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LOCAL TIME DISPLACEMENT AS A SYMMETRY OF NATURE IN FLAT SPACE-TIME

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Abstract

Local time displacement is shown to be a true symmetry of Minkowskian physics, thereby demonstrating the empirical equivalence of different choices of the clock synchronization parameter in generalized Lorentz transformations.

Introduction

As summarized by Spavieri (ref. 1), a question of current interest is whether one-way velocity of light is a physically meaningful concept beyond the nonrelativistic limit. The problem arises because of the failure of the Michelson-Morley and Kennedy-Thorndike interference experiments to uniquely define the coordinate transformation between an inertial laboratory and the isotropic universe relative to which the laboratory moves with velocity $\vec{v}$. Motivated by this question, we will show the more general property that local time displacement is a true symmetry of nature for inertial systems in flat space-time, and thereby demonstrate the impossibility of uniquely determining one-way light velocities from special relativity (SR) considerations.

Discussion

Consider an inertial laboratory moving with speed $\vec{v}$ relative to the isotropic universe (denoted by zero subscripts). One obtains a generalized Lorentz transformation

$$d\vec{r} = d\vec{r}_0 + (\gamma - 1)(\vec{v}\cdot d\vec{r}_0)\,\frac{\vec{v}}{v^2} - \gamma\vec{v}dt_0,$$

$$(1)$$

$$dt = \gamma\left[\left(1 - \frac{\xi v^2}{c^2}\right) dt_0 - \frac{1 - \xi}{c^2}\,\vec{v}\cdot d\vec{r}_0\right],$$

from the invariant interval by letting

$$t = t_0 + \frac{\xi \vec{r} \cdot \vec{v}}{c^2} \tag{2}$$

In terms of the undetermined parameter ξ , one finds the following relation between the one-way particle velocity $\vec{w}(\xi)$ and its Einstein counterpart $\vec{w}(0)$, both referred to the laboratory system:

$$\vec{w}(\xi) = d\vec{r}/dt = \vec{w}(0)\{1 + [\xi\vec{v}\cdot\vec{w}(0)/c^2]\}^{-1}. \tag{3}$$

Within the context of SR, does nature ultimately select ξ and establish a real $\vec{w}(\xi)$, or is the choice entirely arbitrary with no meaningful effect?

Implicit in Eq. (1) is the essential demand that laboratory clocks be so synchronized as to guarantee Eq. (3) whenever one-way velocities are measured using these same clocks. This arrangement is always possible for any ξ merely by using the one-way light speed $u(\theta) = c[1 + (\xi\vec{v}\cdot\hat{e}/c)]^{-1}$ in Einstein's method of synchronizing clocks with light signals. Therefore, at least within the scope of kinematics, $\vec{w}(\xi)$ is a meaningless concept because we can make the relativistic contribution (finite c) anything we want by arbitrarily changing ξ in the equation for $u(\theta)$. The argument that clocks can be synchronized without light signals is irrelevant. Such methods are nothing more than alternative ways of implementing a particular choice of ξ; they add no physics to restrict what we could have chosen. Examples include the implementation of Einstein's convention ($\xi = 0$) by the slow transport of a master clock (ref. 2) and by Spavieri's procedure (ref. 1) based on moving rods kept in contact with a rotating disk. Contrary to his interpretation, Spavieri's procedure forces $\xi = 0$ because of the symmetry introduced by the purely operational requirement that the rod velocities (disk radius times the angular velocity deduced from the ticks of a single clock) be independent of their orientations relative to $\vec{v}$. In short, kinematics puts no limit on our control of the clock synchronization parameter ξ.

But physics is more than kinematics, and no one has ruled out nature's selection of ξ by other means. That is the issue addressed in this paper. We begin by recognizing that all of physics is contained in the complete set of action integrals

$$S = \int L(A^\mu, \partial_\nu A^\mu, x^\mu) d^4x, \tag{4}$$

where L is the Lagrangian spatial field density, A^μ is some space-time field or fields pertinent to the system, and $\partial_\nu A^\mu \equiv \partial A^\mu/\partial x^\nu$. If a given infinitesimal transformation on x^μ and A^μ leaves S unchanged for all possible L, then physics is invariant to that transformation. Such a transformation would identify a true symmetry of nature, and all choices of the symmetry parameter would be empirically equivalent. Any particular choice is therefore strictly conventional. Our question is whether ξ is such a parameter. More generally, since Eq. (2) is of the form

$$t' = t + f(\vec{r}) \ , \tag{5}$$

we ask the more general questions as to whether physics is invariant to local as well as global time displacements.

The most general expression for the change in S is

$$\Delta S = \int L'(A'^\mu, \partial_\nu A'^\mu, x'^\mu) \, d^4x' - \int L(A^\mu, \partial_\nu A^\mu, x^\mu) \, d^4x \ . \tag{6}$$

Notationally, we denote the functional variation at the same argument values θ' as

$$\delta L = L'(\theta') - L(\theta'). \tag{7}$$

Using Eq. (7) and the property (ref. 3) that

$$d^4x' = [1 + \partial_\mu (\delta x^\mu)]\, d^4x \tag{8}$$

in (6) yields

$$\Delta S = \int [\delta L + L\partial_\mu (\delta x^\mu) + L(A'^\mu, \partial_\nu A'^\mu, x'^\mu) - L(A^\mu, \partial_\nu A^\mu, x^\mu)]\, d^4x \tag{9}$$

which can be expanded as

$$\Delta S = \int [\delta L + L\partial_\mu(\delta x^\mu) + \frac{\partial L}{\partial A^\mu}\delta A^\mu + \frac{\partial L}{\partial(\partial_\nu A^\mu)}\partial_\nu(\delta A^\mu) + (\partial_\mu L)\,\delta x^\mu]\, d^4x. \tag{10}$$

If L is the Lagrangian density for a <u>total</u> physical system, the Euler-Lagrange equations

$$\frac{\partial L}{\partial A^\mu} = \partial_\nu \left[\frac{\partial L}{\partial(\partial_\nu A^\mu)} \right] \tag{11}$$

and the energy-momentum conservation laws (obtained from Noether's theorem (ref. 3))

$$\partial_\nu \left[\frac{\partial L}{\partial(\partial_\nu A^\mu)} \partial_\eta A^\mu - L\delta_\eta{}^\nu \right] = 0 \tag{12}$$

must be satisfied. These conditions are valid constraints on the total system, regardless of the particular variation used to derive them and independent of any subsequent variation to be applied. Incorporating Eqs. (11) and (12) into (10), and using the total variation

$$\Delta A^\mu = A'^\mu(x') - A^\mu(x) = \delta A^\mu + (\partial_\nu A^\mu)\, \delta x^\nu \tag{13}$$

yields

$$\Delta S = \int \{\delta L + L\partial_\mu(\delta x^\mu) + \partial_\nu[\frac{\partial L}{\partial(\partial_\nu A^\mu)} \Delta A^\mu] - \frac{\partial L}{\partial(\partial_\nu A^\mu)} (\partial_\eta A^\mu)\partial_\nu(\delta x^\eta)\} d^4x \tag{14}$$

We next consider the infinitesimal transformation (local time displacement)

$$\delta x^{1,2,3} = 0;\ \delta x^0 = f(\vec{r});\ A'^\mu(x') \equiv A^\mu[x(x')] = A^\mu(x), \tag{15}$$

which matches Eq. (5) for $f(\vec{r}) = \xi \vec{v}\cdot\vec{r}/c^2$, makes ΔA^μ vanish, and reduces Eq. (14) to

$$\Delta S = \int [\delta L - \frac{\partial L}{\partial(\partial_j A^\mu)} \frac{\partial A^\mu}{\partial t} \partial_j f] d^4x. \tag{16}$$

Note that Eq. (15) also stipulates that the total variation of any field vanishes identically. This restriction applies because we are interested only in infinitesimal time transformations. Since it is certainly not obvious that ΔS should vanish for all possible L, we look at a few specific Lagrangian densities to see what happens and then generalize from there.

A good first example is an electromagnetic field residing in a source-free region so that

$$L = \frac{1}{2}\left(\varepsilon E^2 - \mu H^2\right) \tag{17}$$

is the total system Lagrangian. Equations (11) and (12) are then respectively Maxwell's source equations and Poynting's theorem. From the field equations

$$\begin{aligned} \vec{B}(x) &= \nabla \times \vec{a}(x) \\ \vec{E}(x) &= -\nabla\phi(x) - \partial\vec{a}(x)/\partial t, \end{aligned} \tag{18}$$

and the differential operator relations

$$\begin{aligned} \frac{\partial}{\partial t} &= \frac{\partial}{\partial t_\tau} \\ \nabla &= \nabla' + \nabla f \frac{\partial}{\partial t_\tau} \end{aligned} \tag{19}$$

we can use Eq. (15) to write

$$\vec{B}(\Theta) = \nabla x \vec{a}(x) = \nabla x \vec{a}'(x')$$

$$= \nabla' x \vec{a}'(x') + \nabla f \; x \frac{\partial \vec{a}'}{\partial t}$$

$$= \vec{B}(\Theta') + \nabla f \; x \frac{\partial \vec{a}}{\partial t} = \vec{B}'(\Theta') \tag{20}$$

which yields the functional variations

$$\delta \vec{B} = \nabla f \; x \; \partial \vec{a}/\partial t \tag{21}$$

In a similar fashion one obtains

$$\delta \vec{E} = -\nabla f \partial \phi / \partial t \tag{22}$$

We now insert these variations into the functional variation of Eq. (17) to write

$$\delta L = -\varepsilon \vec{E} \cdot \delta \vec{E} - \mu \vec{H} \cdot \delta \vec{H}$$

$$= \varepsilon \vec{E} \cdot \nabla f \partial \phi / \partial t - \vec{H} \cdot (\nabla f \; x \; \partial \vec{a}/\partial t)$$

$$= -\nabla f \cdot \left(\frac{\partial \vec{a}}{\partial t} x \vec{H} + \varepsilon \vec{E} \frac{\partial \phi}{\partial t}\right) = \frac{\partial L}{\partial (\partial_j A^\mu)} \frac{\partial A^\mu}{\partial t} \partial_j f. \tag{23}$$

Equation (16) then becomes

$$\Delta S(\text{pure field}) = 0 \tag{24}$$

In a similar manner, the Lagrangian density

$$L = \frac{1}{2}\left[(\nabla\Psi)^2 - \frac{1}{c^2}\left(\frac{\partial\Psi}{\partial t}\right)^2 + \left(\frac{m_o c}{\hbar}\right)^2 \Psi^2\right] \tag{25}$$

gives the Klein-Gordon equation for a free particle through Eq. (11) and has the functional variation

$$\delta L = \frac{1}{2}\left[(\nabla\Psi + \nabla f \partial\Psi/\partial t)^2 - \frac{1}{c^2}\left(\frac{\partial\Psi}{\partial t}\right)^2 + \left(\frac{m_o c}{\hbar}\right)^2 \Psi^2\right] - L$$

$$= (\nabla f)\cdot(\nabla\Psi)\frac{\partial\Psi}{\partial t} = \frac{\partial L}{\partial(\partial_j\Psi)}\frac{\partial\Psi}{\partial t}\partial_j f \tag{26}$$

for the infinitesimal transformation in Eq. (15). Substituting this result into Eq. (16) yields

$$\Delta S(\text{free particle}) = 0. \tag{27}$$

As a final example, we consider the charge (q) - electromagnetic field interaction term $q\Psi^{*}\gamma^{\mu}A_{\mu}\Psi$ in the Lagrangian density of quantum electrodynamics, (ref. 4) where γ^{μ} stands for the four Dirac γ-matrices. Since Ψ, Ψ^{*}, and A_{μ} are regarded as independent variables in the Lagrangian formulation, the interaction term is exactly the same in $L(x)$ and $L'(x)$ and so does not contribute to the functional variation δL. The argument is similar to that used in the derivation of $\delta\vec{B}$ and $\delta\vec{E}$ above, as discussed immediately prior to Eq. (23). Also, since the interaction term contains no derivatives of Ψ, Ψ^*, or A_μ, it does not contribute to the second term in the integrand of Eq. (16). Equations (24) and (27) can thus be combined and generalized to read

$$\Delta S(\text{charged particle + field + interaction}) = 0. \qquad (28)$$

Conclusion

Equation (28) clearly asserts that physics is insensitive to the local time displacement $t \rightarrow t' = t + f(\vec{r})$, at least in inertial laboratories. Clock synchronization with $f(\vec{r}) = \xi\vec{v}\cdot\vec{r}/c^2$ is an example; both it and one-way velocities beyond the nonrelativistic limit are unavoidably conventional and physically meaningless. The velocity $\vec{v}$ of an inertial laboratory relative to the isotropic universe cannot be measured from within, except as the data specifically refer to identifiable signposts in the universe (e.g., the 2.7K cosmic background radiation (ref. 5)). Gravity, of course, may be a different story. The rotating Earth is not an inertial frame, and $\xi\vec{v}\cdot\vec{r}/c^2$ would become a function of time as well as space. Experiments in such laboratories might well select ξ and determine $\vec{v}$ (and they might even confirm or deny Mach's principle), but they would not be interpretable without explicit consideration of the effects in accelerated systems.

We have shown that local time displacement is a symmetry of nature in inertial systems. The question as to whether or not anisotropies in the speed of light actually exist cannot be answered for inertial reference frames. Instead, non-inertial frames must be used.

SYMBOLS

$\vec{a}(x)$	electromagnetic vector potential
A^μ	arbitrary field of space-time system
$\vec{B}(x)$	magnetic field flux density (tesla)
c	round trip velocity of light (2.998×10^8 meters/second)
$\vec{E}(x)$	electric field (volts/meter)
$f(\vec{r})$	arbitrary function of position
$\vec{H}(x)$	magnetic field (amperes/meter)
h	Planck's constant (6.626×10^{-34} Joule-second)
L	Lagrangian density
m_o	particle rest mass (kilogram)
q	electric charge (Coulombs)
$\vec{r}$	position vector (meters)
S	action function
t	time (seconds)
$u(\Theta)$	one-way light speed (meters/second)
$\vec{v}$	velocity of inertial laboratory with respect to isotropic universe (meters/second)
$\vec{w}\ (\xi)$	one-way particle velocity with respect to laboratory reference system (meters/second)
x^μ	μ component of space-time 4-vector
γ	usual relativistic factor $\gamma = (1 - \frac{v^2}{c^2})^{-1/2}$
γ^μ	Dirac γ matrix
∇	gradient operator (meters^{-1})

δ	symbol for functional variation
ε	electrical permittivity (farads/meter)
μ	magnetic permeability (henrys/meter)
ϕ	electrostatic potential (volts)
Ψ	wavefunction (meters$^{-3/2}$)
ξ	clock synchronization parameter

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