in forthcoming papers.

4. Applications and Examples

We briefly describe some classes of systems to which the general framework developed in the previous section applies. In our discussion below we consider theoretical aspects only. Implementation questions will be treated and the results of our numerical studies will be reported on elsewhere.

Example 4.1. Linear regularly dissipative operators. The approximation theory for inverse problems for systems involving linear regularly dissipative operators was treated in detail by Banks and Ito in, and is the central focus of, [2] and [3]. We show here that the linear theory is a special case of the nonlinear theory given in Section 3.

Let the spaces H, V, V^* and $\mathcal{Q}$ and the set Q be as they have been defined above. For each $q \in Q$ let $a(q)(\cdot,\cdot)$ be a sesquilinear form defined on $V \times V$ which satisfies the conditions:

(A') For each $v \in V$ the mapping $q \to a(q)(\cdot,v)$ is continuous from $Q \subset \mathcal{Q}$ into V^*. That is given $\epsilon > 0$ there exists a $\delta > 0$ such that

$$\sup_{\substack{u \in V \\ \|u\|=1}} |a(q_0)(u,v) - a(q)(u,v)| < \epsilon$$

whenever $d(q_0,q) < \delta$ where d denotes the metric on $\mathcal{Q}$.

(B') There exist an $\omega \in R$ and an $\alpha > 0$, both independent of $q \in Q$, for which $a(q)(v,v) + \omega|v|^2 \geq \alpha\|v\|^2$ for every $v \in V$.

(C') There exists a constant $\beta > 0$, independent of $q \in Q$, such that

$$|a(q)(u,v)| \leq \beta\|u\| \|v\| \quad \text{for every } u,v \in V.$$

When conditions (A') - (C') are satisfied it is not difficult to argue that for each $q \in Q$ an operator $A(q) \in \mathfrak{L}(V,V^*)$ can be defined by

$$[A(q)v](u) = \langle A(q)v,u \rangle = a(q)(u,v),$$

$u,v \in V$ and that $A(q)\colon V \to V^*$ satisfies (A) - (C). It then follows from Theorems 2.1 and 3.1 that there exists a unique nonlinear evolution system $\{U_0(t,s;q)\colon 0 \leq s \leq t \leq T\}$ on H corresponding to the initial value problem

$$\dot{u}(t) + A_0(q)u(t) = f(t;q), \quad 0 < t \leq T$$
$$u(0) = u^0(q)$$

where for each $q \in Q$, $f(\cdot;q) \in L_1(0,T;H)$, $u^0(q) \in H$ and $A_0(q)\colon \mathrm{Dom}(A_0(q)) \subset H \to H$ is the restriction of $A(q)$ to the set $\mathrm{Dom}(A_0(q)) = \{v \in V\colon A(q)v \in H\}$. The operator $-A_0(q)$ is the infinitesimal generator of an analytic semigroup $\{T_0(t;q)\colon t \geq 0\}$ on H (see [18]) and for $\phi \in H$

$$U_0(t,s;q)\phi = T_0(t-s;q)\phi + \int_s^t T_0(t-\tau;q)f(\tau;q)d\tau. \tag{4.1}$$

It can be shown that the semigroup $\{T_0(t;q)\colon t \geq 0\}$ admits an extension $\{T(t;q)\colon t \geq 0\}$ which is an analytic semigroup on V^* with generator $A(q)\colon V \subset V^* \to V^*$. Also the restriction of $\{T_0(t;q)\colon t \geq 0\}$ to V, call it $\{\tilde{T}(t;q)\colon t \geq 0\}$, is an analytic semigroup on V with generator $\tilde{A}(q)\colon \mathrm{Dom}(\tilde{A}(q)) \subset V \to V$, the restriction of $A(q)$ to the set $\mathrm{Dom}(\tilde{A}(q)) = \{v \in V\colon A(q)v \in V\}$ (see [3], [18]). Consequently, with appropriate assumptions on $f(\cdot;q)$, the evolution system $\{U_0(t,s;q)\colon 0 \leq s \leq t \leq T\}$ admits an extension $\{U(t,s;q)\colon 0 \leq s \leq t \leq T\}$ which is an evolution system on V^* and a restriction $\{\tilde{U}(t,s;q)\colon 0 \leq s \leq t \leq T\}$ which is an evolution system on V.

It is clear from (4.1) that when $A(q)$ is linear, we may take $f(\cdot;q) \equiv 0$ and consider only the approximation of the semigroup $\{T_0(t;q): t \geq 0\}$. For each $n = 1,2, \ldots$ let the finite dimensional subspaces H_n of H and the corresponding orthogonal projections P_n be as they were defined in Section 3 and assume that condition (D) is satisfied. Denote the Galerkin approximations to $A(q)$ (i.e. the restriction of $A(q)$ to an operator from H_n into $H_n^* = H_n$) by $A_n(q)$ and set $T_n(t;q) = \exp(-tA_n(q))$, $t \geq 0$. Theorem 3.2 then implies that

$$\text{(4.2)} \qquad \lim_{n\to\infty} |T_n(t;q_n)P_n u^0(q_n) - T_0(t;q_0)u^0(q_0)| = 0$$

uniformly in t, for $t \in [0,T]$ whenever $\{q_n\} \subset Q$ with $\lim_{n\to\infty} q_n = q_0 \in Q$, and the mapping $q \to u^0(q)$ is continuous from $Q \subset \mathcal{Q}$ into H. In addition, recalling that we required that $H_n \subset V$ for all $n = 1,2, \ldots$, an inspection of the proof of Theorem 3.2 reveals that in the linear case with the existence of the semigroup $\{\tilde{T}(t;q): t \geq 0\}$ on V, we may apply Theorem 2.2 with $X = V$ and conclude that

$$\text{(4.3)} \qquad \lim_{n\to\infty} \| T_n(t;q_n)P_n u^0(q_n) - \tilde{T}(t;q_0)u^0(q_0)\| = 0$$

uniformly in t for $t \in [0,T]$ whenever $\lim_{n\to\infty} q_n = q_0$, $u^0(q) \in V$ and the map $q \to u^0(q)$ is continuous from Q into V (see also [3]). Then for $\phi \in H$, setting

$$U_n(t,s;q)P_n\phi = T_n(t-s;q)P_n\phi + \int_s^t T_n(t-\tau;q)P_n f(\tau;q)d\tau$$

under appropriate assumptions on $f(\cdot;q)$, (4.2) and (4.3) continue to hold with $T_n(t;q)$, $T_0(t;q)$, and $\tilde{T}(t;q)$ replaced by $U_n(t,s;q)$, $U_0(t,s;q)$, and $\tilde{U}(t,s;q)$, respectively, with the convergence being uniform in t, for $t \in [s,T]$. Hence the linear theory and results of [3] are a special case of the nonlinear theory of Section 3.

We note that in the context of the identification problem, the fact that the stronger V-convergence given in (4.3) can be obtained is significant. Indeed, (4.3) permits the relaxation of the continuity assumption on the performance index Φ to the requirement that for each $z \in Z$, the mapping $u \to \Phi(u,z)$ be continuous from $C([0,T];V)$ into R^+. This can have the effect of significantly enlarging the class of allowable observations. For example, in the case of a one dimensional parabolic system formulated in $H = L_2$ with V in H^1, spatially discrete (i.e. pointwise, as opposed to distributed in space) measurements will suffice (see [3] and [5]).

Among the class of linear regularly dissipative operators which arise from a form satisfying (A') - (C') are the familiar elliptic partial differential operators on L_2. Briefly, let Ω be a region in R^ℓ and let $\mathcal{Q} = \overset{\ell^2+\ell+1}{\underset{n=1}{\times}} L_\infty(\Omega)$. Let Q be a compact subset of $\mathcal{Q}$ with the property that if $q = \{(a_{ij}, b_i, c)\colon i,j=1,\ldots,\ell\} \in Q$, then for some $\alpha > 0$ independent of $q \in Q$.

$$\sum_{i,j=1}^{\ell} a_{ij}(x)\zeta_i\zeta_j \geq \alpha|\zeta|^2$$

for every $x \in \Omega$, and every $\zeta \in R^\ell$. For $q \in Q$ and $u,v \in H^1(\Omega)$ set

$$a(q)(u,v) = \int_\Omega \Big\{ \sum_{i,j=1}^{\ell} a_{ij}(x) \frac{\partial u(x)}{\partial x_i} \frac{\partial v(x)}{\partial x_j} + \sum_{i=1}^{\ell} b_i(x) \frac{\partial u(x)}{\partial x_i} v(x) + c(x)u(x)v(x) \Big\} dx$$

with $H = L_2(\Omega)$ and V any closed subspace of $H^1(\Omega)$ containing $H_0^1(\Omega)$, it can be shown (see [18]) that $a(q)(\cdot,\cdot)$ satisfies (A') - (C'). The operator A(q) is given formally by

$$(4.4) \qquad A(q) = -\sum_{i,j=1}^{\ell} \frac{\partial}{\partial x_j} a_{ij}(x) \frac{\partial}{\partial x_i} + \sum_{i=1}^{\ell} b_i(x) \frac{\partial}{\partial x_i} + c(x).$$

When $\partial\Omega$ is sufficiently smooth, $A(q)$ is the elliptic operator given by (4.4), and V is chosen to be either $H_0^1(\Omega)$ or $H^1(\Omega)$, the equation (3.1) becomes a parabolic partial differential equation with either Dirichlet or Neumann boundary conditions.

For $H = L_2(\Omega)$ and V a subspace of $H^1(\Omega)$, choosing the approximating subspaces to be the span of an appropriate collection of first order spline functions will typically satisfy assumption (D) (see [15] and Example 4.2 below).

Example 4.2. Nonlinear Elliptic Operators. Let Ω be a bounded region in R^{ℓ} with smooth boundary $\Gamma = \partial\Omega$. For $\alpha = (\alpha_1, ..., \alpha_\ell)$ a multi-index, let $|\alpha| = \alpha_1 + \alpha_2 + \cdots + \alpha_\ell$ and denote the αth order generalized, or distributional derivative of a function u by $D^{\alpha}u$; that is,

$$D^{\alpha}u(x) = \frac{\partial^{\alpha_1}}{\partial x_1^{\alpha_1}} \cdots \frac{\partial^{\alpha_\ell}}{\partial x_\ell^{\alpha_\ell}} u(x), \quad x \in \Omega .$$

Let m be a nonnegative integer and let δu denote the vector valued function of length $N = \binom{\ell+m}{\ell}$ whose components are all of the partial derivatives of u of order greater than or equal to zero and less than or equal to m.

For each multi-index α with $|\alpha| \leq m$, let $(x,\zeta) \to a_{\alpha}(x,\zeta)$ be a real valued function defined on $\Omega \times R^N$ which is measurable in x and continuous in ζ. We assume that

(1) there exist a $g \in L_2(\Omega)$ and a positive constant γ such that

$$(4.5) \qquad |a_\alpha(x,\zeta)| \leq \gamma(|\zeta| + g(x))$$

for almost every $x \in \Omega$, each $\zeta \in R^N$ and all α with $|\alpha| \leq m$, and

(2) there exists a positive constant λ such that

$$(4.6) \qquad \sum_{|\alpha|\leq m} (a_\alpha(x,\zeta) - a_\alpha(x,\eta))(\zeta_\alpha - \eta_\alpha) \geq \lambda \sum_{|\alpha|\leq m} |\zeta_\alpha - \eta_\alpha|^2$$

for almost every $x \in \Omega$ and all $\zeta,\eta \in R^N$.

Let $H = L_2(\Omega)$ and let V be any closed subspace of $H^m(\Omega)$ which contains $H_0^m(\Omega)$. Define the operator $A: V \to V^*$ by

$$(4.7) \qquad (Au)(v) = \sum_{|\alpha|\leq m} \int_\Omega a_\alpha(x,\delta u(x))D^\alpha v(x)dx,$$

for $u,v \in V$. The operator A given by (4.7) is the distributional form of the formal differential operator

$$(4.8) \qquad (Au)(x) = \sum_{|\alpha|\leq m} (-1)^\alpha D^\alpha a_\alpha(x,\delta u(x)).$$

A differential operator of the form (4.8) is referred to as a nonlinear elliptic operator and the partial differential equation

$$(4.9) \qquad \frac{\partial u}{\partial t}(t,x) + \sum_{|\alpha|\leq m} (-1)^\alpha D^\alpha a_\alpha(x,\delta u(t,x)) = f(t,x)$$

is said to be of nonlinear parabolic type. When $V = H_0^m(\Omega)$, a solution in V^* to the abstract equation

$$\dot{u}(t) + Au(t) = f(t)$$

with A given by (4.7) corresponds to a variational solution to (4.9) which satisfies Dirichlet boundary conditions. When $V = H^m(\Omega)$, a variational solution to the Neumann problem is obtained. Note that in the linear case we have

$$a_\alpha(x,\delta u(x)) = \sum_{|\beta|\leqslant m} a^{\alpha,\beta}(x)D^\beta u(x).$$

Under the assumptions above, it is not difficult to show that A given by (4.7) is hemicontinuous and satisfies conditions (B) and (C) given in Section 3. With an appropriate choice of the space $\mathcal{Q}$ and the set Q, condition (A) can be satisfied as well.

A quasilinear model for heat conduction or mass transfer in which the heat or mass flux is a function of the temperature or mass fraction gradient discussed in [16] and [17] leads to a nonlinear elliptic operator and a nonlinear parabolic partial differential equation of the forms (4.8) and (4.9), respectively with m = 1. Let Ω be a bounded region in R^ℓ with smooth boundary and let $\mathcal{Q} = L_\infty(\Omega \times R^\ell)$. Let Q be a compact subset of $\mathcal{Q}$ with the property that $q \in Q$ if and only if the mapping $\zeta \to q(x,\zeta)$ is C^1 on R^ℓ for almost every $x \in \Omega$ and there exists a $\lambda > 0$ (which does not depend on q) such that

$$(4.10) \qquad \theta_i\nabla_\xi q(x,\xi)\big|_{\xi=\theta}\cdot(\zeta - \eta) + q(x,\theta)(\zeta_i - \eta_i) \geqslant \lambda(\zeta_i - \eta_i),$$

for $i = 1,2, \ldots, \ell$, almost every $x \in \Omega$ and all $\theta,\zeta,\eta \in R^\ell$. (When $\ell = 1$, the function $q(x,\xi) = q(\xi) = (1 - .5e^{-\xi^2})$ satisfies (4.10).)

Let $H = L_2(\Omega)$ and let V be any closed subspace of $H^1(\Omega)$ which contains $H_0^1(\Omega)$. Then $V \subset H \subset V^*$ and for each $q \in Q$ define $A(q): V \to V^*$ by

$$(4.11) \qquad (A(q)u)(v) = \int_\Omega q(x,\nabla u(x))\nabla u(x)\cdot\nabla v(x)dx$$

for $u,v \in V$. Note that for each $q \in Q$ the operator given by (4.11) is of the form (4.7) with

$$a_\alpha(x,\delta u(x)) = q(x,\nabla u(x))D^\alpha u(x) \tag{4.12}$$

for $x \in \Omega$ and all α with $|\alpha| = 1$ and $a_\alpha = 0$ for $|\alpha| = 0$. The nonlinear parabolic partial differential equation (4.9) takes the form

$$\frac{\partial u}{\partial t}(t,x) - \nabla \cdot q(x,\nabla u(t,x))\nabla u(t,x) = f(t,x), \quad t > 0, \quad x \in \Omega .$$

Taking $\|\cdot\|$ to be the usual norm on $H^1(\Omega)$, it follows that

$$\|A(q_0)u - A(q_1)u\|_* \leq |q_0 - q_1|_{L_\infty}\|u\|$$

for each $u \in V$ and $q_0,q_1 \in Q$. Since Q is a compact subset of $L_\infty(\Omega \times R^\ell)$, it is easily verified that a_α given by (4.12) satisfies a growth condition of the form (4.5) with γ and g independent of $q \in Q$. An application of the mean value theorem together with assumption (4.10) imply the existence of a $\lambda > 0$, independent of $q \in Q$, for which (4.6) holds. Consequently the conditions (A), (B), and (C) given in Section 3 are satisfied, and our general theory can be applied.

With regard to approximation, polynomial spline function based Galerkin subspaces can often be shown to satisfy condition (D). For example, when $\ell = 1$ and $\Omega = (0,1)$ in the nonlinear heat conduction/mass transfer example discussed above, the subspaces H_n can be chosen as the span of the linear B-spline ("hat") functions with respect to the uniform mesh $\{0, 1/n, 2/n, ..., 1\}$ appropriately modified to satisfy stable, or geometric, boundary conditions. Familiar error estimates for interpolation and the Schmidt inequality can then be used to verify that condition (D) is satisfied (see [5]). Generalization to higher dimensions is possible, and can often be achieved via tensor products of one dimensional elements (see [15]).

5. Concluding Remarks

We have developed a general abstract approximation framework for the identification of nonlinear distributed parameter evolution systems. The class of systems to which our theory applies are those whose dynamics can be described by a nonlinear operator which satisfies conditions that are the natural nonlinear extensions, or analogs, of the properties of regularly dissipative, or abstract parabolic, linear operators. The approach we have taken is based upon the defining of a sequence of approximating finite dimensional identification problems in which the systems to be identified are Galerkin approximations to the original, underlying, infinite dimensional nonlinear dynamics. Under a weak continuity assumption with respect to the unknown parameters to be identified, equi-boundedness and equi-monotonicity conditions, and an approximation assumption on the Galerkin subspaces (all of which are readily verified for wide classes of nonlinear distributed systems and finite element subspaces), we are able to demonstrate that solutions to the approximating problems exist, and, in some sense, approximate (i.e. subsequential convergence) solutions to the original infinite dimensional identification problem. We have shown that the linear theory presented in [2] and [3] is a special case of our nonlinear framework and that our results are applicable to a reasonably wide class of nonlinear elliptic operators and corresponding nonlinear parabolic partial differential equations. In particular, we have considered application of our theoretical framework to a quasi-linear model for heat conduction or mass transport.

The general approximation result for nonlinear evolution systems discussed in Section 2 is applicable to a much broader class of nonlinear dynamical systems than we subsequently treated in Section 3. For example, this class of systems would include those with dynamics described by set valued maps or multifunctions, and (after minor modification to the general theory) time dependent or nonautonomous operators. We are currently investigating these features of the general approximation theory in the context of parameter estimation problems. Also, we would like to be able to weaken the somewhat restrictive strong monotonicity condition. Any progress that we might make in these efforts would have the potential to significantly enlarge the class of nonlinear systems to which our theory and framework would apply. Finally, extensive numerical or computational studies designed to demonstrate the feasibility and point out the limitations of our schemes and general approach are currently underway and will be reported on in a forthcoming paper.

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16. Abstract

An abstract approximation framework for the identification of nonlinear distributed parameter systems is developed. Inverse problems for nonlinear systems governed by strongly maximal monotone operators (satisfying a mild continuous dependence condition with respect to the unknown parameters to be identified) are treated. Convergence of Galerkin approximations and the corresponding solutions of finite dimensional approximating identification problems to a solution of the original infinite dimensional identification problem is demonstrated using the theory of nonlienar evolution systems and a nonlinear analog of the Trotter-Kato approximation result for semigroups of bounded linear operators. The nonlinear theory developed here is shown to subsume an existing linear theory as a special case. It is also shown to be applicable to a broad class of nonlinear elliptic operators and the corresponding nonlinear parabolic partial differential equations to which they lead. An application of the theory to a quasilinear model for heat conduction or mass transfer is discussed.

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