

**Generation of Flux Tube Waves in Stellar Convection Zones.  
I. Longitudinal Tube Waves**

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**ABSTRACT**

We derive the source functions and the energy fluxes for wave generation in magnetic flux tubes embedded in an otherwise magnetic field-free, turbulent, and compressible fluid. Specific results for the generation of longitudinal tube waves are presented.

*Subject headings:* hydromagnetics - magnetic flux tubes - stars: turbulence - stars: late-type stars

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## 1. INTRODUCTION

Wave generation by turbulent motions in the outer convection zones of stars has long been thought to be central to the heating of stellar chromospheres and coronae (e.g., Biermann 1946 and Schwarzschild 1948). The early suggestions were followed by number of detailed studies in which the generation of acoustic waves (Lighthill 1952, Proudman 1952, Stein 1967, Renzini *et al.* 1977 and Bohn 1980, 1984) as well as MHD waves (Kulsrud 1955, Osterbrock 1961, Parker 1964, Kuperus 1965, Stein 1981, Ulmschneider and Stein 1982, Musielak and Rosner 1987a, b) was considered. These latter calculations are all based on the assumption of a uniform and weak background magnetic field, an assumption which is contradicted by solar and stellar observational evidence for inhomogeneous and locally strong magnetic fields (cf., Harvey 1977, Stenflo 1978, Robinson *et al.* 1980); thus, at least the solar magnetic field has instead a "flux tube" structure, and flux tube waves carrying the wave energy away from the convection zone may well be responsible for heating at least some portion of the outer atmospheric layers (cf., Spruit and Roberts 1983). In addition, it has been shown that the acoustic and MHD energy fluxes generated in stellar convection zones -- based on calculations which assume homogeneous magnetic fields -- are insufficient to explain the UV and soft X-ray fluxes observed by the *IUE* and *Einstein* Observatories (Linsky 1981, Vaiana *et al.* 1981, Ulmschneider and Bohn 1981, Rosner *et al.* 1985, Musielak and Rosner 1987b).

In this paper, we derive the source function and energy flux for flux tube waves. We consider magnetic flux tubes embedded in a magnetic field-free, turbulent and compressible medium, and assume that the tubes are thin and oriented vertically; the latter assumption allows us to separate the generation of compressional tube waves from incompressional waves. In the present paper (Paper I), we concentrate on the generation of longitudinal tube waves, discuss the wave propagator and the relevant critical frequencies, and finally discuss the dependence of energy fluxes on the parameters which enter into the calculations. The generation of transverse tube waves and the application of the results to late-type stars will be treated in following papers.

The plan of our paper is as follows: The MHD equations and the basic formulation are presented in Section 2; the inhomogeneous wave equation and its solutions are given in Section 3; the energy fluxes for longitudinal tube waves are described in Section 4; and the model parameter dependence of energy fluxes and their discussion are to be found in Section 5. A summary of our new results and our conclusions are given in Section 6. Two appendices contain mathematical details, which amplify discussions in the main text.

## 2. MHD EQUATIONS AND THE BASIC FORMULATION

In this section, we discuss the basic equations of motion, and develop the formalism for calculating the generation rate for magnetic tube waves. In order to simplify the problem to the essentials, we shall assume that the fluid is locally isothermal, that the gas pressure is a scalar, and that displacement currents and electrostatic forces may be neglected; furthermore, it is straightforward to show that (as long as shock formation does not occur) dissipation by molecular viscosity and Ohmic diffusion is negligible for the problem at hand. In the following, we present the linearized magnetohydrodynamic (MHD) wave equations, the basic assumptions for flux tubes, and finally the set of equations used to calculate the rate of tube wave generation.

### (a) The Linearized MHD Equations

Our assumptions lead to the ideal MHD equations, which may be written in the following linearized form:

$$\frac{\partial}{\partial t} \rho + \nabla \cdot (\rho_0 \mathbf{u}) = N_1, \quad (2.1)$$

$$\frac{\partial}{\partial t} p + \mathbf{u} \cdot \nabla p_0 - V_s^2 \left[ \frac{\partial}{\partial t} \rho + \mathbf{u} \cdot \nabla \rho_0 \right] = N_2, \quad (2.2)$$

$$\frac{\partial}{\partial t} \mathbf{u} + \frac{1}{\rho_0} \nabla p - \frac{\rho}{\rho_0} \mathbf{g} - \frac{1}{4\pi\rho_0} [(\nabla \times \mathbf{B}') \times \mathbf{B}_0] = \mathbf{N}_3, \quad (2.3)$$

and

$$\frac{\partial}{\partial t} \mathbf{B}' - \nabla \times (\mathbf{u} \times \mathbf{B}_0) = \mathbf{N}_4, \quad (2.4)$$

where  $\rho_0, p_0, B_0$  refer to the unperturbed atmosphere,  $\rho, p, \mathbf{u}, \mathbf{B}'$  are the perturbations of density, pressure, velocity and magnetic field, respectively, and  $V_s$  ( $\equiv [\gamma RT/\mu]^{1/2}$ ) is the sound velocity; the magnetic field is assumed to be potential, and the equation of motion is simplified by the assumption that the background atmosphere is in static equilibrium.

All terms linear in the perturbations in equations (2.1) - (2.4) are written on the LHS; these terms define the wave propagation operators for MHD waves. However, terms quadratic in the perturbations are collected on the RHS, and are treated as known quantities described by a given flow (Lighthill 1952, Stein 1967). The latter terms determine the source function responsible for the MHD wave generation

(Musielak and Rosner 1987a), and are defined as follows:

$$N_1 = -\nabla \cdot (\rho \vec{u}), \quad (2.5)$$

$$N_2 = -\vec{u} \cdot \nabla p + V_s^2 \vec{u} \cdot \nabla \rho, \quad (2.6)$$

$$\vec{N}_3 = -\frac{\rho}{\rho_0} \frac{\partial}{\partial t} \vec{u} - \frac{\rho}{\rho_0} (\vec{u} \cdot \nabla) \vec{u} + \frac{1}{4\pi\rho_0} (\nabla \times \vec{B}) \times \vec{B}, \quad (2.7)$$

$$\vec{N}_4 = \nabla \times (\vec{u} \times \vec{B}). \quad (2.8)$$

*(b) The Thin Flux Tube Approximation*

In order to calculate the energy fluxes for flux tube waves, we assume that a vertically-oriented magnetic flux tube is embedded in a stratified and non-magnetized medium, and that all unperturbed and perturbed quantities depend on  $z$  and  $t$  alone; the  $z$ -axis is identified with the vertical direction (i.e.,  $\vec{g} = -g\hat{z}$ ) which, in our approach, is just the tube axis. We then obtain

$$\rho = \rho(z, t) + O(\epsilon), \quad (2.9)$$

$$p = p(z, t) + O(\epsilon), \quad (2.10)$$

$$\vec{u} = u_z(z, t) \hat{z} + O(\epsilon), \quad (2.11)$$

$$\vec{B} = B_z(z, t) \hat{z} + O(\epsilon), \quad (2.12)$$

where  $\epsilon$  is defined as the ratio of the tube radius  $R_t$  to the tube length  $L_t$ ; for  $\epsilon \ll 1$ , the tube can be treated in the thin flux tube approximation, an approximation which allows us to consider all unperturbed and perturbed quantities to zeroth-order (see, for example, Roberts and Webb 1978). In this approach, the magnetic field within the flux tube is essentially axial, and is described by  $\vec{B}_0 = B_0(z)\hat{z}$ ; note that the solenoidal condition does not restrict  $B_0(z)$ , but does however allow one to calculate the horizontal components of the magnetic field once the vertical component is known.

We further assume that the cross-section of the tube is always circular, and that the thin flux tube is in temperature equilibrium with the surroundings, so that both density gradients inside  $[\rho_o(z)]$  and outside  $[\rho_e(z)]$  the tube are described by the same density scale height; hence, the vertical density variation is given by

$$\rho_{o,e}(z) = \rho_{o,e} \exp \left[ -\frac{z}{H_p} \right], \quad (2.13)$$

where  $H_p$  ( $\equiv V_s^2/\gamma g$ ) is the density scale height, and is identical to the pressure scale height for an isothermal atmosphere. The vertical magnetic field variation is given by

$$B_o(z) = B_{oo} \exp \left[ -\frac{z}{H_b} \right], \quad (2.14)$$

where the magnetic scale height  $H_b$  can be defined as

$$H_b = \frac{V_A^2}{g} \frac{\rho_o}{\rho_e - \rho_o} = 2H_p, \quad (2.15)$$

and where  $V_A^2$  ( $\equiv B_o^2/4\pi\rho_o$ ) is the square of the Alfvén velocity.

In addition, if the total (gas plus magnetic) pressure is constant across the tube, then we must have (see Roberts and Webb 1978)

$$p_e(z) = p_o(z) + \frac{1}{8\pi} B_o^2(z), \quad (2.16)$$

and

$$\frac{B_o^2(z)}{B_o^2(0)} = \frac{p_o(z)}{p_o(0)} = \frac{p_e(z)}{p_e(0)}, \quad (2.17)$$

which allows us to easily calculate the structure of the tube when the boundary values for the pressure and magnetic field are given. Finally, we assume that the tube is untwisted, so that we ignore processes – such as the generation and propagation of torsional tube waves – as well as instabilities such as the kink instability. Such transverse wave motions will be considered in a subsequent paper.

(c) *The Flux Tube Equations*

Combining the continuity and induction equations, and using equations (2.9) - (2.12) and (2.13) - (2.15), the set of MHD equations (2.1) - (2.4) can be re-written as

$$\frac{\partial}{\partial t} \rho_1 - \frac{\partial}{\partial t} B_1 + \hat{W}_2 u_z = n_1, \quad (2.18)$$

where  $n_1$  contains the nonlinear terms, and is given by  $n_1 = -(1/\rho_o)N_1 - (1/B_o)N_{4z}$ ;

$$\frac{\partial}{\partial t} p_1 - V_s^2 \frac{\partial}{\partial t} \rho_1 + W_z u_z = n_2, \quad (2.19)$$

where  $n_2 = (1/\rho_o)N_2$ ;

$$\frac{\partial}{\partial t} u_z + \hat{W}_1 p_1 + \rho_1 g = n_3; \quad (2.20)$$

where  $n_3 = N_{3z}$ ;  $N_{3z}$  and  $N_{4z}$  are the z-components of  $\vec{N}_3$  and  $\vec{N}_4$  (equations 2.7 and 2.8), respectively. In addition,  $\rho_1 = \rho/\rho_o$ ,  $p_1 = p/\rho_o$ , and  $B_1 = B_z/B_o$  are the new dimensionless perturbations; the operators  $\hat{W}_1$ ,  $\hat{W}_2$  and  $W_z$  are defined by

$$\hat{W}_1 \equiv \frac{\partial}{\partial z} - \frac{1}{H_p}, \quad \hat{W}_2 \equiv \frac{\partial}{\partial z} - \frac{1}{2H_p}, \quad W_z \equiv (\gamma - 1)g. \quad (2.21)$$

The set of equations presented above (plus the horizontal pressure balance equation; see next subsection) fully describes vertical motions inside the flux tube: The non-linear terms  $n_1$ ,  $n_2$  and  $n_3$  can be treated as the source of these motions, and can in principle be determined once the driving turbulent flow is specified. Such an approach has been followed by Stein (1968) for the purely acoustic case, and by Musielak and Rosner (1987a) for the case of homogeneous magnetic fields. As discussed below, we assume in our present (strong magnetic flux tube field) case that these internal sources are suppressed by the internal magnetic field, so that the wave motions inside the tube are driven from the outside alone. If there are no turbulent motions within the tube, the equations describe the propagation of longitudinal waves along the tube (see Roberts and Webb 1978 for comparison), and can be easily solved since all coefficients in these equations are constant. To close this set of equations, we need a relationship between the total pressure (gas and magnetic) inside the tube and the external pressure.

(d) *The Horizontal Pressure Balance*

The relation between the variable fluid pressure applied to the undisturbed tube boundary by the external turbulence (or any other pressure variations) and the internal fluid pressure can be written as (Parker 1979)

$$p_i(z,t) + \frac{1}{8\pi} B_i^2(z,t) = P_e(z,t), \quad (2.22)$$

where  $p_i$  and  $B_i$  are the gas pressure and the magnetic field inside the tube, respectively. The pressure  $P_e$  outside the tube is defined by

$$P_e = p_e(z) + p_{arb}(z,t) + p_{ie}(z,t), \quad (2.23)$$

where  $p_e$  is the gas pressure of the external fluid in the absence of any turbulence;  $p_{arb}(z,t)$  is the pressure of the external turbulence and  $p_{ie}(z,t)$  is the disturbance of the external pressure due to the back-reaction of the moving tube. The last term disappears if the tube is at rest with respect to its surroundings; this term will be neglected in our calculations. Linearizing equation (2.22) and using equation (2.16), one finds

$$p_1 + V_A^2 B_1 + \frac{1}{2} V_A^2 B_1^2 = \frac{1}{\rho_0} p_{arb}, \quad (2.24)$$

where the variable fluid pressure  $p_{arb}$  caused by the external turbulence can be defined as

$$p_{arb} = \frac{1}{2} \rho_e u_t^2, \quad (2.25)$$

and where  $u_t$  is the turbulent velocity.

Using the equipartition energy to calculate  $B_1^2$ , and with the help of equation (2.23), one obtains

$$p_1 + V_A^2 B_1 = n_4, \quad (2.26)$$

where

$$n_4 = \frac{\rho_e}{2\rho_0} (u_x^2 + u_y^2), \quad (2.27)$$

and where  $u_x$  and  $u_y$  are the x- and y-components of the turbulent velocity, respectively.

The set of tube equations (2.18) through (2.21), together with equation (2.26), fully describe the generation (due to the turbulent motions outside and possibly inside of the tube) and the propagation of longitudinal tube waves; this set of equations can be written as a single inhomogeneous wave equation, and can be solved by Fourier-transforming this equation. This problem will be addressed in the next section.

### 3. THE INHOMOGENEOUS WAVE EQUATION

In this section, we derive and solve the inhomogeneous wave equation for longitudinal tube waves. We also calculate the source function, and discuss the cutoff frequencies for longitudinal tube waves.

#### (a) The Wave Propagator

We eliminate the density, magnetic field and velocity perturbations from equations (2.18) - (2.21) and (2.26), and obtain the inhomogeneous wave equation for the pressure perturbations in the form

$$\left[ \frac{\partial^2}{\partial t^2} - V_t^2 \frac{\partial^2}{\partial z^2} + 2 V_t (\omega_{\alpha} - \omega_{br}) \frac{\partial}{\partial z} + \beta_e \omega_{ct}^2 \right] p_1 = S_p(z, t), \quad (3.1)$$

where the characteristic velocity for longitudinal tube waves (Defouw 1976) is given by

$$V_t^2 = \frac{V_A^2 V_s^2}{V_A^2 + V_s^2}, \quad (3.2)$$

and

$$\beta_e \equiv \frac{\rho_e}{\rho_o} = 1 + \frac{\gamma}{2} \frac{V_A^2}{V_s^2}; \quad (3.3)$$

we also define the three critical frequencies  $\omega_{\alpha}$ ,  $\omega_{br}$ , and  $\omega_{ct}$ ,

$$\omega_{\alpha} \equiv \frac{V_t}{2 H_p}, \quad \omega_{br} \equiv \frac{V_t}{2 H_b}, \quad \omega_{ct} \equiv \frac{V_t}{V_A} \frac{g}{V_s} (\gamma - 1)^{1/2} = \frac{V_t}{V_A} \omega_{bv}, \quad (3.4)$$

where  $\omega_{bv}$  is the Brunt-Vaisala frequency. Note that for intense magnetic flux tubes ( $V_s \ll V_A$ ),  $\omega_{ct}$  becomes the magnetic Brunt-Vaisala frequency; however, for  $V_s \gg V_A$ , the characteristic tube velocity  $V_t \approx V_A$  and  $\omega_{ct} \approx \omega_{bv}$ .

In equation (3.1), all the non-linear terms are collected on the RHS, where they become the source function for the longitudinal wave generation. We postpone the calculations of these source terms to the next subsection, and provide here only the definition of the source function

$$S_p(z, t) = \left[ \frac{V_t}{V_s} \right]^2 \left[ \frac{\partial^2}{\partial t^2} - g \hat{W}_2 \right] \left[ \left[ \frac{\partial}{\partial t} \right]^{-1} (V_s^2 n_1 + n_2) + \left[ \frac{V_s}{V_A} \right]^2 n_4 \right] +$$

$$+ (V_r^2 \hat{W}_2 + W_g) \left[ n_3 - g \left[ \frac{\partial}{\partial t} \right]^{-1} n_1 - \frac{g}{V_\lambda^2} n_4 \right]. \quad (3.5)$$

To eliminate the first-order space derivative from the inhomogeneous wave equation (equation 3.1), we make the following transformation

$$p_1 = p_2 \left[ \frac{B_o}{\rho_o} \right]^{1/2}, \quad (3.6)$$

and obtain the inhomogeneous wave equation in the form

$$\left[ \frac{\partial^2}{\partial t^2} - V_r^2 \frac{\partial^2}{\partial z^2} + \Omega_i^2 \right] p_2 = S_i(z, t), \quad (3.7)$$

where

$$S_i(z, t) = \left[ \frac{\rho_o}{B_o} \right]^{1/2} S_p(z, t) \quad (3.8)$$

and

$$\Omega_i^2 = (\omega_{at} - \omega_{bt})^2 + \beta_e \omega_{ct}^2; \quad (3.9)$$

with help of equation (2.15), one obtains instead

$$\Omega_i^2 = \frac{1}{4} \omega_{at}^2 + \beta_e \omega_{ct}^2. \quad (3.10)$$

Finally, after some algebraic manipulations, equation (3.10) can be written in the form given for the first time by Defouw (1976)

$$\Omega_i^2 = \frac{V_r^2}{H_p^2} \left[ \frac{9}{16} - \frac{1}{2\gamma} + \frac{V_r^2}{V_\lambda^2} \frac{\gamma - 1}{\gamma^2} \right]. \quad (3.11)$$

The LHS of equation (3.7) is the propagation operator for the tube waves; this relation allows for cross-sectional variations. In this case, the gas pressure is the principal restoring force. The waves which result are longitudinal tube waves, which may be viewed as acoustic waves propagating along the tube, but modified by the tube geometry. As a result of this modification, the characteristic phase velocity equation (3.2) and the critical frequency for the vertical propagation (equations 3.10 and 3.11) are different from those for acoustic waves. By studying the dispersion relation (which is global in the

approach considered here), it can be shown that  $\Omega_t$  is the cutoff frequency for longitudinal tube waves, so that the waves cannot propagate if their wave frequency is either lower than or equal to this cutoff. As shown by equation (3.10), the cutoff frequency for longitudinal waves is not as simple as for acoustic waves, and depends on both density and magnetic field scale heights, as well as on the Brunt-Vaisala frequency modified by the tube geometry; note, however, that the value of the critical frequency  $\Omega_t$  is always comparable to the acoustic cutoff frequency (see equation 3.11). The tube cutoff frequency reaches a maximum when the tube structure is entirely dominated by the magnetic pressure (e.g., when the gas pressure inside the tube is negligible); in this limit, however, the longitudinal tube waves cannot propagate (this point will be further discussed in section 5b).

(b) *The Source Function*

To calculate the source function defined by equation (3.5), we make one additional assumption, namely that there are no turbulent motions inside the thin flux tube; this *Ansatz* simplifies our calculations substantially since  $n_1 = n_2 = n_3 = 0$ . Under this assumption, it is only the external turbulence which is responsible for the wave generation; this can be justified if we recall that strong magnetic fields (such as those characterizing the thin flux tube we are considering) will tend to suppress turbulent motions within the tube. Equation (3.8) can then be written in the following form

$$S_t(z,t) = \frac{1}{2(\rho_o B_o)^{1/2}} \left[ \frac{V_t}{V_A} \right]^2 \left[ \frac{\partial^2}{\partial t^2} - 2g \left[ \frac{\partial}{\partial z} + \frac{1}{H_b} \right] - \omega_{bt}^2 \right] \rho_e (M_{xx} + M_{yy}), \quad (3.12)$$

where  $M_{xx} \equiv u_x^2$  and  $M_{yy} \equiv u_y^2$ .

We now expand the source function  $S_t(z,t)$  in multipoles (Unno 1964), and obtain

$$S_t(z,t) = \frac{1}{2(B_o)^{1/2}} \beta_s^{1/2} \left[ \beta_t^2 \frac{\partial^2}{\partial t^2} - \frac{4}{\gamma} \beta_s^2 \omega_{as} V_t \frac{\partial}{\partial z} - \omega_{ct}^2 \right] \rho_e^{1/2} (M_{xx} + M_{yy}), \quad (3.13)$$

where  $\beta_t \equiv V_t/V_A$  and  $\beta_s \equiv V_s/V_A$ .

The source function does not depend on the second derivative of  $M_{xx}$  or  $M_{yy}$  (no quadrupole emission!), but does depend, however, on the first derivative with respect to  $z$  (dipole emission) and the monopole source term ( $\omega_{ct}^2$ ), as well as on the time-dependent term. This latter term becomes negligible for stationary or quasi-stationary turbulence. It is interesting that the source function does not depend on the cutoff frequency for longitudinal tube waves, and instead depends on both tube critical frequencies  $\omega_{as}$  and  $\omega_{ct}$ . The latter fact has important consequences for wave generation (see section 5c), and distinguishes tube wave generation from acoustic wave generation (Stein 1967); for purely acoustic waves, both the emission source and the propagation operator depend on the same critical frequencies (the

acoustic cutoff and the Brunt-Vaisala frequency).

(c) *The Solution*

Having obtained the source function, we Fourier-transform equations (3.7) and (3.12) in one dimension, using

$$[p_2(z, t), S_t(z, t)] = \iint dk' d\omega' [p_2(k', \omega'), S_t(k', \omega')] \exp[ i(\omega't - k'z) ], \quad (3.14)$$

where

$$[p_2(k, \omega), S_t(k, \omega)] = \frac{1}{(2\pi)^2} \iint dz' dt' [p_2(z', t'), S_t(z', t')] \exp[ -i(\omega t' - kz') ], \quad (3.15)$$

and obtain the solution for the pressure perturbations emitted by the turbulent motions,

$$p_2(k, \omega) = - \frac{S_t(k, \omega)}{\omega^2 - \Omega_c^2 - k^2 V_t^2}, \quad (3.16)$$

where  $S_t(k, \omega)$  is the Fourier transform of the source function given by equation (3.15). To calculate the explicit form of the source function  $S_t(k, \omega)$ , we integrate equation (3.15) by parts, and obtain

$$S_t(k, \omega) = \frac{1}{8\pi^2} \beta_e \left[ \frac{\rho_o}{B_o} \right]^{1/2} \iint dz' dt' \left[ \beta_t^2 \omega^2 + \frac{4}{\gamma} ik \beta_t^2 \omega_{\alpha} V_t + \omega_{\alpha}^2 \right] \times \\ \times (M_{xx}' + M_{yy}') \exp[ -i(\omega t' - kz') ]. \quad (3.17)$$

The solution of the inhomogeneous wave equation given above will be used to calculate the energy fluxes emitted as a result of the interaction between the turbulent motions and the flux tube. Note that for  $S_t(k, \omega) = 0$ , the solution given by equation (3.16) leads to a dispersion relation which describes the propagation of longitudinal tube waves, and the critical frequency  $\Omega_c$  becomes the cutoff frequency.

#### 4. ENERGY FLUXES FOR LONGITUDINAL TUBE WAVES

In this section, we derive the final expression for the energy fluxes emitted as longitudinal tube waves. We begin with the definition of energy flux for the flux tube geometry and then present and discuss the final results.

##### (a) The Mean Energy Flux

The energy flux can be calculated by using the energy conservation principle, which gives

$$\langle F(z,t) \rangle = \langle \rho u_z A_r \left( \frac{1}{2} u_z^2 + W \right) \rangle, \quad (4.1)$$

where  $A_r$  is the cross-sectional area of the tube and  $W$  is the specific enthalpy; note that the mean energy flux defined above is multiplied by the cross-section of the tube, and therefore has dimension [ $ergs s^{-1}$ ]. It should also be noted that within the thin flux tube approximation, the magnetic terms in equation (4.1) cancel one another, and that the energy flux does not depend on the magnetic energy; this reflects the fact that the gas pressure is the principal restoring force.

Expanding in a perturbation series, and considering second-order quantities only, one obtains

$$\langle F(z,t) \rangle = \frac{5}{2} \frac{p_o}{\rho_o} A_o \langle \rho u_z \rangle + \frac{5}{2} p_o \langle A u_z \rangle + A_o \langle p u_z \rangle, \quad (4.2)$$

where  $A$  is the perturbation of the tube cross-section. The first term in equation (4.2) can be neglected as there is no net flow through the boundary of the tube. One may estimate both remaining terms in equation (4.2) using the solutions given by Herbold *et al.* (1985),

$$\frac{5}{2} \frac{p_o u_z A}{\rho u_z A_o} = \frac{5}{2} \frac{V_r^2}{\gamma V_A^2} = \frac{5}{2\gamma} \beta_r^2. \quad (4.3)$$

Finally, by combining equations (4.2) and (4.3), one obtains

$$\langle F(z,t) \rangle = A_o (1 + \delta) \langle p u_z \rangle, \quad (4.4)$$

where  $\delta = 5\beta_r^2/2\gamma$ .

Using the relation between the pressure perturbations  $p$  and  $p_2$  (equation 3.6) and replacing  $u_z$  by  $u_z^*$ , the energy flux can be expressed in terms of  $p_2$  and written in the form

$$\langle F(z,t) \rangle = (\rho_o B_o)^{1/2} A_o (1 + \delta) \langle p_2 u_z^* \rangle, \quad (4.5)$$

where the velocity perturbation  $u_z^*$  is calculated from equations (2.18) - (2.21) and (2.26), and is given by

$$u_z^* = - \left[ \frac{B_o}{\rho_o} \right]^{1/2} \left[ 1 + \omega_{bv}^2 \left[ \frac{\partial}{\partial t} \right]^{-2} \right]^{-1} \left[ \frac{\partial}{\partial t} \right]^{-1} \left[ \frac{\partial}{\partial z} + \lambda \right] p_2^*, \quad (4.6)$$

where

$$\lambda \equiv \frac{4 - \gamma}{4\gamma} \frac{1}{H_o}. \quad (4.7)$$

Now, we replace  $u_z^*$  in equation (4.5) by (4.6) and obtain

$$\langle F(z,t) \rangle = - \Phi_m (1 + \delta) \langle p_2 \left[ \frac{\partial}{\partial t} \right]^{-1} \frac{\left[ \frac{\partial}{\partial z} + \lambda \right]}{\left[ 1 + \omega_{bv}^2 \left[ \frac{\partial}{\partial t} \right]^{-2} \right]} p_2^* \rangle, \quad (4.8)$$

where  $\Phi_m$  ( $\equiv A_o B_o$ ) is the magnetic flux.

To evaluate the mean energy flux given by the above equation, we express  $p_2(z,t)$  and  $p_2^*(z,t)$  in terms of its Fourier transform (see equation 3.14), and obtain

$$\begin{aligned} \langle F(z,t) \rangle = & \Phi_m (1 + \delta) \iiint \int dk' dk'' d\omega' d\omega'' \omega'' \frac{k'' - i\lambda}{\omega'^2 - \omega_{bv}^2} \times \\ & \times p_2(k', \omega') p_2^*(k'', \omega'') \exp[i(\omega' - \omega'')t - i(k' - k'')z]. \end{aligned} \quad (4.9)$$

We take the time average of the mean energy flux by performing the integration over time  $T_o$ , and then obtain

$$L_t(z, \omega) \equiv \left\langle \frac{\partial}{\partial \omega} \langle F(z,t) \rangle \right\rangle_{T_o} = \frac{2\pi\Phi_m}{T_o} (1 + \delta) \left[ \frac{1}{\omega V_t^4} \right] \beta_{bv}^2 I_{xx}(z, \omega), \quad (4.10)$$

where

$$I_{\alpha}(k, \omega, z) = \iint dk' dk'' (k'' - i\lambda) \frac{S_i(k', \omega)}{k'^2 - k^2} \frac{S_i^*(k'', \omega)}{k''^2 - k^2} \exp[-i(k' - k'')z], \quad (4.11)$$

$$\beta_{bv}^2 = \frac{\omega^2}{\omega^2 - \omega_{bv}^2}, \quad k^2 = \frac{\omega^2 - \Omega_i^2}{V_i^2}. \quad (4.12)$$

and where  $\langle \rangle_{T_o}$  refers to a temporal average over a time scale  $T_o$ . Note that  $L_i(k, \omega, z)$  does not have the standard energy flux dimension, but rather has dimensions [ $ergs s^{-1} Hz^{-1}$ ].

(b) *The Longitudinal Wave Luminosity*

To obtain explicit forms for the energy fluxes emitted in the form of longitudinal waves, we must evaluate the asymptotic Fourier transform at large  $z$  (Appendix A), and calculate the product of the individual source functions and their conjugates. We transform the coordinates in equation (3.17) to average positions and times of the interacting turbulent eddies and then evaluate the turbulence velocity correlations by assuming that the fourth-order correlations can be reduced to second-order correlations (cf., Batchelor 1953). We calculate the one-dimensional convolutions (Appendix B) over the turbulence spectrum by assuming the spectrum to be a separable product of a frequency-independent energy spectrum and a frequency factor (see section 5a and Appendix B). Finally, after some algebra, we obtain the wave luminosity spectrum [ $ergs s^{-1} Hz^{-1}$ ],

$$L_i(k, \omega, z) = \frac{8\pi^3 \Phi_m}{T_o} (1 + \delta) \frac{\beta_{bv}^2}{\omega V_i^4} \frac{k - i\lambda}{k^2} |S_i(k, \omega)|^2 \sin^2(kz - \phi), \quad (4.13)$$

where  $\phi$  is an arbitrary phase (henceforth assumed to be zero). The  $\sin^2(kz - \phi)$  term appears as a result of squeezing the tube at regular intervals of  $z + n\pi/k$  along the tube; at these points, the efficiency of longitudinal wave generation is highest. In addition,  $|S_i(k, \omega)|^2$  is the product of the individual source function and its conjugate, and is given (from 3.17) by

$$|S_i(k, \omega)|^2 = \frac{T_o Z_o}{2\pi^2} \frac{\rho_o}{B_o} \beta_e^2 J_c(k, \omega) \left[ (\beta_i^2 \omega^2 + \omega_{\alpha}^2)^2 + \frac{16}{\gamma^2} \beta_i^4 \omega_{\alpha}^2 k^2 V_i^2 \right], \quad (4.14)$$

and where the convolution integral  $J_c(k, \omega)$  is defined (Appendix B) by

$$J_c(k, \omega) = \frac{1}{16} \int_{-\infty}^{\infty} dk' E_z(k') E_z(q) g(k, k', \omega), \quad (4.15)$$

where  $k'$  is the wave number of a turbulent eddy, and  $q = k - k'$ . As is customary,  $E_2(q_i)$  and  $E_2(q)$  denote the one-dimensional turbulence energy spectra evaluated at the two distinct wave numbers  $q_i$  and  $q$ . Following Heinze (1975), we note that the one-dimensional (1-D) energy spectrum  $E_2(q)$  is connected to the three-dimensional (3-D) energy spectrum  $E(k')$  by the relation

$$E_2(q) = \frac{1}{2} \int_q^{\infty} dk' \frac{E(k')}{k'} \left[ 1 + \frac{q^2}{k'^2} \right], \quad (4.16)$$

where  $k'$  is the wave number of all turbulent motions which contribute to the 1-D energy spectrum. In addition, the function  $g(k, k', \omega)$  is expressed by the frequency factor. Explicit forms for both the energy spectrum  $E(k')$  and the frequency factor are given in subsection 5a below.

The imaginary term in equation (4.13) can be neglected, as it vanishes upon integration over  $\omega$ . In addition, we spatially average  $L_1(z, z)$  over height  $Z_0 \ll 1/k_1$  by performing a spatial integration of equation (4.13), and obtain the height-dependent contribution to the monochromatic wave luminosity [ergs  $s^{-1}$  Hz $^{-1}$ ] for a given flux tube,

$$\begin{aligned} L_{1,z_0}(k, \omega, z) &\equiv \left\langle \frac{\partial}{\partial z_0} L_1(k, \omega, z) \right\rangle_{z_0} = \\ &= 2\pi A_0 (1 + \delta) \frac{\rho_e \beta_e \beta_{bv}^2 \beta_c}{V_t} \left[ \frac{(\beta_{av}^2 \omega^2 + \omega_{ci}^2)^2}{\omega^2 V_t^2} + \frac{16\beta_s^4 \omega_{av}^2}{\gamma^2 \beta_c^2 V_t^2} \right] J_c(k, \omega) \sin^2(kz), \end{aligned} \quad (4.17)$$

where  $\beta_c^2 = \omega^2/(\omega^2 - \Omega_i^2)$ , and a factor of 1/2 was included to take into account the fact that only the outgoing flux is considered.

Separating the dimensional factors by using the turbulent velocity  $u_t$  and the turbulent length scale  $l_t$ , and performing the integration over  $z$  and  $\omega$ , we obtain the total luminosity [ergs  $s^{-1}$ ] due to longitudinal flux tube waves

$$\begin{aligned} L &= N_t \int_0^H dz \int_0^{\infty} d\omega L_{1,z_0}(z, \omega) = \\ &= 4\pi N_t \int_0^H dz \beta_e A_0 E_t \omega_t (1 + \delta) M_t^3 \int_0^{\infty} d\omega \beta_{bv}^2 \beta_c (\alpha_m + \alpha_d) J_c(\bar{k}, \bar{\omega}) \sin^2(kz), \end{aligned} \quad (4.18)$$

where  $N_t$  is the number of flux tubes on the stellar surface, which can be related to the magnetic flux tube surface filling factor;  $E_t (= \rho_t u_t^2/2)$  is the kinetic energy of turbulent motions;  $\omega_t (= u_t/l_t)$  is the

characteristic turbulent frequency;  $M_t$  ( $= u_t/V_t$ ) is a coupling Mach number; and the convolution integral  $J_c(\bar{k}, \bar{\omega})$ ,  $\bar{k}$ , and  $\bar{\omega}$  are dimensionless. Note further that  $H$  is the thickness of the turbulent region in the convection zone, and that in the derivation of equation (4.5a), we have used the dispersion relation to calculate the wave number. In addition,  $\alpha_m$  and  $\alpha_d$  are the coefficients of monopole and dipole emission, respectively, and are defined by

$$\alpha_m \equiv \bar{\omega}_{ct}^2 \left[ 2 \beta_t^2 + \frac{\omega_{ct}^2}{\omega^2} + \frac{4\beta_s^4}{(\gamma-1) \beta_c^2} \right], \quad (4.19)$$

and

$$\alpha_d \equiv \bar{\omega}^2 \beta_t^4, \quad (4.20)$$

where  $\bar{\omega}_{ct}$  is the dimensionless critical frequency (equation 3.4).

Note that both emission coefficients are calculated by including the term  $1/\omega k$  (which comes from the evaluation of the asymptotic Fourier transform, see Appendix A) in the source function (equation 4.14), and by separating all terms that show dependence on  $\bar{\omega}_{ct}^2$  and on  $\bar{\omega}^2$ ; the former and latter terms, respectively, are the coefficients of monopole and dipole emissions with respect to the wave frequency  $\bar{\omega}$ . Note that because of this procedure, the coefficients of monopole and dipole emission, given by equations (4.19) and (4.20), are redefined in comparison to equation (3.13), and that now only the time-dependent term (which derives from non-stationary turbulence) accounts for the coefficient of dipole emission; the latter emission can be neglected if stationary or quasi-stationary turbulence is assumed. Both emission coefficients are dimensionless, and depend on the turbulent energy spectrum, the physical conditions in the region where the waves are generated, and on the critical frequencies for longitudinal waves.

The total wave luminosity for longitudinal tube waves (equation 4.18) shows a dependence on the third power of the Mach number (dipole type of emission with respect to Mach number); this distinguishes our results from those obtained by Stein (1968), who considered the generation of acoustic waves in a non-magnetic stratified medium, and found dependence of the acoustic wave luminosity on the fifth power of the Mach number (quadrupole type of emission). The results presented here are also different from those given by Musielak and Rosner (1987a), who obtained monopole type of emission for the generation of compressible MHD slow waves in a stratified medium with an embedded uniform magnetic field.

To calculate the energy fluxes given by equations (4.18) - (4.20), we have to specify the number of flux tubes on the stellar surface, as well as the values of the magnetic field strength, pressure and density at the level in the atmosphere where the integrations are to take place. In addition, we must determine the wave frequency domain for longitudinal waves, and must describe the turbulence; in the latter case, we must know the shape of the turbulence energy spectrum, the frequency factor, and the

characteristic length scale of the turbulent motions. In the next section, we will show how these parameters are restricted by the observational data or by theoretical studies.

## 5. MODEL PARAMETER DEPENDENCE OF ENERGY FLUXES

In this section, we present preliminary energy fluxes and energy spectra for longitudinal waves, and discuss the dependence of our results on the free parameters. We begin with a description of the turbulence, discuss the dependence of the energy fluxes on the magnetic field and, finally, present the energy spectra for longitudinal tube waves and show how the results depend on the chosen magnetic field strength, the turbulent energy spectrum and the frequency factor.

### (a) The Turbulence Energy Spectra and Turbulent Length Scale

Turbulent motions are characterized by the turbulent velocity and by the turbulent length scale; both these parameters are necessary to calculate the turbulent energy spectra, but have a simple form only for the very special case of isotropic, homogeneous and incompressible turbulence. In this simple case, dimensional analysis shows that the frequency-independent energy spectrum  $E(k)$  has the Kolmogorov form

$$E(k) = \frac{u_t^2}{k_t} \left[ \frac{k}{k_t} \right]^{-5/3}, \quad (k \geq k_t) \quad (5.1)$$

in the inertial range; alternatively, at smaller scales, where viscous effects begin to play a role, the spectrum takes on the exponential form

$$E(k) = \frac{64u_t^2}{k_t} \left[ \frac{k}{k_t} \right]^4 \exp[-4k/k_t], \quad (5.2)$$

where the normalization condition for  $E(k)$  is given by

$$\int_0^{\infty} E(k) dk = \frac{3}{2} u_t^2, \quad (5.3)$$

and where the maxima of the Kolmogorov and exponential spectra occur near  $k_t = 2\pi/l_t$ .

For the frequency factor, either the Gaussian form

$$\Delta(k, \omega) = \frac{2}{\pi^{1/2} k u_k} \exp \left[ -(\omega/k u_k)^2 \right], \quad (5.4)$$

or the exponential form

$$\Delta(k, \omega) = \frac{1}{ku_k} \exp \left[ -(\omega/ku_k) \right], \quad (5.5)$$

can be assumed (see Stein 1967), with

$$u_k = \left[ \int_k^{2k} E(k') dk' \right]^{1/2}. \quad (5.6)$$

In addition,  $\Delta(k, \omega)$  is normalized so that

$$\int_0^{\infty} \Delta(k, \omega) d\omega = 1. \quad (5.7)$$

The shape of the turbulence energy spectrum for stratified atmospheres is not known, and neither observational data nor theoretical considerations can help us to deduce its general properties. The energy spectra presented above certainly do not apply to the stratified turbulent atmosphere, especially to the largest turbulent eddies, whose size is comparable to the atmospheric scale height. Nevertheless, we shall adopt the above forms for the spectra since they span the range of likely behaviors of the actual spectra.

As an aside, we note that Stein (1967, 1968) and Bohn (1980) similarly adopted various *ad hoc* forms for the energy spectra, using the two functional forms just discussed and the so-called Spiegel turbulent spectrum, and three different functional forms for the frequency factors; in all these cases, stratified and magnetic field-free atmospheres were considered, and a turbulence correlation length scale equal to the pressure scale height was assumed. In addition, these authors assumed that the turbulent velocity of the largest eddies is the same as the velocity of convection motions given by the convection zone model. In our approach, the turbulent medium which surrounds a tube is stratified and magnetic field-free, so that in order to describe the turbulence, we have to make the same assumptions as those just mentioned. The difference is, however, that the largest turbulent eddies (with sizes comparable to the density or pressure scale height) do not contribute to the generation of longitudinal tube waves; in the interaction between the tube and turbulence, only eddies with sizes comparable to the tube diameter are important, and these eddies dominate longitudinal wave generation. We will include this effect in future energy flux calculations by computing the diameter of tube  $d_t$  for a given height (using the magnetic flux conservation law) and then, by setting  $l_t = d_t$ , estimate the turbulent velocity of the eddies with size  $d_t$ . Unfortunately, neither the variations of the tube diameter with height in the deep photospheric layers, nor even the diameter of an individual "elementary" flux tube can be estimated with any confidence from the observational data because of current limitations on spatial resolution (cf., Solanki and

Stenflo 1985).

### (b) *The Magnetic Field Strength*

In the approach presented in this paper, the magnetic field strength is a free parameter, but its maximum value is in fact fairly constrained by observational data, as well as by the horizontal pressure balance of the flux tube (see section 2d). From the observational point of view, the magnetic field strength within flux tubes can be estimated for the Sun by analysing the statistical properties of the Stokes  $I$  and  $V$  line profiles (Solanki and Stenflo 1985); this method gives the field strengths in typical network regions ranging from 1,400 G to 1,700 G. From a more theoretical perspective, horizontal pressure balance (based on the thin flux tube approximation) restricts the magnetic field pressure within the tube to be less than or equal to the gas pressure outside the tube (equality only obtains in the unlikely case that the flux tube is purely "magnetic", i.e., that there is no gas inside the tube). For typical published models of the solar photosphere (for example, Vernazza *et al.* 1983), the maximum field strength obtained from horizontal pressure balance also does not exceed 1,700 G.

In our approach, the horizontal balance for pressure and magnetic field perturbations (equation 2.26) restrict  $p_1$  and  $B_1$  to be quantities of the same order as the turbulent pressure outside the tube; note that the turbulent pressure is a small fraction of the gas pressure outside the tube. If the gas pressure inside the tube becomes comparable to the turbulent pressure outside the tube, then perturbed and unperturbed quantities become comparable and the inhomogeneous tube wave equation (3.1) is no longer valid. In order to insure the validity of the perturbation scheme, we therefore consider only the cases when the unperturbed gas pressure inside the tube is at least twice as large as the turbulent pressure defined by equation (2.25); our method of calculation does not allow us to estimate the energy fluxes generated in the form of longitudinal waves for magnetic field strength values close to the maximum value.

### (c) *The Wave Luminosity Spectra*

The wave luminosity spectra presented in this paper are preliminary, and are obtained for  $\log g = 4.5$ , for one fixed tube diameter  $l_t = d_t = 0.5H_p$ , for one fixed Mach number  $M = u_t/V_s = 0.1$  and for magnetic field strengths varying from 1,000 G to 1,550 G; the latter variations of the magnetic field lead to variations of the tube Mach number from 0.15 to 0.08, respectively. These assumptions significantly simplify the problem, and allow us to show dependence of the wave luminosity spectra on the chosen magnetic field, the turbulent energy spectrum and the frequency factor. Note, however, that we do not need to specify the number of flux tubes on the stellar surface (or the filling factor) as we do not calculate the total wave luminosity.

The contribution of monopole and dipole emission to the total wave luminosity spectrum is shown in Figure 1. The results presented in this figure are obtained for the exponential energy spectrum and

the exponential frequency factor, and for values of the magnetic field strength of  $B_0 = 1,500$  G and 1,100 G. In the both cases, monopole emission show a maximum at the same wave frequency ( $\omega \geq 5\Omega_c$ ) as dipole emission; however, the efficiency of wave generation by dipole emission always exceed that of monopole emission; the latter conclusion is valid for both turbulent energy spectra and both frequency factors considered here. However, when the magnetic field strength decreases, the efficiency of monopole emission increases, and becomes comparable to that of dipole emission for  $B_0 < 1,200$  G; this occurs mainly because  $\beta_c$  increases when the field strength decreases.

The dependence of the frequency-integrated energy flux on the magnetic field strength is given in Figure 2, which shows the results obtained for the exponential turbulent energy spectrum and the exponential frequency factor. The results strongly depend on the magnetic field strength: the energy flux decreases when the magnetic field strength increases. The latter effect leads to a decrease of the gas pressure inside the tube, and thus to a decrease of the wave energy flux (as there is not enough gas pressure to support the waves); this effect also places a limit on the longitudinal wave generation rate. The maximum of the wave luminosity spectrum occurs well above the cutoff frequency ( $5 \Omega_c$ ).

Figure 3 shows the dependence of the wave luminosity energy spectra on the form of the chosen turbulent energy spectrum and the frequency factor. As discussed above, we considered Kolmogorov and exponential energy spectra (equations 5.1 and 5.2) and Gaussian and exponential frequency factors (equations 5.4 and 5.5). The results show that the efficiency of wave generation is higher for the exponential energy spectrum, and also that in this case the maximum of the spectrum occurs at higher frequencies.

## 6. DISCUSSION AND CONCLUSIONS

We have derived the source function and the energy fluxes for the generation of longitudinal tube waves in a thin, vertical magnetic flux tube embedded in an otherwise magnetic field-free stellar convection zone; and we have shown that dipole emission dominates in the generation of longitudinal tube waves in such flux tubes; earlier qualitative results given by Stein (1981) and Ulmschneider and Stein (1982) suggested dominance of monopole emission. Our results distinguish tube wave generation from acoustic and MHD wave generation in a stratified medium since in these latter cases, quadrupole and monopole type of emissions are expected to be dominant (Stein 1968, Musielak and Rosner 1987a). The efficiency of longitudinal wave generation decreases significantly when the magnetic field strength increases, mainly the decreasing gas pressure inside the tube cannot support large energy fluxes for such waves. Note also that since the gas pressure within the tube decreases with increasing magnetic field strength, our approximations for the generation rate of longitudinal tube waves become unreliable when the gas pressure inside the tube is comparable to the turbulent pressure outside the tube.

When acoustic waves are generated in a stratified medium, monopole and dipole emissions also appear, and show a narrow maximum in the energy flux for wave frequencies very close to the acoustic cutoff frequency (Stein 1967, Figure 5); this energy flux cannot be carried away efficiently by acoustic waves from the generation region because these waves are almost evanescent. The situation is different for the generation of longitudinal waves; the maximum of the energy flux generated by dipole emission occurs at  $\omega \approx 5\Omega_t$ , and is somewhat broadened for higher frequency waves (mainly because the source function does not depend on the tube cutoff frequency, but instead depends on the Brunt-Vaisala frequency modified by the tube geometry). The latter frequency is significantly lower than the tube cutoff frequency, which is comparable to the acoustic cutoff. Even if the acoustic cutoff frequency  $\omega_{ac}$  (as well as the tube cutoff frequency  $\Omega_t$ ) increases in stellar photospheres (for example,  $\omega_{ac} = 0.024$  at the bottom of the convection zone and  $\omega_{ac} = 0.034$  at the temperature minimum; see Vernazza *et al.* 1976, Model C), we may conclude that the maximum of the longitudinal wave energy flux reaches the temperature minimum region as well as the chromosphere. To estimate however the amount of longitudinal wave energy flux that is available for the heating of the lower and upper chromosphere, further calculations of longitudinal wave propagation, which take radiative damping into account, are necessary.

The energy fluxes generated as longitudinal tube waves strongly depend on the strength of the magnetic field and the number of flux tubes on the stellar surface; any variations of these parameters among late-type stars with the same spectral type will lead to different levels of chromospheric activity observed for the same spectral type. Finally, we note that work corresponding to that discussed here remains to be carried out for transverse tube waves; this will be the aim of a succeeding paper.

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APPENDIX A: *The Asymptotic Fourier Transform*

Let us consider a one-dimensional inhomogeneous partial differential equation with constant coefficients, given in the form

$$P \left[ \frac{\partial^2}{\partial t^2}, \frac{\partial^2}{\partial z^2} \right] u(z, t) = f(z, t), \quad (\text{A1})$$

where  $P$  is a polynomial and  $f(z, t)$  is a function which vanishes outside a restricted region. To solve this equation, we Fourier-transform  $u(z, t)$  and  $f(z, t)$ , substitute these into equation (A1), and obtain

$$U(k, \omega) = \frac{F(k, \omega)}{P(-\omega^2, -k^2)}, \quad (\text{A2})$$

where  $U(k, \omega)$  and  $F(k, \omega)$  are the Fourier transforms of  $u(z, t)$  and  $f(z, t)$ , respectively. The solution for  $u(z, t)$  can be written as

$$u(z, t) = \int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{+\infty} dk \exp[-i(\omega t - kz)] \frac{F(k, \omega)}{P(-\omega^2, -k^2)}; \quad (\text{A3})$$

for the special case of a monochromatic source with frequency  $\omega_0$ ,

$$F(k, \omega) = F(k) \delta(\omega - \omega_0), \quad (\text{A4})$$

we have the result

$$u(z, t) = \exp(-\omega_0 t) \int_{-\infty}^{+\infty} dk \frac{\exp(ikz) F(k)}{P(-\omega_0^2, -k^2)}. \quad (\text{A5})$$

Dropping the subscript  $0$ , and since the integration over  $k$  can be performed with the help of the residue theorem, we find

$$u(z, t) = \exp(-\omega t) 2\pi i \sum_n \frac{\exp(ik_n z) F(k_n)}{\frac{\partial G(k, \omega)}{\partial k} \Big|_{k_n}}, \quad (\text{A6})$$

where  $G(k, \omega) = P(-\omega^2, -k^2)$ ; see also Lighthill (1960).

Using equation (A6), we may evaluate the asymptotic Fourier transform of  $I_{xx}(z, \omega)$  given by equation (4.11), and obtain

$$\begin{aligned} I_{xx}(k, \omega, z) &= -\frac{\pi^2}{k^2} (k - i\lambda) \left[ S_r^*(k, \omega) \exp(ikz) - S_r(k, \omega) \exp(-ikz) \right]^2 = \\ &= 4\pi^2 \frac{k_1 - i\lambda}{k_1^2} |S_r(k_1, \omega)|^2 \sin^2(k_1 z - \phi), \end{aligned} \quad (\text{A7})$$

where  $\phi$  is an arbitrary phase.

**APPENDIX B: The Convolution Integral**

In order to calculate the explicit form of the source function, it is necessary to evaluate a fourth-order velocity correlation. As this cannot as yet be done from first principles, we follow customary procedures (in the present context, see for example Stein 1967) and replace the fourth-order correlation by a sum of products of second-order correlations:

$$\langle (u_x u_x + u_y u_y) (u_x' u_x' + u_y' u_y') \rangle = 8 \langle u_x u_x \rangle \langle u_x' u_x' \rangle, \quad (\text{B1})$$

where we have used the fact that there is no difference between the x and y directions. We thus have to evaluate the convolution integral

$$\begin{aligned} J_{xxxx}(k, \omega) &= \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} dr \int_{-\infty}^{+\infty} d\tau \langle u_x(x, y, z, t) u_x(x, y, z + r, t + \tau) \rangle \times \\ &\quad \times \langle u_x(x, y, z, t) u_x(x, y, z + r, t + \tau) \rangle \exp[i(\omega\tau - kr)] = \\ &= \int_{-\infty}^{+\infty} dk' \int_{-\infty}^{+\infty} d\omega' \Phi_{xx}(k - k', \omega - \omega') \Phi_{xx}(k', \omega'), \end{aligned} \quad (\text{B2})$$

where  $\Phi_{xx}(k, \omega)$  is the Fourier transform of the velocity correlations.

We assume that  $\Phi_{xx}(k, \omega)$  may be factored into the frequency-independent one-dimensional energy spectrum  $E_2(k)$  and the frequency factor  $\Delta(k, \omega)$

$$\Phi_{xx}(k, \omega) = \frac{1}{4} E_2(k) \Delta(k, \omega), \quad (\text{B3})$$

where  $k$  and  $u_k$  are the wave number and the velocity perturbation of the  $k^{\text{th}}$  eddy, and where the factor 1/4 comes from the different normalization of  $\Phi_{xx}(k, \omega)$  as compared to the energy spectrum  $E_2(k)$  and the frequency factor  $\delta$ . Note that  $E_2(k)$  is normalized so that

$$\int_0^{\infty} E_2(k) dk = u_t^2, \quad (\text{B4})$$

and is connected to the three-dimensional energy spectrum (see Heinze 1975).

Using equation (B3), we may evaluate the convolution integral (B2) and obtain

$$J_{xxxx}(k, \omega) = \frac{1}{16} \int_{-\infty}^{\infty} dk' E_2(k - k') E_2(k') g(k, k', \omega), \quad (\text{B5})$$

where

$$g(k, k', \omega) = \int_{-\infty}^{\infty} d\omega' \Delta(k, k', \omega - \omega') \delta(k, \omega'). \quad (\text{B6})$$

Because there is no difference between the x and y-directions, we have

$$J_{xxxx}(k, \omega) = J_{xyxy}(k, \omega) = J_{yyxx}(k, \omega) = J_{yyyy}(k, \omega) = J_c(k, \omega). \quad (\text{B7})$$

## FIGURE CAPTIONS

*Figure 1:* We display the efficiency of monopole and dipole emission for two different values of the magnetic field strength,  $B_0 = 1,500$  G (solid lines) and  $1,100$  G (dashed lines). We assume an exponential turbulent energy spectrum and an exponential frequency factor.

*Figure 2:* The frequency-integrated wave energy flux for longitudinal tube waves is plotted for different values of the magnetic field strength. We assume an exponential turbulent energy spectrum and an exponential frequency factor.

*Figure 3:* The wave luminosity spectra obtained for a Kolmogorov energy spectrum and an exponential frequency factor (KE) are compared to those obtained for an exponential turbulent energy spectrum and an exponential frequency factor (EE), and an exponential energy spectrum and a Gaussian frequency factor (EG). The magnetic field strength is assumed to be  $B_0 = 1,500$  G.

Fig. 1

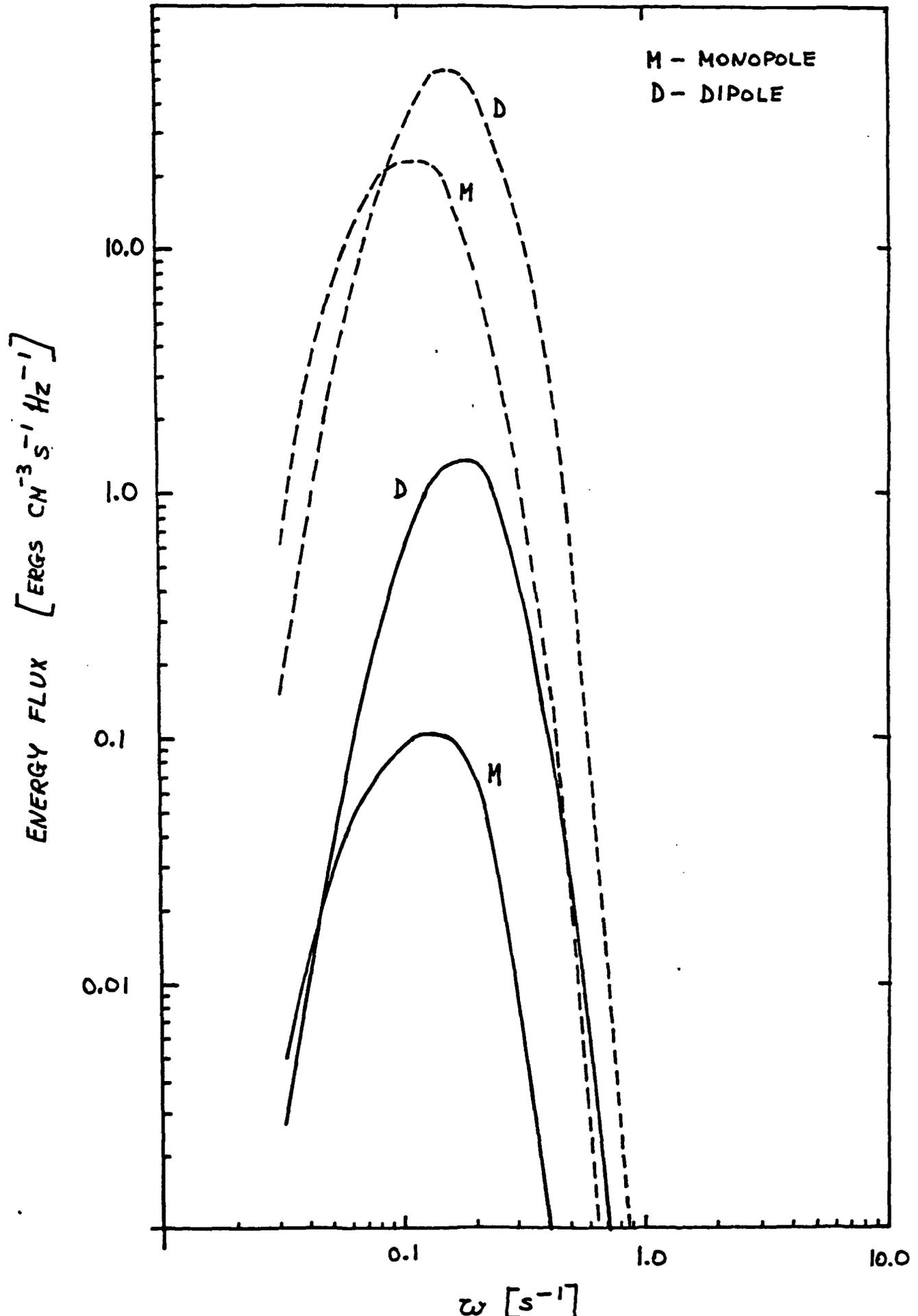


Fig. 2

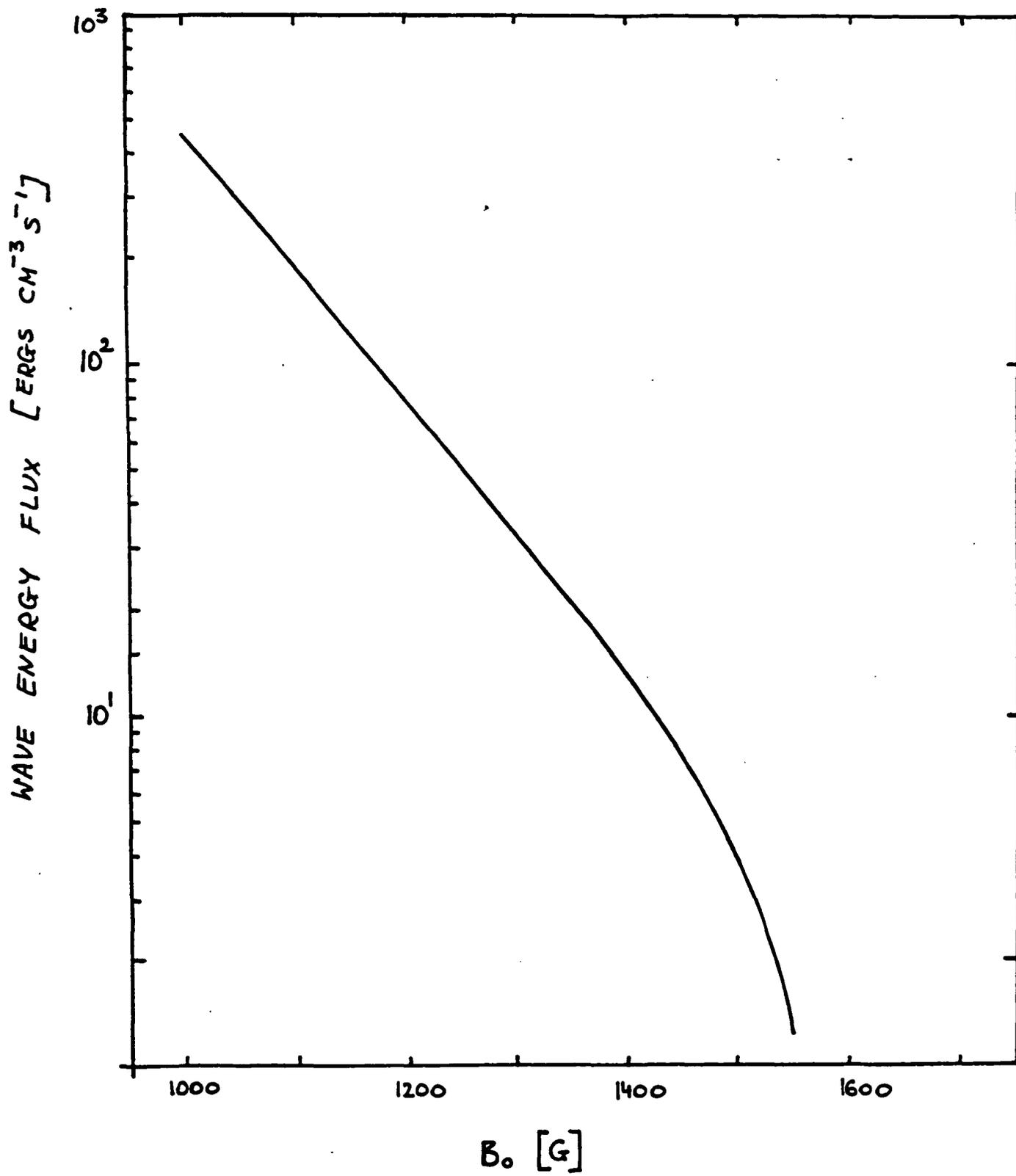


Fig. 3

