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DEPARTMENT OF CIVIL ENGINEERING

EXPERIMENTAL AND THEORETICAL INVESTIGATION OF PASSIVE DAMPING CONCEPTS FOR MEMBER FORCED AND FREE VIBRATION

By

on University Research Found

Zia Razzaq, Principal Investigator

David W. Mykins, Graduate Research Assistant

Progress Report For the period ended December 31, 1987

Prepared for the National Aeronautics and Space Administration Langley Research Center Hampton, Virginia 23665

Under Research Grant NAG-1-336 Harold G. Bush, Technical Monitor SDD-Structural Concepts Branch

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EXPERIMENTAL AND THEORETICAL INVESTIGATION OF PASSIVE DAMPING CONCEPTS FOR MEMBER FORCED AND FREE VIBRATIONS

By

Zia Razzaq¹ and David W. Mykins²

ABSTRACT

The results presented in this research report are the outcome of an ongoing study directed toward the identification of potential passive damping concepts for use in space structures. The effectiveness of copper brush, wool swab, and "silly putty" in chamber dampers is investigated through natural vibration tests on a tubular aluminum member. The member ends have zero translation and possess partial rotational restraints. The silly putty in chamber dampers provide the maximum passive damping efficiency. Forced vibration tests are then conducted with one, two, and three silly putty in chamber dampers. Owing to the limitation of the vibrator used, the performance of these dampers could not be evaluated experimentally until the forcing function was disengaged. Nevertheless, their performance is evaluated through a forced dynamic finite element analysis conducted as a part of this investigation. The theoretical results are based on experimentally obtained damping ratios indicate that the passive dampers are considerably more effective under member natural vibration than during forced vibration. Also, the maximum damping under forced vibration occurs at or near resonance.

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NOMENCLATURE

- [C] = damping matrix for member
- [D] = displacement vector
- [D] = velocity vector
- $\begin{bmatrix} \mathbf{\dot{D}} \end{bmatrix}$ = acceleration vector
- $[D_j]$ = displacement vector at node j
- E = Young's Modulus
- I = moment of inertia
- [K] = global stiffness matrix for member
- [R] = forcing function vector
- $\gamma \beta$ = arbitrary constants for Newmark's method
- Δ_{d}^{*} = dynamic deflection amplitude
- $\Delta_{\rm F}$ = static midspan deflection
- $\boldsymbol{\Delta}_{\!S}$ = static midspan deflection
- $\Delta t = time increment$
- Φ = modal vector
- η = damping efficiency index
- Ω = frequency of applied forcing function
- ω = natural frequency
- ω_{fe} = natural frequency from finite element analysis

ν

- ρ = mass density
- $\zeta = damping ratio$

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1. INTRODUCTION

1.1 Background and Previous Work

The space station designs currently under consideration by NASA are three-dimensional space structures composed of long tubular members. Modules providing the required living and working space for astronauts will be attached to this framework. Such a structure, suspended in a weightless environment, would be subjected to many types of dynamic loading. These include differential heating or cooling of the structure, variations in acceleration or gravitational pull, and impact with a solid object. The ability to expeditiously damp these vibrations before they cause permanent damage is a practical problem worth studying.

The necessarily large slenderness ratio of the average space truss member, combined with the flexible, semi-rigid end restraints cause the dynamic response of these members to be characterized by low frequency, small amplitude vibrations. Active damping techniques utilize electronic sensors and movable masses to reduce vibration of structures. This system, although effective, requires regular maintenance and an external power source. An alternative for mechanically damping a system is the concept of "passive" damping. This method uses a device or material permanently attached to the structure or its components and designed to absorb the energy of vibration thereby providing some damping of the system. Unlike "active" damping, this would require minimal maintenance and no external power.

The challenge to developing a passive damping concept, particularly for a space structure is two-fold. First is the necessity to minimize the mass, for without this constraint one obvious solution would be to provide large

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mass concentrations at the critical nodal points for the vibration modes. Such an approach would be expensive since the cost of transporting the system into space is directly related to the mass. The second challenge is to identify a concept which will provide passive damping without altering the strength or stiffness of the structure. For example, mild compression of the members provides some damping, however, the safe service loads for the structure are altered.

Recently, investigations into passive damping concepts for slender tubular members have been conducted with various end conditions (References 1-5). The most effective concepts found were the mass-string-whiskers assembly, and brushes for electrostatic and frictional damping. In these experiments, only natural flexural vibration was examined.

The previous work was conducted on hollow tubular steel members with an outer diameter of 0.5 inches. The passive damping concepts which were found to be effective for these members may not be as effective if the dimensions are changed. Factors altered by dimensionsal changes may include the damper mass required, the extent of the frictional interaction, and the member dynamic characteristics. Clearly research is needed to identify a viable passive damping concepts for members of different sizes and dymanic properties. In the present study, hollow tubular aluminum members with an outside diameter of 2.0 inches are used. These members more closely resemble the actual size and material which may be used in the future space stations.

1.2 Problem Definition

Figure 1 shows schematically a slender beam of length L with a hollow circular cross section. The outer diameter is D_0 the inner diameter is D_1 ,

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and the material is aluminum with a Young's modulus of 10,000 ksi. An aluminum member is used because the graphite composite tubes which may possibly be used in space structures are not yet available. The member ends are provided with a prototype connection developed by NASA for the space frames. These connections possess partial rotational restraint characteristics in the plane of motion and a more rigid end condition in the orthoganal plane. No axial or lateral movement of the member ends is permitted.

The problem is to identify potential passive damping concepts to absorb the energy of both natural flexural vibration, and harmonic forced flexural vibration, and to study the effectiveness of each concept. The natural vibration is caused by the sudden release of a constant static load. The harmonic forcing function is applied through a mechanical connection to a harmonic vibrator.

1.3 Objective and Scope

The following are the main objectives of this study:

- Identification of potential passive damping concepts for slender tubular structural members. Specifically, the following damping concepts are investigated:
 - a. wool swabs,
 - b. copper brushes,
 - c. silly putty in chambers.
- Evaluation of the damping efficiencies of the various damping concepts.
- 3. Evaluation of the suitability of a theoretical finite element analysis by comparison to experimental results for natural and

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forced vibration, and a previous finite-difference solution for natural vibration.

Only flexural member vibration is considered. The natural vibration study is conducted on each of the three passive damping concepts and for one specific initial deflection. Only the most efficient damping concept is considered for further study under forced vibration. Also, the vibration is induced by load application at the member midspan.

1.4 Assumptions and Conditions

The following assumptions and conditions have been adopted in this study:

- 1. The deflections are small.
- 2. The material of the member is linearly elastic.
- 3. Only planar vibration is considered.
- Damping force is opposite but proportional to the velocity at any location along the member.
- 5. The damping force is uniform along the length of the member.
- 6. The member is tested at 1-g and room temperature.
- 7. The effect of secondary induced forces such as varying axial tension and compression developed in the member during vibration is considered to be negligible.

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2. THEORETICAL FORMULATION

2.1 Finite Element Formulation

The beam shown in Figure 1 may be divided into N finite elements along the length. For the discretized system, the governing equation of motion can be expressed in the following matrix form (Reference 6):

(1)

 $[K] \{D\} + [M] \{D\} + [C] \{D\} = \{R\}$

where:

(D) = displacement vector,

 $\{D\}$ = velocity vector,

 (\ddot{D}) = acceleration vector,

[K] = global stiffness matrix for the "structure",

[M] = global modified lumped mass matrix,

[C] = damping matrix,

 $\{R\}$ = forcing function vector.

The boundary and initial conditions for the problem shown in Figure 1 are given in Reference 2 and are summarized here:

D(0,t) = 0	(2)
D(L,t) = 0	(3)
EI D"(0,t) = $k_1 D'(0,t)$	(4)
EI D"(L,t) = $-k_2$ D'(L,t)	(5)
$\dot{D}(x,0) = 0$	(6)
D(x,0) = O(x; K, EI, L)	(7)

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where primes represent differentiation relative to x, and dots represent time differentiation. The displacement vector at any node j along the member can be written as:

$$(D_j) = \begin{cases} d_j \\ d'_j \end{cases}$$
(8)

where d, and d', represent, respectively, the deflection and slope of j.

Equations 2 to 5 represent the boundary conditions whereas Equations 6 and 7 are the initial conditions. Equation 7 simply states that at time zero, the member deflected shape is dependent on x, K, EI and L.

The first task toward the solution of the matrix equation is the assembly of the three coefficient matrices. The [K] matrix is assembled from the individual element matrices combined in such a way so as to enforce the given boundary and inter-element compatability conditions. To illustrate the procedure, an example of a beam with four elements as shown in Figure 2 is given in Appendix A.

The global mass matrix is a diagonal form of a lumped mass matrix which was developed (Reference 6) for use with elements where translational degrees of freedom are mutually parallel, such as beam or plate elements.

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This matrix may be written as:



where:

m = mass at each degree of freedom = $\rho L(A)$ ρ = mass density (mass/in³) L = length of element (in) A = cross sectional area of element (in²)

In order to calculate the damping matrix [C], it is necessary to first determine the modal shape and natural frequencies of the system. This is accomplished numerically by solving the following eigen value problem using the Jacobi method (Reference 7):

$$([K] - \omega^2 [M] [\{\Phi\}] = \{0\}$$

(10)

where:

 ω = natural frequency,

 $\{\phi\}$ = modal vector.

Once ω and $\{\phi\}$ are known, determination of the damping matrix proceeds as described in Reference 7.

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Once all three coefficient matrices have been assembled, the solution of Equation 1 may proceed using any one of several solution algorithms available.

2.2 Newmark's Method

Newmark's method for solving the dynamic equilibrium equation is sometimes called the trapezoidal method because it is based on a linear interpolation to find succeeding points. This is done by assuming:

$$(D)_{t+}\Delta_{t} = (D)_{t} + \Delta t (\dot{D})_{t} + \Delta t^{2} \left(\left(\frac{1}{2} - \beta \right) (\dot{D})_{t} + \beta (\dot{D})_{t+}\Delta_{t} \right)$$
(11)

and

$$(\dot{D})_{t+}\Delta_t = (\dot{D})_t + \Delta t \left((1-\gamma)(\dot{D})_t + \gamma(\dot{D})_{t+}\Delta_t \right)$$
 (12)

where Δt is a time increment, and β and γ are arbitrary constants. By substituting Equations 11 and 12 into Equation 1 written at time t = t + Δt , one gets (Reference 6):

$$\begin{pmatrix} \left[K \right] + \frac{\gamma}{\beta \Delta_{t}} \left[C \right] + \frac{1}{\beta \Delta_{t}^{2}} \cdot \left[M \right] \end{pmatrix} \begin{pmatrix} D \end{pmatrix}_{t+\Delta t} = \langle R \rangle_{t+\Delta t} +$$

$$\begin{bmatrix} C \end{bmatrix} \begin{pmatrix} \frac{\gamma}{\beta \Delta_{t}} \langle D \rangle_{t} + \frac{\gamma}{\beta} - 1 \langle \dot{D} \rangle_{t} + \langle \Delta_{t} \rangle \frac{\gamma}{2\beta} - 1 \langle \dot{D} \rangle_{t} \end{pmatrix} +$$

$$\begin{bmatrix} M \end{bmatrix} \begin{pmatrix} \frac{1}{\beta \Delta_{t}^{2}} & \left(D \right)_{t} + \frac{1}{\beta \Delta_{t}} \langle \dot{D} \rangle_{t} + \left(\frac{1}{2\beta} - 1 \right) \langle \ddot{D} \rangle_{t} \end{pmatrix}$$

$$\begin{bmatrix} M \end{bmatrix} \begin{pmatrix} \frac{1}{\beta \Delta_{t}^{2}} & \left(D \right)_{t} + \frac{1}{\beta \Delta_{t}} \langle \dot{D} \rangle_{t} + \left(\frac{1}{2\beta} - 1 \right) \langle \ddot{D} \rangle_{t} \end{pmatrix}$$

$$\begin{bmatrix} M \end{bmatrix} \begin{pmatrix} \frac{1}{\beta \Delta_{t}^{2}} & \left(D \right)_{t} + \frac{1}{\beta \Delta_{t}} \langle \dot{D} \rangle_{t} + \left(\frac{1}{2\beta} - 1 \right) \langle \ddot{D} \rangle_{t} \end{pmatrix}$$

For a known loading function we may solve Equation 13 for the deflection at time $t = t + \Delta t$ using the deflection, velocity and acceleration at time t.

The algorithm for Newmark's solution is as follows:

- Compute the coefficient matrices from geometric and material properties.
- 2. At t = 0, set initial conditions by prescribing $\{D\}_{t=0}$ and $\{\dot{D}\}_{t=0}$.
- 3. Use Equation 1 to solve for $\{D\}_{t=0}$.
- 4. Solve Equation 13 for $\{D\}_{t+\Lambda t}$.
- 5. Solve Equation 11 for $\{D\}_{t+\Lambda t}$.
- 6. Solve Equation 12 for $\{D\}_{t+\Lambda t}$.
- 7. Set $(D)_t = (D)_{t+\Lambda t}; (\dot{D})_t = (\dot{D})_{t+\Lambda t}; (\dot{D})_t = (\dot{D})_{t+\Lambda t}.$
- 8. If t < total time desired, go to Step 4.
- 9. Stop.

This method of solution is unconditionally stable if $\gamma > 0.5$ and $\beta > (2\gamma + 1)^2/16$. With $\gamma = 0.5$ and $\beta = 0.25$, there are no amplitude errors in any sine wave motion regardless of its frequency, although the periods are overestimated. The mode shapes of the member in this study, however, are not known exactly. Nevertheless, γ and β values of 0.5 and 0.25 respectively, were tentatively chosen. The suitability of these values is evaluated later in Section 4.

The initial static deflection vector required in Step 2 of the algorithm may be determined using any one of the several classical structural analysis techniques. An approximate shape function for the member due to a specified midpoint displacement Δ_0 at time t = 0 is taken in the following form (Reference 2):

$$d_{j} = A_{j} \left[\sin \frac{\pi x_{j}}{L} + \frac{kL}{4\pi EI} \left(1 - \cos \frac{2\pi x_{j}}{L} \right) \right]$$
(14)

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where:

$$A_{1} = \frac{\Delta_{0}}{\frac{kL}{2\pi EI}}$$

The initial slope of the member at any point is found by differentiating Equation 14 resulting in:

(15)

$$d'_{j} = A_{1} \left[\frac{\pi}{L} \cos \frac{\pi x_{j}}{L} + \frac{k}{2EI} \sin \frac{2\pi x_{j}}{L} \right]$$
(16)

where x_i is the position of node j along the member length.

2.3 Central Difference Formulation

The governing equations and formulation of the coefficient matrices to be used in the central difference method of solution are precisely the same as those previously given for Newmark's method. Once these geometric and physical properties are determined, one proceeds by writing the central difference expressions for both velocity and acceleration at an arbitrary time t:

Equations 17 and 18 may then be substituted into Equation 1 to yield, after some rearrangement:

$$\left(\frac{[M]}{(\Delta t)^{2}} + \frac{[C]}{2(\Delta t)}\right) \{D\}_{t+\Delta t} = \{R\}_{t} - \left(\begin{bmatrix}K\end{bmatrix} - \frac{2[M]}{(\Delta t^{2})}\right) \{D\}_{t}$$

$$\left(\frac{[M]}{(\Delta t)^{2}} - \frac{[C]}{2(\Delta t)}\right) \{D\}_{t-\Delta t}$$
(19)

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The initial conditions $(D)_0$ and $(\dot{D})_0$ are prescribed and $(\dot{D})_0$ is found by solving Equation 1. Once these are known Equations 17 and 18 may be solved simultaneously to yield the displacements $(D)_{-\Delta t}$ required to start the computations.

$$\{D\}_{-\Delta t} = \{D\}_{0} - \Delta t \{\dot{D}\}_{0} + \frac{(\Delta t)^{2}}{2} \dot{(D)}_{0}$$
(20)

The solution algorithm for central difference is as follows:

 Compute the coefficient matrices from geometric and material properties.

2. Set $\Delta t = time step increment.$

- 3. Set initial conditions by prescribing $\{D\}_{t=0}$ and $\{D\}_{t=0}$.
- 4. Solve Equation 1 for $\{D\}_{t=0}$.
- 5. Solve Equation 20 for $\{D\}_{-\Delta t}$.

6. Solve Equation 19 for $\{D\}_{t+\Delta t}$.

7. Set
$$\{D\}_{t-\Lambda t} = \{D\}_{t}$$
, and $\{D\}_{t} = \{D\}_{t+\Lambda t}$.

- 8. If t < total time desired, go to 6.
- 9. Stop.

The central difference method is a conditionally stable, explicit method of solution. Conditionally stable implies that if Δ t is not chosen small enough, the predicted response of the system will grow unbounded. A preliminary numerical study showed that Δ t must be in the range from 0.001 to 0.005, therefore, a Δ t = 0.001 sec. is used in this study.

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3. EXPERIMENTAL STUDY

3.1 Specimen and Connection Details

3.1.1 Specimen

The experimental study consisted of conducting natural and forced vibration tests on a tubular aluminum member. The tests were performed both with and without passive damping devices present inside the member. The tubular member used was 14'-9" long with an outside diameter of 2" and a wall thickness of 0.125", yielding an inside diameter of 1.75". A schematic of the member tested is shown in Figure 1. Note that the member was horizontal for all testing, with gravitational forces acting in the plane of motion.

3.1.2 Connection Details

The prototype end connection used in this study is shown in Figure 3. It is constructed of an aluminum alloy, weights 0.595 lbs. excluding fastener bolts, and has a volume of 3.988 in³. The connection has a total of nine clevis blades, six of which are in the horizontal plane. One of the blades is in the vertical plane (at C) and two are at 45 degrees to the horizontal plane. These two are located at 45 degrees relative to the vertical clevis and in the planes containing the two lower clevis blades shown in Figure 3(a).

The fastner locations for the clevis blades in the horizontal plane are numbered 1 through 12. The member was fastened at locations 3 and 4 shown in Figure 3(a). Fasteners at locations 5 through 11 are used to mount the connection to a fixed base plate. No fastener was installed at location 12 due to an interference problem with the support underneath the base plate. This did not make any difference since the other fasteners provided

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sufficient fixity. Each fastener has a diameter of 0.25 in. and a length of 0.94 in. Washers were used at locations 1 and 2 only.

The ends of the tubular member were threaded to allow one-half of the "snap-lock" connection to be screwed onto it. Small holes were drilled through this threaded connection and pins inserted to prevent rotation and loosening of the connection during testing. The other end of the snap-lock connection had its blade end fit snugly into one of the clevis blades of the prototype end connection and fastened by two bolts. The spring stiffness, k, shown in Figure 1 was determined by a statical analysis using an experimentally determined midspan deflection for a known concentrated load. This value was 53.1 k-in/rad. The assembled connection is shown in Figure 4.

3.2 Passive Damping Concepts

Three different types of passive dampers referred to in Section 1.4 are described in this section.

3.2.1 Copper Brush Dampers

Figure 5 shows a copper brush damper 0.8125 inches in diameter, of total length 3.125 inches and a weight of 13.0 grams. The brush is manufactured by Omack Industries, Onalaska, Wisconsin 54650 with a US Patent 41986 and an inventory control number 07668341989. It has a threaded aluminum piece 1.0 inch long at one end with a twisted wire 2.125 inches in length attached to it. The copper bristles are attached to the entire length of the twisted wire. This type of brush is commonly used in cleaning the bore of a 12 gauge shotgun.

Figures 6 and 7 show schematically the attachments for the passive dampers and their spacing inside the tubular member. As shown in Figure 6,

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the assembly consists of several parts. First, a helical spring with a stiffness of 0.44 lb/in. is attached to the inside of the connection through a hook on the snap-lock connector as shown in Figure 8. A nylon line is tied to the other end of the spring and also connected to the first copper brush damper. The nylon line (sportfisher monofilament line manufactured by K-Mart Corporation, Troy, Michigan 48084, 8013.9, No. EPM-40, inventory control number 04528201391) used in this investigation has a 40 lb. capacity. A series of nylon line and dampers are attached along the member length until the other end of the tubular member is reached. The end of the nylon line is passed through a hole in the snap-lock connector and stretched by an amount of 2.0 inches in the longitudinal direction to induce nominal tension in helical spring. It is then secured to the vertical clevis at the support. The stretched helical spring is shown in Figure 9. The resulting passive damping assembly is aligned with the longitudinal axis of the tubular member due to the small amount of axial tension. No axial compression of the member is induced by the passive damping assembly on the tubular member since both ends of the nylon line are connected to the rigid supports. Since the nylon line is flexible, a significant portion of the stretching is due to elongation of the line itself with the remainder of the stretching taking place in the spring. The dampers are installed equidistantly between the ends of the member.

As a part of the present study, the effect of both number of brushes and presence or absence of tension on the nylon line, on member damping was examined.

In addition to baseline experiments on the specimen with no damping devices, a total of ten different conditions were examined. Tests with 1,

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2, 3, 5, and 7 brushes were conducted both with and without tension in the line.

3.2.2 Wool Swab Dampers

Figure 10 shows a wool swab damper with a 1.0 inch diameter, a total length of 3 inches and a weight of 7.1 grams. The wool swab is manufactured by Omark Industries, Onalaska, Wisconsin 54650 with a US patent 415838 and an inventory control number 076683422187. It has a threaded piece at one end with a twisted wire attached to it to which the wool swab is attached. The aluminum piece is 0.75 inches long while the wool swab has a length of 2.125 inches. This type of brush is commonly used for cleaning 12 gauge shotguns. The dampers are mounted inside the tubular member as shown in Figures 6 and 7. Tests were carried out using 1, 2, 3, 5, and 7 equidistantly spaced wool swab dampers.

3.2.3 Silly Putty in Chamber Dampers

The final device examined was the "silly putty" in chamber damper shown in Figure 11. It consists of a sphere approximately 0.75 inches in diameter made from silly putty placed inside a hollow cylindrical chamber. Silly putty is a trade name for an elasto-plastic material commonly used as a children's toy. It is manufactured by Binney and Smith, Inc., Easton, PA 18042, with an inventory control number of 07166208006. The chamber is made from a 1.0 in. long piece of a "Bristole Pipe" (PVC-1120, Schedule 40, ASTM-D-1785, nominal 1 inch pipe) having an original outer diameter of 1.058 in. and a wall thickness of 0.15 in. Since the damping effect was assumed to be provided by the silly putty, two steps were taken to reduce the mass of the damper thereby improving its efficiency. First, the inside diameter is increased by machining it to 0.914 in. resulting in a wall thickness of 0.07

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in. Its weight is further reduced by drilling a total of seven 0.25 in. diameter holes around its periphery half-way from its ends. The silly putty is held inside the chamber by means of a plastic wrap ("Saran Wrap") stretched over the ends of the chamber and held in place with tape. The silly putty is then free to bounce around inside the chamber. The total weight of the damper including the silly putty, PCV chamber, and plastic wrap is 7.4 gms. The dampers are mounted inside the tubular member as shown in Figures 6 and 7. Tests were conducted with a nominal tension in the spring and with no tension in the spring using 1, 2, 3, 5 and 7 equidistant silly putty in chamber dampers. An additional test was performed with 11 equidistant dampers and a nominal tension in the spring.

3.3 Test Setup and Procedures

The instrumentation used in the tests consisted of a proximity probe, harmonic vibration devices and a deflection-time plotter. This section summarizes the test setup and procedures followed for all the experiments included in this report.

Figure 12 shows a schematic of the member natural vibration test setup. A weight, W = 7.9 lb. was suspended at the member midspan by means of a cord, causing a total midspan deflection of 5/32 in. To induce natural vibration, the cord was cut with a pair of scissors, thereby releasing the member. The time dependent deflection at member midspan is recorded by means of a proximity probe shown in Figure 13, and connected to a deflection-time plotter.

Figure 14 shows the member forced vibration setup, a schematic of which is shown in Figure 15. Forced vibration of the specimen was obtained using a vibrator (Model 203-25-DC) with an oscillator (Model TPO-25). The

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vibrator applies a forcing function of the type:

 $F(t) = F_0 \cos(\Omega t)$

in which $F_0 = 4$ lb., t = time, and Ω = frequency of the forcing function. The applied frequency may be controlled using the oscillator.

(21).

The forcing function F(t) is transmitted from the vibrator to the tubular member through a fabricated vibrator connector as indicated in Figure 14. The details of this mechanical connector are shown in Figure 16. It consists of three main segments FQ, QR and RU interconnected at Q and R by means of pins. End P is connected to the vibrator. The end U is connected to the lower part of a metal hose clamp provided around the tubular member at midspan as indicated in Figure 14. The parts QR and RU can be disengaged at R by pulling out the pin RS instantaneously in the RS direction as indicated by the arrow at S. A string attached at S is used to pull out the pin. Once the pin is pulled, the arm QR drops freely and the beam is free to vibrate without constraints. Both joints Q and R are well lubricated to reduce friction. The vibrator connector in the engaged and the disengaged positions is shown in Figures 17(a) and 17(b), respectively. A record is made of the deflection-time response of the member once the forcing function, F(t), is removed.

3.4 Test Results and Discussion

In this section, the results from the member natural and forced vibration tests are presented and discussed.

3.4.1 Natural Vibration

All passive damping concepts were tested with natural flexural member vibration caused by releasing a weight at midspan as explained in Section 3.3. The initial midspan deflection, Δ_0 , due to the suspended weight is

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0.1563 in. A summary of the test results for the tubular member with no dampers as well as with wool swab, copper brush, and silly putty in chamber dampers is given in Table 1. The number of dampers, the total weight of the damping assembly, the damping ratio and the damping efficiency index are listed for each passive damping assembly. The logarithmic decrement method, as described in Reference 8, was used with the experimentally obtained deflection versus time plots to obtain the damping ratio.

The calculation of the damping ratio for the natural vibration tests was obtained using the first sixteen cycles and reading the amplitudes directly from the experimental deflection versus time plots. Each ζ value in Table 1 was then obtained by taking the average results of three tests for each combination of damping devices.

The efficiency index is defined (Reference 1 and 2):

$$\eta = \frac{\zeta - \zeta_0}{M_A}$$
(22)

in which ζ is the damping ratio with the damping devices, ζ_0 is the damping ratio in the absence of any passive damping device, and M_d is the total mass of the damping assembly.

The natural frequency from all of the experiments was found to be 8.4 Hz. The deflection versus time plots referenced in this section are obtained using the average ζ value and natural frequency from the experiments, and the following Δ -t relationship (Reference 1).

$$\Delta = \Delta_0 e^{-\zeta \omega_t} \left(\frac{\omega \zeta}{\omega_d} \sin \omega_d t + \cos \omega_d t \right)$$
(23)

The damped circular frequency, ω_d , is given by: $\omega_d = \omega \sqrt{1 - \zeta^2}$ (24)

The details including the listing of a computer program utilizing Equation

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23 to produce a deflection versus time plot are given in Reference 1. A baseline plot of deflection versus time for the member with no dampers is shown in Figure 18.

3.4.1.1 Copper Brush Dampers

For the copper brush dampers the maximum $\zeta = 0.0131$ is obtained with an assembly of three damping devices. This assembly produces the maximum η = 16.72 in/lb-sec². Figure 19 shows the corresponding average Δ -t plot for a 10 second duration. Figure 20 shows the effect of the three copper brushes on the deflection time envelopes. The vertical ordinate in this figure is designated by $\Delta_{\mathbf{e}}$ to indicate that the figure represents the envelopes rather than the complete \triangle -t relationship. The damping ratios from the experiments are given in Table 2(a). In addition to the test conducted as described in Section 3.3, a series of tests were made with no tension in the damping assembly. These tests, conducted with 1, 2, 3, 5 and 7 devices in the specimen showed no significant increase in member damping regardless of the number of devices used. The results are summarized in Table 2(b). One plausible explanation for this is as follows. The outer diameter of the copper brush is less than the inside diameter of the member. When there is no tension in the damping assembly, the devices are free to bounce inside the specimen. Because the vibrations are relatively small and the natural frequency low, the assembly with no tension has a tendency to move with the specimen, bouncing slightly inside the member. Due to the relatively negligible mass of the damper as compared to the member this nearly coincident movement produces minimal damping of the vibration. With a slight tension in the assembly, it can have its own natural frequency, different from the specimen. As a result, when vibration of the specimen is

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induced, the impact of the damping assembly with the side of the tube sets the assembly in motion. Two types of motion then contribute to the damping. First, because of the difference in natural frequency of vibration impact of the dampers against the inside of the tubular member acts to damp the vibration. Secondly, the frictional interaction between the dampers and the member inside surface takes place while the dampers vibrate both in plane but out of phase, and axially. When the number of dampers is increased beyond three with nominal tension, the damping ratio decreases.

3.4.1.2 Wool Swab Dampers

For the wool swab dampers the maximum $\zeta = 0.0105$ was obtained with an assembly of three dampers resulting in an efficiency of 9.05 in/lb-sec². The maximum $\eta = 12.34$ was obtained with a single damper assembly yielding a damping ratio of 0.0099. Figures 21 and 22 represent the Δ -t plots for the member with three, and one wool swab damper assemblies, respectively for a 10 second duration. Figures 23 and 24 show the effects of these damping assemblies on the deflection-time envelopes. The damping ratio increased as the number of dampers was increased from one to three. Increasing the number of devices beyond three resulted in a decrease in both damping ratio and efficiency. The small negative efficiency noted for seven devices can be taken as practically zero. It was found that a variation in the method of attachment of the assembly to test specimen from concentric to an eccentric connection had no significant effect on the resulting damping ratio. The results are given in Tables 3(a) and 3(b).

3.4.1.3 Silly Putty in Chambers Dampers

For silly putty in chambers dampers, the maximum $\zeta = 0.0115$ was obtained with an assembly of three dampers resulting in a $\eta = 15.73$ in/lb-

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sec², whereas the maximum $\eta = 21.35$ in/lb-sec² was obtained with an assembly of two dampers corresponding to $\zeta = 0.0113$. Figures 25 and 26 represent the Δ -t plots for the member with three and two silly putty in chamber damper assemblies, respectively, for 10 second duration. Figures 27 and 28 show the effects of these damping assemblies on the deflection-time envelopes. The damping ratio was found to increase as the number of dampers was increased from one to three. Increasing the number of dampers beyond three resulted in a decrease of both damping ratio and efficiency. The tests conducted with no tension in the assembly showed a slight increase in damping ratio up to the three damper assembly. An increase in the number of dampers beyond three with no tension on the assembly showed no increase in damping ratio above the baseline damping ratio for the empty member. The results are given in Tables 4(a) and 4(b). Of all the passive damping devices tested in this study, the assembly of three silly putty in chamber dampers was found to be the most efficient. Therefore, these dampers were chosen for further study under forced harmonic vibration.

3.5 Forced Then Free Vibration

It was discovered during testing that the vibration employed for the forced vibration tests allowed only a limited amount of travel. This meant that the deflection of the member at the location where the vibrator was attached was limited to what the vibrator would allow. Nevertheless, forced vibration tests were conducted on the individual member since it was not known initially whether or not the dynamic deflection would exceed the vibrator capacity. The results presented later in this section indicated that the vibrator constrained the member deflection for a certain range of forcing function frequencies including that which would otherwise have

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constituted a resonance condition. This limitation must be taken into account when evaluating the performance of the dampers on an individual member.

3.5.1.1 Silly Putty in Chamber Dampers

The results of the experimental study of the member under forced then free vibration are summarized in Table 5. Tests were conducted with no dampers, and 1, 2, and 3 dampers inside the member. Each of these assemblies was subjected to a force of 4 lb. at the member midspan, at frequencies of 2, 5, 7, and 9 Hz, corresponding to $\Omega/\omega_{\rm h}$ ratios of 0.238, 0.596, 0.834, and 1.073, respectively. An additional test was conducted on the empty member and the 3 damper assembly using a frequency of 12 Hz (Ω/ω_n - 1.430). The experimental results are shown in Figures 29 through 32. The free vibration part of the deflection-time graph is obtained by disengaging the forcing function from the member midspan as described in Section 3.3. The constrained dynamic deflection amplitude, Δ_D^* , and its dimensionless value, $\Delta_{D}^{*}/\Delta_{S}$, where Δ_{S} is the calculated static midspan deflection due to a 4 lb. load, are listed in Table 5. The constrained dynamic deflection amplitude is the measured amplitude of the initial constrained force part of the deflection-time plots. Also listed in Table 5 are the maximum initial free vibration amplitudes, Δ_F , for each assembly and frequency considered. Two dimensionless quantities are derived from this value as Δ_F / Δ_S and Δ_F / Δ_D^* . The data in Table 5 shows that the $\Delta_{\rm D}^{*}/\Delta_{\rm S}$ values range from 0.59 to 0.95. For all the cases, the maximum value was observed for an applied force frequency of 5 Hz. It was also found that the Δ_F/Δ_S and $\Delta_{\rm F}/\Delta_{\rm D}^{*}$ ratios were gradually increasing for increasing forcing function frequencies. One important consequence of the deflection constraint imposed

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by the vibrator is that no resonance phenomenon could be produced in the vicinity of 8.4 Hz. The average damping ratios were obtained from the free vibration part of the deflection-time curves and are listed in Table 5. As seen from this data, the single silly putty in chamber damper configuration provides the maximum decrease in free vibration amplitude. Another important observation to be made is that the ζ values in Table 5 are significantly less than the corresponding values for the same damping assemblies given in Table 1. This is attributable to the dependence of the damping ratio on the initial velocity which is considerably greater for the results reported in Table 5 than for those in Table 1.

3.5 Comparison of Damping Efficiencies

In Section 3.4, the efficiency index based on Equation 22 was computed for each damping device. The average values of η and the associated damping assembly weight for natural vibration were presented in Table 1. Figure 33 shows the curves between η and the weight of dampers used in the natural vibration tests for various damping concepts. The silly putty in chamber dampers provided the most efficienct damping of the member. It is worth noting that all of the curves have ascending and descending portions which define the maximum attainable damping efficiency. In general, an increase in damping assembly weight beyond 50 grams results in a decline in efficiency.

Figure 34 shows the relationships between the damping efficiency and the number of damping devices for natural vibration using all three concepts. These curves also show that, in general, an assembly of more than three damping devices results in a decline in efficiency. This may indicate that the first and second mode shapes are dominating the dynamic response.

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By applying the dampers to locations in the vicinity of maximum deflection for these mode shapes, the maximum efficiency was realized. Any increase in the number of dampers beyond three adds mass to the system, and is associated with a decrease in damping.

The average damping efficiency indices for the forced then free vibration tests for 1, 2, and 3 silly putty in chamber dampers are given in Table 5. The maximum efficiency was obtained using one silly putty in chamber damper and a forcing function frequency of 5 Hz. No correlation between the maximum efficiency and the initial vibration amplitude was observed. However, the maximum average damping ratio for each device was found to occur near the theoretical resonance of the member (between 7 and 9 Hz) in spite of the inability of the apparatus to allow the resonance to occur.

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4. NUMERICAL STUDY

4.1 Natural Vibration

4.1.1 Finite Element versus Experiment

The formulation and solution algorithms using Newmark's method for computing the dynamic response of a beam was given in Section 2. In this section, a comparison is made of the deflection versus time relations from this finite element analysis to those obtained experimentally.

A preliminary study showed that for $\Delta t = 0.0001 \text{ sec.}$, the central difference formulation described in Section 2.3 gave precisely the same results as Newmark's method. Since Newmark's method provides accurate results even with larger time steps, it was used to produce Figure 35 through 43. Figure 35 shows a comparison of the finite element and experimental Δ -t plots for the member with no dampers. The solid line is the finite element solution and the dashed line is the experimental curve using a frequency of 8.4 Hz and the average damping ratios from Table 1. Figure 36 shows a comparison of the finite element and experimental Δ -t plots for the member brush dampers. In both of these curves, it can be seen that the period of the vibration obtained using finite elements is exagerated by approximately 32%. However, the amplitudes of the vibration are accurate to within 5%.

4.1.2 Finite Element versus Finite-Difference

The \triangle -t curves representing the finite-difference solution are obtained using the computer program developed in Reference 2. Figures 37 and 38 show the comparison of the finite element and the finite-difference solutions for the member with no dampers and three copper brush dampers, respectively. The data for these plots is obtained from Table 1. As indicated in these

- 25 -
figures, the difference in the period calculated by these two methods is approximately 26%. However, the amplitudes of the vibration from the two analyses are within 3% of each other. Figure 39 is a comparison of the finite-element and finite difference solutions for a simply supported beam $(k_1 = k_2 = 0)$. Similar correlation is also observed for a fixed end beam $(k_1 = k_2 = \infty)$. In the presence of end connections of intermediate fixity, the two analyses provide somewhat differing results.

4.2 Forced then Free Vibration

In this section, curves obtained from the finite element solution for various forcing function frequencies are given. Also, a comparison of the theoretical solution to experimental results is made for both the member with no dampers and the member with one silly putty in chamber damper at a forcing function frequency of 2 Hz.

Figure 40 shows the response using Newmark's method for a beam with no dampers and subjected to a 4 lb. force at a frequency of 2 Hz. After 1 second, the forcing function is removed and the beam is allowed to vibrate freely. Figure 41 shows the response of the same system with a forcing function frequency of 6 Hz. This frequency corresponds to a frequency ratio Ω/ω_{fe} of 0.95, where $\omega_{fe} = 6.3$ Hz is the natural frequency of the beam from the finite element solution. Clearly, this represents a nearly resonant condition as expected. After 1 second, the forcing function is removed and the member is allowed to vibrate freely.

Figures 42 shows the finite element and experimental curves for the member with no dampers and subjected to a 4.0 lb. force at a frequency of 2 Hz. Although the forced vibration portions of the two curves at $\Omega = 2$ Hz are quite similar, the free vibration amplitudes differ significantly. The

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reasons for this difference may be as follows. In the experiment, the forcing function was terminated by pulling the pin RS from the vibrator connector shown in Figure 16. During the tiny time interval in which the pin was being pulled out, the contact and frictional forces involved in disengaging the segment QR from RU were unintentionally transferred to the members thereby retarding its initial amplitude in the free vibration range. Consequently, the ensuing envelope of the experimental free vibration Δ -t curve is considerably narrower than the theoretical one. Similar effects are observed in Figure 43 which shows the finite element and experimental results when one silly putty in chamber damper is used.

At larger Ω values such as those of the order of 6 Hz, the Δ -t relations from the finite element analysis do not match the experimental ones even in the forced vibration range. This is primarily due to the constraints imposed by the vibrator on the maximum member deflecting as explained earlier in Section 3.5.

4.3 Finite Element Analysis for Forced Vibration

As mentioned earlier, the vibrator used in the experimental study constrained the motion of the member in the presence of a forcing function. As a result, the actual effect of passing damping could not be observed for this condition. Therefore, a numerical study was conducted to examine the effect of passive damping in the presence of the forcing function. In this section, the theoretical results showing both the extent of damping which would occur during the forced vibration and the effect of the dampers on the deflection-time envelopes are presented and discussed. Figure 44 shows the theoretical dynamic magnification factor (DMF), $\Delta_{\rm p}/\Delta_{\rm S}$, versus the frequency ratio $\Omega/\omega_{\rm n}$ for damping ratios of 0.0094, 0.0131 and 0.50. The first two

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values of the damping ratios were obtained from the member tests with no dampers, and 3 copper brush dampers, respectively. As can be seen in this figure, the copper brush dampers do not change the DMF appreciably for nonresonance frequency ratios. However, the dampers reduce the DMF by approximately 7% at resonance.

Figure 45 shows the deflection versus time relationship for the member with no dampers and with three copper brush dampers, with a forcing function frequency of 6.35 Hz ($\Omega/\omega_{\rm h} = 1.0$) for one second, and allowed to vibrate freely thereafter. These curves show that the passive damping results in a member amplitude reduction in the forced vibration range, however, its most beneficial effect occurs during the free vibration. After 3 seconds of free vibration, the amplitudes of the member with dampers are approximately 40% less than those corresponding to the empty member.

5. CONCLUSIONS AND FUTURE RESEARCH

5.1 Conclusions

The following conclusions are drawn from the research conducted herein:

- The silly putty in chamber concept provides the maximum passive damping efficiency under member natural vibration, as compared to the copper brush or the wool swab concepts.
- 2. The copper brush concept provides the largest damping ratio of the system under natural vibration.
- 3. Due to the limitation of the vibrator used, the effectiveness of the passive damping concepts could not be evaluated until the forcing function was disengaged.
- 4. Frictional and contact forces acting on the member during disengagement from the vibration apparatus caused a reduction of the ensuing free vibration member amplitude.
- 5. The theoretical results indicate that in the presence of a forcing function, the passive damping devices provide the most effective damping in the vicinity of the resonant frequency.
- 6. The theoretical results show that passive dampers are considerably more effective under member natural vibration than during forced vibration.
- 7. Under natural vibration, the finite element solution results in periods which are nearly 30 percent greater than the experimental ones. However, amplitudes are reasonably accurate. The accuracy of the results is improved when the member ends are pinned or fixed.

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5.2 Future Research

The most successful passive damping concepts identified herein should be examined using forced vibration equipment which would allow investigation of their effectiveness at or near resonance. Attempts should be made to identify a means of disengaging an applied force without adversely affecting the dynamic response of the member. These tests should be conducted on both individual members and structure sub-assemblies.

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TABLES

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Table 1. Member natural vibration test results for copper brush, wood swab, and silly putty in chamber dampers.

PASSIVE DAMPING CONCEPT	NUMBER OF DAMPERS	WEIGHT OF DAMPING ASSEMBLY (GM)	AVERAGE DAMPING RATIO ζ	DAMPING EFFICIENCY INDEX (IN/LBSEC ²)
No Dampers		0.00	0.0094	0.00
Copper	1	13.0	0.0098	5.39
Brush	2	26.0	0.0107	8.76
Dampers	3	39.0	0.0131	16.72
	5	65.0	0.0129	9.44
	7	91.0	0.0097	0.48
Wool	1	7.10	0.0099	12.34
Swab	2	14.20	0.0102	9.87
Dampers	3	21.30	0.0105	9.05
	5	35.50	0.0101	3.46
	7	49.70	0.0091	-0,99
Silly	1	7.8	0.0100	13.48
Putty	2	15.6	0.0113	21.35
in	3	23.4	0.0115	15.73
Chamber	5	39.0	0.0109	6.74
Dampers	7	54.6	0.0097	0.96
	11	85.8	0.0094	0.00

Number of Devices	ζ ₁	ζ2	ζ ₃	ζ.avg
1	0.0097	0.0095	0.0095	0.0096
. 2	0.0097	0.0097	0.0097	0.0097
3	0.0098	0.0096	0.0096	0.0097
5	0.0094	0.0096	0.0094	0.0095
7	0.0095	0.0095	0.0096	0.0095

Table 2(a). Damping ratios from natural vibration tests with copper brushes and no cord tension.

Table 2(b). Damping ratios from natural vibration tests with copper brushes and nominal cord tension.

Number of Devices	ζ 1	ζ_2	ζ ₃	ς avg
1	0.0097	0.0102	0.0096	0.0098
2	0.0105	0.0109	0.0108	0.0107
3	0.0133	0.0128	0.0131	0.0131
5	0.0129	0.0131	0.0128	0.0129
7	0.0096	0.0098	0.0095	0.0096

Number of Devices	ζ1	ζ2	ζ ₃	ζ AVG
1	0.0098	0.0097	0.0098	0.0098
2	0.0100	0.0098	0.0101	0.0100
3	0.0107	0.0108	0.0101	0.0105
5	0.0096	0.0100	0.0097	0.0098
7	0.0094	0.0094	0.0094	0.0094

Table 3(a). Damping ratios for natural vibration tests with wool brushes and concentric cord support.

Table 3(b). Damping ratios for natural vibration tests with wool brushes and eccentric cord support.

Number of Devices	ζ, 1	ζ 2	ζ 3	ζ AVG
1	0.0099	0.0099	0.0098	0.0099
2	0.0103	0.0101	0.0101	0.0102
3	0.0105	0.0105	0.0105	0.0105
5	0.0099	0.0102	0.0102	0.0101
7	0.0093	0.0090	0.0090	0.0091

Number of Devices	ζ ₁	ζ2	ζ3	ζ _{avg}
1	0.0096	0.0097	0.0096	0.0096
2	0.0099	0.0099	0.0102	0.0100
3	0.0101	0.0101	0.0101	0.0101
5	0.0095	0.0095	0.0093	0.0094
7	0.0094	0.0092	0.0096	0.0094

Table 4(a). Damping ratios for natural vibration tests with silly putty and no cord tension.

Table 4(b). Damping ratios for natural vibration tests with silly putty and nominal cord tension.

Number of Devices	ζ ₁	ζ2	ζ ₃	ζ _{avg}
1	0.0104	0.0096	0.0100	0.0100
· 2	0.0112	0.0115	0.0113	0.0113
3	0.0112	0.0115	0.0117	0.0115
5	0.0109	0.0109	0.0109	0.0109
7	0.0096	0.0099	0.0096	0.0097
11	0.0094	0.0094	0.0094	0.0094

Passive F Damping F Concept F	orcing unction requency Hz)	Constrained Dynamic Amplitudes Δ_{D}^{*} in.	∆ _D *∕∆ _S	Initial Free Vibration Amplitude ∆ _F max in	Δ _F /Δ _S	∆ _₽ /∆* _D	Äverage Damping Ratio ζ Avg.	η
	2	0.067	0.84	0.030	0.38	0.45	0.0043	1
No Dampers	5	0.073	0.92	0.067	0.84	0.91	0.0072	
	7	0.070	0.88	0.077	0.97	1.10	0.0073	l l
	9	0.067	0.84	0.082	1.03	1.23	0.0069	• • •
	12	0.047	0.59	0.082	1.03	1.75	0.0025	
· .	2	0.062	0.78	0.030	0.38	0.48	0.0058	33.7
1 Silly	5	0.073	0.92	0.063	0.80	0.86	0.0089	38.2
in	7	0.067	0.84	0.070	0.88	1.05	0.0089	36.0
Damper	9	0.067	0.84	0.073	0.92	1.10	0.0080	24.17
	2	0.065	0.82	0.030	0.39	0.48	0.0049	6.7
2 Silly	5	0.075	0.95	0.060	0.76	0.80	0.0069	(-3.4)
in Chambar	7	0.067	0.84	0.053	0.67	0.80	0.0076	3.4
Damper	9	0.065	0.82	0.083	1.05	1.28	0.0080	12.4
	2	0.063	0.80	0.030	0.38	0.47	0.0057	10.5
3 Silly	5	0.075	0.95	0.057	0.71	0.76	0.0055	(-12.7)
in	7	0.068	0.86	0.077	0.97	1.12	0.0064	(-6.7)
Damper	9	0.068	0.86	0.090	1.14	1.32	0.0082	9.7
	12	0.047	0.59	0.082	1.03	1.75	0.0036	8.2

Table 5. Member forced then free vibration test results for silly putty in chamber dampers.

FIGURES

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Figure 1. Schematic of tubular member







Figure 3. Some end connection details

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Figure 4. Member end connection







Figure 6.

. Schematic of attachments for passive damper inside tubular member

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Figure 8. Helical spring attachment at end e











Figure 11. Silly putty in chamber damper







Figure 13. Proximity probe



Figure 14. Member forced vibration setup



Figure 15. Schematic of member forced vibration setup







(a) Vibrator connector in engaged position



(b) Disengaged vibrator connector



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Figure 20. Effect of 3 copper brush dampers on Δ -t envelope.







Figure 23. Effect of 3 wool swab dampers on Δ -t envelope.




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Figure 28. Effect of 2 silly putty in chamber dampers on Δ -t envelope.









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Figure 32. Experimental Δ -t plots for member "constrained" forced then free vibration with 3 silly putty in chamber dampers and forcing function frequencies of 2, 5, 7, and 9 Hz.

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- finite difference (8.0 Hz) t

0. 15 -



Figure 38. Finite element versus finite difference Δ -t plots for member with 3 copper brush dampers.

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Theoretical curve with ideal disengagement.

Experimental curve with mechanical disengagement.

Figure 42. Theoretical and experimental forced then free Δ -t plots for a 4.0 lb. force at 2 Hz for 1 second on member with one silly putty in chamber damper.

Theoretical curve with ideal disengagement.

Experimental curve with mechanical disengagement.

Figure 43. Theoretical and experimental forced then free Δ -t plots for a 4.0 lb. force at 2 Hz for 1 second on member with no dampers.

APPENDICES

APPENDIX A

EXAMPLE OF FOUR ELEMENT BEAM STIFFNESS MATRIX

In this appendix the procedure used to assemble the beam stiffness matrix using a beam composed of four elements is presented.

The typical element stiffness matrices for Elements b and c as shown in Figure 2 are given as (Reference 6):

$$[K]_{b,c} = \begin{cases} \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ & \frac{4EI}{L} & \frac{-6EI}{L^2} & \frac{2EI}{L^2} \\ & & \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ & & & \frac{4EI}{L} \end{cases}$$
(A.1)

Since only planar motion is considered, axial effects are negligible and, therefore, not included in the element stiffness matrix.

Derivation of the stiffness matrix for Element a as shown in Figure 2 is as follows. The flexibility matrix for the element is given by: $[F] = [H]^{t}[F]_{c1}[H] + [F]_{m} + [F]_{c2}$ (A.2)

in which [H] is the equilibrium matrix given by:

$$[H] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & L \\ 0 & 0 & 1 \end{bmatrix}$$
(A.3)

and $[F]_{c1}$ represents the flexibility of the connection at end one, $[F]_m$ is the flexibility of the element itself and $[F]_{c2}$ is the flexibility of the connection at end two. These are defined as follows:

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$$[F]_{c1} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{L}{4EIk} \end{bmatrix}$$
(A.4)
$$[F]_{m} = \begin{bmatrix} \frac{L^{3}}{3EI} & \frac{L^{2}}{2EI} \\ \frac{L^{2}}{2EI} & \frac{L}{EI} \end{bmatrix}$$
(A.5)
$$[F]_{c2} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{L}{4EI} \end{bmatrix}$$
(A.6)

therefore the flexibility matrix in full can be written as

$$[F] = \begin{bmatrix} \frac{L^{3}}{EI}\begin{pmatrix} 1 & 1 \\ 3 & 4k \end{pmatrix} & \frac{L^{2}}{EI}\begin{pmatrix} 1 & 1 \\ 2 & 4k \end{pmatrix} \\ \\ \frac{L^{2}}{EI}\begin{pmatrix} 1 & 1 \\ 2 & 4k \end{pmatrix} & \frac{L}{EI}\begin{pmatrix} 1 & 1 \\ 1 & +\frac{1}{4k} \end{pmatrix} \end{bmatrix}$$
(A.7)

The inverse of [F] is given by:

$$[K_{22}]_{a} = [F]^{-1} = \begin{bmatrix} \frac{3EI}{L^{3}} & \frac{(4k+1)}{(k+1)} & \frac{-3EI}{L^{2}} & \frac{(2k+1)}{(k+1)} \\ \frac{-3EI}{L^{2}} & \frac{(2k+1)}{(k+1)} & \frac{EI}{L} & \frac{(4k+3)}{(k+1)} \end{bmatrix}$$
(A.8)

The other stiffness matrices now follow from:

$$\begin{bmatrix} K_{11} \end{bmatrix}_{a} = \begin{bmatrix} \frac{3EI}{L^{3}} & \frac{(4k+1)}{(k+1)} & \frac{3EI}{L^{2}} & \frac{(2k)}{(k+1)} \\ \end{bmatrix}$$

$$\begin{bmatrix} H \end{bmatrix} \begin{bmatrix} K_{22} \end{bmatrix} \begin{bmatrix} H \end{bmatrix}^{t} = \begin{bmatrix} \frac{3EI}{L^{2}} & \frac{(2k)}{(k+1)} & \frac{EI}{L} & \frac{(4k)}{(k+1)} \end{bmatrix}$$

$$(A.9)$$

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$$\begin{bmatrix} K_{12} \end{bmatrix}_{a} = \begin{bmatrix} -3EI \\ L^{3} \\ (k+1) \end{bmatrix} = \begin{bmatrix} -3EI \\ L^{3} \\ (k+1) \end{bmatrix} = \begin{bmatrix} -3EI \\ L^{2} \\ (k+1) \end{bmatrix} = \begin{bmatrix} -3EI \\$$

Therefore, the total stiffness matrix for Element a is:

$$[K]_{a} = \begin{bmatrix} \frac{3EI}{L^{3}} & \frac{(4k+1)}{(k+1)} & \frac{3EI}{L^{2}} & \frac{(2k)}{(k+1)} & \frac{-3EI}{L^{3}} & \frac{(4k+1)}{(k+1)} & \frac{3EI}{L^{2}} & \frac{(2k+1)}{(k+1)} \\ & \frac{EI}{L} & \frac{(4k)}{(k+1)} & \frac{-3EI}{L^{2}} & \frac{(2k)}{(k+1)} & \frac{EI}{L} & \frac{(2k)}{(k+1)} \\ & & \frac{3EI}{L^{3}} & \frac{(4k+1)}{(k+1)} & \frac{-3EI}{L^{2}} & \frac{(2k+1)}{(k+1)} \\ & & \frac{EI}{L} & \frac{(4k+3)}{(k+1)} \end{bmatrix}$$
(A.11)

Similarly, Element d shown in Figure 2:

C - 2

$$\begin{bmatrix} K_{22} \end{bmatrix}_{d} = \begin{bmatrix} \frac{3EI}{L^{3}} \frac{(4k+1)}{(1+k)} & \frac{-6EI}{L^{2}} \frac{(k)}{(1+k)} \\ \frac{-6EI}{L^{2}} \frac{(k)}{(1+k)} & \frac{4EI}{L} \frac{(k)}{(1+k)} \end{bmatrix}$$
(A.12)
$$\begin{bmatrix} K_{11} \end{bmatrix}_{d} = \begin{bmatrix} \frac{3EI}{L^{2}} \frac{(4k+1)}{(1+k)} & \frac{3EI}{L^{2}} \frac{(2k+1)}{(1+k)} \\ \frac{3EI}{L^{2}} \frac{(2k+1)}{(1+k)} & \frac{EI}{L} \frac{(4k+3)}{(1+k)} \end{bmatrix}$$
(A.13)

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$$\begin{bmatrix} K_{12} \end{bmatrix}_{d} = \begin{bmatrix} K_{21} \end{bmatrix}^{t}_{d} = \begin{bmatrix} \frac{3EI}{L^{3}} (4k+1) \\ L^{3} (1+k) \end{bmatrix} \xrightarrow{\begin{array}{c} 6EI}{L^{2}} \frac{k}{(1+k)} \\ \\ \\ - \begin{bmatrix} H \end{bmatrix} \begin{bmatrix} K \end{bmatrix} \\ \\ \frac{-3EI}{L^{2}} \frac{(2k+1)}{(1+k)} \\ \\ \end{array} \xrightarrow{\begin{array}{c} 2EI \\ L \end{array} \xrightarrow{\begin{array}{c} (k) \\ L \end{array} \xrightarrow{\begin{array}{c} (k+1) \\ \end{array}} \xrightarrow{\begin{array}{c} (k+1) \\ \end{array}}$$

$$\begin{bmatrix} K \end{bmatrix}_{d} = \begin{bmatrix} K \end{bmatrix}_{d} = \begin{bmatrix} \frac{3EI}{L^{3}} & \frac{(4k+1)}{(1+k)} & \frac{3EI}{L^{2}} & \frac{(2k+1)}{(1+k)} & \frac{-3EI}{L^{3}} & \frac{(4k+1)}{(k+1)} & \frac{6EI}{L^{2}} & \frac{(k)}{(k+1)} \\ & \frac{EI}{L} & \frac{(4k+3)}{(1+k)} & \frac{-3EI}{L^{2}} & \frac{(2k+1)}{(1+k)} & \frac{2EI}{L} & \frac{(k)}{(1+k)} \\ & & 3EI & (4+1) & -6EI & (k) \end{bmatrix}$$
(A.15)

Symmetric
$$\frac{3EI}{L^3} \frac{(4+1)}{(1+k)} - \frac{-6EI}{L^2} \frac{(k)}{(1+k)}$$
$$\frac{4EI}{L} \frac{(k)}{(1+k)}$$

Using the above element matrices, the following global matrix is assembled:

$$\begin{bmatrix} [K_{11}]_{a} & [K_{22}]_{a} & [0] & [0] & [0] \\ [K_{21}]_{a} & [K_{22}]_{a} + [K_{11}]_{b} & [K_{12}]_{b} & [0] & [0] \\ [0] & [K_{21}]_{b} & [K_{22}]_{b} + [K_{11}]_{c} & [K_{12}]_{c} & [0] \\ [0] & [0] & [K_{21}]_{c} & [K_{22}]_{c} + [K_{11}]_{d} & [K_{12}]_{d} \\ [0] & [0] & [0] & [0] & [K_{21}]_{d} & [K_{22}]_{d} \end{bmatrix}$$
(A.16)

This is an n x n matrix where n = 2N + 2, and N = the number of elements. The first two boundary conditions are enforced by putting 1.0 in the diagonal corresponding to the translational degrees of freedom at the supports and setting all other entries in that row and column equal to zero. The last two boundary conditions are accounted for in the derivation of the individual stiffness matrices.

Note that an adjustment to the stiffness matrix must be made when the

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stiffness of the rotational restraint at each end approaches zero. For this case the diagonal term corresponding to the rotational stiffness at the support should be set equal to unity.

APPENDIX B

COMPUTER PROGRAMS

As a part of this study, two computer programs were developed to solve the dynamic equilibrium matrix equation given in Chapter 2. A brief description of these programs along with their listings and sample outputs are given in this appendix.

B.1 NEWMARK

This program is based on the analysis described in Section 2.2. A description of the required input data is given at the beginning of the program listing. Data is input by means of the data statements in lines 48 to 53 of the program listing. The output consists of the time in seconds and corresponding midspan deflection in inches.

B.2 CENDIF

Program CENDIF is based on the analysis described in Section 2.3. Data input and output are the same on NEWMARK.

FILE: NEWMARK FORTRAN A OLD DOMINION UNIVERSITY -- CMS -- 4.2

			NEW00010
	IMPLICIT REAL*8 (A-H,O-Z)	•	NEW00020
			NEW00030
	DOUBLE PRECISION L,K1,K2,K(70,70),M(70,70),C(70,70),U(7	'O),UDT(70),NEW00040
	<pre>&RT (70), C1 (70, 70), C2 (70, 70), C9 (70, 70), C4 (70), KEL (4, 4), F (</pre>	(70),C5(70),NEW00050
	&DUM (70), UT (70), MINV (70, 70), C8 (70, 70), UDTP (70), UDDT (70),	UDDTP (70), UNEW00060
	&TP (70), UTN (70), C3 (70), C6 (70), FREQ (5), KINV (70, 70), X (70, 7	'O),A(70	,70) NEW00070
	&,B(70,70),D(70),EIGV(70),DAMRAT(70),ANS(70,70),RES(70,7	'O),XT (7	D,70NEW00080
	δ)		NEW00090
	IFPR=0		NEW00100
C****	****************** INPUT DATA ************************	****	NEW00110
C**		**	NEW00120
C**	IFPR = PRINT REQUEST VARIABLE FOR JACOBI	たた	NEW00130
C**	0 = 00 NOT PRINT INTERMEDIATE VALUES	**	NEW00140
C * *	1 = PRINT INTERMEDIATE VALUES	**	NEW00150
C * *		**	NEW00160
C**	L = LENGTH (IN)	* *	NEW00170
C * *		* *	NEW00180
C**	NUMEL = NUMBER OF ELEMENTS (MUST BE AN EVEN NUMBER)	**	NEW00190
C**		**	NEW00200
C * *	TS = TIME STEP; DELTA 'T' (SEC)	**	NEW00210
C**		**	NEW00220
C**	ROW = MASS PER UNIT LENGTH (KIP*SEC**2/IN**2)	**	NEW00230
C**		* *	NEW00240
C * *	E = MODULUS OF ELASTICITY (KSI)	**	NEW00250
C**		**	NEW00260
C * *	XI = MOMENT OF INERTIA (IN**4)	**	NEW00270
C**		**	NEW00280
C * *	AR = AREA (IN**2)	* *	NEW00290
C * *		* *	NEW00300
C**	K1 = ROTATIONAL STIFFNESS AT END 1 (K*IN/RAD)	* *	NEW00310
C**	· · · · · · · · · · · · · · · · · · ·	**	NEW00320
C**	K2 = ROTATIONAL STIFFNESS AT END 2 (K*IN/RAD)	* *	NEW00330
C**		* *	NEW00340
C * *	ZETA = DAMPING RATIO	**	NEW00350
Cxx		**	NEW00360
Cxx	TT = TOTAL TIME FOR PROGRAM EXECUTION (SEC)	x x	NEW00370
C**		**	NEW00380
Cxx	PO = MAGNITUDE OF THE FORCING FUNCTION (KIPS)	**	NEW00390
Can		**	NEW00400
Cxx	UMEGA = FREQUENCY OF THE FUNCTING FUNCTION (RAD)	**	NEW00410
(xx		**	NEW00420
Cxx or t	DELO = PRESCRIBED INITIAL DEFLECTION AT MEMBER MIDSPAN	**	NEW00430
Can	(IN)	XX AA	NEW00440
(TR		**	NEW00450
Casas	***************************************	*****	NEW00460
			NEW004/0
			NEW00460
	UATA L,NUMEL, IS, KUW/ 1//, IU, 0.000500, 202.1454E-09/		NEW00490
	DATA E,XI,AK/10000.,0.32500000,0./363/		NEWUU5UU
	UATA RI,RZ,11/53.1100,53.1100,0.00/		NEWUU510
	UATA PU,UMEGA,ZETA/U.UU222036/,12.5663/U62,U.00/200/		NEWUU52U
	DATA GAMA, BETA, DELU/0.50,0.25,0.00/914/		NEWUU53U
			NEWUU54U
•			NEWUU55U

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		NEW00560
	TIME=0.0	NEW00570
	WRITE (2,1)	NEW00580
	H=L/NUMEL	NEW00590
1	FORMAT (/1X. 'THIS IS NEWMARKS SOLUTION')	NEW00600
•	WRITE (2, 1059) K1	NEW00610
1059	FORMAT(/1) + STIFFNESS = - + D16 Q	NEW00620
1099	WPITE (2, 1060) 7ETA	NEW00630
1060	EODMAT(/1) = DAMPINC = + D16 O	NEW00610
1000	WELTE (2 10(1) DO	NEW00040
10(1	WRITE(2, 1001) PO	NEW00050
1061	FURMAT (/ IX, FURCE , DID.9)	NEWOUGOU
	WRITE (2, 1062) UMEGA	NEWOU670
1062	FORMAT (/TX, 'FREQUENCY ', DT6.9)	NEWOO680
	· · · · · · · · · · · · · · · · · · ·	NEW00690
·	N=2*NUMEL+2	NEW00700
		NEW00710
С	WRITE (2,176)	NEW00720
С	WRITE (2,177)	NEW00730
		NEW00740
	DO 10 I=1,N	NEW00750
	DO 10 J=1.N	NEW00760
10	K(I,J) = 0.0	NEW00770
		NEW00780
		NEW00790
	K(1, 1) = 1000	NEW00800
	K(N-1, N-1) = 1000	NEW00810
	(((1; N 1) = 1000	NEW00820
		NEW00830
	x (2, 2) - FAXIAL AV1 / (14 / V111))	NEW00030
	(2,2) = (-1) +	NEW00040
	$K(2, 3) = (-1,)\pi 3, \pi E \pi X \pi 2, \pi K / ((1 \pi \pi 2) \pi (K + 1,))$	NEW00050
	$K(2,4) = E^{*}X[*2.*K[/(H^{*}(K]+1.))]$	NEWOUDBU
	K(3,2) = K(2,3)	NEWOUB/U
	K(3,3) = 3.*E*XI*(4.*KI+1.)/((H**3)*(KI+1.))	NEW00880
	K(3,4) = (-3.) *E *X * (2.*K + 1.) / ((H * 2) * (K + 1.))	NEW00890
	K(4,2) = K(2,4)	NEW00900
	K(4,3) = K(3,4)	NEW00910
	K (4,4) =E*X *(4.*K1+3.)/(H*(K1+1.))	NEW00920
		NEW00930
		NEW00940
	K (N-3,N-3)=3.*E*X *(4.*K2+1.)/((H**3)*(K2+1.))	NEW00950
	K(N-3, N-2) = (3.) *E*X * (2.*K2+1.) / ((H**2) * (K2+1.))	NEW00960
	$K(N-3,N) = 6 \times E \times X \times K2 / ((H \times 2) \times (K2+1,))$	NEW00970
	K(N-2, N-3) = K(N-3, N-2)	NEW00980
	$K(N-2, N-2) = F \times X + X (\Delta \times K 2 + 3) / (H \times (K 2 + 1))$	NEW00990
	$K(N-2, N) = 2 \times F \times X + K \times 2 / (H \times (K 2 + 1)))$	NEW01000
	K(N = 2, N) = 2.000 mm(N = 2, N)	NEW01010
	K(N, N-2) = K(N-2, N)	NEW01070
	ハ (N) N) - F & Y A A - Z , N / K (N) N) - F & Y A A + K) / (Ц & (K) + 1))	NEW01020
	R (N,N) ~ E^AIA4+^AR2/ (N* (R271+))	NEWOIOJO
		NEW01040
		NEWO1050
		NEWU 1060
	$KEL(1, 1) = \frac{12.*E*X1}{(H**3)}$	NEW01070
,	KEL(1,2) = 6.*E*X /(H**2)	NEW01080
	KEL(1,3) = (-1.) * KEL(1,1)	NEW01090
	KEL(1,4) = KEL(1,2)	NEW01100

	KEL(2,2) = 4.*E*X1/(H) $KEL(2,3) = (-1.)*KEL(1,2)$ $KEL(2,4) = KEL(2,2)/2.$ $KEL(3,3) = KEL(1,1)$ $KEL(3,4) = KEL(2,3)$ $KEL(4,4) = KEL(2,2)$	NEW01110 NEW01120 NEW01130 NEW01140 NEW01150 NEW01160
30	IF (K (2,2).LE.0.00001) K (2,2)=1.0 IF (K (N,N).LE.0.00001) K (N,N)=1.0 D0 30 1=1,4 D0 30 J=1,4 IF (J.GT.1) KEL (J,1)=KEL (1,J)	NEW01170 NEW01180 NEW01200 NEW01210 NEW01220
	DO 50 JK =2,NUMEL-1 =JK*2-2 JJ=!! DO 45 I=1,4 DO 40 J=1,4	NEW01230 NEW01240 NEW01250 NEW01260 NEW01270 NEW01280
	K (+ ,JJ+J) = K (+ ,JJ+J) +KEL (,J)	NEW01290 NEW01300
		NEW01310
40 1. E		NEW01320
47 50	CONTINUE	NEW01350
<i>J</i> U		NEW01350
66	DO 75 =1,N	NEW01360
	DO 70 J=1,N	NEW01370
	M(I,J)=0.0	NEW01380
	C (I, J) = 0.0	NEW01390
70	CONTINUE	NEW01400
75	CONTINUE	NEW01410
	r ,	NEW01420
	M(1,1)≈39.	NEW01430
	$M(2,2) = H^{*} 2$	NEWO1440
	M(N-1, N-1) = 39	NEWO1450
	$M(\mathbf{N},\mathbf{N}) = \mathbf{M}\mathbf{\pi}\mathbf{\pi}\mathbf{Z}$	NEW01460
	DO(8O) = 2 N - 2 2	NEW01470
	j=j+1	NEW01400
	M(1,1) = 78.	NEW01500
	M(J,J) = 2 * (H**2)	NEW01510
80	CONTINUE	NEW01520
		NEW01530
	DO 90 I=1,N	NEW01540
90	M(1,1) = M(1,1) * (ROW * H / 78.)	NEW01550
	CALL JACOBI (K,M,N,IFPR,X,EIGV)	NEW01560
	D0 95 1=1,N	NEW01570
95	DAMRAT(I)=ZETA	NEW01580
		NEWO1590
	LALL DAMP (N, EIGV, X, M, DAMRAI, C)	NEWO1600
		NEWOIDIU
	DDINT# IN STADT	NEWO 1020
	ENTRE", IN START	NEWOIADO
	DD 300 L=1.N	NEWO1650
		112 10 10 50

FILE: NEWMARK FORTRAN A OLD DOMINION UNIVERSITY -- CMS -- 4.2

300	RT(1)=0.0	NEW01660
-	PI=ACOS (-1.0)	NEW01670
	RT(N/2) = PO*(DCOS(OMEGA*TIME))	NEW01680
		NEW01690
	CALL INVERT (M, MINV, N)	NEW01700
		NEW01710
	DO 333 I=1,NUMEL+1	NEW01720
	DUM1=K1*L/(4*P1*E*X1)	NEW01730
	Z=P1*H*(1-1)/L	NEW01740
	UT(2*I) = (DELO/(1.+DUM1*2))*((PI/L*DCOS(Z))+(DUM1*2*PI/L*DSIN(2*Z))	NEW01750
	((3	NEW01760
	UT (2*1-1) = (DELO/(1.+DUM1*2)) * (DSIN(Z)+DUM1*(1DCOS(2*Z)))	NEW01770
333	CONTINUE	NEW01780
	D0 302 I=1,N	NEW01790
	SUM=0.0	NEW01800
	DO 301 J=1,N	NEW01810
301	SUM=SUM+K(I,J)*UT(J)*(-1.)	NEW01820
302	DUM(I)=SUM	NEW01830
		NEW01840
	DO 306 I=1,N	NEW01850
	SUM=0.0	NEW01860
	DO 305 J=1,N	NEW01870
305	SUM=SUM+MINV(1,J)*DUM(J)	NEW01880
306	UDDT(I)=SUM	NEW01890
		NEW01900
	DO 310 I=1,N	NEW01910
	UDT(1)=0.0	NEW01920
310	CONTINUE	NEW01930
		NEW01940
	PRINT*, 'OUT START'	NEW01950
		NEW01960
	DO 320 I=1,N	NEW01970
	DO 320 J=1,N	NEW01980
320	C1(I, J) = K(I, J) + GAMA*C(I, J) / (BETA*TS) + M(I, J) / (BETA*(TS**2))	NEW01990
	CALL INVERT (C1, C2, N)	NEW02000
120	DO 122 =1,N	NEW02010
	C3(I) = (GAMA/(BETA*TS)) *UT(I) + (GAMA/BETA-1.0) *UDT(I) +TS*((GAMA/(BE	NEW02020
1	&TA*2.))-1.0)*UDDT(1)	NEW02030
		NEW02040
	L4(I)=UI(I)/(BETA*(IS**2))+UDI(I)/(BETA*TS)+((I./(2.*BETA))-I.U)*(JNEW02050
100		NEW02060
122		NEW02070
		NEWU2000
	DU 130 1=1,N	NEW02090
		NEWUZIUU
125	$\frac{1}{125} J=1, N$	NEWUZIIU
125	$SUM=SUM+U(1, J) \times US(J)$	NEWU2120
130	(5(1)=SUM	NEWU2130
		NEWUZ14U
		NEWU2150
		NEWU2160
1.00	$\begin{array}{c} UU = 4UU = 1, \\ CUM = CUM + CUM + CL + (1) \\ \end{array}$	NEWUZI/U
400	SUM=SUM+m (1, J) *L4 (J)	NEWU210U
410	LO(I)=3UM	NEWU2190
		NEWUZZUU

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	DO 420 =1,N	NEW02210
420	F(I) = RT(I) + C5(I) + C6(I)	NEW02220
		NEW02230
	DO 430 1=1 N	NEWOZZJU
		NEW02240
		NEWUZZ50
		NEW02260
425	SUM=SUM+C2(1, J) *F(J)	NEW02270
430	UTP(I)=SUM	NEW02280
		NEW02290
	ICOUNT=ICOUNT+1	NEW02300
	TIME=TIME+TS	NEW02310
_	IF (TIME.GT.1.00) GD TO 199	NEW02320
²	RT(N/2) = PO*(DCOS(OMEGA*TIME))	NEW02330
		NEW02310
100	PT(N/2) = 0.0	NEW02340
200	$R \models (N/2) = 0.0$	NEWU2350
200		NEWU2300
		NEW023/0
	IF (ICOUNT.EQ.IO) GO TO 141	NEW02380
	GO TO 143	NEW02390
141	WRITE (2, 175) TIME, UTP (N/2), JEST	NEW02400
	ICOUNT=0	NEW02410
		NEW02420
143	DO 150 =1,N	NEW02430
	UDDTP(I) = (UTP(I) - UT(I) - (TS*UDT(I)) - ((TS**2) * (0.5-BETA) * UDDT	(I)))*NEW02440
	&(1./((TS**2)*BETA))	NEW02450
150	CONTINUE	NEW02460
-		NEW02470
	DO 161 I=1.N	NEW02480
	IIDTP(I) = IIDT(I) + TS*(((I) O - GAMA) * IIDDT(I)) + (GAMA* IIDDTP(I)))	NEW02490
		NEW02500
		NEW02500
		NEW02510
161		
101	CUNTINUE	NEW02530
	IF (IIME.GI.II) GU IU 500	NEW02540
		NEW02550
	GO TO 120	NEW02560
175	FORMAT (F10.8,1X,F10.8,1X,11)	NEW02570
176	FORMAT(/1X,' TIME DEFLECTION AT L/2')	NEW02580
177	FORMAT ('	NEW02590
	§')	NEW02600
500	STOP	NEW02610
	END	NEW02620
		NEW02630
		NEW02640
	SUBBOUTINE INVERT (AO. A. N)	NEW02650
	DOUBLE PRECISION A $(70, 70)$ AO $(70, 70)$	NEW02650
	DODLE TREGISTOR # (70,70); NO (70,70)	NEW02000
		NEWO2070
		NEW02000
,		NEWUZDYU
I	A (I, J) = AU (I, J)	NEW02/00
		NEW02710
	NP=N+1	NEW02720
	A (1,NP) = 1.0	NEW02730
	DO 10 I=2,N	NEW02740
10	A (I,NP) =0.0	NEW02750

		NEW02760
	DO 40 J=1,N	NEW02770
	DO 20 LX=1,N	NEW02780
20	A(NP, LX) = A(1, LX+1) / A(1, 1)	NEW02790
	DO 30 KX=2,N	NEW02800
	DO 30 $LX=1,N$	NEW02810
30	A(KX-1,LX) = A(KX,LX+1) - A(KX,1) * A(NP,LX)	NEW02820
10	DO 40 LX=1, N	NEW02830
40	A(N, LX) = A(NP, LX)	NEW02840
	DETUDN	NEWU2050
		NEW02000
	CIRPOLITINE JACOBI (K. M. N. JEPP, Y. ELCV)	NEW02070
r	SUBROUTINE ACORI	NEW02000
C	$\frac{1}{1} \frac{1}{1} \frac{1}$	NEW02090
	$\begin{array}{c} \text{POUBLE PRECISION } \mathbf{A} (70, 70) \\ \mathbf{B} (70, 70) \\ \mathbf{X} (70, 70) \\ \mathbf{EIGV} (70) \\ \mathbf{D} (70) \\ \mathbf{D} (70) \\ \mathbf{C} (70) \\$	NEW02900
2	\$K (70, 70) . M (70, 70)	NEW02920
		NEW02930
С	COMMON/K.M/	NEW02940
č	WRITE (2, 1051)	NEW02950
C1051	FORMAT (/1X, ' INPUT DATA ')	NEW02960
c	READ(1,*)N, IFPR	NEW02970
С	WRITE (2,1001) N, IFPR	NEW02980
C	D0 1010 I=1,N	NEW02990
С	READ(1, *) (A(I, J), J=1, N)	NEW03000
С	WRITE (2,1110) (A (I, J), J=1, N)	NEW03010
C1010	CONTINUE	NEW03020
С	D0 1020 =1,N	NEW03030
С	READ $(1, *)$ (B $(1, J), J=1, N$)	NEW03040
С	WRITE (2,1110) (B(I,J),J=1,N)	NEW03050
C1020	CONTINUE	NEW03060
C1001	FORMAT (2110)	NEW03070
C1110	FURMAT (8FT0.4)	NEW03080
		NEW03090
	DU = J = I, N	NEW03100
	P(I = I) = P(I = I)	NEW03110
1		NEW03120
2	CONTINUE	NEW03140
2	NSM2X=15	NEW03150
C	WRITE(2, 1980)	NEW03160
1980	FORMAT (/1X. ' FIGENVALUES ')	NEW03170
	RTOL=1, $D-12$	NEW03180
	IOUT=2	NEW03190
	DO 10 I=1.N	NEW03200
	IF (A (I, I) .GT.O.AND.B (I, I) .GT.O.) GO TO 4	NEW03210
	WRITE (IOUT, 2020)	NEW03220
	STOP	NEW03230
4	D(1) = A(1, 1) / B(1, 1)	NEW03240
10	E GV(1) = D(1)	NEW03250
	DO 30 I=1,N	NEW03260
	DO 20 J=1,N	NEW03270
20	X (I, J)=0.	NEW03280
30	X(1,1) = 1.0	NEW03290
	IF (N.EO.1) RETURN	NEW03300

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С NEW03310 С INITIALIZE SWEEP COUNTER AND EIGEN ITERATION NEW03320 С NEW03330 NSWEEP=0 NEW03340 NR = N - 1NEW03350 40 NSWEEP=NSWEEP+1 NEW03360 IF (IFPR.EQ.1) WRITE (IOUT, 2000) NSWEEP NEW03370 PRINT*, ' SWEEP NUMBER... ', NSWEEP NEW03380 С NEW03390 C CHECK IF PRESENT OFF DIAGONAL ELEMENT IS TOO LARGE NEW03400 С NEW03410 EPS=(0.01**NSWEEP)**2 NEW03420 DO 210 J=1,NR NEW03430 JJ=J+JNEW03440 DO 210 K1=JJ,N NEW03450 IF (DABS (A (J,K1)).LT.1.D-20) GO TO 211 NEW03460 EPTOLA= (A (J,K1) *A (J,K1)) / (A (J,J) *A (K1,K1)) NEW03470 NEW03480 GO TO 212 EPTOLA=0.0 211 NEW03490 212 EPTOLB = (B(J,K1) * B(J,K1)) / (B(J,J) * B(K1,K1))NEW03500 IF ((EPTOLA.LT.EPS).AND. (EPTOLB.LT.EPS)) GO TO 210 NEW03510 AKK=A(K1,K1) *B(J,K1) -B(K1,K1) *A(J,K1)NEW03520 AJJ=A(J,J) *B(J,K1) -B(J,J) *A(J,K1)NEW03530 AB=A(J, J) *B(K1, K1) - A(K1, K1) *B(J, J)NEW03540 NEW03550 $CHECK = (AB * AB + 4 \cdot AKK * AJJ) / 4$. IF (CHECK) 50,60,60 NEW03560 50 NEW03570 WRITE (10UT, 2020) NEW03580 STOP 60 NEW03590 SOCH=DSORT (CHECK) NEW03600 D1=AB/2.+SQCHD2=AB/2.-SQCH NEW03610 DEN=D1 NEW03620 IF (DABS (D2).GT.DABS (D1)) DEN=D2 NEW03630 NEW03640 IF (DEN) 80,70,80 70 CA=0. NEW03650 NEW03660 CG = (-1.) *A (J,K1) / A (K1,K1)NEW03670 GO TO 90 80 CA=AKK/DEN NEW03680 CG = (-1.) * AJJ/DENNEW03690 90 IF (N-2) 100, 190, 100 NEW03700 100 JP1=J+1NEW03710 NEW03720 JM1=J-1 KP1=K1+1 NEW03730 NEW03740 KM1=K1-1 NEW03750 IF (JM1-1) 130, 110, 110 110 DO 120 I=1, JM1 NEW03760 AJ=A(I,J)NEW03770 NEW03780 BJ=B(I,J)NEW03790 AK=A(I,K1)NEW03800 BK=B(I,K1)NEW03810 A(I, J) = AJ + CG * AKNEW03820 B(I,J) = BJ + CG * BKNEW03830 A(I,KI) = AK + CA * AJ97 NEW03840 B(I,K1) = BK + CA * BJ120 NEW03850 130 IF (KP1-N) 140, 140, 160
140	DO 150 I=KP1,N	NEW03860
	AJ=A (J, 1)	NEW03870
	BJ=B(J,I)	NEW03880
	AK=A (K1,1)	NEW03890
	BK=B(K1,1)	NEW03900
	A(J,I) = AJ + CG * AK	NEW03910
	$B(J,I) = BJ + CG \times BK$	NEW03920
	A(K1, 1) = AK + CA * AJ	NEW03930
150	B(K1, I) = BK + CA * BJ	NEW03940
160	IF (JP1-KM1) 170, 170, 190	NEW03950
170	DO 180 I=JP1.KM1	NEW03960
•	AJ=A(J,I)	NEW03970
	BJ=B(J, I)	NEW03980
	AK=A(I,K1)	NEW03990
	BK=B(I,K1)	NEW04000
	A(J,I) = AJ + CG * AK	NEW04010
	B(J,I) = BJ + CG * BK	NEW04020
	$A(I,KI) = AK + CA \times AJ$	NEW04030
180	$B(I,K1) = BK + CA \times BJ$	NEW04040
190	AK=A(K1,K1)	NEW04050
	BK=B(K1,K1)	NEW04060
	A(K1, K1) = AK+2, *CA*A(J, K1) + CA*CA*A(J, J)	NEW04070
	B(K1,K1) = BK+2, *CA*B(J,K1) + CA*CA*B(J,J)	NEW04080
	A(J,J) = A(J,J) + 2 * CG*A(J,K1) + CG*CG*AK	NEW04090 .
	$B(J,J) = B(J,J) + 2 \cdot *CG * B(J,K) + CG * CG * BK$	NEW04100
	A(J,K1) = 0.	NEW04100
·	B(J,K1) = 0	NEW04120
С		NEW04120
C UPD	ATE FIGENVECTOR MATRIX	NEW04150
с С		NEW04140
•	DO 200 L=1 N	NEW04150
	X.J=X (1)	NEW04100
	XK=X (1,K1)	NEW04170
	x(1 - 1) = x 1 + CC + xK	NEW04100
200	$X(1, k_1) = X K + C A * X I$	NEW04130
210		NEW04200
с С	our ruse	
C UP	DATE ELGENVALUES	NEW04220
C C		NEW04250
U I	DO 220 1=1.N	NEW04240
	IE(A(1,1)) GT () AND $B(1,1)$ GT () GD TO 220	NEW04250
	WRITE (1011, 2020)	NEW04200
		NEW04270
220	F(GV(1) = A(1, 1) / B(1, 1)	NEW04200
220	E (15PP = 0.0) co To 230	NEWO4230
	$H_{1}(1) = (10$	NEW04300
	WRITE(1001, 2030)	NEW04310
r	WRITE (TUUT, 2010) (ETGV (T), T=T,N)	NEW04320
	FCK FOR CONVERCENCE	NEW04330
C LH	EUR FUR LUNVERGENLE	NEWU434U
L 120		NEWU435U
0ر 2	UU 240 1=1,N	NEWU430U
		NEW043/0
	UIF=UABS (EIGV (I) -D (I))	NEW04380
	IF (UIF.GI.TOL) GO TO 280	NEW04390
240	CONTINUE	NEW04400

С NEW04410 С CHECK ALL OFF DIAG ELEMENTS TO SEE IF ANOTHER SWEEP IS REQ'D NEW04420 С NEW04430 EPS=RTOL**2 NEW04440 DO 250 J=1,NR NEW04450 JJ≈J+1 NEW04460 DO 250 K1=JJ,N NEW04470 IF (DABS (A (J,K1)).LT.1.D-30) G0 T0 251 NEW04480 EPSA = (A(J,K1) * A(J,K1)) / (A(J,J) * A(K1,K1))NEW04490 GO TO 252 NEW04500 251 EPSA=0.0 NEW04510 EPSB = (B(J,K1) * B(J,K1)) / (B(J,J) * B(K1,K1))252 NEW04520 IF ((EPSA.LT.EPS).AND. (EPSB.LT.EPS)) GO TO 250 NEW04530 GO TO 280 NEW04540 250 CONTINUE NEW04550 С NEW04560 C FILL OUT BOTTOM TRIANGLE OF RESULTANT MATRICES & SCALE EIGENVECTORS. NEW04570 С NEW04580 255 DO 260 |=1,N NEW04590 DO 260 J=1,N NEW04600 A(J, I) = A(I, J)NEW04610 260 B(J,I) = B(I,J)NEW04620 DO 270 J=1,N NEW04630 BB=DSORT (B(J,J)) NEW04640 DO 270 $K_{1=1,N}$ NEW04650 270 X(K1,J) = X(K1,J) / BBNEW04660 С WRITE (10UT, 310) NEW04670 С DO 300 I=1,N NEW04680 C300 WRITE (10UT, 2010) (X (I, J), J=1, N) NEW04690 310 FORMAT (/1X, ' THE EIGENVECTORS ARE ') NEW04700 NEW04710 NEW04720 NEW04730 NEW04740 RETURN NEW04750 С NEW04760 C UPDATE THE 'D' MATRIX AND START NEW SWEEP IF ALLOWED NEW04770 NEW04780 C NEW04790 280 D0 290 I=1,N 290 D(I) = E I G V(I)NEW04800 IF (NSWEEP.LT.NSMAX) GO TO 40 NEW04810 NEW04820 GO TO 255 2000 FORMAT (/1X, ' SWEEP NUMBER IN JACOBI = ', 14) NEW04830 NEW04840 2010 FORMAT (/1X,6E20.12) 2020 FORMAT (/1X, ' **** ERROR SOLUTION STOP / MATRICES NOT POSITIVE NEW04850 NEW04860 &DEFINITE') 2030 FORMAT (/1X, ' CURRENT EIGENVALUES IN JACOBI ARE ') NEW04870 NEW04880 END NEW04890 SUBROUTINE DAMP (N, EIGV, X, M, DAMRAT, C) NEW04900 IMPLICIT REAL*8(A-H, 0-Z) NEW04910 DOUBLE PRECISION X (70,70), T (70,70), M (70,70), C (70,70), EIGV (70), DAMRNEW04920 NEW04930 &AT (70) NEW04940 NEW04950 DO 10 |=1,N

	•	
	$E \mid GV(I) = DSQRT(E \mid GV(I))$	NEW04960
	D0 10 J=1,N	NEW04970
10	C(I,J)=0.0	NEW04980
		NEW04990
	DO 20 =1,N	NEW05000
	DA=2.*DAMRAT(11)*EIGV(11)	NEW05010
	DO 20 =1,N	NEW05020
	D0 20 J=1,N	NEW05030
20	C (, J) =C (, J) +X (,) *X (J,) *DA	NEW05040
		NEW05050
	D0 30 I=1,N	NEW05060
	DO 30 J=1,N	NEW05070
	T(I,J)=0.0	NEW05080
	DO 30 KI=1,N	NEW05090
30	T(I, J) = T(I, J) + M(I, K1) * C(K1, J)	NEW05100
		NEW05110
	DO 40 =1,N	NEW05120
	D0 40 J=1,N	NEW05130
	C(I,J)=0.0	NEW05140
	D0 40 K1=1,N	NEW05150
40	C(I,J)=C(I,J)+T(I,K1)*M(K1,J)	NEW05160
		NEW05170
С	DO 50 I=1,N	NEW05180
C50	WRITE (2,120) (C (I,J),J=1,N)	NEW05190
120	FORMAT (6014.4)	NEW05200
	RETURN	NEW05210
		NEW05220
	END	NEW05230

FILE: DAVE

OUT

THIS IS NEWMARKS SOLUTION
STIFFNESS 0.531100006D+02
DAMPING 0.130999982D-01
FORCE 0.40000066D-02
FREQUENCY 0.251300049D+02
TIME DEFLECTION AT L/2
0.00500000 0.07560742 1
0.01000000 0.07411487 1
0.01500000 0.07822479 1
0.02000000 0.07755833 1
0.02500000 0.07321689 1
0.03000000 0.07439081 1
0.03500001 0.07596175 1
0.04000001 0.070300831
0.04900001 0.004000991 0.0000000000000000000000000000000
0.05500001 0.05838368 1
0.06000001 0.04628549 1
0.06500001 0.03770042 1
0.07000001 0.03044851 1
0.07500001 0.01549482 1
0.08000001 -0.00155219 1
0.08500001 -0.01381828 1
0.09500002 = 0.04917751 1
0.10500002 - 0.08058453 1
0.11000002 -0.09807491 1
0.11500002 -0.11601054 1
0.12000002 -0.12788046 1
0.12500002 -0.13797468 1
0.13000002 -0.14964225 1
0.13500002 -0.15657816 1
0.14000002 = 0.15/30855 1
0.14500002 = 0.15770900 1
0.15500003 - 0.14753221 1
0.16000003 -0.13531903 1
0.16500003 -0.12327482 1
0.17000003 -0.10672159 1
0.17500003 -0.08461688 1
0.1800003 -0.06275240 1
0.18500003 -0.04085973 1
0.19500003 0.01269540 1
0.20500003 0.05075040 1
0.21000003 0.08652612 1

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IMPLICIT REAL*8 (A-H.O-Z) CEN00010 CEN00020 CEN00030 С THIS PROGRAM SOLVES FOR THE DEFLECTIONS OF A BEAM CEN00040 CEN00050 С SUBJECT TO A FORCING FUNCTION AT THE MIDPOINT CEN00060 **CEN00070** С USING CENTRAL DIFFERENCE METHOD CEN00080 **CEN00090 CENO0100** CEN00110 DOUBLE PRECISION L,K1,K2,K(70,70),M(70,70),C(70,70),U(70),UTN(70),CEN00120 &RT (70), C1 (70, 70), C2 (70, 70), C3 (70, 70), C4 (70), KEL (4, 4), B (70), C5 (70), CENO0130 &DUM (70), UT (70), MINV (70, 70), C6 (70, 70), X (70, 70), EIGV (70), DAMRAT (70), CENO0140 &UDDT (70) CEN00150 IFPR=0 CEN00160 ******* CEN00170 ** C ** CEN00180 ** C * * L = LENGTH(IN)CEN00190 ** C ** **CEN00200** ** C * * NUMEL = NUMBER OF ELEMENTS (MUST BE AN EVEN NUMBER) CEN00210 C ** ** **CEN00220** ** C×× TS = TIME STEP: DELTA 'T' (SEC) CEN00230 C ** ** CEN00240 (K | P*SEC**2/1N**2) ** C * * ROW = MASS PER UNIT LENGTH CEN00250 C ** ** CEN00260 C ** E = MODULUS OF ELASTICITY(KSI) ** · CEN00270 C** ** CEN00280 ** C ** XI = MOMENT OF INERTIA(IN**4) CEN00290 ** C * * CEN00300 C** AR = AREA (1N**2) ** CEN00310 C** ** CEN00320 C** K1 = ROTATIONAL STIFFNESS AT END 1(K*IN/RAD)** CEN00330 х× C** CEN00340 C * * K2 = ROTATIONAL STIFFNESS AT END 2 (K*IN/RAD)** CEN00350 х× CEN00360 C×× C ** ZETA = DAMPING RATIO** CEN00370 C ** ** ** CEN00380 ** CEN00390 C ** TT = TOTAL TIME FOR PROGRAM EXECUTION (SEC) хx CEN00400 C * * C * * PO = MAGNITUDE OF THE FORCING FUNCTION (KIPS) ** CEN00410 C * * ** CEN00420 C * * OMEGA = FREQUENCY OF THE FORCING FUNCTION (HZ)** **CEN00430** ste ste C** CEN00440 ** ** CEN00450 C** CEN00460 **CEN00470 CEN00480** CEN00490 DATA L, NUMEL, TS, ROW/177., 12,0.00010, 181.9527D-09/ CEN00500 DATA E,XI,AR/10000.,.325,.7363/ CEN00510 CEN00520 DATA K1,K2,TT/00.000,00.000,4.00/ CEN00530 DATA P0,0MEGA,ZETA/0.00,0.000000000,0.0000/ CEN00540 I COUNT=0 CEN00550

	TIME=0.0	CEN00560
		CEN00570
	H-L/NUMEL	CEN00580
	N=2*NIIME1 - 2	CEN00590
		CEN00600
	DO 10 I=1.N	CEN00010
	DO 10 J=1.N	CEN00630
10	K(1, J) = 0.0	CEN00640
		CEN00650
	K (1,1)=3.*E*XI*(4.*K1+1.)/((H**3)*(K1+1.))	CEN00660
	K(1, 2) = (-1.) * 3. * E * X * (2. * K 1 + 1.) / ((H * * 2) * (K 1 + 1.))	CEN00670
	K(2, 1) = K(1, 2)	CEN00680
	K (2,2) = E * X I * (4. * K I + 3.) / (H * (K I + I.))	CEN00690
		CEN00700
	K (N-1,N-1)=3.*E*X *(4.*K2+1.)/((H**3)*(K2+1.))	CEN00710
	K (N-1,N) =3.*E*X *(2.*K2+1.)/((H**2)*(K2+1.))	CEN00720
	K(N, N-1) = K(N-1, N)	CEN00730
	K (N,N) = E * X I * (4. * K2+3.) / (H * (K2+1.))	CEN00740
		CEN00750
	KEL(1,1) = 12.*E*X1/(H**3)	CEN00760
	KEL(1,2) = 6.*E*X1/(H**2)	CEN00770
	KEL(1,3) = (-1.) * KEL(1,1)	CEN00780
	KEL(1,4) = KEL(1,2)	CEN00790
	$KEL(2,2) = 4.\pi E^* X I / (H)$ $KEL(2,2) = (-1) \times KEL(1,2)$	CEN00800
	$NEL(2, 5) = (-1.) \times NEL(1, 2)$ $KEL(2, 5) = KEL(2, 3) / 2$	CENOUSIO
	KEL(2,4) = KEL(2,2)/2.	
	KE[(3, 5)] = KE[(2, 3)]	CENOO840
	KE(0, 4) = KE(2, 3)	CEN00040
	REL(F)F' = REL(Z)A'	CEN00860
	DO 30 =1.4	CEN00870
	$DO_{30} J=1.4$	CEN00880
30	IF (J.GT.I) KEL (J,I) = KEL (I,J)	CEN00890
•		CEN00900
	DO 50 JK =1,NUMEL-2	CEN00910
•	11=JK*2-2	CEN00920
	JJ=	CEN00930
	DO 45 1=1,4	CEN00940
	DO 40 J=1,4	CEN00950
		CEN00960
	K (+ ,JJ+J) = K (+ ,JJ+J) +KEL (,J)	CEN00970
		CEN00980
40		CEN00990
45		CENCIOUC
50	CUNTINUE	CENCIUIU
C * * *	******	**CEN01020
0.000		CENOIOLO
C I	THIS IS A TEST OF THE STIFFNESS MATRIX FOR THE STATIC LOAD CASE	CEN01050
•		CEN01060
С	DO 61 1=1.N	CEN01070
C61	RT(1) = 0.0	CEN01080
C	RT(N/2) = .10	CEN01090
С	CALL INVERT (K, C6, N)	CEN01100

С	DO 63 I=1,N	CEN01110
С	SUM=0.0	CEN01120
С	DO 62 J=1,N	CEN01130
C62	SUM=SUM+C6(1,J) *RT(J)	CEN01140
C63	U(1)=SUM	CEN01150
C	DO 65 I=1,N	CEN01160
C	PRINT*, 'DEFLECTION AT NODE ', I, ' IS ', U(I)	CEN01170
C65	WRITE (2,64) I, U(I)	CEN01180
C64	FORMAT (/1X, 'STATIC SOLUTION $U(', 12, ') = ', D23.16$)	CEN01190
С	GO TO 500	CEN01200
С	-	CEN01210
C****	***********	CEN01220
		CEN01230
66	DO 75 I=1,N	CEN01240
	DO 70 J=1,N	CEN01250
	M(I, J) = 0.0	CEN01260
	C(i,J)=0.0	CEN01270
70	CONTINUE	CEN01280
75	CONTINUE	CENO1290
	·	CENO1300
		CEN01310
	$DO \ 80 \ =1, N-1, 2$	CENO1320
	J= +]	CEN01330
	M(I,I)=78.	CENO1340
	M(J, J) = 2.*(H**2)	CENO1350
80	CONTINUE	CEN01360
		CEN01370
	DO 90 =1,N	CENO1380
90	M(1,1) = M(1,1) * (ROW * H / 78.)	CEN01390
		CENO1400
С	DO 100 (=1,N	CEN01410
	IFPR=0	CENO1420
	CALL JACOBI (K, M, N, IFPR, X, EIGV)	CENO1430
		CENO1440
	DO 100 I=1,N	CEN01450
100	DAMRAT (1) =ZETA	CEN01460
		CEN01470
	CALL DAMP(N,EIGV,X,M,DAMRAT,C)	CENO1480
		CEN01490
		CEN01500
		CEN01510
C100	C (1,1)=ZETA	CEN01520
		CEN01530
C ****	* PRINT STIFFNESS, MASS, AND DAMPING MATRICES ***********	CEN01540
С	· ·	CEN01550
С	WRITE (2,220) N/2	CEN01560
С	DO 210 I=1,N	CEN01570
C210	WRITE (2,215) (K (I, J), J=1, N/2)	CEN01580
С	WRITE (2,221) N/2	CEN01590
С	DO 211 I=1,N	CEN01600
C211	WRITE (2,215) (K (I,J), J=N/2+1,N)	CEN01610
С	WRITE (2,222) N/2	CEN01620
С	DO 212 =1,N	CEN01630
C212	WRITE(2,215)(M(I,J),J=1,N/2)	CEN01640
С	WRITE (2,223) N/2	CEN01650

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С DO 213 |=1,N CEN01660 C213 WRITE (2,215) (M(I,J), J=N/2+1, N)CEN01670 С WRITE (2,224) N/2 CEN01680 С DO 214 |=1.N CEN01690 C214 WRITE (2,215) (C(I,J), J=1, N/2) CEN01700 С WRITE (2,225) N/2 CEN01710 С DO 216 I=1,N CEN01720 C216 WRITE (2,215) (C(I,J), J=N/2+1,N) CEN01730 CEN01740 215 FORMAT (/1X,5D14.7) CEN01750 CEN01760 FORMAT (/1X, 'THESE ARE THE FIRST ',13,' COLUMNS OF K ') FORMAT (/1X, 'THESE ARE THE LAST ',13,' COLUMNS OF K ') 220 CEN01770 221 CEN01780 222 FORMAT (/1X, 'THESE ARE THE FIRST ', 13, ' COLUMNS OF M ') CEN01790 FORMAT (/1X, 'THESE ARE THE LAST ', 13, ' COLUMNS OF M ') 223 CEN01800 FORMAT (/1X, 'THESE ARE THE FIRST ', 13, ' COLUMNS OF C ') 224 CEN01810 FORMAT (/1X, 'THESE ARE THE LAST ', 13, ' COLUMNS OF C ') 225 CEN01820 CEN01830 **CENO1840** PRINT*, 'IN START' CEN01850 CEN01860 DO 300 I=1.N CEN01870 300 RT(I) = 0.0**CENO1880** CEN01890 С RT(N/2) = PO*(DSIN(OMEGA*TIME))CEN01900 CEN01910 CALL INVERT (M, MINV, N) CEN01920 CEN01930 CEN01940 P = ACOS(-1.0)CEN01950 DO 333 I=1,NUMEL CEN01960 Z=P|*H*|/L CEN01970 UT (2*1) = (.1563/(1.+DUM1*2)) * ((P1/L*DCOS(Z)) + (DUM1*2*P1/L*DS1N(2*Z) CEN01980 8)) CEN01990 UT(2*I-1) = (.1563/(1.+DUM1*2))*(DSIN(Z)+DUM1*(1.-DCOS(2*Z)))**CEN02000** CONTINUE 333 CEN02010 С DO 334 I=1,N CEN02020 ٦ PRINT*, 'UT(', 1, ') = ', UT(1)C334 CEN02030 CEN02040 CEN02050 CEN02060 DO 302 I=1,N CEN02070 SUM=0.0 CEN02080 DO 301 J=1,N CEN02090 301 SUM=SUM+K(I,J)*UT(J)*(-1.0)CEN02100 302 DUM(I) = SUMCEN02110 CEN02120 DO 306 I=1,N CEN02130 SUM=0.0 CEN02140 CEN02150 DO 305 J=1,N SUM=SUM+MINV(I,J)*DUM(J) 305 CEN02160 306 UDDT(I) = SUMCEN02170 CEN02180 DO 303 I=1,N CEN02190 303 UTN(I)=UT(I)+UDDT(I)*(TS**2)/2. **CEN02200**

105

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	· · · · · · · · · · · · · · · · · · ·	CEN02210
		CEN02220
C	DO 302 I=I,N	CEN02230
С	SUM=0.0	CENO2240
С	DO 301 J=1,N	CEN02250
C301	SUM=SUM+MINV(I,J)*RT(J)	CENO2260
C302	UTN (I) = SUM	CEN02270
		CEN02280
С	DO 303 (=1,N	CEN02290
C303	UTN (1) =UTN (1) * (TS**2) / (2.0)	CEN02300
		CEN02310
	DO 310 1=1,N	CEN02320
	U(I)=0.0	CEN02330
С	UT(1) = 0.0	CEN02340
310	CONTINUE	CEN02350
		CEN02360
	PRINT*. 'OUT START'	CEN02370
		CEN02380
	DO 320 1=1.N	CEN02390
	DO(315) = 1 N	CEN02500
	$C_1(1 1) = M(1 1) / (TS + + 2) + C(1 1) / (2 0 + TS)$	CEN02400
	$C_2(1, 1) = K(1, 1) = (2 + W(1, 1) / (T_5 + 2))$	CENO2470
	$C_2(1, J) = (M(1, J) - (2.871(1, J) / (13882))$ $C_2(1, J) = (M(1, J) / (T_{5,5,5,2})) = (C(1, J) / (2.575))$	CENO2420
- 15	CONTINUE	CENU2430
315		
320	CUNTINUE	
		CEN02460
	LALL INVERI (LI, LO, N)	
-		CEN02480
C	DO 122 1 = 1, N	CEN02490
C122	WRITE (2, 123) 1, C6 (1, 1)	CEN02500
123	FORMAT (/1X, 'THIS IS C6 (', 12, ') ', D23.16)	CEN02510
С	WRITE (2, 176)	CEN02520
С	WRITE (2,177)	CEN02530
130	DO 440 I=1,N	CEN02540
	SUM=0.0	CEN02550
	DO 410 J=1,N	CEN02560
С	WRITE (2,131) TIME, C2 (1, J), UT (1)	CEN02570
410	SUM=SUM+C2(1,J)*UT(J)	CEN02580
440	C4(1)=SUM	CEN02590
131	FORMAT(/1X,'TIME ',F5.3,' C2 ',D18.10,' UT ',D18.10)	CEN02600
	DO 442 I=1,N	CEN02610
	SUM=0.0	CEN02620
	DO 411 J=1,N	CEN02630
411	SUM=SUM+C3(I,J)*UTN(J)	CENO2640
442	C5(1)=SUM	CEN02650
		CEN02660
	RT(N/2) = PO*(DSIN(OMEGA*TIME))	CEN02670
		CENO2680
	DO 140 I=1.N	CEN02690
140	B(1) = BT(1) - CL(1) - CL(1)	CEN02700
140		CEN02710
r	DO 142 1=1 N	CEN02710
C145	$U_{0} = (1 - 1) (1 -$	CEN02/20
11.0	WILLE (2,147) 1,0(1) EDDWAT //19 (TUIC IC 0/1 10 1) _ 1 000 14)	
149	runmi(/1A, 1D) = (, 12, 1) = (, 02, 10)	· CENU2/40
		LENU2/50

.

С	WRITE (2, 176)	CEN02760
С	WRITE (2,177)	CENO2770
	DO 542 I=1,N	CEN02780
	SUM=0.0	CEN02790
	DO 511 J=1,N	CEN02800
511	SUM=SUM+C6(I,J)*B(J)	CEN02810
542	U(1)=SUM	CEN02820
		CEN02830
	ICOUNT=ICOUNT+1	CEN02840
	TIME=TIME+TS	CEN02850
		CEN02860
С	SUM=0.0	CEN02870
С	DO 199 I=1,7,2	CENO2880
С	X=1	CENO2890
С	D1=((E*X1/(ROW*(L**4)))**0.5)*(X**2)*(9.869604404)	CEN02900
С	D2=((DSIN((X)*3.14592654/2.0))**2)	CEN02910
С	D3=1.0-(DCOS(D1*TIME))	CEN02920
C199	SUM=SUM+D2*D3/(D1**2)	CEN02930
C199	SUM=SUM+D3/(D1**2)	CEN02940
С	EXACT=(2.*PO/(ROW*L))*SUM	CEN02950
		CEN02960
С	DI=2.*PO*(DSIN(OMEGA*TIME))*(L**3)/((3.14592654**4)*E*XI)	CEN02970
С	SUM=0.0	CEN02980
С	DO 199 I=1,NUMEL-1,2	CEN02990
С	X=1	CEN03000.
C199	SUM=SUM+(1./((X**4)-0.25))	CEN03010
С	EXACT=SUM*D1	CEN03020
		CEN03030
	LINE=3	CEN03040
	IF (ICOUNT.EQ.50) GO TO 141	CEN03050
	GO TO 143	CEN03060
141	WRITE (2, 175) TIME, U (N/2), LINE	CEN03070
С	DIF = (U(N/2) - EXACT) / EXACT	CEN03080
С	IF (DABS (DIF) .LE.O.15) WRITE (2,177)	CEN03090
	I COUNT=O	CEN03100
		CEN03110
143	DO 150 I=1,N	CEN03120
	UTN(I) = UT(I)	CEN03130
150	UT (I) =U (I)	CEN03140
		CEN03150
	IF (TIME.GT.TT) GO TO 500	CEN03160
		CEN03170
	GO TO 130	CEN03180
175	FORMAT (F10.8,1X,F10.8,1X,11)	CEN03190
176	FORMAT(/1X,' TIME DEFLECTION AT L/2',15X,'EXACT')	CEN03200
177	FORMAT ('	CEN03210
	ξ')	CEN03220
500	STOP	CEN03230
	END	CEN03240
		CEN03250
		CEN03260
	SUBROUTINE INVERT (AO, A, N)	CEN03270
	DOUBLE PRECISION A (70,70), AO (70,70)	CEN03280
		CEN03290
		CEN03200

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FILE: CENDIF

	DO 1 J=1,N	CEN03310
1	A (I, J) = AO (I, J)	CEN03320
		CEN03330
	NP=N+1	CEN03340
	$\Delta(1, NP) = 1.0$	CEN033E0
	$D_{0} = 10 + 20$	CEN03350
10	A(1, NP) = 0, 0	CEN03300
10	A (1,NP) -0.0	CENU33/U
		CEN03380
	DO 40 J=1,N	CEN03390
	DO 20 LX=1,N	CEN03400
20	A(NP, LX) = A(1, LX+1) / A(1, 1)	CEN03410
	DO 30 KX=2,N	CEN03420
	DO 30 LX=1,N	CEN03430
30	A (KX-1,LX) = A (KX,LX+1) - A (KX,1) * A (NP,LX)	CEN03440
	DO 40 LX=1,N	CEN03450
40	A(N, LX) = A(NP, LX)	CEN03460
		CEN03470
	RETURN	CEN03480
	FND	CEN03490
		CEN03500
	CHERONITINE FACORT (K M N FERR Y ELCV)	CEN02510
r	SUDROUTINE JACODI (N, M, M, M, FR, A, EIGV)	CEN03510
L	SUDRUUTINE JACUDI	CEN03520
	$[MPL(U)] = REAL*O(A^{-H}, U^{-Z})$	CENU 35 30
	DOUBLE PRECISION $A(/0, /0), B(/0, /0), X(/0, /0), E(GV(/0), D(/0))$	CEN03540
	εκ (70, 70), Μ (70, 70)	CEN03550
	IFPR=0	CEN03560
С	COMMON/K,M/	CEN03570
С	WRITE (2, 1051)	CEN03580
C1051	FORMAT(/1X,' INPUT DATA ')	CEN03590
С	READ (1,*) N, IFPR	CEN03600
С	WRITE (2, 1001) N, IFPR	CEN03610
С	D0 1010 I=1,N	CEN03620
С	READ $(1, *)$ (A $(1, J), J=1, N$)	CEN0 36 30
С	WRITE $(2, 1110)$ (A(1, J), J=1, N)	CEN03640
c1010	CONTINUE	CEN03650
C	DO = 1020 =1 N	CEN03660
r	$BEAD(1 \pm 1) = 1 = 1$	CEN03670
c c	$W_{P}(T_{F}(2, 1)) = (1, 0) = (1, 0)$	CEN03680
C1020	$\frac{1}{2} = \frac{1}{2} = \frac{1}$	CEN03000
01020		CEN03090
C1001	FURMAT (2110)	
CINO	FURMAT (OF 10.4)	CENU3/10
	UU 2 (=1,N	CEN03/20
	DO 1 J=1,N	CEN03730
	A(I, J) = K(I, J)	CEN03740
	B(I, J) = M(I, J)	CEN03750
1	CONTINUE	CEN03760
2	CONTINUE	CEN03770
	NSMAX=15	CEN03780
С	WRITE (2, 1980)	CEN03790
1980	FORMAT (/1X.' EIGENVALUES ')	CEN03800
	RTO[=1, D-12]	CEN03810
	I NIT=2	CEN03820
		CEN03020
r		CENO 2810
C	$F_{\mathbf{A}}(\mathbf{A}_{1}, \mathbf{A}_{2}, \mathbf{A}_$	CEN03040
		していいうつうい

	WRITE (IOUT, 2020)	CEN03860
	STOP	CEN03870
4	D(1) = A(1,1) / B(1,1)	CEN03880
10	F[GV(1) = D(1)	CEN03890
	$D_{0} = 30 l = 1.N$	CEN03900
	DO = 20 + 100	CEN03910
20	$X(1, 1) \sim 0$	CEN03910
20	X(1, 3) = 0	CEN03920
30		CENU3930
~	(F (N.EQ.I) KEIUKN	CENU3940
		CEN03950
CIN	IIIIALIZE SWEEP COUNTER AND EIGEN ITERATION	CEN03960
C		CEN03970
	NSWEEP=0	CEN03980
	NR=N-1	CEN03990
40	NSWEEP=NSWEEP+1	CEN04000
	IF (IFPR.EQ.1) WRITE (IOUT, 2000) NSWEEP	CEN04010
	PRINT*,' SWEEP NUMBER ',NSWEEP	CENO4020
С		CEN04030
C C	HECK IF PRESENT OFF DIAGONAL ELEMENT IS TOO LARGE	CENO4040
С		CEN04050
	EPS= (0.01**NSWEEP) **2	CEN04060
	DO 210 J=1,NR	CEN04070
	[+L=LL	CEN04080
	DO 210 K1=JJ.N	CEN04090
	IF (DABS (A (J.K1)), LT.1, D-20) G0 T0 211	CEN04100
	$FPTO(A = (A(. .K)) \times A(. .K)) / (A(. .) \times A(K K))$	CENO4110
		CENO4120
211		CEN04120
211	EPTOLR-(R(K1) *R(K1)) / (R(I) *R(K K1))	CENO4130
212	$EFICED = (D(3, KT) \times D(3, KT)) / (D(3, 3) \times D(KT, KT))$ $EF((EPTOLA T EPS) AND (EPTOLB T EPS)) CO TO 210$	CENO4140
	$\frac{1}{1} \left(\left(\frac{1}{1} \right) \left(\frac{1}{1} \right) + \frac{1}{1} \left(\frac{1}{1} \right) \left(\frac{1}$	CEN04150
	A(X - A(X + X + Y) - D(X + X + Y) - D(X + Y) + A(X +	CENO4180
	AJJ=A (J, J) * D (J, KI) = D (J, J) * A (J, KI)	CENO4170
	$AD = A (J, J) \land D (N , N) = A (N , N) \land D (J, J)$ $CUPCV = (AD + AD + J) \land AVV + A J \rangle (J)$	CEN04180
~	CHEUK=(AB*AB+4.*AKK*AJJ)/4.	LENU4190
L	PRINIA, THIS IS CHECK ', CHECK	CEN04200
	IF (CHECK) 50,60,60	CEN04210
50	WRITE (10UT, 2020)	CEN04220
	STOP	CEN04230
60	SQCH=DSQRT (CHECK)	CEN04240
	D1=AB/2.+SQCH	CEN04250
	D2=AB/2SQCH	CEN04260
	DEN=D1	CEN04270
•	IF (DABS (D2).GT.DABS (D1))DEN≈D2	CEN04280
	IF (DEN) 80, 70, 80	CEN04290
70	CA=O.	CEN04300
	CG= (-1.) *A (J,K1) /A (K1,K1)	CEN04310
	GO TO 90	CEN04320
80	CA=AKK/DEN	CEN04330
	CG = (-1.) * AJJ/DEN	CEN04340
90	IF (N-2) 100, 190, 100	CEN04350
100	JP1=J+1	CEN04360
	.IM1=J-1	CEN04370
	KP1=K1+1	CENOT 380
	KM1=K1-1	CENUT300
	IE (IM1_1) 130 110 110	CENO4330
	IF (JRI=1/120,110,110	LENU4400

110	D0 120 [=].JM1	CEN04410
	$A = A \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	CENO4410
		CEN04420 CEN04420
		CEN04430
	AK=A(1,K1)	CEN04440
	$BK=B\left(I,K\right)$	CEN04450
	A(l,J)=AJ+CG*AK	CENO4460
	B(,J)=BJ+CG*BK	CEN04470
	A(I,K1) = AK + CA + AJ	CEN04480
120	B(I,K1) = BK + CA * BI	CEN04490
130	F(KB)-N 140 140 160	CENO4490
140		CEN04500
140	150 + RPT, N	CEN04510
	AJ=A(J,T)	CEN04520
	$BJ=B\left(J,I\right)$	CEN04530
	AK=A (K1,1)	CEN04540
	BK=B(K1, I)	CEN04550
	A(J,I) = AJ + CG * AK	CEN04560
	$B(J,I) = BJ + CG \times BK$	CEN04570
	$A(K1, I) = AK + CA \times A I$	CEN04580
150	$\mathbf{R}(\mathbf{K}) = \mathbf{R}\mathbf{K} + \mathbf{C}\mathbf{A} + \mathbf{R}\mathbf{I}$	CENOLEOO
140	(1, 1, 1) = 0	CEN04530
100	IF (JPI-MI) 1/0, 1/0, 190	
170	UU IOU I=JPI,KMI	CEN04610
	AJ=A(J,I)	CEN04620
	BJ=B(J,)	CEN04630
	AK=A (I,K1)	CEN04640
	BK=B(1,K1)	CEN04650
	A (J,I)=AJ+CG*AK	CEN04660
	B(J, I) = BJ + CG * BK	CEN04670
	$\Delta (I KI) = \Delta K + C \Delta \star \Delta I$	CEN04680
180		CEN04000
100	$D(I, NI) = DN = CA^{D} DJ$	CEN04890
190	$A\mathbf{N} = A (\mathbf{N} \mathbf{I}, \mathbf{N} \mathbf{I})$	
		LEN04/ID
	A(K1,K1) = AK+2.*CA*A(J,K1) + CA*CA*A(J,J)	CEN04720
	B(K1,K1)=BK+2.*CA*B(J,K1)+CA*CA*B(J,J)	CEN04730
	A (J,J) =A (J,J) +2.*CG*A (J,K1) +CG*CG*AK	CEN04740
	B(J,J)=B(J,J)+2.*CG*B(J,K1)+CG*CG*BK	CEN04750
	A (J,K1) =0.	CEN04760
	B(J,K1) = 0	CEN04770
C		CEN04780
	ATE FICENVECTOD MATDIX	CEN04700
C UFD	ATE ETGENVECTOR MATRIX	CEN04/30
L		
	D0 200 I=I,N	CEN04810
	XJ=X(,J)	CEN04820
	XK=X (I,K1)	CEN04830
	X (I,J) =XJ+CG*XK	CEN04840
200	X(I,KI) = XK + CA * XJ	CEN04850
210	CONTINUE	CEN04860
r		CEN04870
		CEN04070
	DATE EIGENVALUES	CEN04800
U		CEN04890
	DO 220 I=1,N	CEN04900
	IF (A (1,1).GT.O.AND.B (1,1).GT.O) GO TO 220	CEN04910
	WRITE (IOUT, 2020)	CEN04920
	STOP	CEN04930
220	E[GV(1) = A(1, 1) / B(1, 1)	CENO4940
	1F (1FPR_F0_0) G0_T0_230	CENOLOSO

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. •
      WRITE (10UT, 2030)
                                                                              CEN04960
      WRITE (IOUT, 2010) (EIGV (I), I=1,N)
                                                                              CEN04970
С
                                                                              CEN04980
C
   CHECK FOR CONVERGENCE
                                                                              CEN04990
С
                                                                              CEN05000
 230 D0 240 I=1,N
                                                                              CEN05010
      TOL=RTOL*D(1)
                                                                              CEN05020
      DIF=DABS(EIGV(I) - D(I))
                                                                              CEN05030
      IF (DIF.GT.TOL) GO TO 280
                                                                              CEN05040
 240 CONTINUE
                                                                              CEN05050
С
                                                                              CEN05060
   CHECK ALL OFF DIAG ELEMENTS TO SEE IF ANOTHER SWEEP IS REQ'D
С
                                                                              CEN05070
С
      PRINT*, ' RTOL ', RTOL
                                                                              CEN05080
      EPS=RTOL**2
                                                                              CEN05090
      DO 250 J=1,NR
                                                                              CEN05100
      JJ=J+1
                                                                              CEN05110
      DO 250 K1=JJ,N
                                                                              CEN05120
      IF (DABS (A (J,K1)).LT.1.D-30) G0 T0 251
                                                                              CEN05130
      EPSA = (A (J,K1) * A (J,K1)) / (A (J,J) * A (K1,K1))
                                                                              CEN05140
      GO TO 252
                                                                              CEN05150
 251
      EPSA=0.0
                                                                              CEN05160
      PRINT*, ' EPSA ', EPSA, ' EPS ', EPS
С
                                                                              CEN05170
      EPSB = (B(J,K1) * B(J,K1)) / (B(J,J) * B(K1,K1))
                                                                              CEN05180
 252
С
      PRINT*, ' EPSB ', EPSB, ' EPS ', EPS
                                                                              CEN05190
      IF ((EPSA.LT.EPS) AND. (EPSB.LT.EPS)) GO TO 250
                                                                              CEN05200
      GO TO 280
                                                                              CEN05210
       CONTINUE
 250
                                                                              CEN05220
С
                                                                              CEN05230
  FILL OUT BOTTOM TRIANGLE OF RESULTANT MATRICES & SCALE EIGENVECTORS
                                                                              CEN05240
С
                                                                              CEN05250
С
      DO 260 |=1,N
                                                                              CEN05260
 255
      DO 260 J=1,N
                                                                              CEN05270
      A(J,I) = A(I,J)
                                                                              CEN05280
 260 B(J,I) = B(I,J)
                                                                              CEN05290
      DO 270 J=1,N
                                                                              CEN05300
                                                                              CEN05310
      BB=DSQRT(B(J,J))
                                                                              CEN05320
      DO 270 K1=1.N
 270 X(K1,J) = X(K1,J) / BB
                                                                              CEN05330
С
      WRITE (10UT, 310)
                                                                              CEN05340
                                                                              CEN05350
С
      DO 300 I=1,N
C300 WRITE (IOUT, 2010) (X (I, J), J=1, N)
                                                                              CEN05360
 310 FORMAT (/1X.' THE EIGENVECTORS ARE ')
                                                                              CEN05370
                                                                              CEN05380
                                                                              CEN05390
                                                                              CEN05400
                                                                              CEN05410
                                                                              CEN05420
      RETURN
С
                                                                              CEN05430
    UPDATE THE 'D' MATRIX AND START NEW SWEEP IF ALLOWED
                                                                              CEN05440
С
                                                                              CEN05450
С
                                                                              CEN05460
 280 D0 290 I=1,N
                                                                              CEN05470
 290 D(1) = E | GV(1)
       IF (NSWEEP.LT.NSMAX) GO TO 40
                                                                              CEN05480
      GO TO 255
                                                                              CEN05490
 2000 FORMAT (/1X, ' SWEEP NUMBER IN JACOBI = ', 14)
                                                                              CEN05500
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.

2010	FORMAT (/1X,6E20.12)	CEN05510
2020	FURMAL(/IA, ***** ERRUR SULUTION STUP / MAIRTLES NUT PUSITIVE	CEN05520
2020	GUEFINIEL) EODMAT(/1Y / CHIDDENT ELCENVALHES IN LACORE ADE /)	CEN05530
2030	FURMAL (/ IX, CURRENT ETGENVALUES IN SACODI ARE)	CEN05540
		CENOSSO
•	SUBDOUTINE DAMP (N ELGV X M DAMPAT C)	CEN05500
	$IMPIICIT REAL *8 (A-H \Omega-7)$	CEN05570
	DOUBLE PRECISION X (70, 70) . T (70, 70) . M (70, 70) . C (70, 70) . ELGV (70) DAM	RCEN05590
	δΑΤ (70)	CEN05600
		CEN05610
	DO 10 =1,N	CEN05620
	$E \mid GV(I) = DSQRT(E \mid GV(I))$	CEN05630
	DO 10 J≑1,N	CEN05640
10	C(I,J)=0.0	CEN05650
		CEN05660
	DO 20 =1,N	CÉN05670
	DA=2.*DAMRAT()*E GV()	CEN05680
	DO 20 =1,N	CEN05690
	DO 20 J=1,N	CEN05700
20	C(I,J) = C(I,J) + X(I,II) * X(J,II) * DA	CEN05710
		CEN05720
	DO 30 I=1,N	CEN05730
	DO 30 J=1,N	CEN05740
	T (I, J) = 0.0	CEN05750
	DO 30 K1=1,N	CEN05760
30	T(I, J) = T(I, J) + M(I, KI) * C(KI, J)	CEN05770
		CEN05780
	DO 40 =1,N	CEN05790
	DO 40 J≈1,N	CEN05800
	C(I, J) = 0.0	CEN05810
	DO 40 K1=1,N	CEN05820
40	C(I, J) = C(I, J) + T(I, K1) * M(K1, J)	CEN05830
-		CEN05840
С	DO 50 I=1,N	CEN05850
C50	WRITE(2, 120)(C(I, J), J=1, N)	CEN05860
120	FORMAT (6014.4)	CEN05870
	RETURN	CEN05880
		CEN05890
	END	CEN05900

FILE: DAVE

THIS	15	CENTRAL	DIFFERENCE	SOLUTION
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STI	FFN	FSS	 ٥.	53	11	00	າດເ	17()+(22
J I I	E E IN.	LJJ	υ.	71	11	υu	////		1.0	

DAMPING ---- 0.130999982D-01

FORCE ----- 0.40000066D-02

FREQUENCY -- 0.2513000490+02

TIME DEFLECTION AT L/2

0.00500000	0.07560742	1
0.01000000	0.07411487	1
0.01500000	0.07822479	1
0.02000000	0.07755833	1
0.02500000	0.07321689	1
0.03000000	0.07439081	1
0.03500001	0.07596175	1
0.04000001	0.07030083	1
0.04500001	0.06480695	1
0.05000001	0.06403831	1
0.05500001	0.05838368	1
0.0600001	0.04628549	1
0.06500001	0.03770042	1
0.07000001	0.03044851	1
0.07500001	0.01549482	1
0.08000001	-0.00155219	1 ·
0.08500001	-0.01381828	1
0.0900001	-0.02866951	1
0.09500002	-0.04917751	1
0.1000002	-0.06667677	1
0.10500002	-0.08058453	1
0.11000002	-0.09807491	1
0.11500002	-0.11601054	1
0.1200002	-0.12788046	1
0.12500002	-0.13797468	1
0.13000002	-0.14964225	1
0.13500002	-0.15657816	1
0.14000002	-0.15730855	1 ·
0.14500002	-0.15770900	1
0.15000002	-0.15621028	1
0.15500003	-0.14753221	1
0.16000003	-0.13531903	1
0.16500003	-0.12327482	1
0.17000003	-0.10672159	1
0.17500003	-0.08461688	1
0.18000003	-0.06275240	1
0.18500003	-0.04085973	1
0.19000003	-0.01462197	1
0.19500003	0.01269540	1
0.2000003	0.03673648	1
0.20500003	0.06085743	1
0.21000003	0.08652612	1