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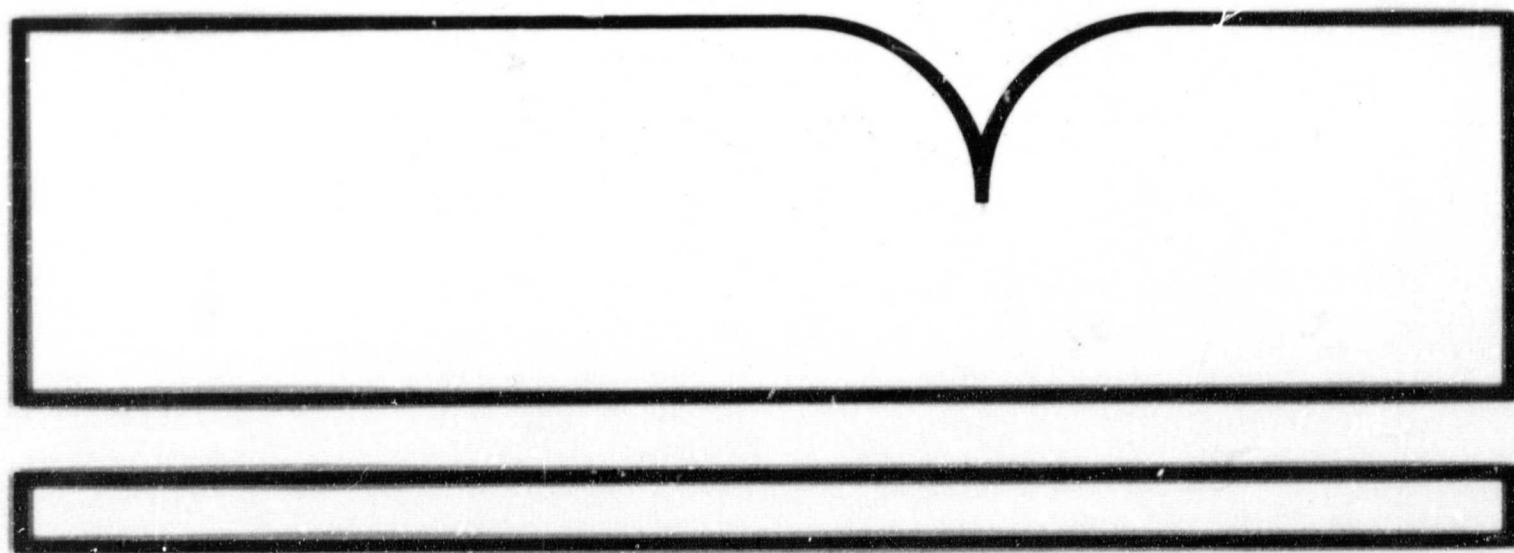
Complex Space Monofilar Approximation of
Diffraction Currents on a
Conducting Half Plane

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ABSTRACT
Simple approximation of diffraction surface currents on a conducting half plane, due to an incoming plane wave, is obtained with a line current (monofile) in complex space. When compared to an approximating current at the edge, the diffraction pattern is seen to improve by an order of magnitude for a minimal increase of computation effort. Thus, the inconvenient Fresnel integral functions can be avoided for quick calculations of diffracted fields and the accuracy is good in other directions than along the half plane. The method can be applied to general problems involving planar metal edges.

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INTRODUCTION

The problem of wave diffraction by a conducting half plane is a classical one, and its first non-asymptotic solution was given by Sommerfeld in 1896 [1]. The exact result for the surface current induced on the half plane involvs Fresnel integral functions, which although straightforward to compute, are inconvenient for quick calculations. Hence, there have been attempts to use approximate expressions for the surface currents. Since the surface current has a singularity at the edge, it is most natural to try to use a line current at the edge as an approximation for the diffraction current function (true current minus the geometrical optics current). The accuracy is, however, not too good. In fact, since a line current radiates omnidirectionally in the plane transverse to the line, the radiation pattern of the diffraction current is only poorly approximated. The common technique in improving the accuracy of the GTD is to use nonphysical, so-called equivalent edge currents, which depend on the point where the field is to be calculated. These techniques are compared in the reference [2] but it is out of scope to delve into any comparisons in this short study. However, the equivalent currents can obviously be expressed in terms of a multifilar series expression (soon to be defined), the first term of which is the monofile, or line c rent. The problem studied here is to find an optimal location for the monofile such that the next term has the least effect, to make the approximation through the monofile most efficient. The idea has its origin in the corresponding three-dimensional approximation of scatterers by multipole series, [3], [4].

MULTIFILAR EXPANSION OF SOURCES

In references [3], [4] the multipole expansion in complex space has been applied to source approximation in three dimensions. The theory can be modified to two dimensions so that instead of pointlike multipoles, threadlike *multifilar* elements are used. The problem need not be truly two dimensional, since the multifilar currents need not be constant.

Let us consider a current source $\bar{J}(\bar{r})$ and its vector potential $\bar{A}(\bar{r})$ function

$$\bar{A}(\bar{r}) = \mu \int_V G(D)\bar{J}(\bar{r}')dV'. \tag{1}$$

Here, D is the distance function

$$D = \sqrt{(\bar{r} - \bar{r}') \cdot (\bar{r} - \bar{r}')} \tag{2}$$

and it can be complex if $\bar{r}'$ is given complex values. $G(D)$ is the Green function

$$G(D) = \frac{e^{-jkD}}{4\pi D}. \tag{3}$$

If the source is almost axial, i.e., of small extension in th direction transverse to the axis, and we are interested in approximations giving good accuracy for the far field calculation, the Green function can be approximated in two dimensions (x, y) through the Taylor expansion:

$$G(D) = G(\rho) - \bar{\rho}' \cdot \nabla G(\rho) + \frac{1}{2}\bar{\rho}'\bar{\rho}' : \nabla\nabla G(\rho) - \dots . \tag{4}$$

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Here, $G(\rho)$ is also a function of $z - z'$, although not explicitly shown. When substituted in (1), after partial integration, we have the expansion

$$A(\bar{r}) = \mu \int_{-\infty}^{\infty} \left(\bar{I}_0 G(\rho) + \bar{\bar{I}}_1 \cdot \nabla G(\rho) + \frac{1}{2}\bar{\bar{\bar{I}}}_2 : \nabla\nabla G(\rho) + ... \right) dz'. \tag{5}$$

This is equivalent with the following two-dimensional multipole or *multifile* expansion for the current function:

$$\bar{J}(\bar{r}) = \bar{I}_0(z)\delta(\bar{\rho}) + \bar{\bar{I}}_1(z) \cdot \nabla\delta(\bar{\rho}) + \frac{1}{2}\bar{\bar{\bar{I}}}_2(z) : \nabla\nabla\delta(\bar{\rho}) + ... \tag{6}$$

The first (zeroth order) term corresponds to a *monofilar* current component, the next one (first order term) a *bifilar*, the following one a *quadrifilar* component and so on. The expansion (6) follows a similar pattern as given in reference [5] for three-dimensional sources and it can be formally obtained directly by expanding the function $\delta(\bar{\rho} - \bar{\rho}')$ within the integral $\bar{J}(\bar{r}) = \bar{J}(\bar{\rho}, z) = \int_S \delta(\bar{\rho} - \bar{\rho}')\bar{J}(\bar{\rho}', z)dS'$. In (5) and (6), the I_i's are polyadic z-dependent current moments defined by

$$\bar{I}_0 = \int_S \bar{J}(\bar{r})dS, \tag{7}$$

$$\bar{\bar{I}}_1 = \int_S \bar{\rho}\bar{J}(\bar{r})dS, \tag{8}$$

$$\bar{\bar{\bar{I}}}_2 = \int_S \bar{\rho}\bar{\rho}\bar{J}(\bar{r})dS, \tag{9}$$

and the series (6) could be further extended [3], [4]. If the axis of the multifile is moved from the z axis to an axis going through a point $\bar{\rho} = \bar{a}$, instead of (6), we can write, suppressing the z dependence,

$$\bar{J}(\bar{\rho}) = \bar{P}_0(\bar{a})\delta(\bar{\rho} - \bar{a}) + \bar{\bar{I}}_1(\bar{a}) \cdot \nabla\delta(\bar{\rho} - \bar{a}) + \frac{1}{2}\bar{\bar{\bar{I}}}(\bar{a}) : \nabla\nabla\delta(\bar{\rho} - \bar{a}) + ..., \tag{10}$$

with the new current moments

$$\bar{I}_0(\bar{a}) = \int_S \bar{J}(\bar{r})dS = \bar{I}_0, \tag{11}$$

$$\bar{\bar{I}}_1(\bar{a}) = \int_S (\bar{\rho} - \bar{a})\bar{J}(\bar{r})dS = \bar{\bar{I}}_1 - \bar{a}\bar{I}_0, \tag{12}$$

$$\bar{\bar{\bar{I}}}_2(\bar{a}) = \int_S (\bar{\rho} - \bar{a})(\bar{\rho} - \bar{a})\bar{J}(\bar{r})dS = \bar{\bar{\bar{I}}}_2 - \bar{a}\bar{\bar{I}}_1 - \sum \bar{b}_i\bar{a}\bar{c}_i + \bar{a}\bar{a}\bar{I}_0. \tag{13}$$

In (13), we have applied the notation $\bar{\bar{I}}_1 = \sum \bar{b}_i\bar{c}_i$. For sources sufficiently small in the transverse plane and of sufficiently slow variation, the series (5) for the radiation field converges quickly and the first few terms will give a good approximation. To find a good first-term approximation, we can try to move our multifilar current to a point $\bar{a}$ which makes the first term most dominating. This amounts to finding a point $\bar{a}$ such that the first moment in (12) is minimized. Following the method given in [3], [4], assuming $P_0 \neq 0$, we find

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$$\bar{a} = \frac{\bar{\bar{I}}_1 \cdot \bar{I}_0^*}{\bar{I}_0 \cdot \bar{I}_0^*}. \tag{14}$$

In this paper, we study currents that have constant polarization. Thus, the vector potential will also be of the same constant polarization. Denoting this polarization by a unit vector w, we can write for $\bar{I}_0 = I_0 w, \bar{I}_1 = \bar{I}_1 w$, etc. Thus, the rank of the current moment polyadics can be lowered by one, which makes the formulas appear simpler. For, example, (14) can be written as

$$\bar{a} = \frac{\bar{I}_1}{I_0}. \tag{14a}$$

Also, it is easy to see, that the choice (14a) not only minimizes the norm of (12), but actually makes $\bar{I}_1(\bar{a}) = 0$. Further, if the zeroth moment is zero, the following value for $\bar{a}$ will render the first moment the most dominant one by making the second moment (13) vanish:

$$\bar{a} = \frac{\bar{\bar{I}}_2 \cdot \bar{I}_1^*}{|\bar{I}_1|^2} - \frac{\bar{I}_1}{2|\bar{I}_1|^4} \bar{\bar{I}}_2 : \bar{I}_1^* \bar{I}_1^*. \tag{15}$$

For a current depending on a single coordinate x_i, (15) can be reduced to the simple formula

$$\bar{a} = \bar{u}_i \frac{I_2}{2I_1}. \tag{15a}$$

If a planar surface current is replaced by the multifilar expansion, it can be easily seen that the monofilar term gives the far field radiation correctly in the direction perpendicular to the plane. By choosing the location of the monofile as given in (14a), it can further be shown, that also the derivative of the radiation pattern becomes correct in the same direction. To see this, we consider the far field in the direction of the unit vector $\bar{u}_r$, as $A(\bar{r}) = \mu G(r) f(\bar{u}_r)$, with the radiation function

$$f(\bar{u}_r) = \int_V e^{jk\bar{u}_r \cdot \bar{r}'} J(\bar{r}') dV'. \tag{16}$$

If $\bar{n}$ is the unit vector normal to the plane, obviously $\bar{n} \cdot \bar{r} = 0$ when $\bar{r}$ is on the plane, and the origin lies on the plane. For a direction $\bar{u}_r$ differing from $\bar{n}$ only a little, the term in the exponent in (16) is small and we may write

$$f(\bar{u}_r) \approx \int_S (1 + jk\bar{u}_r \cdot \bar{r}') J(\bar{r}') dS'$$

$$= I_0 + jk\bar{u}_r \cdot \bar{I}_1 = (1 + jk\bar{u}_r \cdot \bar{a}) I_0 \approx e^{jk\bar{u}_r \cdot \bar{a}} I_0. \tag{17}$$

The last expression is equal to the radiation function due to a monofilar current at the location $\bar{p} = \bar{a}$. Thus, it gives a two-term approximation for the radiation pattern at the direction $\bar{u}_r = \bar{n}$, or both the radiation and its angular derivative correctly. The monofile at $\bar{p} = 0$ gives only the first term correctly.

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THE HALF-PLANE CURRENT PROBLEM

Let us consider first the simple case of plane wave coming perpendicularly ($\theta_0 = 0$ and $\phi_0 = 0...180^\circ$, Fig.1) to the edge with the field polarized so that either the electric or the magnetic field is in the direction of the edge (z direction). These two cases are sometimes called , respectively, the E wave and the H wave. We prefer calling them the TM wave and the TE wave, where "T" denotes transverse to the direction of the edge (z coordinate). For the induced surface current we can write

$$J_s(x) = \bar{J}_{go}(x) + \bar{J}_d(x), \tag{18}$$

where the geometrical optics surface current is ($\bar{n} = \bar{u}_y$)

$$\bar{J}_{go}(x) = 2\bar{u}_y \times \bar{H}_i e^{jkx\cos\phi_o}. \tag{19}$$

According to the reference [6, p.151], the diffraction current can be expressed for the TM wave as

$$\bar{J}_d^{TM}(x) = -2\bar{J}_{go}^{TM}(x)\left(F_-(v) - \frac{e^{-jv^2}}{2jv}\right), \tag{20}$$

and for the TE wave as

$$\bar{J}_d^{TE}(x) = -2\bar{J}_{go}^{TE}(x)\sqrt{\frac{j}{\pi}}F_-(v). \tag{21}$$

Here, $F_-(v)$ is a Fresnel integral function and with

$$v = \sqrt{kx(1 + \cos\phi_0)}, \tag{22}$$

$$F_-(v) = \int_v^\infty e^{-jt^2}dt, \tag{23}$$

and with positive branch for the square root in (22). The Fresnel integral has the properties

$$F_-(0) = \frac{\sqrt{\pi}}{2\sqrt{j}}, \tag{24}$$

$$F_-(v) = \frac{e^{-jv^2}}{2jv}\left(1 + 2\sum_1^\infty \frac{(2m-1)!j^m}{(m-1)!(2v)^{2m}}\right). \tag{25}$$

(25) converges quickly for large v values. It is easy to see that, for large v, the function (25), and hence (21), decay as $1/\sqrt{x}$, whereas the function in brackets in (20) obeys the law $1/x\sqrt{x}$. Hence, the diffraction current in the TM case is more concentrated near the edge.

LINE CURRENT APPROXIMATIONS

For the approximation of the previous surface currents (20), (21), we have to evaluate the current moments (7), (8), which for the TM wave are of the form $I_0^{TM} = u \cdot I_0^{TM}, I_1^{TM} = u_x u_z I_1^{TM}$, and for the TE wave of the form $I_0^{TE} = u_x I_0^{TE}, I_1^{TE} = u_x u_z I_1^{TE}$. The moments can be evaluated invoking the following integral expressions:

$$\int_0^{\infty} \frac{e^{-px}}{\sqrt{x}}\, dx = \sqrt{\frac{\pi}{p}}, \tag{26}$$

$$\int_0^{\infty} x^{n-\frac{1}{2}} e^{-px}\, dx = \frac{\sqrt{\pi}(2n-1)!}{(n-1)!2^{2n-1}p^{n+\frac{1}{2}}}, \tag{27}$$

$$\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x^{3/2}}\, dx = \sqrt{\pi}(\sqrt{a} - \sqrt{b}). \tag{28}$$

Thus, we can write for the monofilar approximation of the surface diffraction currents

$$J_d^{TM}(x) \approx \frac{4}{\eta}\bar{E}_i I_0^{TM}\delta(x - a^{TM}), \tag{29}$$

$$J_d^{TE}(x) \approx -4u_y \times \bar{H}_i I_0^{TE}\delta(x - a^{TE}). \tag{30}$$

After a number of algebraic operations, the normalized current moments can be found in the following form:

$$I_0^{TM} = \frac{\sqrt{\pi}}{2j^{3/2}k}\frac{\sin\phi_0}{\sqrt{1 + \cos\phi_0}(1 + \sqrt{1 + \cos\phi_0})}, \tag{31}$$

$$I_0^{TE} = \frac{j}{2k}\frac{1}{1 + \sqrt{1 + \cos\phi_0}}, \tag{32}$$

$$I_1^{TM} = \frac{\sqrt{\pi}}{4\sqrt{j}k^2}\frac{\sin\phi_0}{\sqrt{1 + \cos\phi_0}(1 + \sqrt{1 + \cos\phi_0})^2}, \tag{33}$$

$$I_1^{TE} = \frac{1}{4k^2}\frac{2 + \sqrt{1 + \cos\phi_0}}{(1 + \sqrt{1 + \cos\phi_0})^2}, \tag{34}$$

The quantities a^{TM} and a^{TE} can be solved from (31)-(34) and (14a), with the result as in the following expressions:

$$a^{TM} = -\frac{j}{2k}\frac{1}{1 + \sqrt{1 + \cos\phi_0}}, \tag{35}$$

$$a^{TE} = -\frac{j}{2k}\frac{2 + \sqrt{1 + \cos\phi_0}}{1 + \sqrt{1 + \cos\phi_0}}. \tag{36}$$

It is seen that the best location for the monofile is on the negative x axis at a distance dependent on the angle ϕ_0 of the incoming wave.

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DIFFRACTION FIELDS

To see the effect of the shift of the monofile from the origin to the complex point, let us consider the diffracted far field and the far fields arising from the monofile located both at the edge and in complex space. The far field arising from the exact diffraction current (20) can be evaluated in the form

$$\bar{E}_d^{TM} = \bar{E}_i \frac{e^{-jk\rho}}{\sqrt{8j\pi k\rho}} \frac{2\sqrt{\pi}}{\sqrt{j}} \left(\frac{sin\frac{\phi_0}{2}}{sin\frac{\phi}{2} + cos\frac{\phi_0}{2}} \right). \tag{37}$$

The line current I_0^{TM} at the edge gives rise to the following field:

$$\bar{E}_d^{TM} = -\bar{E}_i \frac{e^{-jk\rho}}{\sqrt{8j\pi k\rho}} \frac{2\sqrt{\pi}}{\sqrt{j}} \left(\frac{sin\frac{\phi_0}{2}}{\frac{1}{\sqrt{2}} + cos\frac{\phi_0}{2}} \right), \tag{38}$$

which is seen to coincide with (37) at $\phi = \pi/2$. The line current at the point $x = a^{TM}$ defined by (35), radiates the field

$$\bar{E}_d^{TM} = -\bar{E}_i \frac{e^{-jk\rho}}{\sqrt{8j\pi k\rho}} \frac{2\sqrt{\pi}}{\sqrt{j}} \left(\frac{sin\frac{\phi_0}{2}}{\frac{1}{\sqrt{2}} + cos\frac{\phi_0}{2}} \right) exp \left(\frac{1}{2\sqrt{2}} \frac{cos\phi}{\frac{1}{\sqrt{2}} + cos\frac{\phi_0}{2}} \right). \tag{39}$$

By expanding both (37) and (39) at $\phi = \frac{\pi}{2} + \delta$, in power series in terms of δ, which is a small quantity, it can be seen that the first term is coincident with (38), and the second terms in (37) and (38) are equal. This shows that (39) approximates (37) more closely near the radiation direction $\phi = 90^o$. Examples of direction patterns for these fields can be seen in figures 2, 3 for the incidence angles $\phi_0 = 60^o, 120^o$. It is seen that while the current at the edge only gives omnidirectional radiation, the complex space monofile gives quite good estimate for the diffraction patterns for angles about $45^o - 180^o$, wheras the diffraction field in the direction of the half plane is poorly approximated. From the previous we conclude, that the complex space line current gives the diffraction pattern and its derivative correctly for the angle 90°. It is obvious that this is the best that can be done with an approximation by a single monofile. The value of a^{TM} could be chosen also to meet some other requirement. For example, we might have wanted the correct field in the direction $\phi = 0$, for which the coefficient of $cos\phi$ within the exponent function of (39) could have been accordingly chosen. In this case, however, it is obvious that the radiation in the direction $\phi = 180^o$ would be greatly deteriorated.

Corresponding expressions for the TE field can be written, respectively,

$$\bar{E}_d^{TE} = \bar{u}_\phi \eta H_{iz} \frac{e^{-jk\rho}}{\sqrt{8j\pi k\rho}} \left(\frac{2 cos\frac{\phi}{2}}{sin\frac{\phi}{2} + cos\frac{\phi_0}{2}} \right), \tag{40}$$

$$\bar{E}_d^{TE} = \bar{u}_\phi \eta H_{iz} \frac{e^{-jk\rho}}{\sqrt{8j\pi k\rho}} \left(\frac{\sqrt{2}sin\phi}{\frac{1}{\sqrt{2}} + cos\frac{\phi_0}{2}} \right), \tag{41}$$

$$\bar{E}_d^{TE} = \bar{u}_\phi \eta H_{iz} \frac{e^{-jk\rho}}{\sqrt{8j\pi k\rho}} \left(\frac{\sqrt{2}sin\phi}{\frac{1}{\sqrt{2}} + cos\frac{\phi_0}{2}} \right) exp \left(\frac{\sqrt{2} + cos\frac{\phi_0}{2}}{2(\frac{1}{\sqrt{2}} + cos\frac{\phi_0}{2})} cos\phi \right). \tag{42}$$

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Again. after some algebra, it can be seen that the two-term approximations of (40) and (42) coincide for $\phi = \frac{\pi}{2} + \delta$ for small δ and the first term equals that of (41). Figures 4. 5 demonstrate that the complex space monofile (actually a current strip with transverse current flow) gives quite good approximation, but only for angles $60°...180°$. This is mainly because a finite dipolar current cannot produce a non-null radiation field in its own direction. whereas the infinite original current can. The current strip at the edge gives now a sinusoidal transverse pattern, and the complex location distorts that pattern to make the pattern and its derivative match that of the exact solution.

GENERAL INCIDENCE

The approximation can be generalized to the plane wave incident obliquely to the edge. The waves with TE and TM polarizations are still considered separately. If the angle with the edge is θ_0, the previous theory applies with the z dependence $exp(-jkzcos\theta_0)$ added to the currents and fields. and with k everywhere in the formulas replaced by $ksin\theta_0$, [6. p.127]. Thus, the normalized line currents (31), (32) and the locations of the monofiles. (35), (36) must be divided by $sin\theta_0$

The incoming plane wave must be decomposed in two plane waves of TE and TM polarization. This can be done with the following expansion for the unit dyadic:

$$\bar{I} = \frac{\bar{k}\bar{k}}{k^2} + \frac{(\bar{k} \times \bar{u}_z)(\bar{k} \times \bar{u}_z)}{(\bar{k} \times \bar{u}_z)^2} + \frac{(\bar{k} \times (\bar{k} \times \bar{u}_z))(\bar{k} \times (\bar{k} \times \bar{u}_z))}{k^2(\bar{k} \times \bar{u}_z)^2}. \tag{43}$$

Here, the first term does not affect when multiplying the fields, since $\bar{k} \cdot E = 0$ and $\bar{k} \cdot \bar{H} = 0$. Multiplying (43) by $\cdot E$ gives us the decomposition $E = 0 + \bar{E}^{TE} + \bar{E}^{TM}$. Multiplying by $\cdot \bar{H}$ we have the decomposition $\bar{H} = 0 + \bar{H}^{TM} + \bar{H}^{TE}$. Thus, any plane wave not incident along the edge can be decomposed and the monofilar currents for each partial field can be summed.

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FIGURE CAPTIONS

Figure 1 Basic geometry of plane wave incidence on a conducting half plane. The angle of incidence ϕ_0 is between 0 and 180° and the angle θ_0 between 0 and 90°.

Figure 2 TM polarized diffraction field patterns for $\theta_0 = 0$ and $\phi_0 = 60^o$. Solid line: exact diffraction field, equation (37), dot line: monofilar current at the edge, equation (38), dashed line: monofilar current in complex space, equation (39). The quantity displayed is actually the bracketed term in the field expressions.
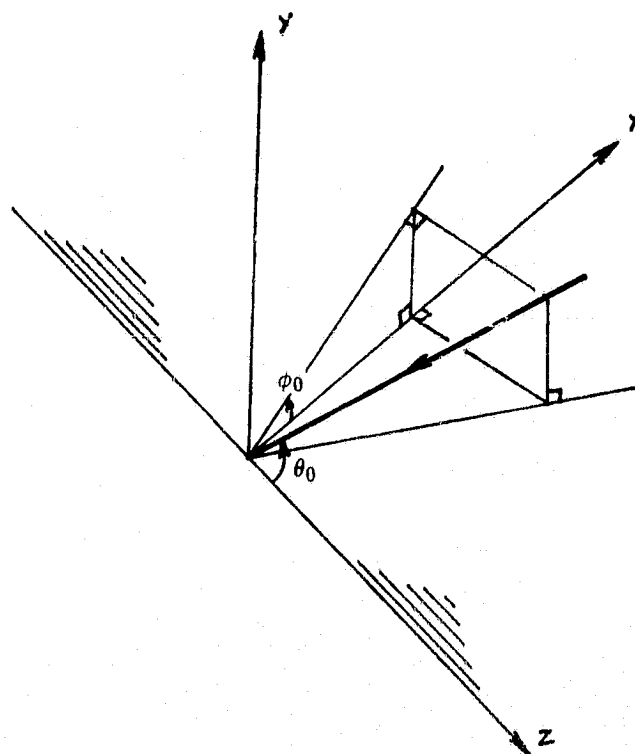
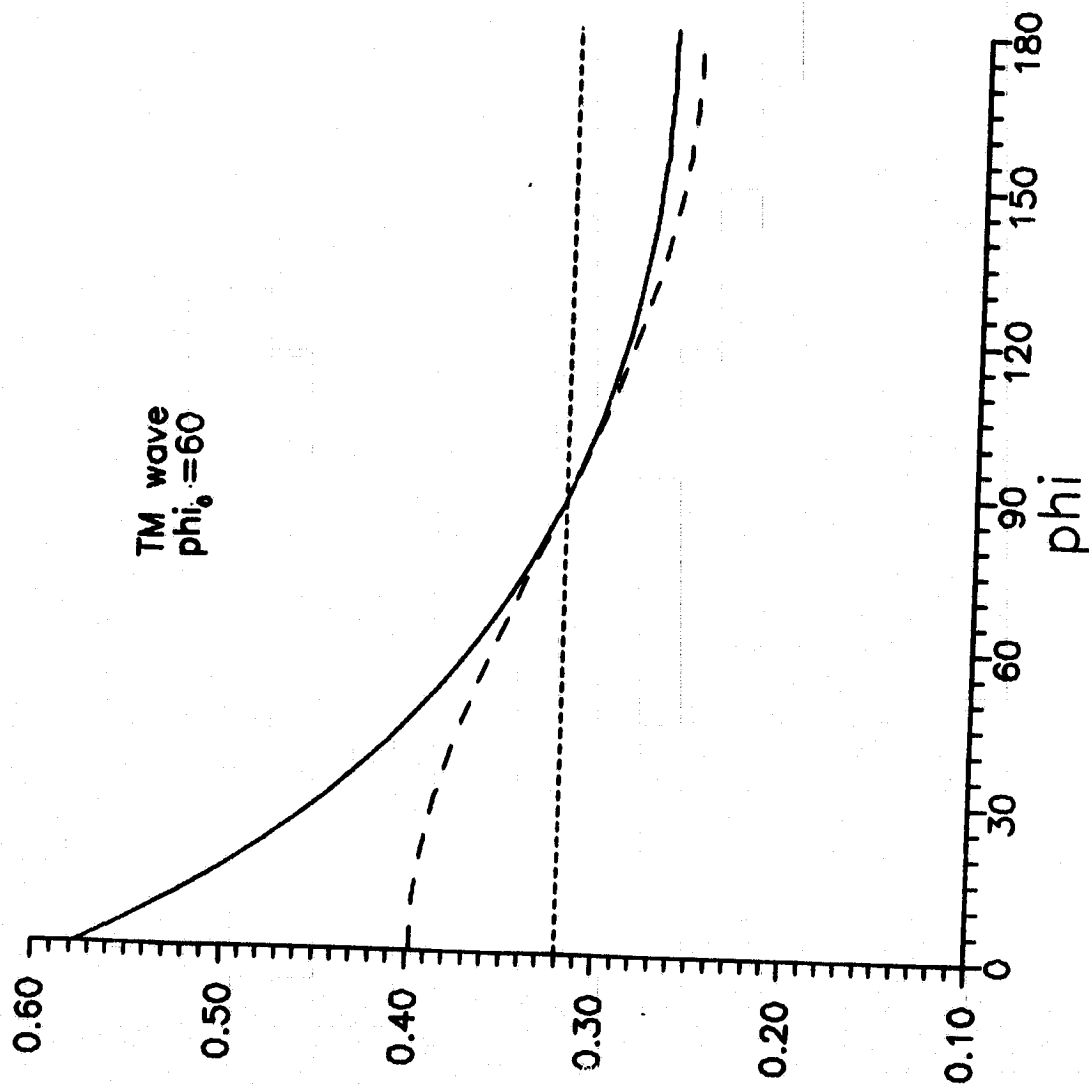*Figure 3* Same as Fig.2, but with $\phi_0 = 120^o$.

Figure 4 TE polarized diffraction field patterns for $\theta_0 = 0$ and $\phi_0 = 60^o$. Solid line: exact diffraction field, equation (40), dot line: monofilar current at the edge, equation (41), dashed line: monofilar current in complex space, equation (42). The quantity displayed is actually the bracketed term in the field expressions.

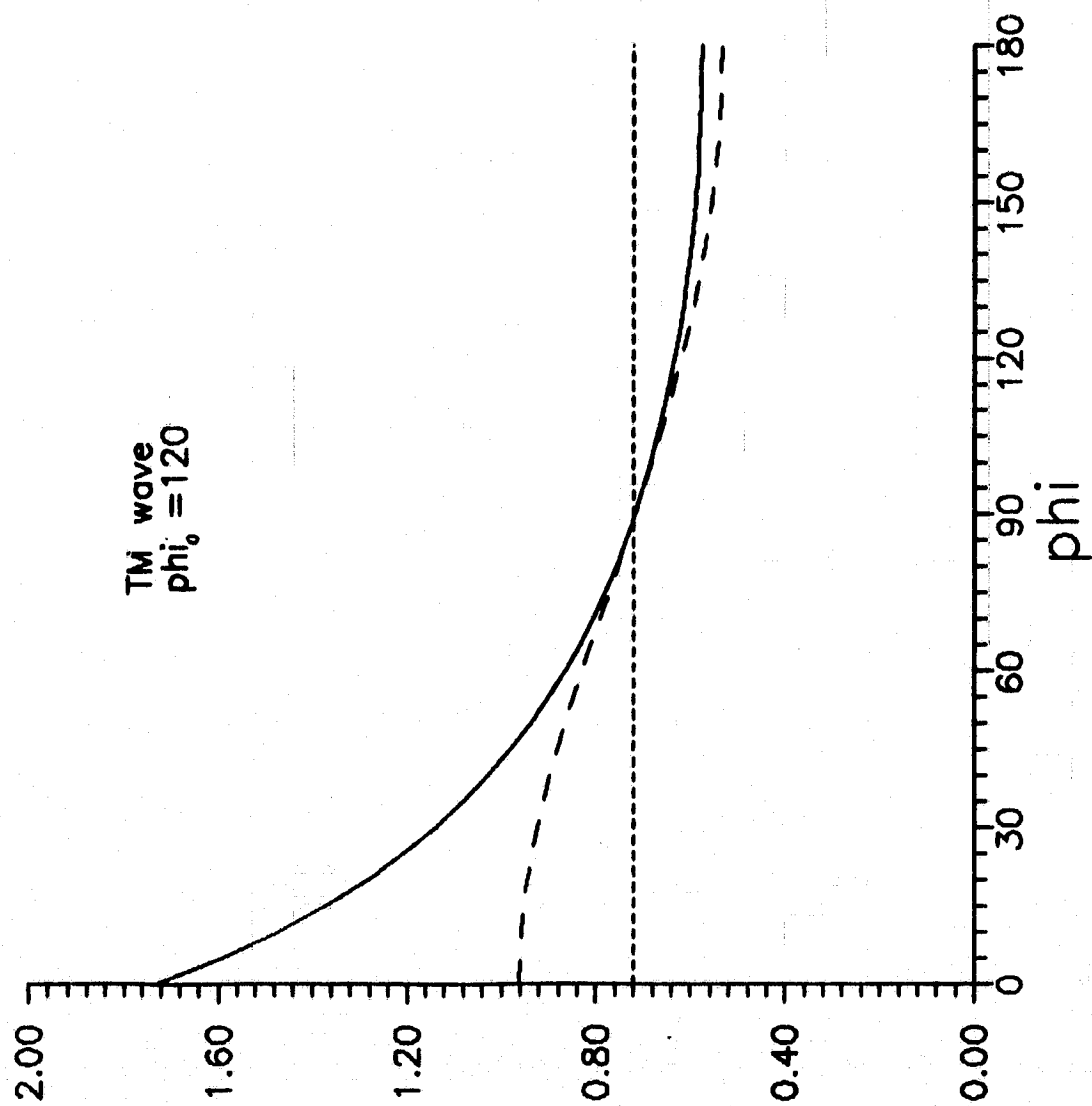*Figure 5* Same as Fig.4, but with $\phi_0 = 120^o$.

Figure 1

TM wave
phi$_0$=60

phi

Figure 2

12

TM wave
phi$_0$ =120

phi

Figure 3

13

TE wave
phi$_o$ =60

phi

Figure 4

14

TE wave
phi$_0$=120

Figure 5

15