FLIGHT PENETRATION OF WIND SHEAR : CONTROL STRATEGIES

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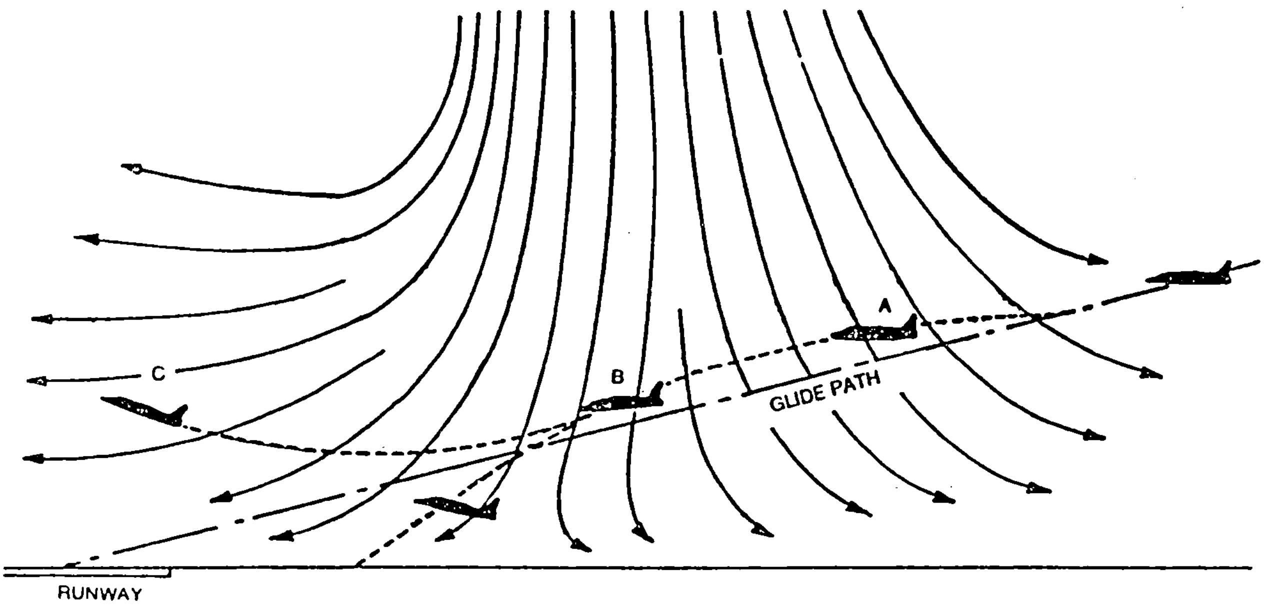
A Typical Microburst Encounter

Wind shear is a dangerous condition where there is a sharp change in the direction and magnitude of the wind velocity over a short distance or time.

This condition is especially dangerous to aircraft during landing and take off and can cause a sudden loss of lift and thereby height at a critical time.



Problem Formulation

Wind shear is a condition of changing speed and/or direction of the wind rapidly over a short distance. A microburst is a special kind of a wind shear in which a downward blast of air hits the ground. Wind shears, especially microbursts, are very hazardous to aircraft manueuvering close to the ground. If unopposed by the pilot, there is a sudden gain in height and then an equally sudden loss in height which can lead to a crash. Microbursts have caused a number of crashes during take off and landing.

Wind Shear

A change in wind velocity in a brief time so as to cause a rapid change in the speed of the air flowing over the wing

Microburst

A downward blast of air which spreads on hitting the ground

Effects of Wind Shear

Loss in height and or position due to changes in lift causing severe hazard to the airplane

Problems

Could cause crashes during landing, take off or other maneuvers close to the ground

Linear Quadratic Regulator

An aircraft represents a nonlinear system; hence the general problem of its control is nonlinear. It is possible to linearize about the flight path to allow control strategies developed for linear systems to be applied. One approach is to use a Linear Quadratic Regulator, in which a control law that minimizes a quadratic cost function is found. Minimization leads to a linear state feedback strategy giving a stable closed-loop system.

x ==> states = { velocity flight-path-angle pitch-rate angle-of-attack height thrust }

u ==> controls = { elevator throttle }

w ==> disturbances = { horizontal-wind vertical-wind }

Δ ( ) ==> perturbation about nominal

Q, M, R ==> cost weighting matrices

This approach hinges on choosing good cost weights. This is clear from the results. The choice can be difficult, and it is dependent on the aircraft; this is a major drawback of this method.

Given linear system

$$\Delta \dot{\underline{x}} = F \Delta \underline{x} + G \Delta \underline{u} + L \Delta \underline{w}$$

Define a " cost"

$$J = \frac{1}{2} \int_0^\infty \left[\Delta \underline{x}^T \; \Delta \underline{u}^T \right] \begin{bmatrix} Q & M \\ M^T & R \end{bmatrix} \begin{bmatrix} \Delta \underline{x} \\ \Delta \underline{u} \end{bmatrix} dt$$

Minimizing J leads to

$$\Delta \underline{u} = -R^{-1} (G^T S + M^T) \Delta \underline{x} = - C \Delta \underline{x}$$

Where S is the solution of the Riccati Equation

$$(F - GR^{-1}M^T)^T S + S(F - GR^{-1}M^T) - SGR^{-1}G^T S + Q - MR^{-1}M^T = 0$$

Variation of Cost Weights

The cost weights for the cost function were obtained by using a combination of state-rate and direct state and control weighting. No direct cross weighting between the states and the controls was used. The cost weights were varied by varying the direct cost weights on the controls.

A sinusoidal model of the microburst developed by Mark Psiaki was used.It had a maximum headwind/tailwind of 10.7 m/s and a maximum downdraft of 6 m/s.

State rate weighting used :
$$\begin{bmatrix} 100 & & & & & \\ & 10 & & & \varnothing & \\ & & 10 & & & \\ & & & 1 & & \\ & \varnothing & & & 100 & \\ & & & & & 1 \end{bmatrix}$$

Q0 (direct state cost weighting) = Identity

R0 (direct control weighting) =

$$\begin{bmatrix} 1000 & 0 \\ 0 & 10 \end{bmatrix} \quad \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \quad \begin{bmatrix} 10 & 0 \\ 0 & 1000 \end{bmatrix}$$

Microbursts :

Range : Headwind/tailwind begins 0 m and ends 3000 m

Strength : 10.7 m/s

Range : Downdraft begins 1050 m and ends 1950 m

Strength : 6 m/s

Gains with Varying Cost Weights

By varying the cost weight associated with the elevator, different gains were obtained for a Linear Quadratic Regulator. The aim of this figure is to show that different weights lead to very different control requirements.

Gains set #1 :

$$C = \begin{bmatrix} -3.407E-2 & -9.228 & -7.403 & -4.496 & -2.801E-2 & -8.644E-2 \\ 4.268E-1 & 3.260E+1 & 1.926E+1 & 1.299E+1 & 1.468E-2 & 1.983 \end{bmatrix}$$

Gains set #2 :

$$C = \begin{bmatrix} -1.662E-1 & -6.336E+1 & -2.581E+1 & -2.499E+1 & -3.081E-1 & -2.966E-1 \\ 3.372E-1 & 8.2 & 9.741E-1 & 1.104 & 7.082E-2 & 1.762 \end{bmatrix}$$

Gains set #3 :

$$C = \begin{bmatrix} -1.783E-1 & -6.367E+1 & -2.585E+1 & -2.503E+1 & -3.108E-1 & -3.309E-1 \\ 4.100E-2 & 6.429E-1 & 1.806E-3 & 8.513E-3 & 5.802E-3 & 3.414E-1 \end{bmatrix}$$

Effects of Cost Weighting

Variations on Altitude and Controls

Simulation results for a simplified model of a Boe727 (representing a typical jet aircraft) follow. All the simulations are for a take off condition. The nominal filght conditions are

Airspeed	= 71.628 m / s	flight path angle	= 0.0523 rad
pitch rate	= 0.0 rad / s	angle of attack	= 0.0611 rad
altitude	= 3.0 m	thrust	= 0.8713 * max thrust
elevator	= -0.0518 rad	throttle	= 0.8713 * max thrust

These conditions apply to all the simulation results. SI units are used on all the plots.

The altitude vs range plot shows the dramatic improvement due to the control law over the openloop performance. It also shows that a choice of high elevator cost weight leads to a poorer performance.

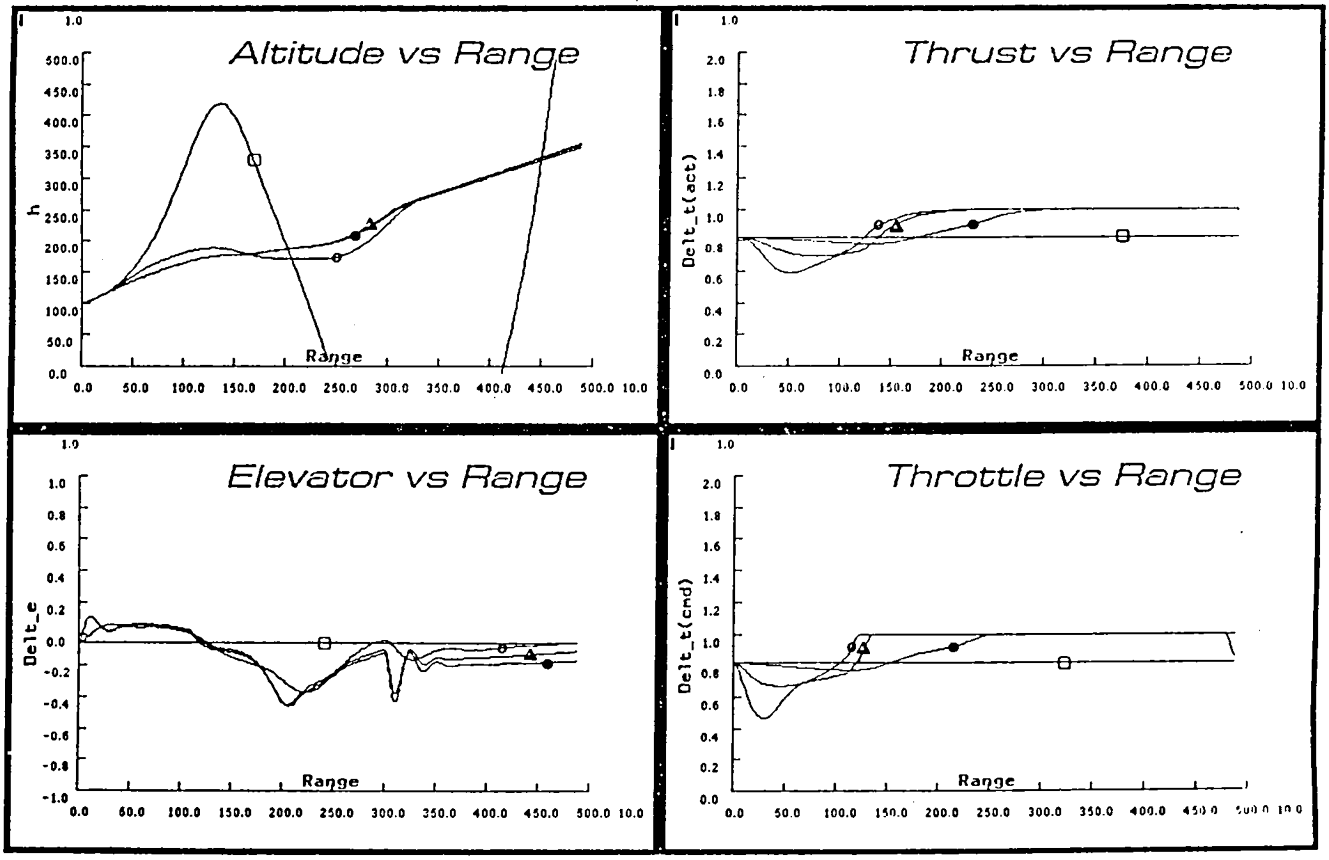
The difference between the throttle and the thrust plots is due to modelling of a lag between the generation of the thrust and the throttle command. The throttle vs range plot shows that as the cost weight of the throttle is increased, the throttle saturates later. The result is that the control activity of the elevator goes up. In general, though, the saturation of the throttle sooner or later means that the elevator is the main control remaining. This is seen from the similarity between all the elevator plots.

□ = open loop
▲ = gain set #2

Effects of Cost Weighting

Variations on Altitude and Controls

o = gain set #1
● = gain set #3



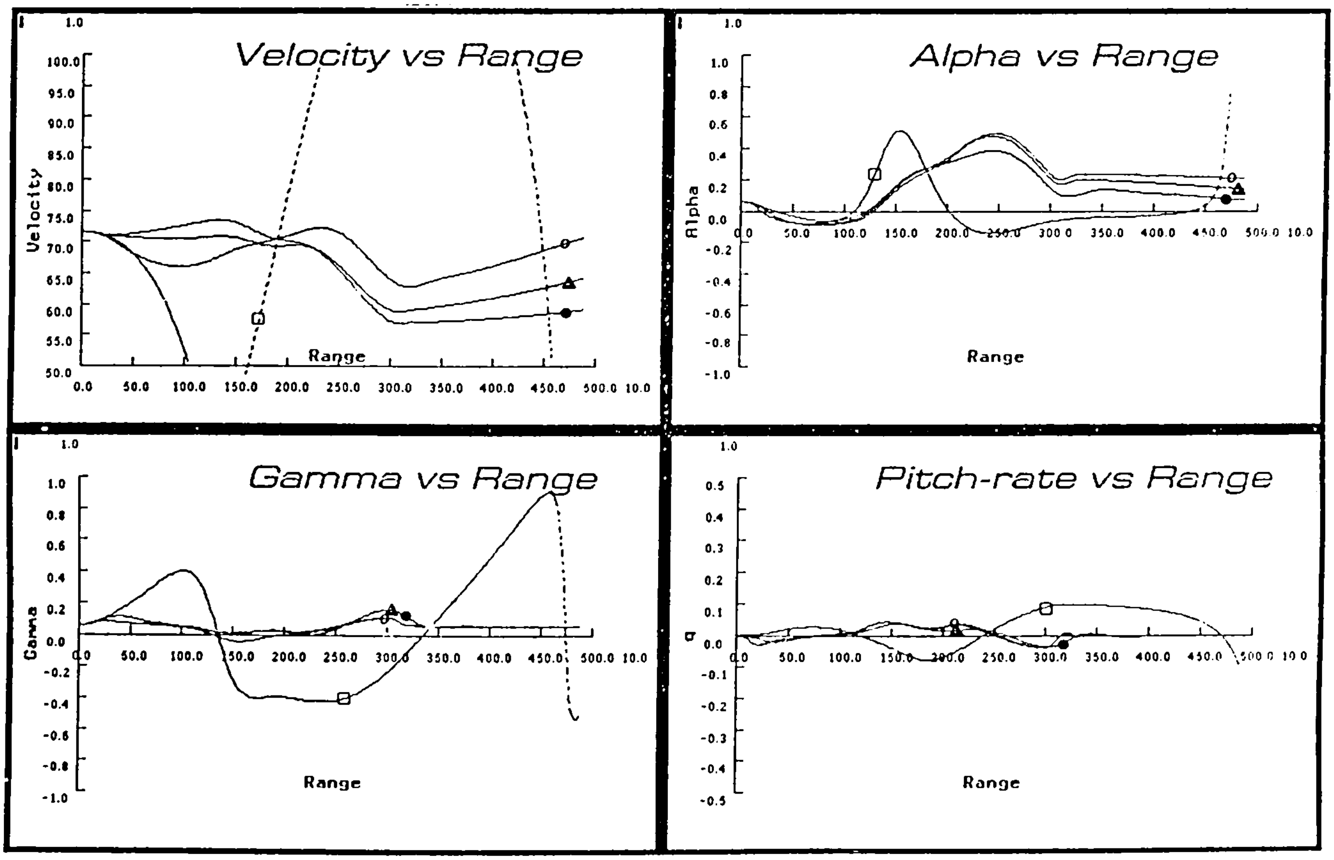
Effects of Cost Weights

Variations on Velocity and Angular Controls

These set of plots show the velocity, angle of attack (alpha), flight path angle (gamma), and the pitch rate as functions of the range. Again there is a dramatic improvement over the open-loop case. The flight path angle and velocity variations are reduced considerably, leading to the improvement in the altitude vs range seen earlier. The pitch rate and the angle of attack are affected much less. It can be seen that the angle of attack reaches quite high values.

□ = open loop
▲ = gain set #2

o = gain set #1
● = gain set #3



Nonlinear Inverse Dynamics

The aircraft is a nonlinear system, and we can tackle the nonlinear control problem directly using a nonlinear inverse dynamics (NID) approach.

In this approach we asume that we have a nonlinear system with $\underline{\mathbf{x}}$, $\underline{\mathbf{u}}$, $\underline{\mathbf{w}}$ the states, controls, and the disturbances as for the LQR. Additionally we define an output $\underline{\mathbf{y}}$ as linear combination of the states. Since an exact inverse of the nonlinear system cannot be found, we calculate an approximate one.

We differentiate the output 'd' times until some controls appear in each of the outputs. It is assumed that there is a function defining the desired output. Setting the derivatives of the output equal to those of the desired output gives us a set of nonlinear algebraic equations. When solved for the control with some simplifications, we get the set of controls giving the desired output behavior.

Given a nonlinear system

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, \underline{w})$$

$$\underline{y} = C\underline{x}$$

Differentiate $\underline{y}$ 'd' times until the control appears in the output to get

$$\underline{y}^{[d]} = g(\underline{x}, \underline{v})$$

$$\text{where } \underline{v}^T = (\underline{u}^T, \dot{\underline{u}}^T, \ddot{\underline{u}}^T, \dddot{\underline{u}}^T \cdots)$$

Now let the desired output be $\underline{y}_{desired}$

Now set $\dot{\underline{u}} = \ddot{\underline{u}} = \ldots \underline{0}$ and solve

$$\underline{y}^{[d]} = g(\underline{x}, \underline{u}) = \underline{y}^{[d]}_{desired}$$

Nonlinear Inverse Dynamics

In the NID approach, we are free to choose either a functional form of the desired output or choose the dth derivatives of the desired output and then specify a dynamics for the output.

Choosing $\underline{y}_{desired}$ we choose the desired dynamics for y

Alternatively choosing a dynamics for $\underline{y}^{[d]}_{desired}$ gives us the $\underline{y}^{[d]}$

Conclusions

* The results of the simulation show the effective performance of the LQR and the NID controllers. The major conclusions that one can draw from these results are

* The LQR seems to try to keep the variation in the velocity and the flight path angle to the minimum. This is obvious in hindsight when it is realized that these are the major factors controlling deviations from the desired flight trajectory.

* There is more variation in the angle of attack and the pitch rate from the nominal values.

* Thrust almost always saturates for the LQR type of control law and a reasonably large microburst. There is an initial reduction in the thrust as the aircraft enters the microburst ; then there is sharp increase until it saturates. Finally the thrust comes back to normal as the aircraft gets out of the region of the microburst.

* The elevator shows a very different behaviour. High cost weights associated with the elevator lead to lesser elevator use. However, with the thrust saturated, the elevator is effectively the only control and this shows in the elevator behaviour.

Linear Quadratic Regulators lead to a good performance with a good choice of costs.

LQR s can require very high feedback gains for a good performance.

Nonlinear Inverse Dynamics with complete solution of nonlinear equations promises to give excellent performance.
NID with complete solution of nonlinear equations would have the penalty of :

1) Time required
2) Possibility of none or multiple solutions.