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PARAMETER ESTIMATION FOR BOUNDARY VALUE PROBLEMS
BY INTEGRAL EQUATIONS OF THE SECOND KIND

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ABSTRACT

This paper is concerned with the parameter estimation for boundary integral equations of the second kind. The parameter estimation technique by using the spline collocation method is proposed. Based on the compactness assumption imposed on the parameter space, the convergence analysis for the numerical method of parameter estimation is discussed. The results obtained here are applied to a boundary parameter estimation for 2-D elliptic systems.

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1. INTRODUCTION

Recently, there is growing interest in the problem of identification for distributed parameter systems (IDPS) both from the theoretical and numerical points of views (e.g. [4~8], [11],[15],[21],[22]). The minimization of the output least square criterion (OLS) is one of the most popular methods for IDPS (see [16] and the references therein). The numerical determination of parameters by minimizing OLS involves some theoretical difficulties. Namely, numerical methods proposed must be equipped with convergence properties, such that the solution of discretized optimization problem implemented on the computer converges to the optimal solution of the original problem in infinite dimensions when we take the limit with respect to the number of dimensions. The compactness idea in the context of parameter estimation provides us a useful theoretical framework for the convergence and stability arguments in computer implementations of the discretized problems (see [4~8]).

In this paper, an effort for parameter estimation is directed to the numerical method of parameter identification on the boundary, equipped with convergence properties for the optimal solution of discretized parameter estimation problem. Let G and ∂G be the bounded domain and its boundary curve such that

$$\partial G = \{x = \xi(t) | \xi(t) = (\xi_1(t), \xi_2(t)),\ \xi_k(t)\ (k = 1,2) \quad \text{are a} \\ \text{1-periodic} \quad C^{\{r+1\}}\text{- class function on } [0,1] \text{ with } \quad |d\xi/dt| \neq 0\}$$

where $\{r\} := -[-r]$ and $[r]$ stands for the greatest integer $\leq r$. Throughout this paper, we assume $r \geq 2$. We consider the boundary integral equation of the second kind,

$$\phi(s) - \int_0^1 k(s,t;\theta)\phi(t)\left|\frac{d\xi}{dt}\right|dt = f(t) \tag{1}$$

where θ is the unknown parameter to be identified. A large number of classes of 2-D elliptic boundary value problems can be reduced to systems of integral equations of the second kind as described in (1). Such integral equations are found in many applications to the field of engineering which appear in thermal diffusivity, viscous flow, electrostatics, acoustics, elasticity, etc. (See [10] and the references therein).

Motivated by this fact, we consider the following parameter estimation problem.

(IP) Given the measurement data ψ_i at $t_i^p \in [0,1]$ $(i = 1,2,\cdots,m)$, find $\theta^* \in \Theta$ which minimizes the OLS

$$J(\theta) = \frac{1}{2}\sum_{i=1}^{m} |\phi(t_i^p,\theta) - \psi_i|^2 \tag{2}$$

subject to (1).

In Section 2, we discuss the existence property of an optimal solution for the problem (IP). In Section 3, the finite approximation technique for parameter estimation is proposed. Theoretical convergence proofs of the approximation method proposed are given. In Section 4, the method proposed in Section 3 is applied to the boundary parameter estimation of 2-D steady state thermal diffusivity. Finally, some numerical results will be demonstrated in Section 5.

The notation used throughout this paper is standard and closely follows that explained in [17]. For norms, we use $\|\cdot\|_X$ where X is an appropriate Banach or Hilbert space. We denote by $H^r(Y)$ the periodic Sobolev space of arbitrary real order r on Y. The notation $C^{\{t\}}(Y)$ stands for the space of $\{t\}$-times Hölder continuously differentiable functions on Y.

2. PRELIMINARIES

The equation (1) is rewritten symbollically as

$$[(I + K(\theta))\phi](s) = f(s) \quad \text{for} \quad 0 \leq s \leq 1 \tag{3}$$

with the integral operator

$$K(\theta) := -\int_0^1 k(s,t;\theta)(\cdot)\left|\frac{d\xi}{dt}\right| dt.$$

Throughout this paper we assume that:

(H-1) $$\theta \in C^{\{t\}}(\overline{T};R^{(\ell)})$$

where $\overline{T}$ is a closed interval in $[0,1]$, t is a real number such that $t < r - \frac{1}{2}$, and ℓ is a natural number, respectively;

(H-2) $$\alpha\|\phi\|_{H^r(\partial G)} \leq \|(I + K(\theta))\phi\|_{H^r(\partial G)}$$

$$\text{for each} \quad \theta \quad \text{and} \quad \forall\psi \in H^r(\partial G);$$

(H-3) $K(\theta)$ is a compact linear operator in $H^r(\partial G)$

for each θ;

(H-4) $\|(K(\theta_1) - K(\theta_2))\psi\|_{H^r(\partial G)} \leq \beta \|\theta_1 - \theta_2\|_{C^{\{t\}}(T;R^{(\ell)})} \|\psi\|_{H^r(\partial G)}$

for $\forall\phi \in H^r(\partial G)$;

(H-5) $f \in H^r(\partial G)$.

The admissible parameter set Θ in (IP) is then defined as follows:

Θ is a compact subset of $H^r(T;R^{(\ell)})$ and its metric space (Θ,d) is compact with respect to the distance function

$$d := \rho(x,y) = \|x - y\|_{C^{\{t\}}(\overline{T};R^{(\ell)})} \quad \text{for} \quad x,y \in \Theta.$$

Theorem 1: Under hypotheses (H-1) to (H-5), there exists at least one solution $\theta^* \in \Theta$ of (IP).

To prove Theorem 1, we need the preliminary results:

Proposition 1: Let $\phi_1 = \phi(\theta_1)$ and $\phi_2 = \phi(\theta_2)$ be the solutions of (2) corresponding to the parameters θ_1 and θ_2. Then, under (H-1) to (H-5), there exists a positive constant α such that, for $\forall_{\theta_1,\theta_2} \in \Theta$,

$$\|\phi_1 - \phi_2\|_{H^r(\partial G)} \leq \frac{\beta}{\alpha^2} \rho(\theta_1,\theta_2)\|f\|_{H^r(\partial G)}. \tag{4}$$

Proof: From (3), it follows that

$$(I + K(\theta_1))(\phi_1 - \phi_2) = (K(\theta_2) - K(\theta_1))\phi_2.$$

By taking the norm in $H^r(\partial G)$ and by virtue of (H-2) and (H-4),

$$\|\phi_1 - \phi_2\|_{H^r(\partial G)} \leq \frac{\beta}{\alpha} \rho(\theta_1,\theta_2)\|\phi_2\|_{H^r(\partial G)}.$$

From this fact and noting that

$$\|\phi_2\|_{H^r(\partial G)} \leq \frac{1}{\alpha} \|(I + K(\theta_2))\phi_2\|_{H^r(\partial G)} = \frac{1}{\alpha} \|f\|_{H^r(\partial G)}, \tag{5}$$

we obtain the a priori estimate (4).

Proposition 2: For (2), the following inequality holds:

$$\begin{aligned} |J(\theta_1) - J(\theta_2)| &\leq \frac{\sqrt{2}m}{2} \|\phi(\theta_1) - \phi(\theta_2)\|_{H^1(\partial G)} \\ &\times (\|\phi(\theta_1)\|^2_{H^1(\partial G)} + \|\phi(\theta_2)\|^2_{H^1(\partial G)} + 4 \sum_{i=1}^{m} |\psi_i|^2)^{1/2}. \end{aligned} \tag{6}$$

By virtue of Sobolev imbedding (see [17, p. 46]), we find that, for any $\psi \in H^1(\partial G)$

$$\sum_{i=1}^{m} |\psi(t_i^p)|^2 \leq \|\psi\|_{C(\partial G)} \leq \|\phi\|_{H^1(\partial G)}.$$

By applying this, we obtain the inequality (6).

Proof of Theorem 1: From the compactness property, we may extract a subsequence $\{\theta_{n'}\}$ of a minimizing sequence $\{\theta_n\}$ in Θ such that $\theta_{n'} \to \theta^*$ as $n' \to \infty$. From Propositions 1 and 2 and using (5), we have

$$|J(\theta_{n'}) - J(\theta)| \leq \frac{\beta m}{\alpha^2} \rho(\theta_{n'},\theta^*) \| f \|_{H^r(\partial G)}$$

$$\times \{\frac{1}{\alpha} \| f \|^2_{H^r(\partial G)} + 2 \sum_{i=1}^{m} |\psi_i|^2\}^{1/2} \leq C\, \rho(\theta_{n'},\theta^*).$$

Hence we argue that

$$J(\theta_{n'}) \to J(\theta^*) = \inf_{\theta \in \Theta} J(\theta) \quad \text{as} \quad \theta_{n'} \to \theta^*.$$

The proof has been completed.

3. PARAMETER ESTIMATION TECHNIQUE AND CONVERGENCE ANALYSIS

In order to solve (IP) on the computer, we consider a discretized optimization problem. Many numerical methods have been proposed for solving the integral equation (e.g., see [1]~[3],[20]). Here we adopt the collocation methods with spline functions. We select an increasing sequence of mesh points,

$$t_j = \frac{j}{N} = jh \quad \text{with} \quad j = 0,1,\cdots,N; \ \Delta^N := \{t_j\}_{j=0}^{N}$$

and the nodal points

$$\tilde{t}_j = (j - \frac{1}{2})h \quad \text{with} \quad j = 1,2,\cdots,N.$$

We denote by $S_k(\Delta^N)$ the space of all 1-periodic (k-1)-times continuously differentiable splines of degree k with knot sequence Δ^N. The spline collocation method for (3) is stated as follows [1], [20]:

Find $\phi^N \in S_k(\Delta^N)$ such that the collocation equations

$$[(I + K(\theta))\phi^N](\tilde{t}_j) = f(\tilde{t}_j), \; j = 1,2,\cdots,N \tag{7}$$

are satisfied.

To formulate the discretized optimization problem, we approximate both the solution ϕ and the admissible parameter θ by a parabolic B-spline [9]. Let $\{B_i^N\}_{i=0}^{N+1}$ and $\{\tilde{B}_i^M\}_{i=0}^{M+1}$ be basic elements with the knot sequence Δ^N and Δ^M $(\subset T)$, respectively. Then we approximate ϕ and θ by

$$\phi^N(t) = \sum_{i=0}^{N+1} w_i^N B_i^N(t)$$

$$\theta^M(t) = \sum_{i=1}^{\ell(M+2)} \lambda_i^M \beta_i^M(t)$$

where

$$\beta_i^M(t) := \begin{cases} (\tilde{B}_i^M, 0,0,\cdots,0) & \text{for } i = 0,\cdots,M+1 \\ (0,\tilde{B}_{i-M-2}^M,0,\cdots,0) & \text{for } i = M+2,\cdots,2(M+1) \\ \vdots & \\ (0,\cdots,0,\tilde{B}_{i-(\ell-1)(M+2)}^M) & \text{for } i = \ell M-M+\ell,\cdots,\ell(M+1). \end{cases}$$

The collocation equation (7) yields the linear equation for the unknown coefficient vector $w^N = \mathrm{col}(w_1^N, w_2^N, \cdots, w_N^N)$,

$$(C^N + K^N(\theta^M))w^N = f^N \tag{8}$$

where C^N, $K^N(\theta^M)$ and f^N are N-dimensional element matrices and vector, respectively. Since $\phi^N(t)$ claims a 1-periodic function on [0,1], we note that $w_0^N = w_N^N$ and $w_{N+1}^N = w_1^N$. We introduce the mapping I^M of $C^1(\overline{T};R^{(\ell)})$ into $R^{(\ell(M+2))}$ such that $I^M\Theta$ is a compact subset of $R^{(\ell(M+2))}$. By using this, let us define the admissible parameter class Θ^M for the discretized parameter estimation problem $(AIP)^{N,M}$

$$\Theta_\lambda^M = \left\{ \theta^M \,\middle|\, \begin{array}{l} \theta^M(t,\lambda^M) = \sum_{i=1}^{\ell(M+2)} \lambda_i^M \beta_i^M(t), \\ \lambda^M = \mathrm{col}(\lambda_1^M, \lambda_2^M, \cdots, \lambda_{\ell(M+2)}^M) \in I^M\Theta \end{array} \right\}.$$

Thus we implement the parameter estimation numerically by solving the problem

$(AIP)^{N,M}$ Find $\theta^M(\overline{\lambda}^M) \in \Theta_\lambda^M$ which minimizes the approximate OLS,

$$J^N(\theta^M) = \frac{1}{2} \sum_{i=1}^{m} |\phi^N(t_i^P, \theta^M) - \psi_i|^2 \tag{9}$$

subject to (8).

In the sequel, we assume that all hypotheses stated in Section 2 hold only for the case $r = 2$. Our next results play a fundamental role in establishing the convergence proof.

Lemma 1 ([20, p. 104, Theorem 3.5]): For each $\theta \in C^1(\overline{T};R^{(\ell)})$, the following error estimate holds:

$$\|\phi(\theta) - \phi^N(\theta)\|_{H^1(\partial G)} \leq \gamma h \|\phi(\theta)\|_{H^2(\partial G)}.$$

Lemma 2 ([1, p. 353]): For any $\theta \in H^2(T;R^{(\ell)})$ and $\epsilon \in (0,\frac{1}{2})$, there exists $\theta^M \in S_2(\Delta^M)$ such that

$$\|\theta - \theta^M\|_{C^1(\overline{T};R^{(\ell)})} \leq \delta \overline{h}^{\epsilon} \|\theta\|_{H^2(T;R^{(\ell)})}$$

where $\overline{h} = \max_i |\overline{t}_{i+1} - \overline{t}_i|$ for $\overline{t}_i, \overline{t}_{i+1} \in \Delta_M$.

Theorem 2: For each N and M, there exists an optimal solution of $(AIP)^{N,M}$.

To prove Theorem 2, we require

Proposition 3: Let $\theta_i = \theta^M(\lambda_i^M) \in \Theta_\lambda^M$ $(i = 1,2)$. Then

$$\|\phi^N(\theta_i)\|_{H^1(\partial G)} \leq \frac{1}{\alpha} (\gamma h + 1)\|f\|_{H^2(\partial G)} \quad \text{for } i = 1,2, \tag{10}$$

$$\|\phi^N(\theta_1) - \phi^N(\theta_2)\|_{H^1(\partial G)} \leq \frac{\beta}{\alpha^2} (\gamma h + 1)\|f\|_{H^2(\partial G)} \|\theta_1 - \theta_2\|_{C^1(\overline{T};R^{(\ell)})}. \tag{11}$$

Proof: Applying lemma 1, it follows that

$$\|\phi^N(\theta_i)\|_{H^1(\partial G)} \leq \|\phi^N(\theta_i) - \phi(\theta_i)\|_{H^1(\partial G)} + \|\phi(\theta_i)\|_{H^1(\partial G)}$$
$$\leq (\gamma h + 1)\|\phi(\theta_i)\|_{H^2(\partial G)}. \tag{12}$$

Since $\theta_i \in C^1(\overline{T};R^{(\ell)})$, we note that, from (H-2),

$$\|\phi(\theta_i)\|_{H^2(\partial G)} \leq \frac{1}{\alpha} \|f\|_{H^2(\partial G)}. \tag{13}$$

Combining (12) with (13), we obtain the inequality (10). By using the same procedure as in Proposition 2, it follows that

$$\|\phi^N(\theta_1) - \phi^N(\theta_2)\|_{H^1(\partial G)} \leq \frac{\beta}{\alpha} \|\theta_1 - \theta_2\|_{C^1(\overline{T};R^{(\ell)})} \|\phi^N(\theta_2)\|_{H^1(\partial G)}. \tag{14}$$

From the inequality (10) to (14), the inequality (11) can be obtained.

Proof of Theorem 2: Since $I^M\mathbb{H}$ is compact, there exists a convergent subsequence of $\{\theta^M(\lambda^M_n)\}$ such that

$$\theta^M(\lambda^M_{n'}) \to \theta^M(\overline{\lambda}^{N,M})$$

where $\overline{\lambda}^{N,M}$ is the solution of

$$\inf_{\theta^M \in \mathbb{H}^M_\lambda} J^N(\theta^M(\lambda^M)) = J^N(\theta^M(\overline{\lambda}^{N,M})).$$

Hence it suffices to show that, for each N and M, $\theta^M(\lambda^M) \to J^N(\theta^M(\lambda^M))$ is continuous. From Proposition 2, we have

$$|J^N(\theta^M(\lambda_1^M)) - J^N(\theta^M(\lambda_2^M))|$$

$$\leq \frac{\sqrt{2}m}{2} \|\phi^N(\theta^M(\lambda_1^M)) - \phi^N(\theta^M(\lambda_2^M))\|_{H^1(\partial G)}$$

$$\times(\|\phi^N(\theta^M(\lambda_1^M))\|^2_{H^1(\partial G)} + \|\phi^N(\theta^M(\lambda_2^M))\|^2_{H^2(\partial G)} + 4 \sum_{i=1}^{m} |\psi_i|^2)^{1/2}.$$

Applying Proposition 3, we obtain

$$|J^N(\theta^M(\lambda_1^M)) - J^N(\theta^M(\lambda_2^M))|$$

$$\leq \frac{m\beta}{\alpha^2} (\gamma h + 1)\|\theta^M(\lambda_1^M) - \theta^M(\lambda_2^M)\|_{C^1(\overline{T};R^{(\ell)})} \tag{15}$$

$$\times\|f\|_{H^2(\partial G)}\{(\frac{\gamma h+1}{\alpha})^2\|f\|^2_{H^2(\partial G)} + 2 \sum_{i=1}^{m} |\psi_i|^2\}^{1/2} .$$

The statement of Theorem 2 directly follows from (15).

The next theorem shows that the solution of $(AIP)^{N,M}$ converges to the optimal solution of (IP).

Theorem 3: <u>The sequence</u> $\{\theta^M(\overline{\lambda}^{N,M})\}$ <u>admits a convergent subsequence</u> $\{\theta^M(\overline{\lambda}^{N_j,M_k})\}$ <u>such that</u>

$$J^N(\theta^M(\overline{\lambda}^{N_j,M_k})) \to J(\theta^*) = \inf_{\theta\in\Theta} J(\theta)$$

<u>as</u> $N_j,M_k \to \infty$.

Proof: Since $\overline{\lambda}^{N_j,M_k} \in I^M \Theta$, there always exists $\hat{\theta}_\lambda^{N_j,M_k} \in \Theta$ such that $\hat{\lambda}^{N_j,M_k} = I^M \hat{\theta}_\lambda^{N_j,M_k}$. By using this, it follows that

$$\|\theta^M(\overline{\lambda}^{N_j,M_k}) - \theta^*\|_{C^1(\overline{T};R^{(\ell)})}$$

$$\leq \|\theta^M(\overline{\lambda}^{N_j,M_k}) - \hat{\theta}_\lambda^{N_j,M_k}\|_{C^1(\overline{T};R^{(\ell)})} + \|\hat{\theta}_\lambda^{N_j,M_k} - \theta^*\|_{C^1(\overline{T};R^{(\ell)})}.$$

Applying Lemma 2, we have

$$\begin{aligned} &\|\theta^M(\overline{\lambda}^{N_j,M_k}) - \theta^*\|_{C^1(\overline{T};R^{(\ell)})} \\ &\leq \delta \overline{h}^\varepsilon \|\hat{\theta}_\lambda^{N_j,M_k}\|_{H^2(T;R^{(\ell)})} + \|\hat{\theta}^{N_j,M_k} - \theta^*\|_{H^2(T;R^{(\ell)})}. \end{aligned} \tag{16}$$

Noting that Θ is a compact set in $H^2(T;R^{(\ell)})$ and that $\overline{h}^\varepsilon \to 0$ as $M_k \to \infty$, the inequality (16) asserts there exists convergent subsequence $\{\theta^M(\overline{\lambda}^{N_j,M_k})\}$ with $\theta^M(\overline{\lambda}^{N_j,M_k}) \to \theta^*$ as N_j, $M_k \to \infty$. The remainder half of this proof is to show $J^N(\theta^M(\overline{\lambda}^{N_j,M_k})) \to J(\theta^*)$ as $\theta^M(\overline{\lambda}^{N_j,M_k}) \to \theta^*$, N_j, $M_k \to \infty$. Applying Proposition 2, we derive

$$|J^N(\theta^M(\overline{\lambda}^{N_j,M_k})) - J(\theta^*)|$$

$$\leq |J^N(\theta^M(\overline{\lambda}^{N_j,M_k})) - J^N(\theta^*)| + |J^N(\theta^*) - J(\theta^*)|$$

$$\leq \frac{\sqrt{2}m}{2} \|\phi^N(\theta^M(\overline{\lambda}^{N_j,M_k})) - \phi^N(\theta^*)\|_{H^1(\partial G)}$$

$$\times(\|\phi^N(\theta^M(\overline{\lambda}^{N_j,M_k}))\|^2_{H^1(\partial G)} + \|\phi^N(\theta^*)\|^2_{H^1(\partial G)} + 4\sum_{i=1}^{m} |\psi_i|^2)^{1/2}$$

$$+ \frac{\sqrt{2}m}{2} \|\phi^N(\theta^*) - \phi(\theta^*)\|_{H^1(\partial G)}$$

$$\times(\|\phi^N(\theta^*)\|^2_{H^1(\partial G)} + \|\phi(\theta^*)\|^2_{H^1(\partial G)} + 4 \sum_{i=1}^{m} |\psi_i|^2)^{1/2}.$$

By virtue of Lemma 1 and Proposition 3, we obtain

$$|J^N(\theta^M(\bar{\lambda}^{N_j,M_k})) - J(\theta^*)|$$

$$\leq \frac{\beta m}{\alpha^2} (\gamma h + 1)\|\theta^M(\bar{\lambda}^{N_j,M_k}) - \theta^*\|_{C^1(T;R^{(\ell)})}$$

$$\times\|f\|_{H^2(\partial G)} \{(\frac{\gamma h+1}{\alpha})^2\|f\|^2_{H^2(\partial G)} + 2 \sum_{i=1}^{m} |\psi_i|^2\}^{1/2}$$

$$+ h \cdot \frac{\sqrt{2}m\gamma}{2\alpha} \|f\|_{H^2(\partial G)} \{\frac{(\gamma h+1)^2+ 1}{\alpha^2} \|f\|^2_{H^2(\partial G)} + 4 \sum_{i=1}^{m} |\psi_i|^2\}^{1/2}.$$

From the fact that $h \to 0$ as $N_j \to \infty$, we conclude that

$$|J^N(\theta^M(\bar{\lambda}^{N_jM_k})) - J(\theta^*)| \to 0 \quad \text{as} \quad N_j,M_k \to \infty.$$

The proof has been completed.

4. APPLICATION TO PARAMETER ESTIMATION OF ELLIPTIC SYSTEMS

We consider a 2-D steady state diffusion system

$$\Delta u = 0 \quad \text{in} \quad G \tag{17.a}$$

with the boundary condition

$$\frac{\partial u}{\partial n} + c(u - g) = 0 \qquad \text{on} \quad \partial G_1 \tag{17.b}$$

$$\frac{\partial u}{\partial n} + \theta u = 0 \qquad \text{on} \quad \partial G_2 \tag{17.c}$$

where the domain G represents some geometrical shape of the material and one part of the boundary ∂G_1 means the surface of system structure. The given input g stands for the external excitation through the surface ∂G_1 and c is a known parameter on ∂G_1. The problem considered here is to identify the parameter θ on ∂G_2. One possible application in the problem above mentioned arises in material testing, in particular, in the area of nondestructive evaluation approach in space technology (see [13]). The parameter θ on ∂G_2 represents the conductivity and/or heat transfer which characterize the structural flaws of unvisual section of the boundary ∂G. The decomposition of ∂G into ∂G_1 and ∂G_2 is taken as $\partial G_1 = \{\xi(t) | t \in [0,\bar{t}]\}$ and $\partial G_2 = \{\xi(t) | t \in [\bar{t},1]\}$. By using Green's formula, Eq. (17) is reduced to an integral equation on the boundary curve (see [10, Ch. 2],[12])

$$\begin{aligned}
&\phi(s) - (\lambda(s))^{-1} \Big[\int_0^{\bar{t}} c(t)\ell n|\xi(s) - \xi(t)|\phi(t)\left|\frac{d\xi}{dt}\right|dt \\
&+ \int_{\bar{t}}^1 \theta(t)\ell n|\xi(s) - \xi(t)|\phi(t)\left|\frac{d\xi}{dt}\right|dt \\
&+ \int_0^1 \frac{\partial}{\partial n}\ell n|\xi(s) - \xi(t)|\phi(t)\left|\frac{d\xi}{dt}\right|dt\Big] \\
&= -(\lambda(s))^{-1} \int_0^{\bar{t}} c(t)\ell n|\xi(s) - \xi(t)|g(t)\left|\frac{d\xi}{dt}\right|dt
\end{aligned} \tag{18}$$

where

$$\phi(s) := u(\xi(s)) \quad \text{for} \quad 0 \le s \le 1. \tag{19}$$

In the above equation (18), $\lambda(s)$ is given by

$$\lambda(s) := \begin{cases} \pi & \text{for} \quad s \in [0,1]/\{t_b^i\}_{i=1}^{\ell} \\ \kappa_i & \text{for} \quad s \in t_b^i \quad i = 1,2,\cdots,\ell \end{cases}$$

where κ_i is the internal angle of the boundary at t_b^i. By setting as

$$k(s,t;\theta) = (\lambda(s))^{-1}(\tilde{c}(t,\theta)\ln|\xi(s) - \xi(t)| + \frac{\partial}{\partial n}\ln|\xi(s) - \xi(t)|)|\frac{d\xi}{dt}|$$

and

$$f(s) = -(\lambda(s))^{-1}\int_0^1 \ln|\xi(s) - \xi(t)|\tilde{g}(t)|\frac{d\xi}{dt}|dt,$$

we have the representation (1) where

$$\tilde{c}(t;\theta) := \begin{cases} c(t) & \text{for} \quad 0 \le t < \bar{t} \\ \theta(t) & \text{for} \quad \bar{t} \le t \le 1 \end{cases}$$

$$\tilde{g}(t) := \begin{cases} c(t)g(t) & \text{for} \quad 0 \le t < \bar{t} \\ 0 & \text{for} \quad \bar{t} \le t \le 1 \end{cases}$$

respectively. Although each integral in (18) becomes singular, its limit is computable in terms of its principal value. Without loss of generality, we assume that

$$\sup_{0 \le t, s \le 1} |\xi(t) - \xi(s)| < 1.$$

Moreover, the following conditions are assumed:

(A-1) $$g \in H^1(0,1) \quad \text{and} \quad \text{supp}(g) \subset \partial G_1;$$

(A-2) $$c \in H^2(0,\bar{t}) \quad \text{and}$$

$$0 < \eta_1 \le c(t) \le \eta_2 < \infty \qquad \text{a.e. in} \quad (0,\bar{t});$$

(A-3) $$\theta \in H^2(\bar{t},1) \quad \text{and}$$

$$0 < \eta_1 \le \theta(t) \le \eta_2 < \infty$$

$$\theta(\bar{t}) = c(\bar{t}) \qquad \theta(1) = c(0)$$

$$\theta'(\bar{t}) = c'(\bar{t}) \qquad \theta'(1) = c'(0).$$

Then it can be checked that the operator $K(\theta)$ and the input f satisfy the hypotheses (H-2) to (H-5) in Section 2. Hence the results of Section 3 can be applied to this problem. Since the unknown function θ in (17.c) takes its value in $H^2(0,\bar{t};R^1)$, $\theta^M(t)$ in Section 3 is simply rewritten by

$$\theta^M(t) = \sum_{i=0}^{M+1} \lambda_i^M \tilde{B}_i^M(t).$$

In order to assert the assumption (A.3), we require the following linear and inequality constraints:

$$
\begin{aligned}
\lambda_0^M &= c(\bar{t}) - \frac{h_M}{2} c'(\bar{t}) \\
\lambda_1^M &= c(\bar{t}) + \frac{h_M}{2} c'(\bar{t}) \\
\lambda_M^M &= c(0) - \frac{h_M}{2} c'(0) \\
\lambda_{M+1}^M &= c(0) + \frac{h_M}{2} c'(0) \\
0 < \beta_1 \le \lambda_i^M &\le \beta_2 < \infty \quad \text{for} \quad i = 2,\cdots,M-1,
\end{aligned}
\tag{20}
$$

where

$$h_M = \frac{1-\bar{t}}{M} .$$

The problem $(AIP)^{N,M}$ in this section is to solve

$$\min_{\lambda^M} J^N(\lambda^M) = J^N(\hat{\lambda}^{N,M}) \tag{21}$$

subject to

$$(C^N + K^N(\lambda^M))w^N = f^N \tag{22}$$

with the constraints (20), where

$$\lambda^M = (\lambda_0^M, \lambda_1^M, \cdots, \lambda_{M+1}^M).$$

The corresponding coefficient matrices C^N, $K^N(\lambda^M)$ and vector f^N in (22) are explicitly given by

$$[C^N]_{ij} := \begin{cases} B_1^N(\tilde{t}_i) + B_{N+1}^N(\tilde{t}_i) & \text{for } i = 1,\cdots,N; j = 1 \\ B_j^N(\tilde{t}_i) & \text{for } i = 1,\cdots,N; j = 2,\cdots,N-1 \\ B_N^N(\tilde{t}_i) + B_0^N(\tilde{t}_i) & \text{for } i = 1,\cdots,N; j = N \end{cases}$$

$$[K^N(\lambda^M)]_{ij} := \begin{cases} -\dfrac{1}{\lambda(\tilde{t}_i)}\displaystyle\int_0^1 T(\tilde{t}_i,t;\lambda^M)\{B_1^N(t) + B_{N+1}^N(t)\}dt \\ \qquad \text{for } i = 1,\cdots,N; j = 1 \\ -\dfrac{1}{\lambda(\tilde{t}_i)}\displaystyle\int_0^1 T(\tilde{t}_i,t;\lambda^M)B_j^N(t)dt \\ \qquad \text{for } i = 1,\cdots,N; j = 2,\cdots,N-1 \\ -\dfrac{1}{\lambda(\tilde{t}_i)}\displaystyle\int_0^1 T(\tilde{t}_i,t;\lambda^M)\{B_N^N(t) + B_0^N(t)\}dt \\ \qquad \text{for } i = 1,2,\cdots,N; j = N \end{cases}$$

$$[f^N]_j := -\frac{1}{\lambda(\tilde{t}_j)}\int_0^1 S(\tilde{t}_j,t)\tilde{g}(t)dt$$

$$\text{for } j = 1,2,\cdots,N$$

where

$$T(s,t;\lambda^M) := \{\tilde{c}(t,\lambda^M)\ell n|\xi(s) - \xi(t)|$$

$$+ \frac{\partial}{\partial n} \ell n|\xi(s) - \xi(t)|\} \left|\frac{d\xi}{dt}\right|$$

$$S(s,t) := \ell n|\xi(s) - \xi(t)| \left|\frac{d\xi}{dt}\right|$$

and where

$$\tilde{c}(t,\lambda^M) := \begin{cases} c(t) & \text{for } 0 \leq t \leq \bar{t} \\ \sum_{i=0}^{M+1} \lambda_i^N \tilde{B}_i^M(t) & \text{for } \bar{t} \leq t \leq 1 \end{cases}.$$

For a fixed N and M, the necessary condition for the optimality of $(AIP)^{N,M}$ can be obtained.

Theorem 4: Set the matrix H^M and vector ψ as

$$[H^N]_{ij} := \begin{cases} B_1^N(t_i^P) + B_{N+1}^N(t_i^P) & \text{for } i = 1,\cdots,m; j = 1 \\ B_j^N(t_i^P) & \text{for } i = 1,\cdots,m; j = 2,\cdots,N-1 \\ B_N^N(t_i^P) + B_0^N(t_i^P) & \text{for } i = 1,\cdots,m; j = N \end{cases}$$

$$[\psi]_j := \psi_j \qquad \text{for } j = 1,\cdots,N.$$

Then the necessary condition for $\hat{\lambda}^{N,M}$ to be optimal is characterized by

$$\sum_{k=0}^{M+1} v^N(\hat{\lambda}^{N,M})'[\nabla_{\lambda_k} K^N(\hat{\lambda}^{N,M})]'w^N(\hat{\lambda}^{N,M})(\lambda_k^M - \hat{\lambda}_k^{N,M}) \geq 0 \tag{23}$$

where

$$[C^N + K^N(\hat{\lambda}^{N,M})]w^N = f^N \tag{24}$$

$$[C^N + K^N(\hat{\lambda}^{N,M})]'v^N = -(H^N)'(H^N w^N(\hat{\lambda}^{N,M}) - \psi) \tag{25}$$

and where λ_k^M is an any element of $R^{(M+2)}$ with the constraint (20).

Proof: As is well-known, the necessary condition for $\hat{\lambda}^{N,M}$ to be optimal is characterized by

$$\nabla_\lambda J^N(\hat{\lambda}^{N,M}) \cdot (\lambda^M - \hat{\lambda}^{N,M}) \geq 0 \tag{26}$$

for all λ^M with the constraint (20). We note that, from (9),

$$[\nabla_\lambda J^N(\lambda^M)]_k = (w_{\lambda_k}^N(\lambda^M))'(H^N)'(H^N w^N(\lambda^M) - \psi) \qquad \text{for } k = 0,1,\cdots,M+1 \tag{27}$$

where

$$[w_{\lambda_k}^N]_j := \nabla_{\lambda_k} w_j^N(\lambda^M) \qquad \text{for } j = 1,2,\cdots,N.$$

The sensitivity equations for (20) with respect to λ_k are given by

$$[C^N + K^N(\lambda^M)]w^N_{\lambda_k} = -[\nabla_{\lambda_k} K^N(\lambda^M)]w^N(\lambda^M) \qquad (28)$$

$$\text{for} \quad k = 0,1,\cdots,M+1.$$

Hence, by introducing the adjoint equation

$$[C^N + K^N(\lambda^M)]'v^N = -(H^N)'(H^N w^N(\lambda^M) - \psi), \qquad (29)$$

we can evaluate the gradient vector by

$$[\nabla_\lambda J^N(\lambda^M)]_k = (w^N(\lambda^M))'[\nabla_{\lambda_k} K^N(\lambda^M)]'v^N(\lambda^M). \qquad (30)$$

Setting as $\lambda^M = \hat{\lambda}^{N,M}$ in (30), the substitution of (30) into (26) yields the variational inequality (23). The proof has been completed.

In the sequel, we consider numerical procedures for solving $(AIP)^{N,M}$. From Theorem 4, we can compute the gradient of the cost function using (30). Hence many optimization techniques for the constrained problem are readily applicable to our problem (see [14],[18] and their references therein). We can represent the constraint (20) as the linear equality and inequality:

$$A_1^M \lambda^M = b_1^M \qquad (31)$$

$$A_2^M \lambda^M \leq b_2^M \qquad (32)$$

where

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & \cdot & \cdot & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdot & \cdot & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdot & \cdot & 0 & 1 & 0 \\ 0 & 0 & 0 & \cdot & \cdot & 0 & 0 & 1 \end{bmatrix} \qquad b_1 = \begin{bmatrix} c(\bar{t}) - \dfrac{h_M c'(\bar{t})}{2} \\ c(\bar{t}) + \dfrac{h_M c'(\bar{t})}{2} \\ c(0) - \dfrac{h_M c'(0)}{2} \\ c(0) + \dfrac{h_M c'(0)}{2} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & & & & 0 \\ -1 & & & & \\ & 1 & & & \\ & -1 & \cdot & & \\ & & & \cdot & \\ & & & & \cdot\ 1 \\ & 0 & & & -1 \end{bmatrix} \qquad b_2 = \begin{bmatrix} \beta_2 \\ -\beta_1 \\ \vdots \\ \beta_2 \\ -\beta_1 \end{bmatrix} .$$

For the numerical results reported in this paper, we adopted the gradient projection method which is a particularly useful technique for the optimization problem with linear constraints as described in (31) and (32) (see [14],[18],[19]). The iterative algorithm by the gradient projection method can be stated as follows:

<<Numerical Algorithm>>

Step 0: Fix the number of dimensions N and M for the problem $(AIP)^{N,M}$. Set an initial value vector $\lambda^{N,M}(0)$ satisfying (31) and (32) and $i = 0$.

Step 1: Compute the gradient vector $\nabla_\lambda J^N(\lambda^{N,M}(i))$ by

$$[\nabla_\lambda J^N(\lambda^{N,M}(i))]_k = w^{N\prime}[\nabla_{\lambda_k} K^N(\lambda^{N,M}(i))]' v^N$$
$$\text{for} \quad k = 0,1,\cdots,M+1 \tag{33}$$

$$[C^N + K^N(\lambda^{N,M}(i))]w^N = f^N \tag{34}$$

$$[C^N + K^N(\lambda^{N,M}(i))]' v^N = -(H^N)'(H^N w^N(\lambda^{N,M}(i)) - \psi). \tag{35}$$

Step 2: If $A_2^M \lambda^{N,M}(i) \leq b_2^M$, set

$$\eta^{N,M}(i) = -\nabla_\lambda J^N(\lambda^{N,M}(i)) \tag{36}$$

and proceed to Step 5; otherwise, proceed to Step 3.

Step 3: Compute the current direction by

$$\eta^{N,M}(i) = -\frac{P^M \nabla_\lambda J^N(\lambda^{N,M}(i))}{|P^M \nabla_\lambda J^N(\lambda^{N,M}(i))|} \tag{37}$$

where

$$P^M = I - A_p^{M\prime}(A_p^M A_p^{M\prime})^{-1} A_p$$

and A_p^M includes the gradient of all currently active constraints associated with matrix A_2^M. If $\eta^{N,M}(i) \neq 0$, proceed to Step 4; otherwise, proceed to Step 5.

Step 4: Compute $\gamma_{min}^{(i)}$ satisfying

$$J^N(\lambda^{N,M}(i) + \gamma_{min}^{(i)}\eta^{N,M}(i)) = \min_{\gamma\in[0,\hat{\gamma}]} J^N(\lambda^{N,M}(i) + \gamma\eta^{N,M}(i)) \tag{38}$$

where $\hat{\gamma}$ is the largest step that may be taken from $\lambda^{N,M}(i)$ along $\eta^{N,M}(i)$ without violating any constraint. If $\gamma_{min}^{(i)} = \hat{\gamma}$, then add the new constraint to the matrix A_p^M and proceed to Step 5; otherwise, the new approximation to the solution is given by

$$\lambda^{N,M}(i + 1) = \lambda^{N,M}(i) + \gamma_{min}^{(i)}\eta^{N,M}(i) \tag{39}$$

and proceed to Step 6.

Step 5: Compute the vector $\delta(\lambda^{N,M})$ by

$$\delta(\lambda^{N,M}(i)) = -(A_p^M A_p^{M\prime})^{-1} A_p^M \nabla_\lambda J^N(\lambda^{N,M}(i)). \tag{40}$$

If all components of δ are nonnegative, then set

$$\hat{\lambda}^{N,M} = \lambda^{N,M}(i)$$

and terminate the computation; otherwise, delete the column of A_p^M corresponding to the smallest components of $\delta(\lambda^{N,M}(i))$.

Step 6: If $i \leq i_{max}$, then replace $i + 1$ by i and return to Step 1; otherwise, print the statement "iteration over" and stop the computation.

5. NUMERICAL EXPERIMENTS

This section is devoted to a report of our efforts on computer implementation of our techniques for the problem $(AIP)^{N,M}$ outlined in the previous section. In numerical experiments, we set the rectangular domain G with the boundary curve $\xi(t)$ given by

$$\xi(t) = (\xi_1(t), \xi_2(t)) = \begin{cases} (2t, 0) & \text{for } 0 \le t < \frac{1}{4} \\ (\frac{1}{2}, 2t - \frac{1}{2}) & \text{for } \frac{1}{4} \le t < \frac{1}{2} \\ (-2t + \frac{3}{2}, \frac{1}{2}) & \text{for } \frac{1}{2} \le t < \frac{3}{4} \\ (0, -2t + 2) & \text{for } \frac{3}{4} \le t \le 1. \end{cases}$$

Clearly, we have

$$\left|\frac{d\xi}{dt}\right| = ((\frac{d\xi_1}{dt})^2 + (\frac{d\xi_2}{dt})^2)^{1/2} = 2 \quad \text{for} \quad 0 \le t \le 1 \tag{42}$$

and we decompose the boundary ∂G into ∂G_1 and ∂G_2 by $\bar{t} = \frac{3}{4}$. The known function $c(t)$ was preassigned as

$$c(t) = 10 \qquad \text{for} \quad 0 \le t \le \frac{3}{4}. \tag{43}$$

The test input was set as

$$g(t) = 10 \qquad \text{for} \quad 0 \le t \le \frac{3}{4}. \tag{44}$$

The unknown function $\theta_0(t)$ in the numerical experiments was given by

$$\theta_0(t) = \theta_0(\tfrac{7}{4} - t) \qquad \text{for } \tfrac{3}{4} \leq t \leq 1 \tag{45}$$

$$\theta_0(t) = \begin{cases} -800t^2 + 1200t - 440 & \text{for } \frac{3}{4} \leq t < \frac{31}{40} \\ -40t + \frac{81}{2} & \text{for } \frac{31}{40} \leq t < \frac{17}{20} \\ 800t^2 - 1400t + \frac{1237}{2} & \text{for } \frac{17}{20} \leq t \leq \frac{7}{8} . \end{cases}$$

Solving the boundary element model (34) and (35), the computations of element matrix K^N and vector f^N must be carefully treated since, as indicated in Section 4, K^N and f^N involve the integrands with the weak singular kernel. In the numerical experiments, we computed these in the following way. The (i,j)-component of K^N $(i = 1,2,\cdots,N; j = 2,\cdots,N-1)$ is rewritten as

$$-\frac{4}{\pi}\Big[\sum_{\ell=1}^{N}\int_{t_{\ell-1}}^{t_\ell} \tilde{c}(t,\lambda^M)\ln|\xi(\tilde{t}_i) - \xi(t)|B_j^N(t)dt$$
$$+ \sum_{\ell=1}^{N}\int_{t_{\ell-1}}^{t_\ell} \frac{\partial}{\partial n}\ln|\xi(\tilde{t}_i) - \xi(t)|B_j^N(t)dt\Big]. \tag{46}$$

For the first integral, we decompose it into

$$\sum_{\substack{\ell=1 \\ \ell\neq i}}^{N}\int_{t_{\ell-1}}^{t_\ell} \tilde{c}(t,\lambda^M)\ln|\xi(\tilde{t}_i) - \xi(t)|B_j^N(t)dt$$

$$+\int_{t_{i-1}}^{t_i} \tilde{c}(t,\lambda^M)\ell n|\xi(\tilde{t}_i) - \xi(t)|B_j^N(t)dt.$$

The first integrand becomes well-behaved and each integral can be approximated using an appropriate Gaussian quadrature formula. On the other hand, the second integral becomes singular. In this case, we further decompose it into

$$\int_{t_{i-1}}^{t_i} \tilde{c}(t,\lambda^M)\ell n|\tilde{t}_i - t|B_j^N(t)dt$$

$$+\int_{t_{i-1}}^{t_i} \tilde{c}(t,\lambda^M)\ell n \frac{|\xi(\tilde{t}_i)-\xi(t)|}{|\tilde{t}_i - t|} B_j^N(t)dt.$$

We can compute the first term analytically. The second integrand has a removable singularity at the point $\tilde{t}_i$ since $|\frac{d\xi}{dt}| \neq 0$. Therefore, the Gaussian quadrature can again be used to approximate this integral. Similarly, the second integral in (46), which is the so-called "double layer potential," has a removable singularity at point $\tilde{t}_i$ (see [12] for more details). The element vector f^N can be computed in the same way.

Our numerical experiments were carried out as follows: First, the observed data $\psi = (\psi_1,\psi_2,\cdots,\psi_m)'$ were generated by solving the boundary element model, i.e.,

$$\psi_i = \phi^{N'}(t_i^p) \qquad (i = 1,2,\cdots,m)$$

$$[C^{N'} + K^{N'}(\theta)]\phi^{N'} = f^{N'}$$

with the true parameter $\theta = \theta_0$ for $N' \geq N$. Secondly, we implemented the numerical scheme proposed in Section 4. Experiments were carried out for the dimension $N = 40$ and $M = 10$. Namely, we solved the problem $(AIP)^{40,10}$. The initial guesses were given by

$$[\lambda^{10}(0)]_j = 10 \quad \text{for} \quad j = 1,\cdots,10.$$

In Step 1, the numerical integration of K^N and f^N can be accomplished by using the eight point Gauss-Legendre formula. In Step 2, parameters of the constraint vector b_2 in (32) were chosen as

$$\beta_1 = 2 \qquad \beta_2 = 12.$$

In Step 4, the golden section search was used for a method of optimization along a line [14]. In Step 6, we set $i_{max} = 64$.

In the numerical example, we checked the sensitivity of our estimation algorithm with respect to the number of sensors by testing experiments for a different number of sensors. From several numerical findings, we suggest that, for a couple of fixed dimension (N,M) of $(AIP)^{N,M}$, the proposed method requires at least M sensors, i.e., $m \geq M$, in order to assert the effectiveness of the proposed method. To demonstrate this, we present numerical results for the following three typical cases;

(Case 1) $m = 20$

$$t_i^p = \frac{2i+9}{80} \quad \text{for} \quad i = 1,2,\cdots,20$$

(Case 2) $m = 10$

$$t_i^p = \begin{cases} \dfrac{4i+7}{80} & \text{for } i = 1,\cdots,5 \\[2ex] \dfrac{33}{80} & \text{for } i = 6 \\[2ex] \dfrac{4i+9}{80} & \text{for } i = 7,\cdots,10 \end{cases}$$

(Case 3) $m = 4$

$$t_i^p = \begin{cases} \dfrac{12i-1}{80} & \text{for } i = 1,2 \\[2ex] \dfrac{12i+1}{80} & \text{for } i = 3,4. \end{cases}$$

The corresponding sensor locations are illustrated in Fig. 1. Our results are given in Tables 1, 2, and 3. For convenience of comparative discussions, the estimated parameter function $\theta^M(t,\lambda^{N,M})$ and true function $\theta_0(t)$ are shown in Figs. 2, 3, and 4, respectively.

6. CONCLUDING REMARKS

The computational costs in solving the identification of distributed parameter systems often tend to be quite expensive. For some class of boundary value problems (BVP), computational savings can be achieved in the use of the boundary integral equation method (BIE). Since the BIE method makes it possible to replace the BVP by certain integral equations, the application of the BIE method to the related identification problems has the effect of reducing the dimension of the problems. A couple of effective estimation algorithms by BIE methods have been proposed for identifying boundary parameters without the theoretical convergence proofs [21],[22]. In this paper, we developed the numerical method with emphasis on a theoretical framework for the boundary parameter estimation technique. To assert the convergence property, we claim the regularity of the solution for an integral equation model and also require compactness of the set of parameter functions to be identified. A spline collocation method of even degree was used to obtain a spline-based parameter space. Following the compactness ideas for parameter estimation in [4]~[8], a convergence property for the discretized solutions of $(AIP)^{N,M}$ was shown in Section 4. The efficiency of the theoretical idea was demonstrated by the numerical experiments. Our approach can be readily extended to treat optimal shape design, optimal shape control, etc.

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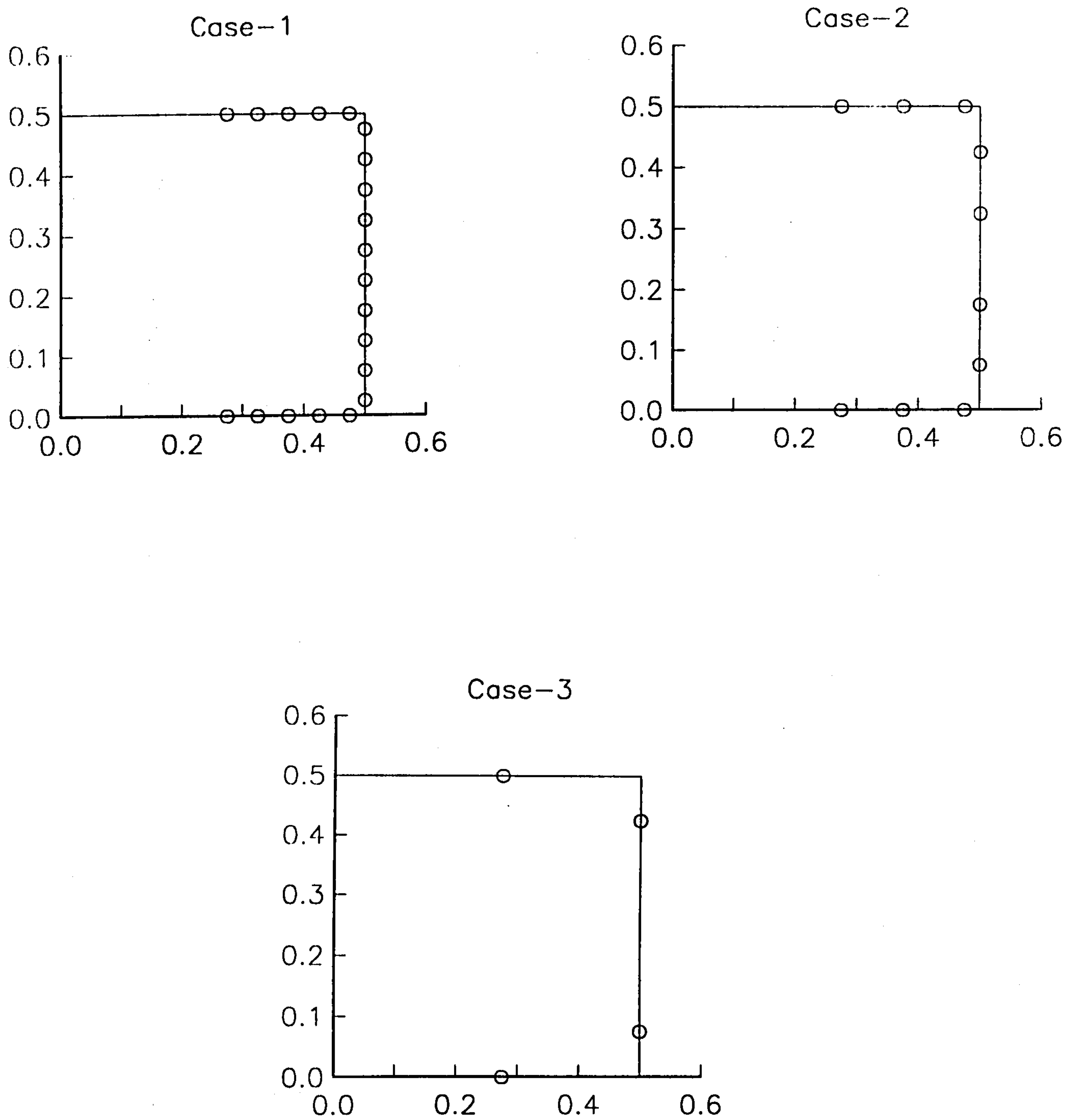

Fig. 1. An illustration of sensor locations in Cases 1, 2, and 3.

Table 1. Estimated Value of $\hat{\lambda}^{N,M}$ in Case 1.

	initial guess	iteration 8	iteration 16	iteration 32	$\hat{\lambda}^{N,M}$ (iteration 64)
$\lambda_2^{N,M}$	10.000	8.7072	8.7377	8.8241	8.8812
$\lambda_3^{N,M}$	10.000	7.7119	7.7159	7.7352	7.7494
$\lambda_4^{N,M}$	10.000	6.9597	6.9432	6.9102	6.8905
$\lambda_5^{N,M}$	10.000	6.5553	6.5277	6.4654	6.4263
$\lambda_6^{N,M}$	10.000	6.5553	6.5267	6.4655	6.4262
$\lambda_7^{N,M}$	10.000	6.9539	6.9406	6.9106	6.8902
$\lambda_8^{N,M}$	10.000	7.7028	7.7117	7.7360	7.7491
$\lambda_9^{N,M}$	10.000	8.6924	8.7319	8.8250	8.8807
Value of Cost Function	6.1742 $\times 10^{-3}$	1.5297 $\times 10^{-7}$	1.3093 $\times 10^{-7}$	7.6534 $\times 10^{-8}$	5.2906 $\times 10^{-8}$

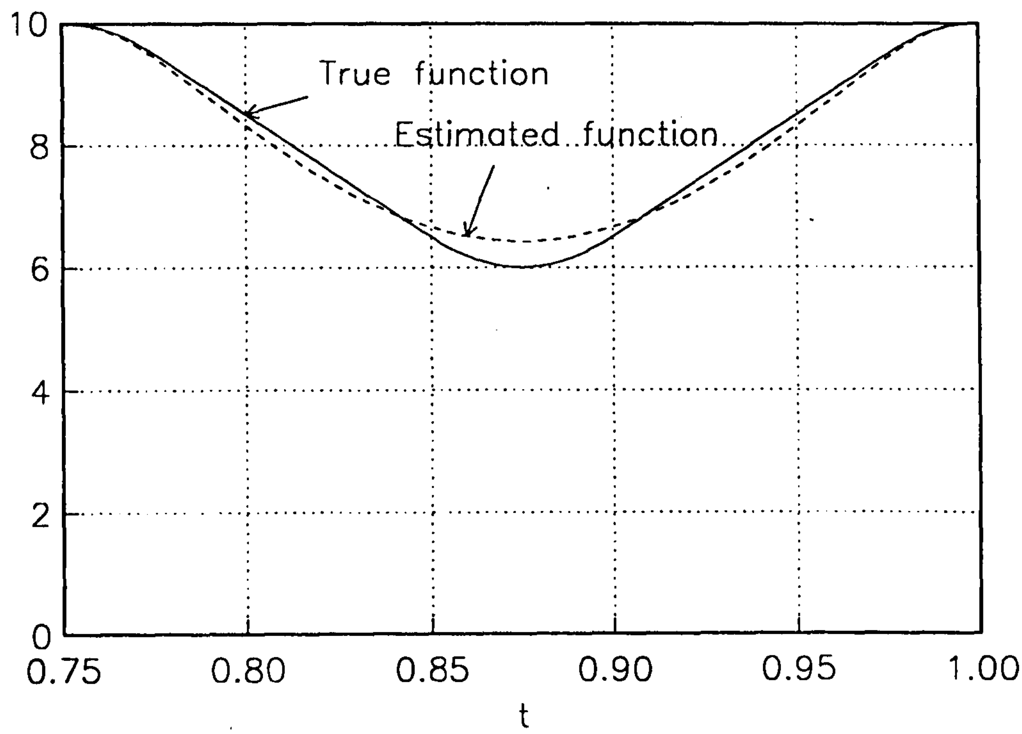

Fig. 2. True and estimated function in Case 1.

Table 2. Estimated Value of $\hat{\lambda}^{N,M}$ in Case 2.

	initial guess	iteration 8	iteration 16	iteration 32	$\hat{\lambda}^{N,M}$ (iteration 64)
$\lambda_2^{N,M}$	10.000	7.8689	8.0285	8.1654	8.3297
$\lambda_3^{N,M}$	10.000	7.5169	7.5837	7.6076	7.6438
$\lambda_4^{N,M}$	10.000	7.2489	7.2387	7.1752	7.1108
$\lambda_5^{N,M}$	10.000	7.1080	7.0479	6.9372	6.8164
$\lambda_6^{N,M}$	10.000	7.1215	7.0452	6.9370	6.8159
$\lambda_7^{N,M}$	10.000	7.2887	7.2311	7.1749	7.1094
$\lambda_8^{N,M}$	10.000	7.5817	7.5716	7.6076	7.6421
$\lambda_9^{N,M}$	10.000	7.9562	8.0133	8.1664	8.3289
Value of Cost Function	3.6011×10^{-3}	7.2368×10^{-7}	5.4085×10^{-7}	4.1514×10^{-7}	2.7980×10^{-7}

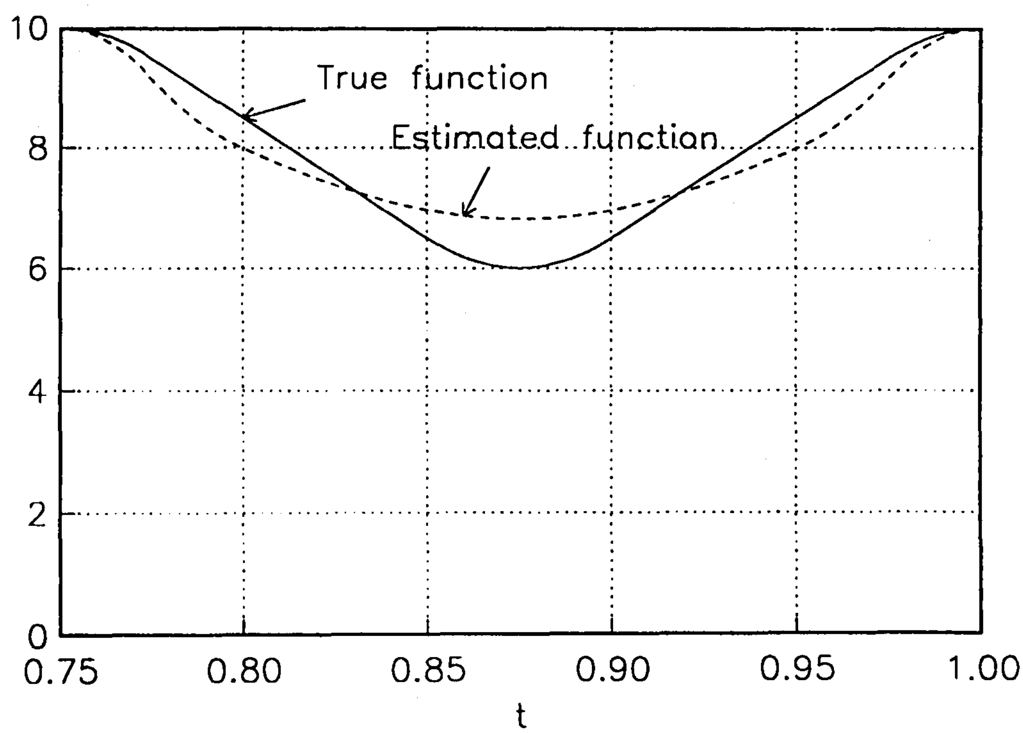

Fig. 3. True and estimated function in Case 2.

Table 3. Estimated Value of $\hat{\lambda}^{N,M}$ in Case 3.

	initial guess	iteration 8	iteration 16	iteration 32	$\hat{\lambda}^{N,M}$ (iteration 64)
$\lambda_2^{N,M}$	10.000	7.2629	7.7044	7.7634	7.9721
$\lambda_3^{N,M}$	10.000	7.4443	7.5174	7.5326	7.5681
$\lambda_4^{N,M}$	10.000	7.5814	7.3820	7.3639	7.2961
$\lambda_5^{N,M}$	10.000	7.6516	7.3108	7.2726	7.1076
$\lambda_6^{N,M}$	10.000	7.6511	7.3110	7.2722	7.1073
$\lambda_7^{N,M}$	10.000	7.5800	7.3853	7.3626	7.2684
$\lambda_8^{N,M}$	10.000	7.4419	7.5227	7.5307	7.5673
$\lambda_9^{N,M}$	10.000	7.2596	7.7119	7.7611	7.9714
Value of Cost Function	2.3216 $\times 10^{-3}$	4.8032 $\times 10^{-7}$	2.8174 $\times 10^{-7}$	2.6762 $\times 10^{-7}$	1.8701 $\times 10^{-7}$

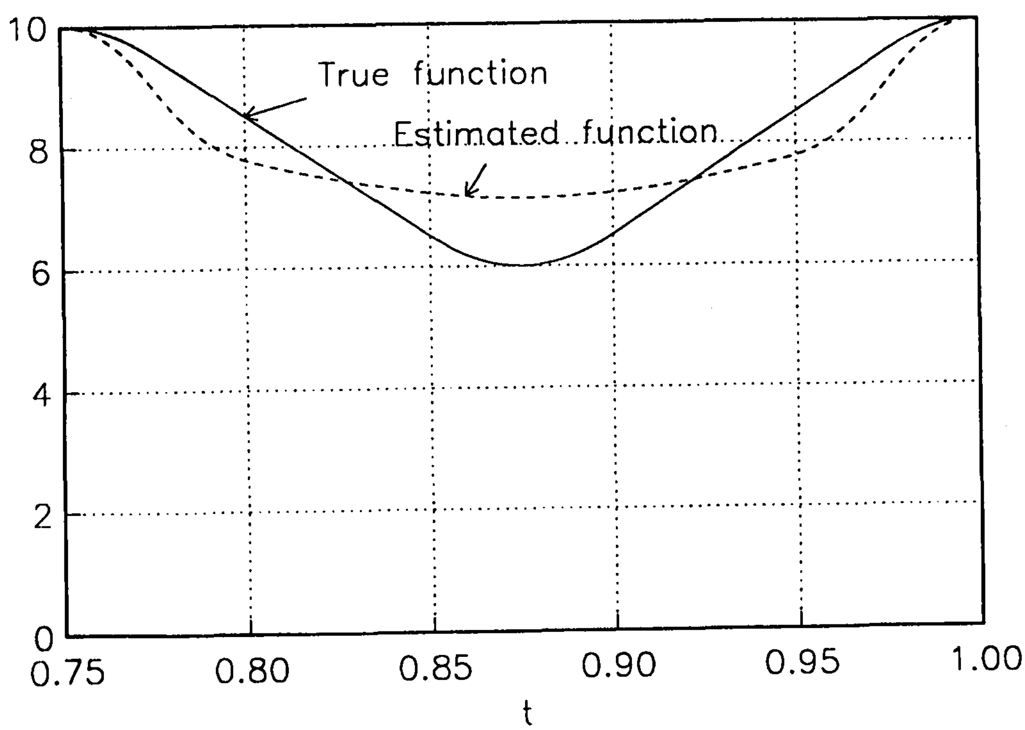

Fig. 4. True and estimated function in Case 3.

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16. Abstract

This paper is concerned with the parameter estimation for boundary integral equations of the second kind. The parameter estimation technique by using the spline collocation method is proposed. Based on the compactness assumption imposed on the parameter space, the convergence analysis for the numerical method of parameter estimation is discussed. The results obtained here are applied to a boundary parameter estimation for 2-D elliptic systems.

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