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DEPARTMENT OF CIVIL ENGINEERING
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OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA 23529

EXPERIMENTAL AND THEORETICAL INVESTIGATION OF PASSIVE DAMPING CONCEPTS FOR MEMBER FORCED AND FREE VIBRATION

By

Zia Razzaq, Principal Investigator

David W. Mykins, Graduate Research Assistant

Progress Report
For the period ended December 31, 1987

Prepared for the
National Aeronautics and Space Administration
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Hampton, Virginia 23665

Under
Research Grant NAG-1-336
Harold G. Bush, Technical Monitor
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EXPERIMENTAL AND THEORETICAL INVESTIGATION OF PASSIVE DAMPING CONCEPTS FOR MEMBER FORCED AND FREE VIBRATIONS

By

Zia Razzaq[1] and David W. Mykins[2]

ABSTRACT

The results presented in this research report are the outcome of an ongoing study directed toward the identification of potential passive damping concepts for use in space structures. The effectiveness of copper brush, wool swab, and "silly putty" in chamber dampers is investigated through natural vibration tests on a tubular aluminum member. The member ends have zero translation and possess partial rotational restraints. The silly putty in chamber dampers provide the maximum passive damping efficiency. Forced vibration tests are then conducted with one, two, and three silly putty in chamber dampers. Owing to the limitation of the vibrator used, the performance of these dampers could not be evaluated experimentally until the forcing function was disengaged. Nevertheless, their performance is evaluated through a forced dynamic finite element analysis conducted as a part of this investigation. The theoretical results are based on experimentally obtained damping ratios indicate that the passive dampers are considerably more effective under member natural vibration than during forced vibration. Also, the maximum damping under forced vibration occurs at or near resonance.

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NOMENCLATURE

[C] = damping matrix for member

[D] = displacement vector

$[\dot{D}]$ = velocity vector

$[\ddot{D}]$ = acceleration vector

$[D_j]$ = displacement vector at node j

E = Young's Modulus

I = moment of inertia

[K] = global stiffness matrix for member

[R] = forcing function vector

$\gamma\,\beta$ = arbitrary constants for Newmark's method

Δ^*_d = dynamic deflection amplitude

Δ_F = static midspan deflection

Δ_S = static midspan deflection

Δt = time increment

Φ = modal vector

η = damping efficiency index

Ω = frequency of applied forcing function

ω = natural frequency

ω_{fe} = natural frequency from finite element analysis

ρ = mass density

ζ = damping ratio

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1. INTRODUCTION

1.1 Background and Previous Work

The space station designs currently under consideration by NASA are three-dimensional space structures composed of long tubular members. Modules providing the required living and working space for astronauts will be attached to this framework. Such a structure, suspended in a weightless environment, would be subjected to many types of dynamic loading. These include differential heating or cooling of the structure, variations in acceleration or gravitational pull, and impact with a solid object. The ability to expeditiously damp these vibrations before they cause permanent damage is a practical problem worth studying.

The necessarily large slenderness ratio of the average space truss member, combined with the flexible, semi-rigid end restraints cause the dynamic response of these members to be characterized by low frequency, small amplitude vibrations. Active damping techniques utilize electronic sensors and movable masses to reduce vibration of structures. This system, although effective, requires regular maintenance and an external power source. An alternative for mechanically damping a system is the concept of "passive" damping. This method uses a device or material permanently attached to the structure or its components and designed to absorb the energy of vibration thereby providing some damping of the system. Unlike "active" damping, this would require minimal maintenance and no external power.

The challenge to developing a passive damping concept, particularly for a space structure is two-fold. First is the necessity to minimize the mass, for without this constraint one obvious solution would be to provide large mass concentrations at the critical nodal points for the vibration modes. Such an approach would be expensive since the cost of transporting the system into space is directly related to the mass. The second challenge is to identify a concept which will provide passive damping without altering the strength or stiffness of the structure. For example, mild compression of the members provides some damping, however, the safe service loads for the structure are altered.

Recently, investigations into passive damping concepts for slender tubular members have been conducted with various end conditions (References 1-5). The most effective concepts found were the mass-string-whiskers assembly, and brushes for electrostatic and frictional damping. In these experiments, only natural flexural vibration was examined.

The previous work was conducted on hollow tubular steel members with an outer diameter of 0.5 inches. The passive damping concepts which were found to be effective for these members may not be as effective if the dimensions are changed. Factors altered by dimensionsal changes may include the damper mass required, the extent of the frictional interaction, and the member dynamic characteristics. Clearly research is needed to identify a viable passive damping concepts for members of different sizes and dymanic properties. In the present study, hollow tubular aluminum members with an outside diameter of 2.0 inches are used. These members more closely resemble the actual size and material which may be used in the future space stations.

1.2 Problem Definition

Figure 1 shows schematically a slender beam of length L with a hollow circular cross section. The outer diameter is D_0 the inner diameter is D_i, and the material is aluminum with a Young's modulus of 10,000 ksi. An aluminum member is used because the graphite composite tubes which may possibly be used in space structures are not yet available. The member ends are provided with a prototype connection developed by NASA for the space frames. These connections possess partial rotational restraint characteristics in the plane of motion and a more rigid end condition in the orthoganal plane. No axial or lateral movement of the member ends is permitted.

The problem is to identify potential passive damping concepts to absorb the energy of both natural flexural vibration, and harmonic forced flexural vibration, and to study the effectiveness of each concept. The natural vibration is caused by the sudden release of a constant static load. The harmonic forcing function is applied through a mechanical connection to a harmonic vibrator.

1.3 Objective and Scope

The following are the main objectives of this study:

1. Identification of potential passive damping concepts for slender tubular structural members. Specifically, the following damping concepts are investigated:
   a. wool swabs,
   b. copper brushes,
   c. silly putty in chambers.
2. Evaluation of the damping efficiencies of the various damping concepts.
3. Evaluation of the suitability of a theoretical finite element analysis by comparison to experimental results for natural and

forced vibration, and a previous finite-difference solution for natural vibration.

Only flexural member vibration is considered. The natural vibration study is conducted on each of the three passive damping concepts and for one specific initial deflection. Only the most efficient damping concept is considered for further study under forced vibration. Also, the vibration is induced by load application at the member midspan.

1.4 Assumptions and Conditions

The following assumptions and conditions have been adopted in this study:

1. The deflections are small.
2. The material of the member is linearly elastic.
3. Only planar vibration is considered.
4. Damping force is opposite but proportional to the velocity at any location along the member.
5. The damping force is uniform along the length of the member.
6. The member is tested at 1-g and room temperature.
7. The effect of secondary induced forces such as varying axial tension and compression developed in the member during vibration is considered to be negligible.

2. THEORETICAL FORMULATION

2.1 Finite Element Formulation

The beam shown in Figure 1 may be divided into N finite elements along the length. For the discretized system, the governing equation of motion can be expressed in the following matrix form (Reference 6):

$$[K]\{D\} + [M]\{D\} + [C]\{D\} = \{R\} \qquad (1)$$

where:

$\{D\}$ = displacement vector,

$\{\dot{D}\}$ = velocity vector,

$\{\ddot{D}\}$ = acceleration vector,

$[K]$ = global stiffness matrix for the "structure",

$[M]$ = global modified lumped mass matrix,

$[C]$ = damping matrix,

$\{R\}$ = forcing function vector.

The boundary and initial conditions for the problem shown in Figure 1 are given in Reference 2 and are summarized here:

$$D(0,t) = 0 \qquad (2)$$

$$D(L,t) = 0 \qquad (3)$$

$$EI\, D''(0,t) = k_1\, D'(0,t) \qquad (4)$$

$$EI\, D''(L,t) = -k_2\, D'(L,t) \qquad (5)$$

$$\dot{D}(x,0) = 0 \qquad (6)$$

$$D(x,0) = 0(x;\ K,\ EI,\ L) \qquad (7)$$

where primes represent differentiation relative to x, and dots represent time differentiation. The displacement vector at any node j along the member can be written as:

$$\{D_j\} = \begin{Bmatrix} d_j \\ d'_j \end{Bmatrix} \tag{8}$$

where d_j and d'_j represent, respectively, the deflection and slope of j.

Equations 2 to 5 represent the boundary conditions whereas Equations 6 and 7 are the initial conditions. Equation 7 simply states that at time zero, the member deflected shape is dependent on x, K, EI and L.

The first task toward the solution of the matrix equation is the assembly of the three coefficient matrices. The [K] matrix is assembled from the individual element matrices combined in such a way so as to enforce the given boundary and inter-element compatability conditions. To illustrate the procedure, an example of a beam with four elements as shown in Figure 2 is given in Appendix A.

The global mass matrix is a diagonal form of a lumped mass matrix which was developed (Reference 6) for use with elements where translational degrees of freedom are mutually parallel, such as beam or plate elements.

This matrix may be written as:

$$[M] = \begin{bmatrix} \frac{m}{2} & & & & & & & \\ & \frac{mL^2}{78} & & & & & & \\ & & m & & & & & \\ & & & \frac{mL^2}{39} & & & & \\ & & & & \cdot & & & \\ & & & & & \cdot & & \\ & & & & & & \frac{m}{2} & \\ & & & & & & & \frac{mL^2}{78} \end{bmatrix} \qquad (9)$$

where:

m = mass at each degree of freedom = $\rho L(A)$

ρ = mass density (mass/in^3)

L = length of element (in)

A = cross sectional area of element (in^2)

In order to calculate the damping matrix [C], it is necessary to first determine the modal shape and natural frequencies of the system. This is accomplished numerically by solving the following eigen value problem using the Jacobi method (Reference 7):

$$([K] - \omega^2 [M][\{\Phi\}] = \{0\} \qquad (10)$$

where:

ω = natural frequency,

$\{\Phi\}$ = modal vector.

Once ω and $\{\Phi\}$ are known, determination of the damping matrix proceeds as described in Reference 7.

Once all three coefficient matrices have been assembled, the solution of Equation 1 may proceed using any one of several solution algorithms available.

2.2 Newmark's Method

Newmark's method for solving the dynamic equilibrium equation is sometimes called the trapezoidal method because it is based on a linear interpolation to find succeeding points. This is done by assuming:

$$\{D\}_{t+\Delta t} = \{D\}_t + \Delta t\{\dot{D}\}_t + \Delta t^2\left(\left(\frac{1}{2}-\beta\right)\{\ddot{D}\}_t + \beta\{\ddot{D}\}_{t+\Delta t}\right) \quad (11)$$

and

$$\{\dot{D}\}_{t+\Delta t} = \{\dot{D}\}_t + \Delta t\left((1-\gamma)\{\ddot{D}\}_t + \gamma\{\ddot{D}\}_{t+\Delta t}\right) \quad (12)$$

where Δt is a time increment, and β and γ are arbitrary constants. By substituting Equations 11 and 12 into Equation 1 written at time $t = t + \Delta t$, one gets (Reference 6):

$$\left([K] + \frac{\gamma}{\beta\Delta_t}[C] + \frac{1}{\beta\Delta_{t^2}}[M]\right)\{D\}_{t+\Delta t} = \{R\}_{t+\Delta t} +$$

$$[C]\left(\frac{\gamma}{\beta\Delta t}\{D\}_t + \frac{\gamma}{\beta} - 1\,\{\dot{D}\}_t + (\Delta t)\ \frac{\gamma}{2\beta} - 1\,\{\ddot{D}\}_t\right) + \quad (13)$$

$$[M]\left(\frac{1}{\beta\Delta t^2}\{D\}_t + \frac{1}{\beta\Delta t}\{\dot{D}\}_t + \left(\frac{1}{2\beta} - 1\right)\frac{\{\ddot{D}\}_t}{2}\right)$$

For a known loading function we may solve Equation 13 for the deflection at time $t = t + \Delta t$ using the deflection, velocity and acceleration at time t.

The algorithm for Newmark's solution is as follows:

1. Compute the coefficient matrices from geometric and material properties.
2. At t = 0, set initial conditions by prescribing $\{D\}_{t=0}$ and $\{\dot{D}\}_{t=0}$.
3. Use Equation 1 to solve for $\{\ddot{D}\}_{t=0}$.
4. Solve Equation 13 for $\{D\}_{t+\Delta t}$.
5. Solve Equation 11 for $\{\ddot{D}\}_{t+\Delta t}$.
6. Solve Equation 12 for $\{\dot{D}\}_{t+\Delta t}$.
7. Set $\{D\}_t = \{D\}_{t+\Delta t}$; $\{\dot{D}\}_t = \{\dot{D}\}_{t+\Delta t}$; $\{\ddot{D}\}_t = \{\ddot{D}\}_{t+\Delta t}$.
8. If $t \leq$ total time desired, go to Step 4.
9. Stop.

This method of solution is unconditionally stable if $\gamma \geq 0.5$ and $\beta \geq (2\gamma + 1)^2/16$. With $\gamma = 0.5$ and $\beta = 0.25$, there are no amplitude errors in any sine wave motion regardless of its frequency, although the periods are overestimated. The mode shapes of the member in this study, however, are not known exactly. Nevertheless, γ and β values of 0.5 and 0.25 respectively, were tentatively chosen. The suitability of these values is evaluated later in Section 4.

The initial static deflection vector required in Step 2 of the algorithm may be determined using any one of the several classical structural analysis techniques. An approximate shape function for the member due to a specified midpoint displacement Δ_0 at time t = 0 is taken in the following form (Reference 2):

$$d_j = A_1 \left[\sin \frac{\pi x_j}{L} + \frac{kL}{4\pi EI} \left(1 - \cos \frac{2\pi x_j}{L} \right) \right] \tag{14}$$

where:

$$A_1 = \frac{\Delta_0}{1 + \dfrac{kL}{2\pi EI}} \tag{15}$$

The initial slope of the member at any point is found by differentiating Equation 14 resulting in:

$$d'_j = A_1 \left[\frac{\pi}{L} \cos \frac{\pi x_j}{L} + \frac{k}{2EI} \sin \frac{2\pi x_j}{L} \right] \tag{16}$$

where x_j is the position of node j along the member length.

2.3 Central Difference Formulation

The governing equations and formulation of the coefficient matrices to be used in the central difference method of solution are precisely the same as those previously given for Newmark's method. Once these geometric and physical properties are determined, one proceeds by writing the central difference expressions for both velocity and acceleration at an arbitrary time t:

$$\{\dot{D}\}_t = \frac{1}{2(\Delta t)} \left[\{D\}_{t+\Delta t} - \{D\}_{t-\Delta t} \right] \tag{17}$$

$$\{\ddot{D}\}_t = \frac{1}{(\Delta t)^2} \left[\{D\}_{t+\Delta t} - 2\{D\}_t + \{D\}_{t-\Delta t} \right] \tag{18}$$

Equations 17 and 18 may then be substituted into Equation 1 to yield, after some rearrangement:

$$\left(\frac{[M]}{(\Delta t)^2} + \frac{[C]}{2(\Delta t)} \right) \{D\}_{t+\Delta t} = \{R\}_t - \left([K] - \frac{2[M]}{(\Delta t^2)} \right) \{D\}_t$$
$$\left(\frac{[M]}{(\Delta t)^2} - \frac{[C]}{2(\Delta t)} \right) \{D\}_{t-\Delta t} \tag{19}$$

The initial conditions $\{D\}_0$ and $\{\dot{D}\}_0$ are prescribed and $\{\ddot{D}\}_0$ is found by solving Equation 1. Once these are known Equations 17 and 18 may be solved simultaneously to yield the displacements $\{D\}_{-\Delta t}$ required to start the computations.

$$\{D\}_{-\Delta t} = \{D\}_0 - \Delta t\{\dot{D}\}_0 + \frac{(\Delta t)^2}{2}\{\ddot{D}\}_0 \tag{20}$$

The solution algorithm for central difference is as follows:

1. Compute the coefficient matrices from geometric and material properties.
2. Set Δt = time step increment.
3. Set initial conditions by prescribing $\{D\}_{t=0}$ and $\{\dot{D}\}_{t=0}$.
4. Solve Equation 1 for $\{\ddot{D}\}_{t=0}$.
5. Solve Equation 20 for $\{D\}_{-\Delta t}$.
6. Solve Equation 19 for $\{D\}_{t+\Delta t}$.
7. Set $\{D\}_{t-\Delta t} = \{D\}_t$, and $\{D\}_t = \{D\}_{t+\Delta t}$.
8. If $t \leq$ total time desired, go to 6.
9. Stop.

The central difference method is a conditionally stable, explicit method of solution. Conditionally stable implies that if Δt is not chosen small enough, the predicted response of the system will grow unbounded. A preliminary numerical study showed that Δt must be in the range from 0.001 to 0.005, therefore, a Δt = 0.001 sec. is used in this study.

3. EXPERIMENTAL STUDY

3.1 Specimen and Connection Details

3.1.1 Specimen

The experimental study consisted of conducting natural and forced vibration tests on a tubular aluminum member. The tests were performed both with and without passive damping devices present inside the member. The tubular member used was 14'-9" long with an outside diameter of 2" and a wall thickness of 0.125", yielding an inside diameter of 1.75". A schematic of the member tested is shown in Figure 1. Note that the member was horizontal for all testing, with gravitational forces acting in the plane of motion.

3.1.2 Connection Details

The prototype end connection used in this study is shown in Figure 3. It is constructed of an aluminum alloy, weights 0.595 lbs. excluding fastener bolts, and has a volume of 3.988 in^3. The connection has a total of nine clevis blades, six of which are in the horizontal plane. One of the blades is in the vertical plane (at C) and two are at 45 degrees to the horizontal plane. These two are located at 45 degrees relative to the vertical clevis and in the planes containing the two lower clevis blades shown in Figure 3(a).

The fastner locations for the clevis blades in the horizontal plane are numbered 1 through 12. The member was fastened at locations 3 and 4 shown in Figure 3(a). Fasteners at locations 5 through 11 are used to mount the connection to a fixed base plate. No fastener was installed at location 12 due to an interference problem with the support underneath the base plate. This did not make any difference since the other fasteners provided sufficient fixity. Each fastener has a diameter of 0.25 in. and a length of 0.94 in. Washers were used at locations 1 and 2 only.

The ends of the tubular member were threaded to allow one-half of the "snap-lock" connection to be screwed onto it. Small holes were drilled through this threaded connection and pins inserted to prevent rotation and loosening of the connection during testing. The other end of the snap-lock connection had its blade end fit snugly into one of the clevis blades of the prototype end connection and fastened by two bolts. The spring stiffness, k, shown in Figure 1 was determined by a statical analysis using an experimentally determined midspan deflection for a known concentrated load. This value was 53.1 k-in/rad. The assembled connection is shown in Figure 4.

3.2 Passive Damping Concepts

Three different types of passive dampers referred to in Section 1.4 are described in this section.

3.2.1 Copper Brush Dampers

Figure 5 shows a copper brush damper 0.8125 inches in diameter, of total length 3.125 inches and a weight of 13.0 grams. The brush is manufactured by Omack Industries, Onalaska, Wisconsin 54650 with a US Patent 41986 and an inventory control number 07668341989. It has a threaded aluminum piece 1.0 inch long at one end with a twisted wire 2.125 inches in length attached to it. The copper bristles are attached to the entire length of the twisted wire. This type of brush is commonly used in cleaning the bore of a 12 gauge shotgun.

Figures 6 and 7 show schematically the attachments for the passive dampers and their spacing inside the tubular member. As shown in Figure 6, the assembly consists of several parts. First, a helical spring with a stiffness of 0.44 lb/in. is attached to the inside of the connection through a hook on the snap-lock connector as shown in Figure 8. A nylon line is tied to the other end of the spring and also connected to the first copper brush damper. The nylon line (sportfisher monofilament line manufactured by K-Mart Corporation, Troy, Michigan 48084, 8013.9, No. EPM-40, inventory control number 04528201391) used in this investigation has a 40 lb. capacity. A series of nylon line and dampers are attached along the member length until the other end of the tubular member is reached. The end of the nylon line is passed through a hole in the snap-lock connector and stretched by an amount of 2.0 inches in the longitudinal direction to induce nominal tension in helical spring. It is then secured to the vertical clevis at the support. The stretched helical spring is shown in Figure 9. The resulting passive damping assembly is aligned with the longitudinal axis of the tubular member due to the small amount of axial tension. No axial compression of the member is induced by the passive damping assembly on the tubular member since both ends of the nylon line are connected to the rigid supports. Since the nylon line is flexible, a significant portion of the stretching is due to elongation of the line itself with the remainder of the stretching taking place in the spring. The dampers are installed equidistantly between the ends of the member.

As a part of the present study, the effect of both number of brushes and presence or absence of tension on the nylon line, on member damping was examined.

In addition to baseline experiments on the specimen with no damping devices, a total of ten different conditions were examined. Tests with 1, 2, 3, 5, and 7 brushes were conducted both with and without tension in the line.

3.2.2 **Wool Swab Dampers**

Figure 10 shows a wool swab damper with a 1.0 inch diameter, a total length of 3 inches and a weight of 7.1 grams. The wool swab is manufactured by Omark Industries, Onalaska, Wisconsin 54650 with a US patent 415838 and an inventory control number 076683422187. It has a threaded piece at one end with a twisted wire attached to it to which the wool swab is attached. The aluminum piece is 0.75 inches long while the wool swab has a length of 2.125 inches. This type of brush is commonly used for cleaning 12 gauge shotguns. The dampers are mounted inside the tubular member as shown in Figures 6 and 7. Tests were carried out using 1, 2, 3, 5, and 7 equidistantly spaced wool swab dampers.

3.2.3 **Silly Putty in Chamber Dampers**

The final device examined was the "silly putty" in chamber damper shown in Figure 11. It consists of a sphere approximately 0.75 inches in diameter made from silly putty placed inside a hollow cylindrical chamber. Silly putty is a trade name for an elasto-plastic material commonly used as a children's toy. It is manufactured by Binney and Smith, Inc., Easton, PA 18042, with an inventory control number of 07166208006. The chamber is made from a 1.0 in. long piece of a "Bristole Pipe" (PVC-1120, Schedule 40, ASTM-D-1785, nominal 1 inch pipe) having an original outer diameter of 1.058 in. and a wall thickness of 0.15 in. Since the damping effect was assumed to be provided by the silly putty, two steps were taken to reduce the mass of the damper thereby improving its efficiency. First, the inside diameter is increased by machining it to 0.914 in. resulting in a wall thickness of 0.07

in. Its weight is further reduced by drilling a total of seven 0.25 in. diameter holes around its periphery half-way from its ends. The silly putty is held inside the chamber by means of a plastic wrap ("Saran Wrap") stretched over the ends of the chamber and held in place with tape. The silly putty is then free to bounce around inside the chamber. The total weight of the damper including the silly putty, PCV chamber, and plastic wrap is 7.4 gms. The dampers are mounted inside the tubular member as shown in Figures 6 and 7. Tests were conducted with a nominal tension in the spring and with no tension in the spring using 1, 2, 3, 5 and 7 equidistant silly putty in chamber dampers. An additional test was performed with 11 equidistant dampers and a nominal tension in the spring.

3.3 Test Setup and Procedures

The instrumentation used in the tests consisted of a proximity probe, harmonic vibration devices and a deflection-time plotter. This section summarizes the test setup and procedures followed for all the experiments included in this report.

Figure 12 shows a schematic of the member natural vibration test setup. A weight, W = 7.9 lb. was suspended at the member midspan by means of a cord, causing a total midspan deflection of 5/32 in. To induce natural vibration, the cord was cut with a pair of scissors, thereby releasing the member. The time dependent deflection at member midspan is recorded by means of a proximity probe shown in Figure 13, and connected to a deflection-time plotter.

Figure 14 shows the member forced vibration setup, a schematic of which is shown in Figure 15. Forced vibration of the specimen was obtained using a vibrator (Model 203-25-DC) with an oscillator (Model TPO-25). The vibrator applies a forcing function of the type:

$$F(t) = F_0 \cos(\Omega t) \tag{21}$$

in which F_0 = 4 lb., t = time, and Ω = frequency of the forcing function. The applied frequency may be controlled using the oscillator.

The forcing function F(t) is transmitted from the vibrator to the tubular member through a fabricated vibrator connector as indicated in Figure 14. The details of this mechanical connector are shown in Figure 16. It consists of three main segments PQ, QR and RU interconnected at Q and R by means of pins. End P is connected to the vibrator. The end U is connected to the lower part of a metal hose clamp provided around the tubular member at midspan as indicated in Figure 14. The parts QR and RU can be disengaged at R by pulling out the pin RS instantaneously in the RS direction as indicated by the arrow at S. A string attached at S is used to pull out the pin. Once the pin is pulled, the arm QR drops freely and the beam is free to vibrate without constraints. Both joints Q and R are well lubricated to reduce friction. The vibrator connector in the engaged and the disengaged positions is shown in Figures 17(a) and 17(b), respectively. A record is made of the deflection-time response of the member once the forcing function, F(t), is removed.

3.4 Test Results and Discussion

In this section, the results from the member natural and forced vibration tests are presented and discussed.

3.4.1 Natural Vibration

All passive damping concepts were tested with natural flexural member vibration caused by releasing a weight at midspan as explained in Section 3.3. The initial midspan deflection, Δ_o, due to the suspended weight is

0.1563 in. A summary of the test results for the tubular member with no dampers as well as with wool swab, copper brush, and silly putty in chamber dampers is given in Table 1. The number of dampers, the total weight of the damping assembly, the damping ratio and the damping efficiency index are listed for each passive damping assembly. The logarithmic decrement method, as described in Reference 8, was used with the experimentally obtained deflection versus time plots to obtain the damping ratio.

The calculation of the damping ratio for the natural vibration tests was obtained using the first sixteen cycles and reading the amplitudes directly from the experimental deflection versus time plots. Each ζ value in Table 1 was then obtained by taking the average results of three tests for each combination of damping devices.

The efficiency index is defined (Reference 1 and 2):

$$\eta = \frac{\zeta - \zeta_0}{M_d} \tag{22}$$

in which ζ is the damping ratio with the damping devices, ζ_0 is the damping ratio in the absence of any passive damping device, and M_d is the total mass of the damping assembly.

The natural frequency from all of the experiments was found to be 8.4 Hz. The deflection versus time plots referenced in this section are obtained using the average ζ value and natural frequency from the experiments, and the following Δ-t relationship (Reference 1).

$$\Delta = \Delta_0 e^{-\zeta\omega t}\left(\frac{\omega\zeta}{\omega_d}\sin\omega_d t + \cos\omega_d t\right) \tag{23}$$

The damped circular frequency, ω_d, is given by:

$$\omega_d = \omega\sqrt{1 - \zeta^2} \tag{24}$$

The details including the listing of a computer program utilizing Equation

23 to produce a deflection versus time plot are given in Reference 1. A baseline plot of deflection versus time for the member with no dampers is shown in Figure 18.

3.4.1.1 **Copper Brush Dampers**

For the copper brush dampers the maximum $\bar{\zeta} = 0.0131$ is obtained with an assembly of three damping devices. This assembly produces the maximum $\eta =$ 16.72 in/lb-sec^2. Figure 19 shows the corresponding average Δ-t plot for a 10 second duration. Figure 20 shows the effect of the three copper brushes on the deflection time envelopes. The vertical ordinate in this figure is designated by Δ_e to indicate that the figure represents the envelopes rather than the complete Δ-t relationship. The damping ratios from the experiments are given in Table 2(a). In addition to the test conducted as described in Section 3.3, a series of tests were made with no tension in the damping assembly. These tests, conducted with 1, 2, 3, 5 and 7 devices in the specimen showed no significant increase in member damping regardless of the number of devices used. The results are summarized in Table 2(b). One plausible explanation for this is as follows. The outer diameter of the copper brush is less than the inside diameter of the member. When there is no tension in the damping assembly, the devices are free to bounce inside the specimen. Because the vibrations are relatively small and the natural frequency low, the assembly with no tension has a tendency to move with the specimen, bouncing slightly inside the member. Due to the relatively negligible mass of the damper as compared to the member this nearly coincident movement produces minimal damping of the vibration. With a slight tension in the assembly, it can have its own natural frequency, different from the specimen. As a result, when vibration of the specimen is induced, the impact of the damping assembly with the side of the tube sets the assembly in motion. Two types of motion then contribute to the damping. First, because of the difference in natural frequency of vibration impact of the dampers against the inside of the tubular member acts to damp the vibration. Secondly, the frictional interaction between the dampers and the member inside surface takes place while the dampers vibrate both in plane but out of phase, and axially. When the number of dampers is increased beyond three with nominal tension, the damping ratio decreases.

3.4.1.2 Wool Swab Dampers

For the wool swab dampers the maximum $\zeta = 0.0105$ was obtained with an assembly of three dampers resulting in an efficiency of 9.05 in/lb-sec^2. The maximum $\eta = 12.34$ was obtained with a single damper assembly yielding a damping ratio of 0.0099. Figures 21 and 22 represent the Δ-t plots for the member with three, and one wool swab damper assemblies, respectively for a 10 second duration. Figures 23 and 24 show the effects of these damping assemblies on the deflection-time envelopes. The damping ratio increased as the number of dampers was increased from one to three. Increasing the number of devices beyond three resulted in a decrease in both damping ratio and efficiency. The small negative efficiency noted for seven devices can be taken as practically zero. It was found that a variation in the method of attachment of the assembly to test specimen from concentric to an eccentric connection had no significant effect on the resulting damping ratio. The results are given in Tables 3(a) and 3(b).

3.4.1.3 Silly Putty in Chambers Dampers

For silly putty in chambers dampers, the maximum $\zeta = 0.0115$ was obtained with an assembly of three dampers resulting in a $\eta = 15.73$ in/lb-

sec^2, whereas the maximum $\eta = 21.35$ in/lb-sec^2 was obtained with an assembly of two dampers corresponding to $\zeta = 0.0113$. Figures 25 and 26 represent the Δ-t plots for the member with three and two silly putty in chamber damper assemblies, respectively, for 10 second duration. Figures 27 and 28 show the effects of these damping assemblies on the deflection-time envelopes. The damping ratio was found to increase as the number of dampers was increased from one to three. Increasing the number of dampers beyond three resulted in a decrease of both damping ratio and efficiency. The tests conducted with no tension in the assembly showed a slight increase in damping ratio up to the three damper assembly. An increase in the number of dampers beyond three with no tension on the assembly showed no increase in damping ratio above the baseline damping ratio for the empty member. The results are given in Tables 4(a) and 4(b). Of all the passive damping devices tested in this study, the assembly of three silly putty in chamber dampers was found to be the most efficient. Therefore, these dampers were chosen for further study under forced harmonic vibration.

3.5 Forced Then Free Vibration

It was discovered during testing that the vibration employed for the forced vibration tests allowed only a limited amount of travel. This meant that the deflection of the member at the location where the vibrator was attached was limited to what the vibrator would allow. Nevertheless, forced vibration tests were conducted on the individual member since it was not known initially whether or not the dynamic deflection would exceed the vibrator capacity. The results presented later in this section indicated that the vibrator constrained the member deflection for a certain range of forcing function frequencies including that which would otherwise have constituted a resonance condition. This limitation must be taken into account when evaluating the performance of the dampers on an individual member.

3.5.1.1 Silly Putty in Chamber Dampers

The results of the experimental study of the member under forced then free vibration are summarized in Table 5. Tests were conducted with no dampers, and 1, 2, and 3 dampers inside the member. Each of these assemblies was subjected to a force of 4 lb. at the member midspan, at frequencies of 2, 5, 7, and 9 Hz, corresponding to Ω/ω_n ratios of 0.238, 0.596, 0.834, and 1.073, respectively. An additional test was conducted on the empty member and the 3 damper assembly using a frequency of 12 Hz (Ω/ω_n = 1.430). The experimental results are shown in Figures 29 through 32. The free vibration part of the deflection-time graph is obtained by disengaging the forcing function from the member midspan as described in Section 3.3. The constrained dynamic deflection amplitude, Δ^*_D , and its dimensionless value, Δ^*_D/Δ_S, where Δ_S is the calculated static midspan deflection due to a 4 lb. load, are listed in Table 5. The constrained dynamic deflection amplitude is the measured amplitude of the initial constrained force part of the deflection-time plots. Also listed in Table 5 are the maximum initial free vibration amplitudes, Δ_F, for each assembly and frequency considered. Two dimensionless quantities are derived from this value as Δ_F/Δ_S and Δ_F/Δ^*_D. The data in Table 5 shows that the Δ^*_D/Δ_S values range from 0.59 to 0.95. For all the cases, the maximum value was observed for an applied force frequency of 5 Hz. It was also found that the Δ_F/Δ_S and Δ_F/Δ^*_D ratios were gradually increasing for increasing forcing function frequencies. One important consequence of the deflection constraint imposed by the vibrator is that no resonance phenomenon could be produced in the vicinity of 8.4 Hz. The average damping ratios were obtained from the free vibration part of the deflection-time curves and are listed in Table 5. As seen from this data, the single silly putty in chamber damper configuration provides the maximum decrease in free vibration amplitude. Another important observation to be made is that the ζ values in Table 5 are significantly less than the corresponding values for the same damping assemblies given in Table 1. This is attributable to the dependence of the damping ratio on the initial velocity which is considerably greater for the results reported in Table 5 than for those in Table 1.

3.5 Comparison of Damping Efficiencies

In Section 3.4, the efficiency index based on Equation 22 was computed for each damping device. The average values of η and the associated damping assembly weight for natural vibration were presented in Table 1. Figure 33 shows the curves between η and the weight of dampers used in the natural vibration tests for various damping concepts. The silly putty in chamber dampers provided the most efficienct damping of the member. It is worth noting that all of the curves have ascending and descending portions which define the maximum attainable damping efficiency. In general, an increase in damping assembly weight beyond 50 grams results in a decline in efficiency.

Figure 34 shows the relationships between the damping efficiency and the number of damping devices for natural vibration using all three concepts. These curves also show that, in general, an assembly of more than three damping devices results in a decline in efficiency. This may indicate that the first and second mode shapes are dominating the dynamic response.

By applying the dampers to locations in the vicinity of maximum deflection for these mode shapes, the maximum efficiency was realized. Any increase in the number of dampers beyond three adds mass to the system, and is associated with a decrease in damping.

The average damping efficiency indices for the forced then free vibration tests for 1, 2, and 3 silly putty in chamber dampers are given in Table 5. The maximum efficiency was obtained using one silly putty in chamber damper and a forcing function frequency of 5 Hz. No correlation between the maximum efficiency and the initial vibration amplitude was observed. However, the maximum average damping ratio for each device was found to occur near the theoretical resonance of the member (between 7 and 9 Hz) in spite of the inability of the apparatus to allow the resonance to occur.

4. NUMERICAL STUDY

4.1 Natural Vibration

4.1.1 Finite Element versus Experiment

The formulation and solution algorithmn using Newmark's method for computing the dynamic response of a beam was given in Section 2. In this section, a comparison is made of the deflection versus time relations from this finite element analysis to those obtained experimentally.

A preliminary study showed that for $\Delta t = 0.0001$ sec., the central difference formulation described in Section 2.3 gave precisely the same results as Newmark's method. Since Newmark's method provides accurate results even with larger time steps, it was used to produce Figure 35 through 43. Figure 35 shows a comparison of the finite element and experimental Δ-t plots for the member with no dampers. The solid line is the finite element solution and the dashed line is the experimental curve using a frequency of 8.4 Hz and the average damping ratios from Table 1. Figure 36 shows a comparison of the finite element and experimental Δ-t plots for the member with 3 copper brush dampers. In both of these curves, it can be seen that the period of the vibration obtained using finite elements is exagerated by approximately 32%. However, the amplitudes of the vibration are accurate to within 5%.

4.1.2 Finite Element versus Finite-Difference

The Δ-t curves representing the finite-difference solution are obtained using the computer program developed in Reference 2. Figures 37 and 38 show the comparison of the finite element and the finite-difference solutions for the member with no dampers and three copper brush dampers, respectively. The data for these plots is obtained from Table 1. As indicated in these figures, the difference in the period calculated by these two methods is approximately 26%. However, the amplitudes of the vibration from the two analyses are within 3% of each other. Figure 39 is a comparison of the finite-element and finite difference solutions for a simply supported beam ($k_1 = k_2 = 0$). Similar correlation is also observed for a fixed end beam ($k_1 = k_2 = \infty$). In the presence of end connections of intermediate fixity, the two analyses provide somewhat differing results.

4.2 Forced then Free Vibration

In this section, curves obtained from the finite element solution for various forcing function frequencies are given. Also, a comparison of the theoretical solution to experimental results is made for both the member with no dampers and the member with one silly putty in chamber damper at a forcing function frequency of 2 Hz.

Figure 40 shows the response using Newmark's method for a beam with no dampers and subjected to a 4 lb. force at a frequency of 2 Hz. After 1 second, the forcing function is removed and the beam is allowed to vibrate freely. Figure 41 shows the response of the same system with a forcing function frequency of 6 Hz. This frequency corresponds to a frequency ratio Ω/ω_{fe} of 0.95, where $\omega_{fe} = 6.3$ Hz is the natural frequency of the beam from the finite element solution. Clearly, this represents a nearly resonant condition as expected. After 1 second, the forcing function is removed and the member is allowed to vibrate freely.

Figures 42 shows the finite element and experimental curves for the member with no dampers and subjected to a 4.0 lb. force at a frequency of 2 Hz. Although the forced vibration portions of the two curves at $\Omega = 2$ Hz are quite similar, the free vibration amplitudes differ significantly. The reasons for this difference may be as follows. In the experiment, the forcing function was terminated by pulling the pin RS from the vibrator connector shown in Figure 16. During the tiny time interval in which the pin was being pulled out, the contact and frictional forces involved in disengaging the segment QR from RU were unintentionally transferred to the members thereby retarding its initial amplitude in the free vibration range. Consequently, the ensuing envelope of the experimental free vibration Δ-t curve is considerably narrower than the theoretical one. Similar effects are observed in Figure 43 which shows the finite element and experimental results when one silly putty in chamber damper is used.

At larger Ω values such as those of the order of 6 Hz, the Δ-t relations from the finite element analysis do not match the experimental ones even in the forced vibration range. This is primarily due to the constraints imposed by the vibrator on the maximum member deflecting as explained earlier in Section 3.5.

4.3 Finite Element Analysis for Forced Vibration

As mentioned earlier, the vibrator used in the experimental study constrained the motion of the member in the presence of a forcing function. As a result, the actual effect of passing damping could not be observed for this condition. Therefore, a numerical study was conducted to examine the effect of passive damping in the presence of the forcing function. In this section, the theoretical results showing both the extent of damping which would occur during the forced vibration and the effect of the dampers on the deflection-time envelopes are presented and discussed. Figure 44 shows the theoretical dynamic magnification factor (DMF), Δ_D/Δ_S, versus the frequency ratio Ω/ω_n for damping ratios of 0.0094, 0.0131 and 0.50. The first two values of the damping ratios were obtained from the member tests with no dampers, and 3 copper brush dampers, respectively. As can be seen in this figure, the copper brush dampers do not change the DMF appreciably for non-resonance frequency ratios. However, the dampers reduce the DMF by approximately 7% at resonance.

Figure 45 shows the deflection versus time relationship for the member with no dampers and with three copper brush dampers, with a forcing function frequency of 6.35 Hz ($\Omega/\omega_n = 1.0$) for one second, and allowed to vibrate freely thereafter. These curves show that the passive damping results in a member amplitude reduction in the forced vibration range, however, its most beneficial effect occurs during the free vibration. After 3 seconds of free vibration, the amplitudes of the member with dampers are approximately 40% less than those corresponding to the empty member.

5. CONCLUSIONS AND FUTURE RESEARCH

5.1 Conclusions

The following conclusions are drawn from the research conducted herein:

1. The silly putty in chamber concept provides the maximum passive damping efficiency under member natural vibration, as compared to the copper brush or the wool swab concepts.
2. The copper brush concept provides the largest damping ratio of the system under natural vibration.
3. Due to the limitation of the vibrator used, the effectiveness of the passive damping concepts could not be evaluated until the forcing function was disengaged.
4. Frictional and contact forces acting on the member during disengagement from the vibration apparatus caused a reduction of the ensuing free vibration member amplitude.
5. The theoretical results indicate that in the presence of a forcing function, the passive damping devices provide the most effective damping in the vicinity of the resonant frequency.
6. The theoretical results show that passive dampers are considerably more effective under member natural vibration than during forced vibration.
7. Under natural vibration, the finite element solution results in periods which are nearly 30 percent greater than the experimental ones. However, amplitudes are reasonably accurate. The accuracy of the results is improved when the member ends are pinned or fixed.

5.2 Future Research

The most successful passive damping concepts identified herein should be examined using forced vibration equipment which would allow investigation of their effectiveness at or near resonance. Attempts should be made to identify a means of disengaging an applied force without adversely affecting the dynamic response of the member. These tests should be conducted on both individual members and structure sub-assemblies.

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TABLES

Table 1. Member natural vibration test results for copper brush, wood swab, and silly putty in chamber dampers.

PASSIVE DAMPING CONCEPT	NUMBER OF DAMPERS	WEIGHT OF DAMPING ASSEMBLY (GM)	AVERAGE DAMPING RATIO ζ	DAMPING EFFICIENCY INDEX (IN/LB.-SEC^2)
No Dampers		0.00	0.0094	0.00
Copper	1	13.0	0.0098	5.39
Brush	2	26.0	0.0107	8.76
Dampers	3	39.0	0.0131	16.72
	5	65.0	0.0129	9.44
	7	91.0	0.0097	0.48
Wool	1	7.10	0.0099	12.34
Swab	2	14.20	0.0102	9.87
Dampers	3	21.30	0.0105	9.05
	5	35.50	0.0101	3.46
	7	49.70	0.0091	-0.99
Silly	1	7.8	0.0100	13.48
Putty	2	15.6	0.0113	21.35
in	3	23.4	0.0115	15.73
Chamber	5	39.0	0.0109	6.74
Dampers	7	54.6	0.0097	0.96
	11	85.8	0.0094	0.00

Table 2(a). Damping ratios from natural vibration tests with copper brushes and no cord tension.

Number of Devices	ζ_1	ζ_2	ζ_3	ζ_{AVG}
1	0.0097	0.0095	0.0095	0.0096
2	0.0097	0.0097	0.0097	0.0097
3	0.0098	0.0096	0.0096	0.0097
5	0.0094	0.0096	0.0094	0.0095
7	0.0095	0.0095	0.0096	0.0095

Table 2(b). Damping ratios from natural vibration tests with copper brushes and nominal cord tension.

Number of Devices	ζ_1	ζ_2	ζ_3	ζ_{AVG}
1	0.0097	0.0102	0.0096	0.0098
2	0.0105	0.0109	0.0108	0.0107
3	0.0133	0.0128	0.0131	0.0131
5	0.0129	0.0131	0.0128	0.0129
7	0.0096	0.0098	0.0095	0.0096

Table 3(a). Damping ratios for natural vibration tests with wool brushes and concentric cord support.

Number of Devices	ζ_1	ζ_2	ζ_3	ζ_{AVG}
1	0.0098	0.0097	0.0098	0.0098
2	0.0100	0.0098	0.0101	0.0100
3	0.0107	0.0108	0.0101	0.0105
5	0.0096	0.0100	0.0097	0.0098
7	0.0094	0.0094	0.0094	0.0094

Table 3(b). Damping ratios for natural vibration tests with wool brushes and eccentric cord support.

Number of Devices	ζ_1	ζ_2	ζ_3	ζ_{AVG}
1	0.0099	0.0099	0.0098	0.0099
2	0.0103	0.0101	0.0101	0.0102
3	0.0105	0.0105	0.0105	0.0105
5	0.0099	0.0102	0.0102	0.0101
7	0.0093	0.0090	0.0090	0.0091

Table 4(a). Damping ratios for natural vibration tests with silly putty and no cord tension.

Number of Devices	ζ_1	ζ_2	ζ_3	ζ_{AVG}
1	0.0096	0.0097	0.0096	0.0096
2	0.0099	0.0099	0.0102	0.0100
3	0.0101	0.0101	0.0101	0.0101
5	0.0095	0.0095	0.0093	0.0094
7	0.0094	0.0092	0.0096	0.0094

Table 4(b). Damping ratios for natural vibration tests with silly putty and nominal cord tension.

Number of Devices	ζ_1	ζ_2	ζ_3	ζ_{AVG}
1	0.0104	0.0096	0.0100	0.0100
2	0.0112	0.0115	0.0113	0.0113
3	0.0112	0.0115	0.0117	0.0115
5	0.0109	0.0109	0.0109	0.0109
7	0.0096	0.0099	0.0096	0.0097
11	0.0094	0.0094	0.0094	0.0094

Table 5. Member forced then free vibration test results for silly putty in chamber dampers.

Passive Damping Concept	Forcing Function Frequency (Hz)	Constrained Dynamic Amplitudes Δ_D^* in.	Δ_D^*/Δ_S	Initial Free Vibration Amplitude Δ_F max in	Δ_F/Δ_S	Δ_F/Δ^*_D	Average Damping Ratio ζ Avg.	η
No Dampers	2	0.067	0.84	0.030	0.38	0.45	0.0043	
	5	0.073	0.92	0.067	0.84	0.91	0.0072	
	7	0.070	0.88	0.077	0.97	1.10	0.0073	
	9	0.067	0.84	0.082	1.03	1.23	0.0069	
	12	0.047	0.59	0.082	1.03	1.75	0.0025	
1 Silly Putty in Chamber Damper	2	0.062	0.78	0.030	0.38	0.48	0.0058	33.7
	5	0.073	0.92	0.063	0.80	0.86	0.0089	38.2
	7	0.067	0.84	0.070	0.88	1.05	0.0089	36.0
	9	0.067	0.84	0.073	0.92	1.10	0.0080	24.17
2 Silly Putty in Chamber Damper	2	0.065	0.82	0.030	0.39	0.48	0.0049	6.7
	5	0.075	0.95	0.060	0.76	0.80	0.0069	(-3.4)
	7	0.067	0.84	0.053	0.67	0.80	0.0076	3.4
	9	0.065	0.82	0.083	1.05	1.28	0.0080	12.4
3 Silly Putty in Chamber Damper	2	0.063	0.80	0.030	0.38	0.47	0.0057	10.5
	5	0.075	0.95	0.057	0.71	0.76	0.0055	(-12.7)
	7	0.068	0.86	0.077	0.97	1.12	0.0064	(-6.7)
	9	0.068	0.86	0.090	1.14	1.32	0.0082	9.7
	12	0.047	0.59	0.082	1.03	1.75	0.0036	8.2

FIGURES

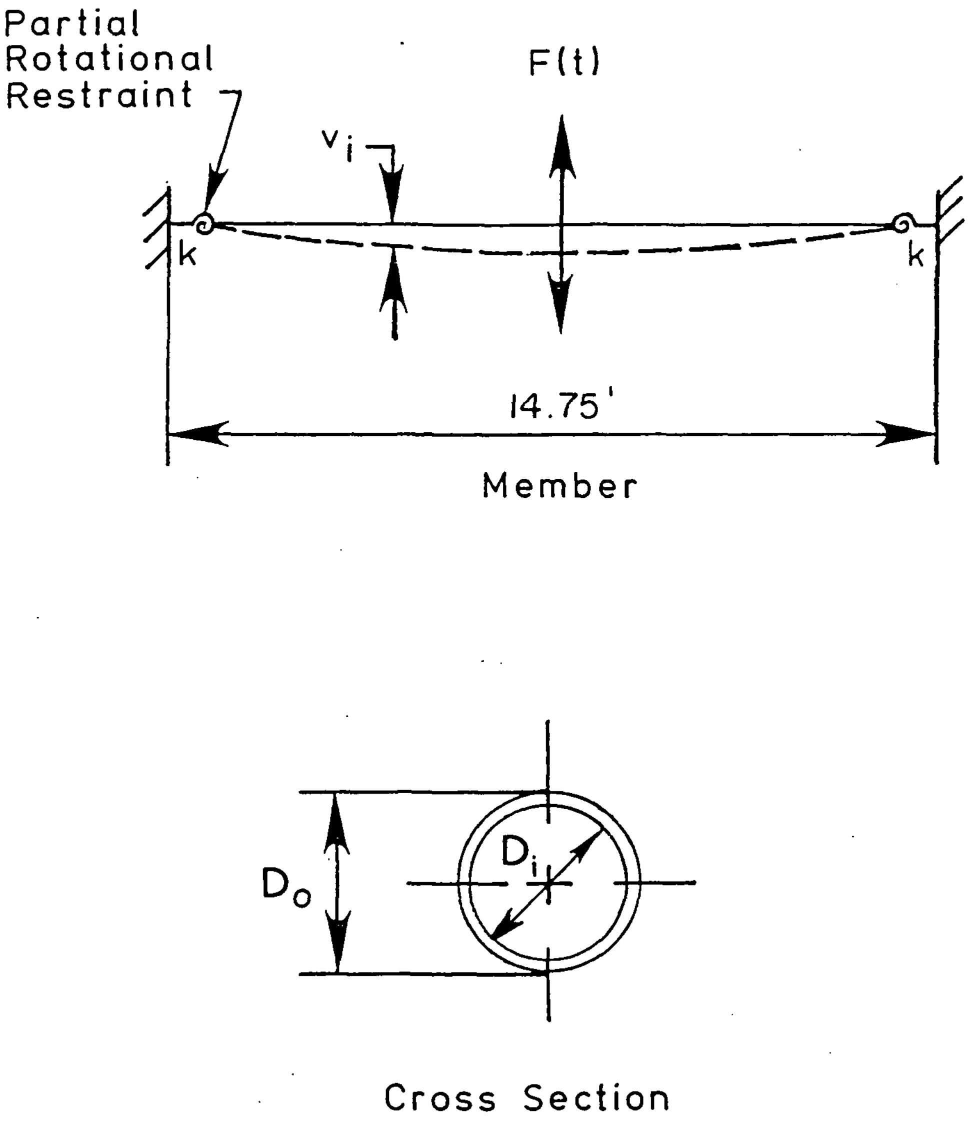

Figure 1. Schematic of tubular member

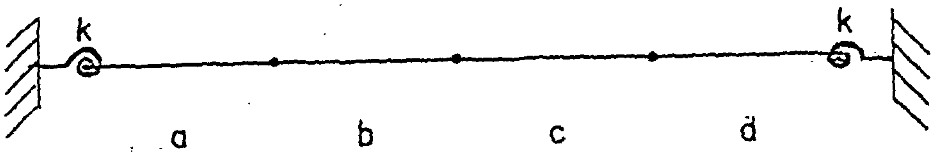

Figure 2. Example of finite element model for the beam.

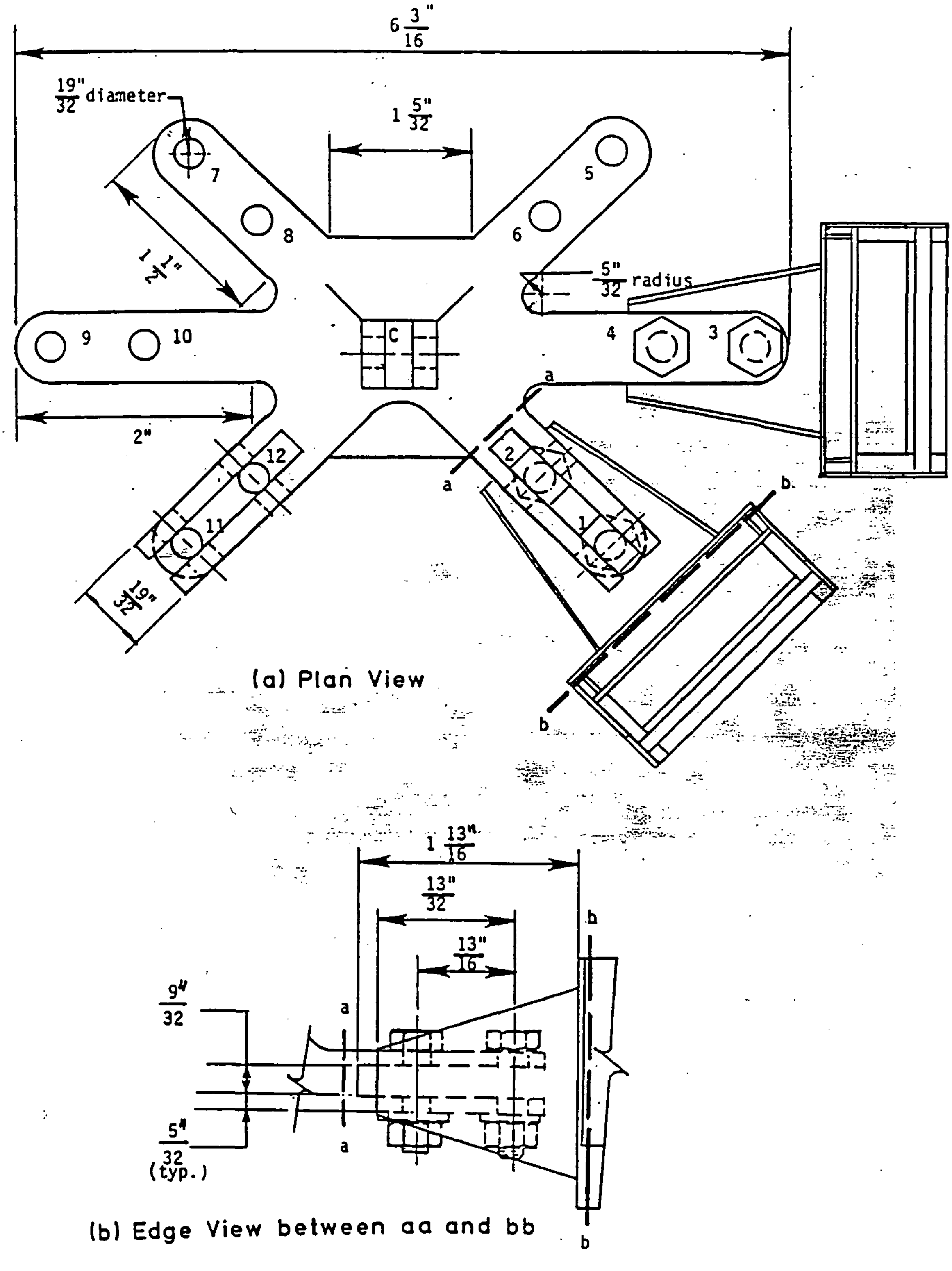

Figure 3. Some end connection details

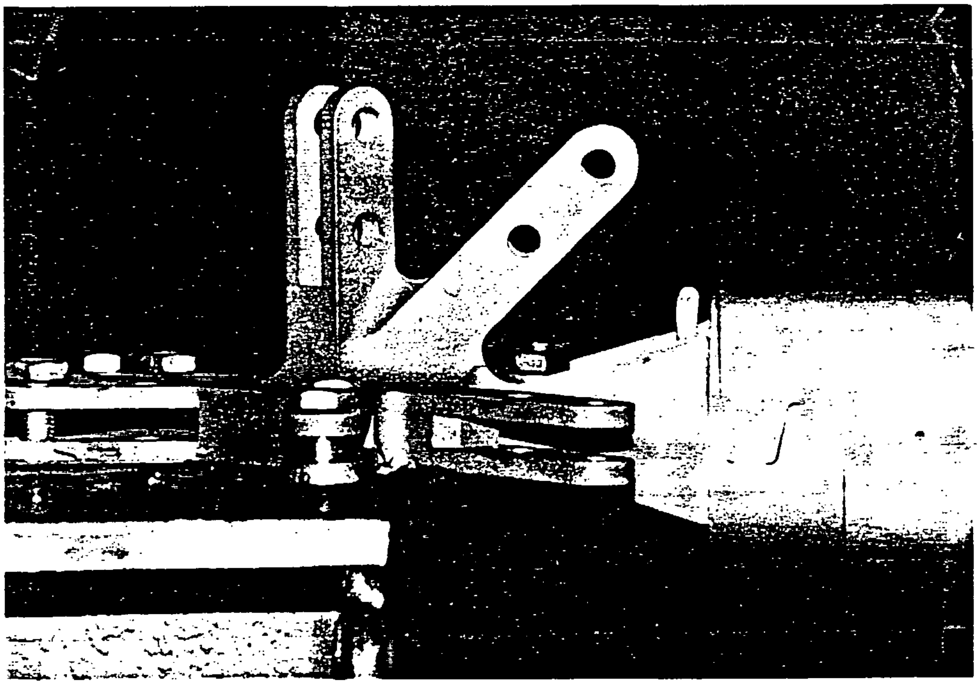

Figure 4. Member end connection

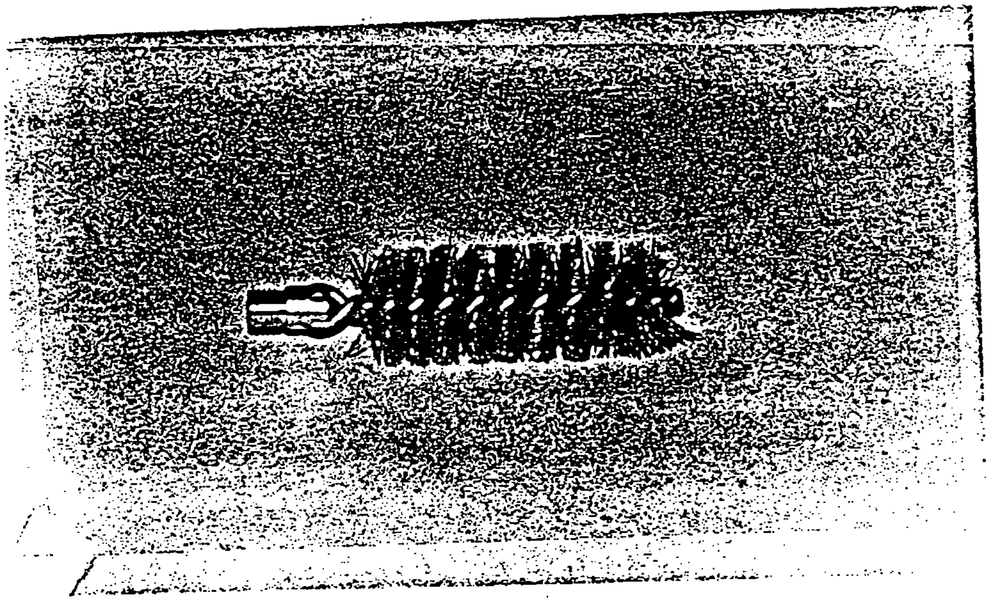

Figure 5. Copper brush damper

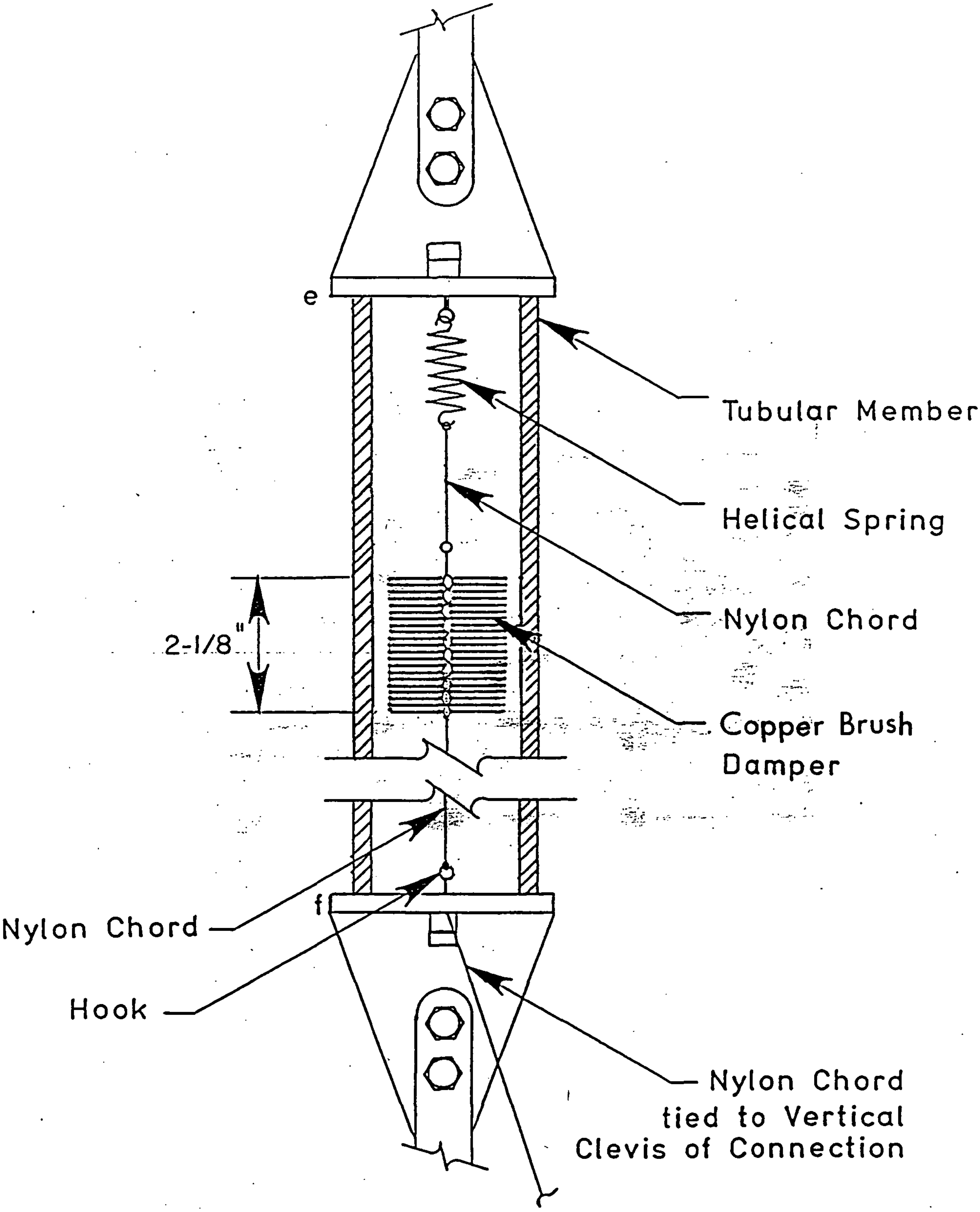

Figure 6. Schematic of attachments for passive damper inside tubular member

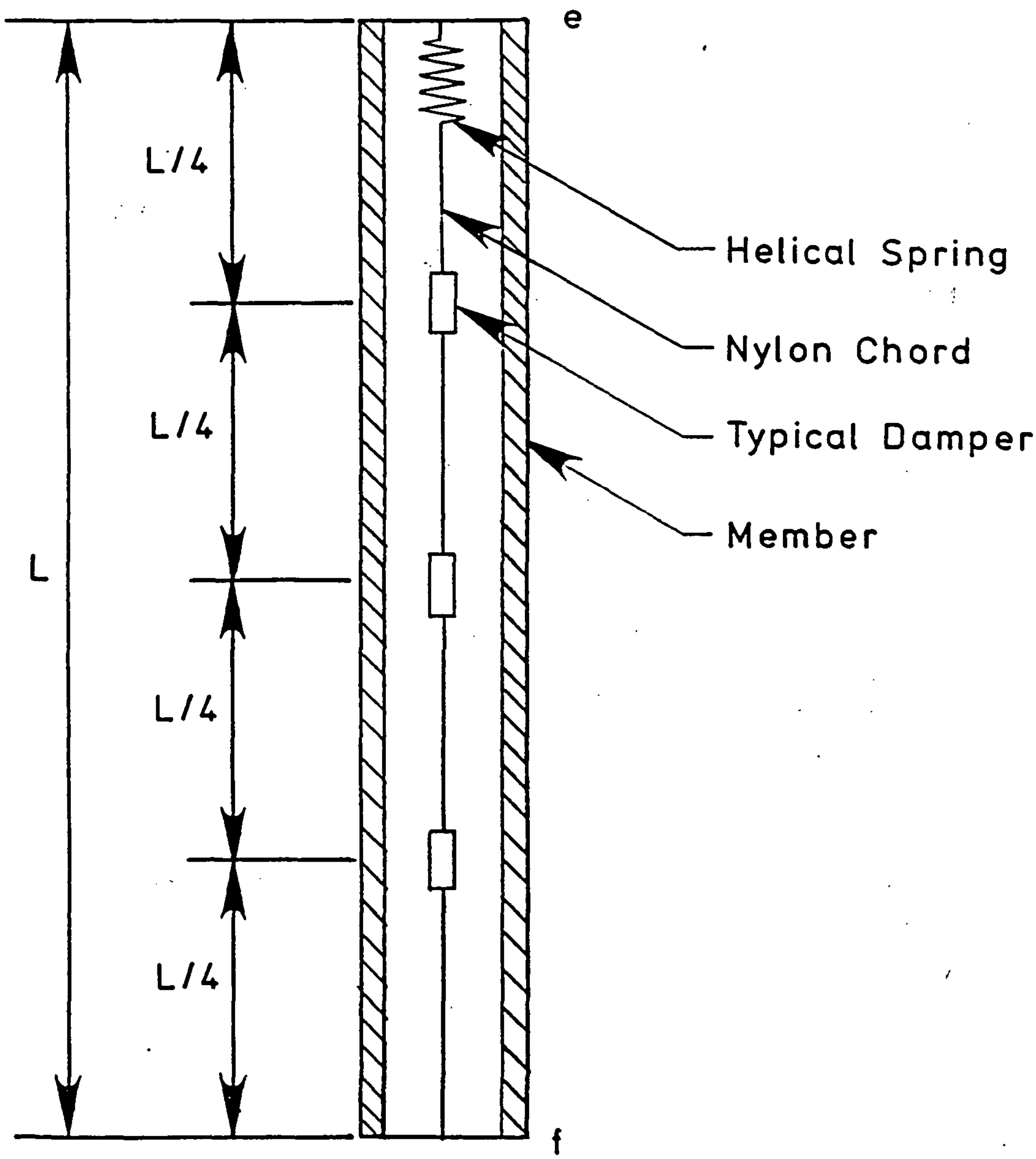

Figure 7. Schematic for spacing of passive dampers

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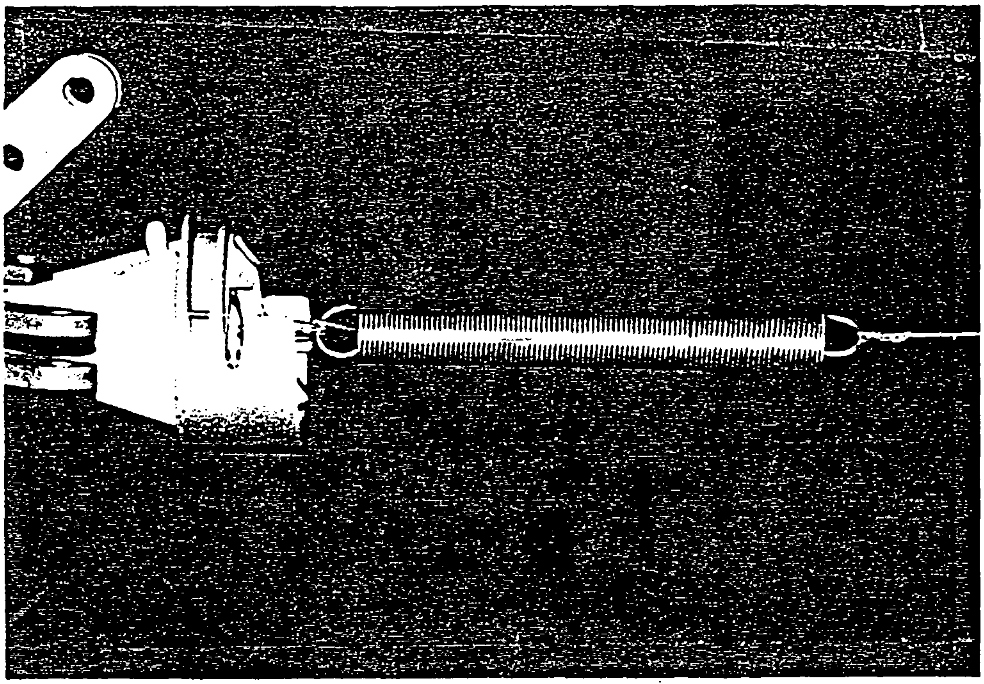

Figure 8. Helical spring attachment at end e

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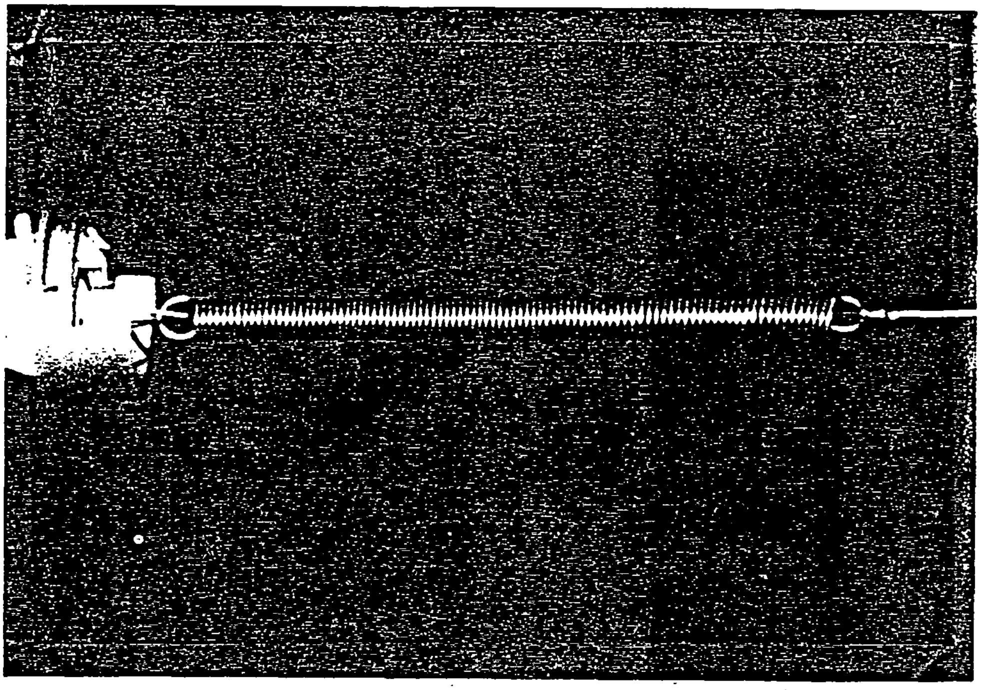

Figure 9. Stretched helical spring at end e

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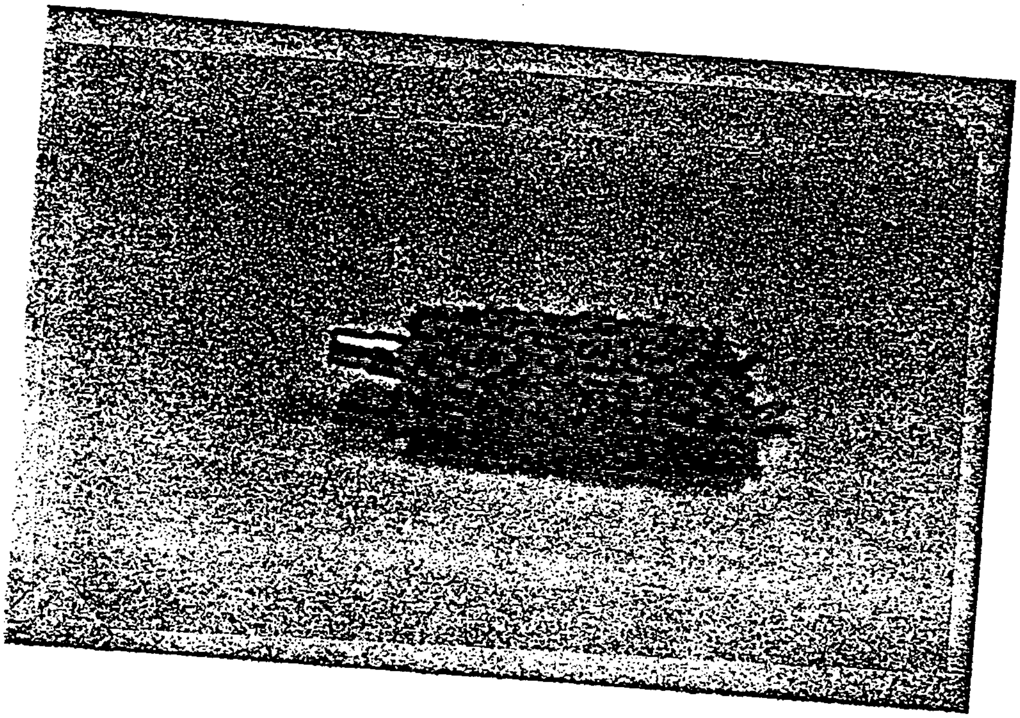

Figure 10. Wool swab damper

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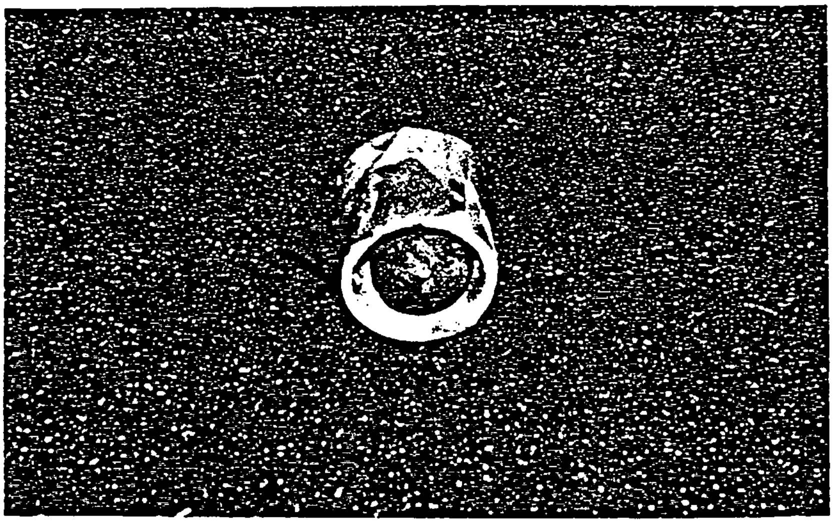

Figure 11. Silly putty in chamber damper

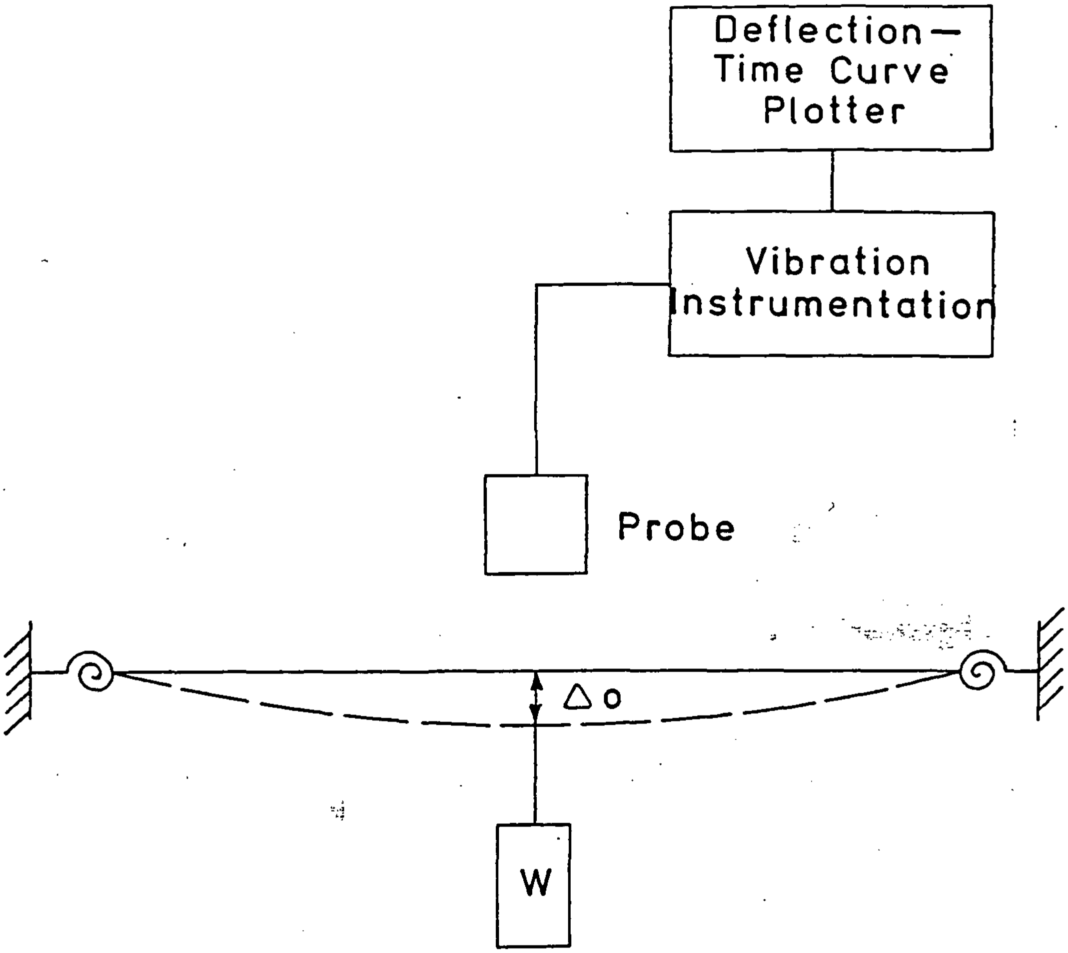

Figure 12. Schematic of member natural vibration setup

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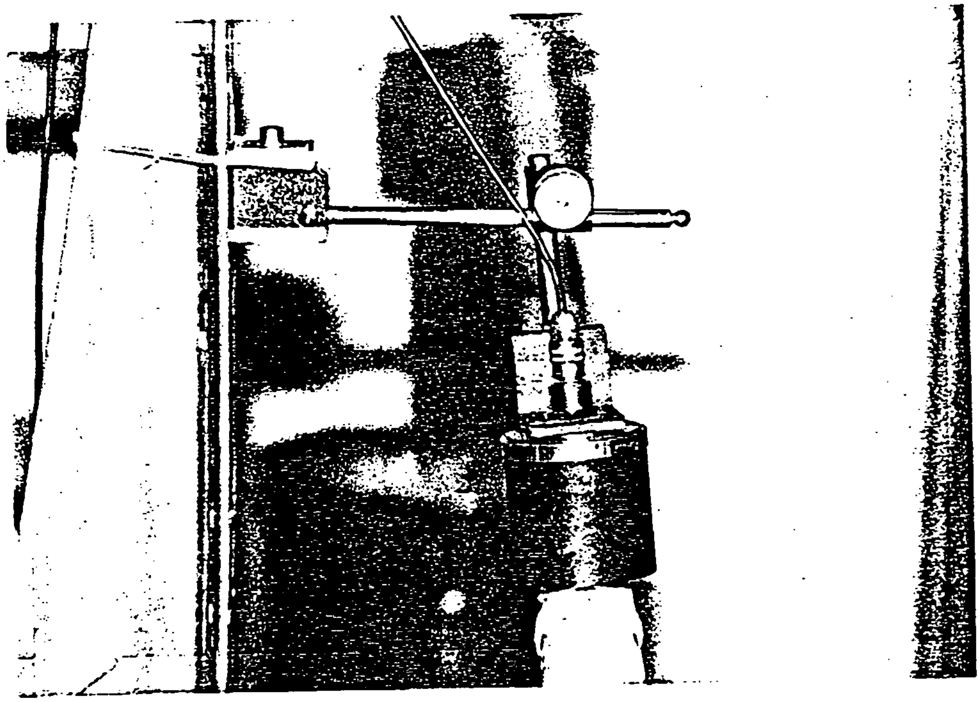

Figure 13. Proximity probe

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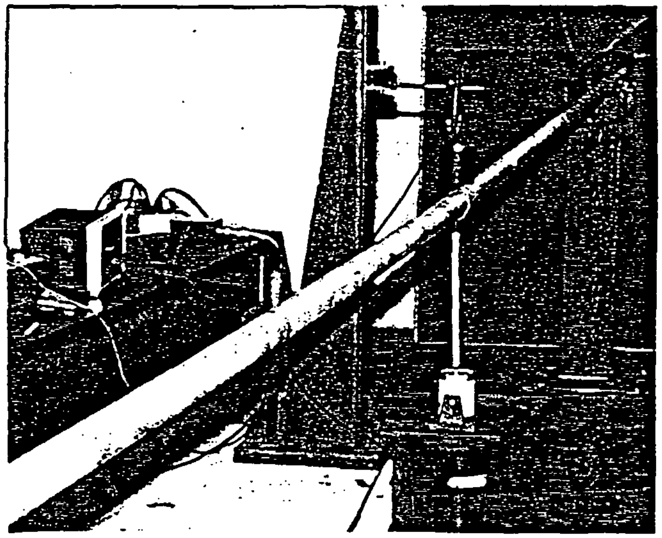


Figure 14. Member forced vibration setup

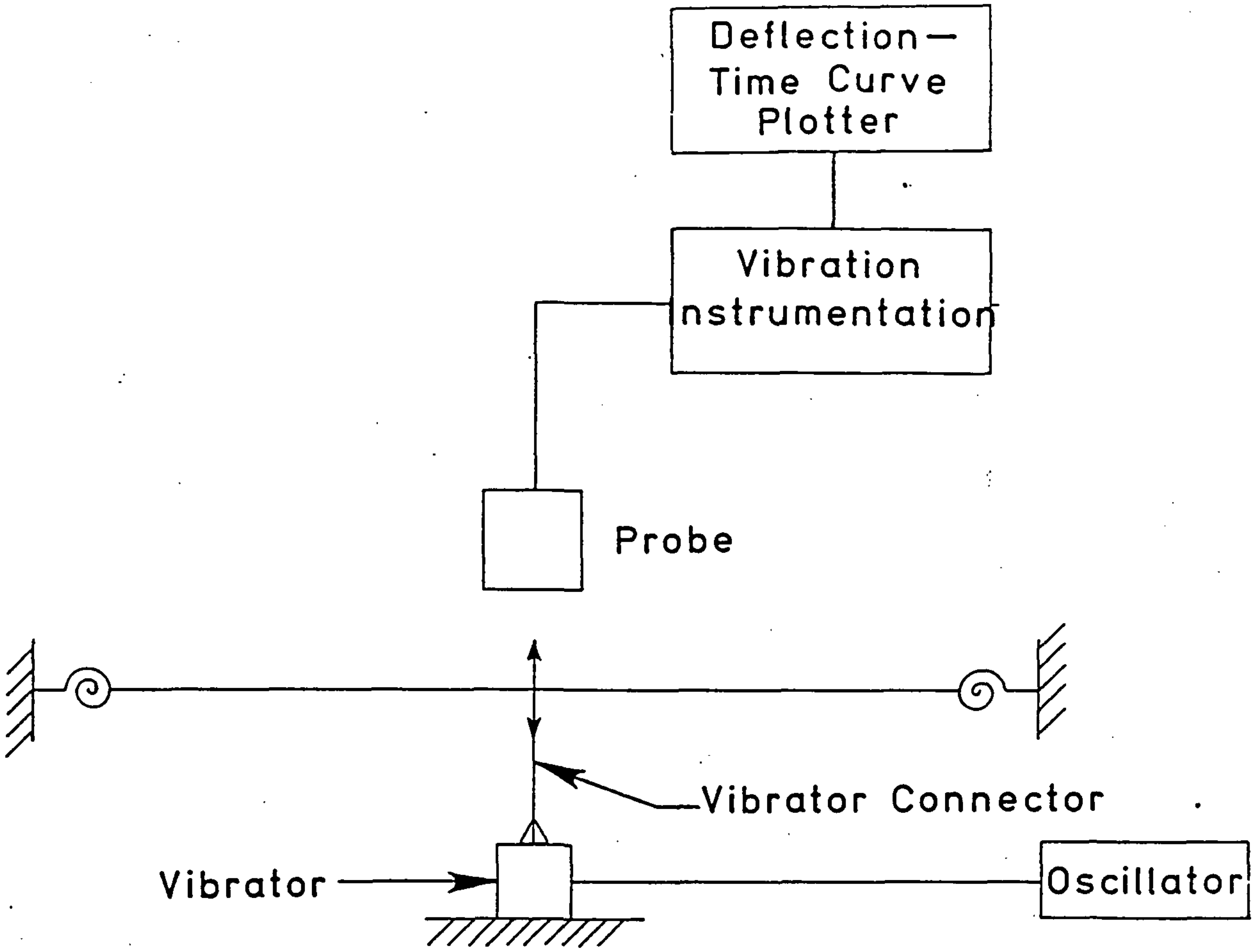


Figure 15. Schematic of member forced vibration setup

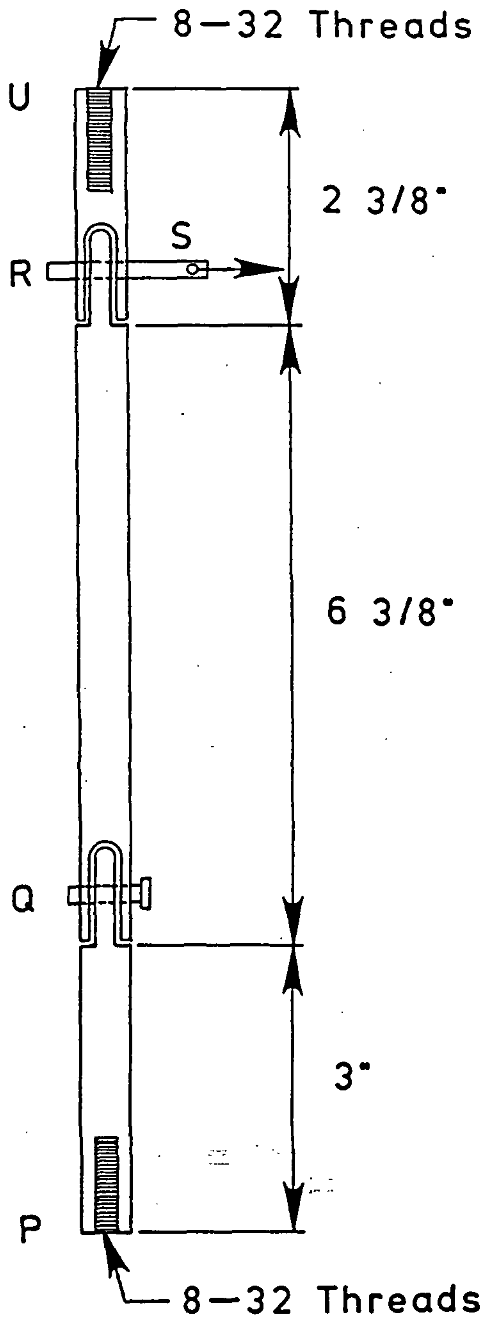
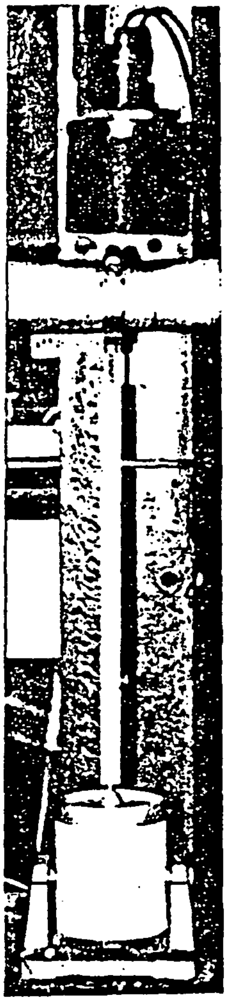


Figure 16. Vibrator connector details



(a) Vibrator connector in engaged position

(b) Disengaged vibrator connector

Figure 17. Vibrator connector in engaged and disengaged positions

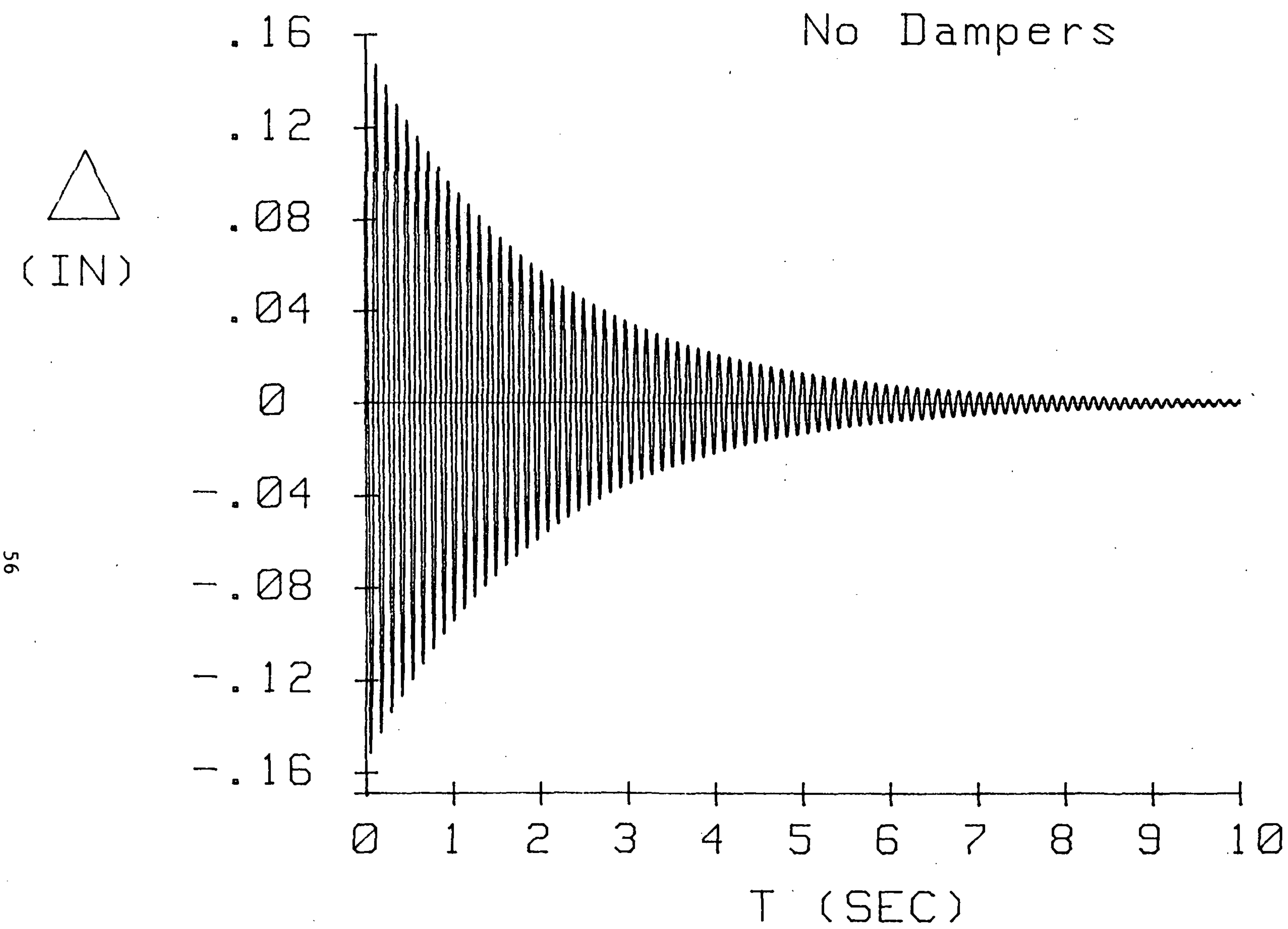


Figure 18. Average Δ-t plot for member with no dampers.

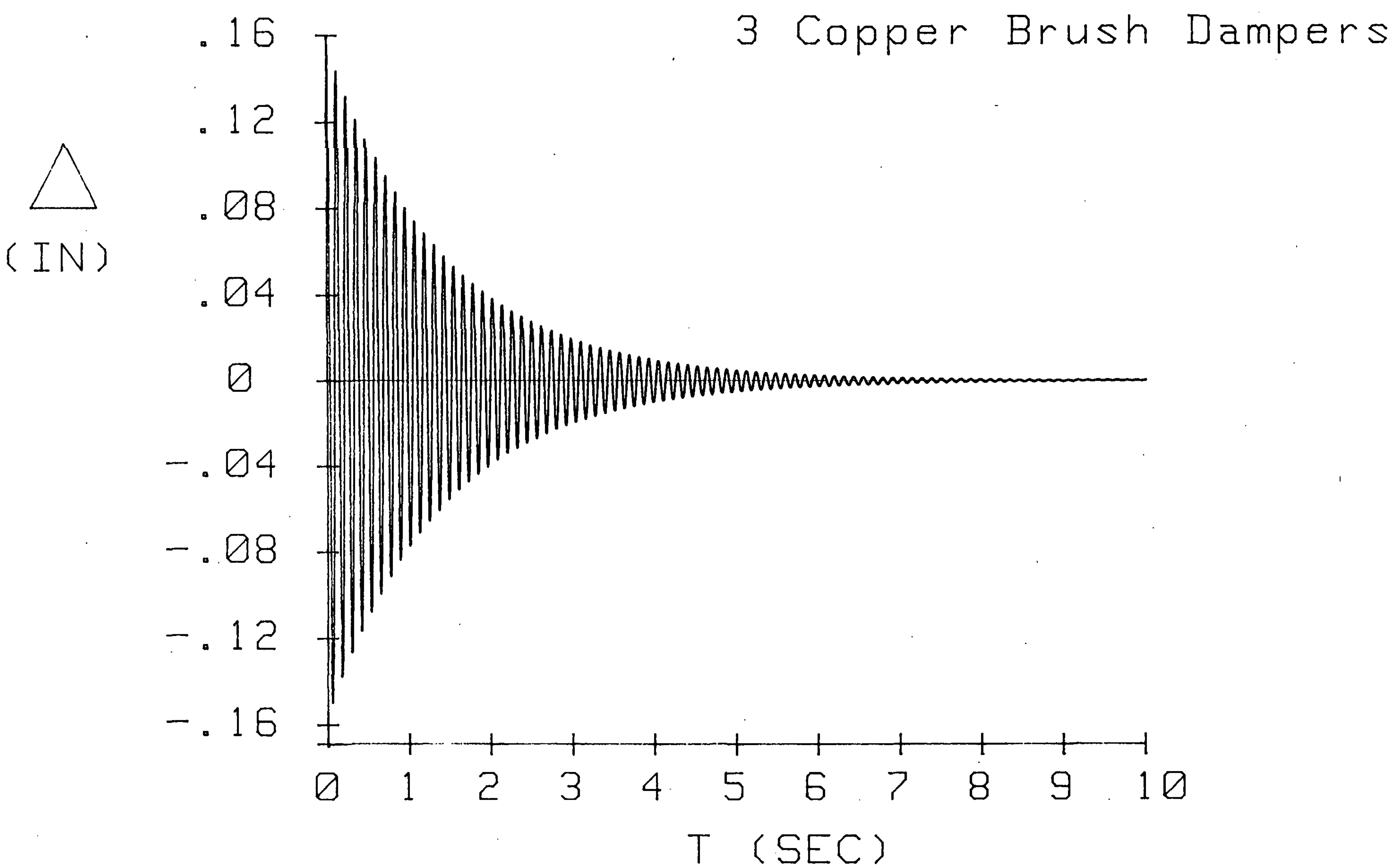


Figure 19. Average Δ-t plot for member with 3 copper brush dampers.

Best 3 & most eff.

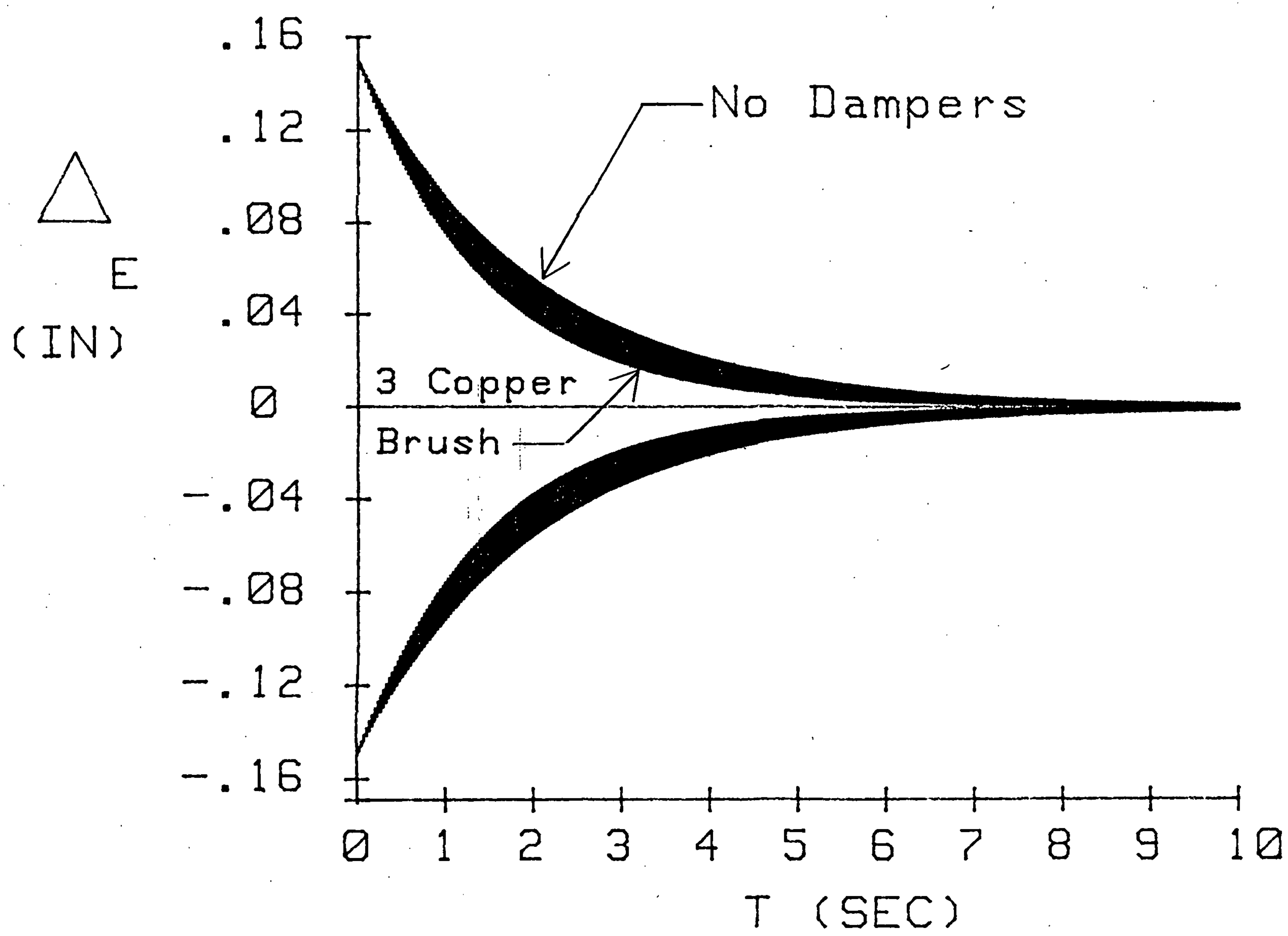


Figure 20. Effect of 3 copper brush dampers on Δ-t envelope.

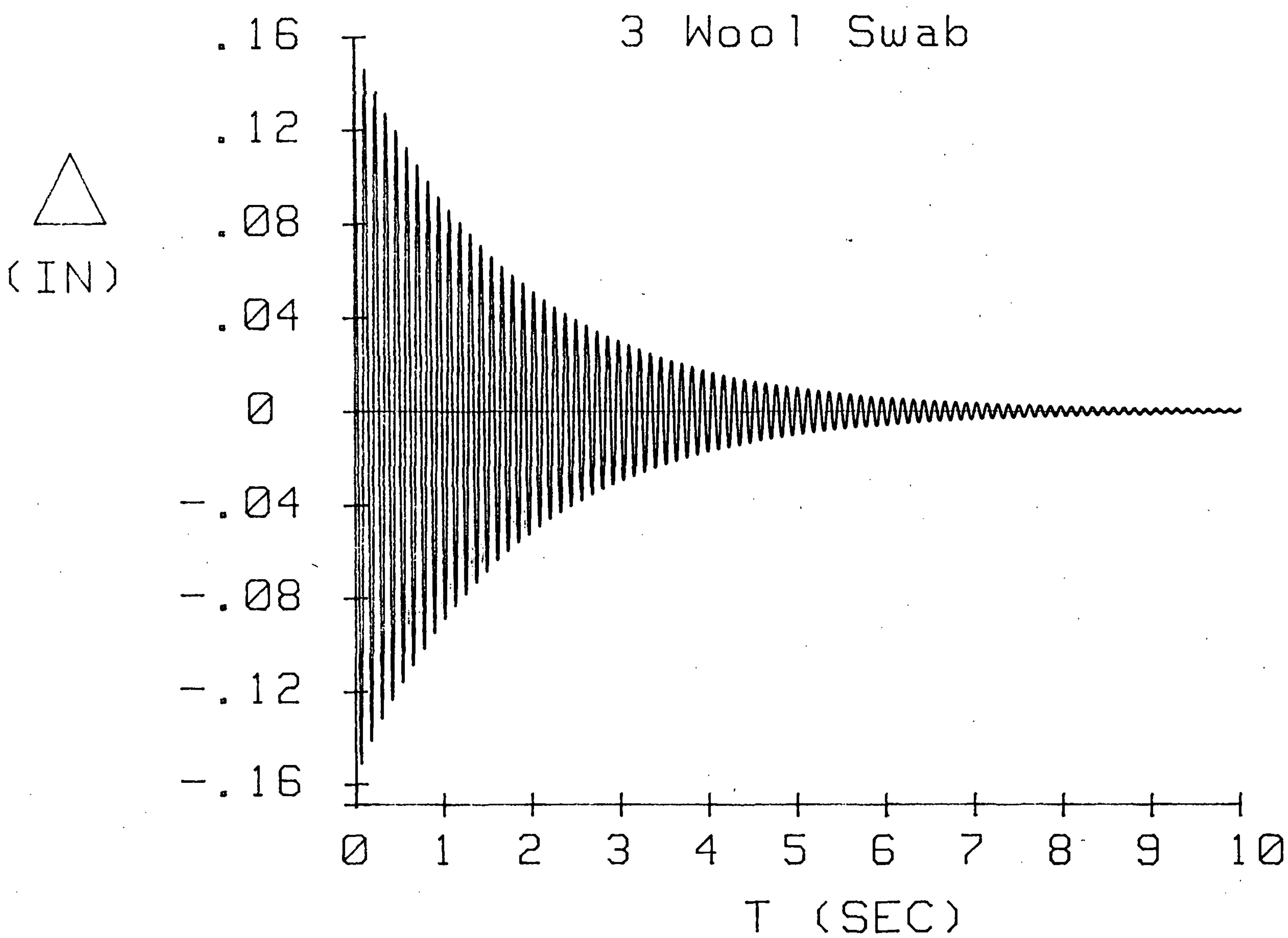


Figure 21. Average Δ -t plot for member with 3 wool swab.

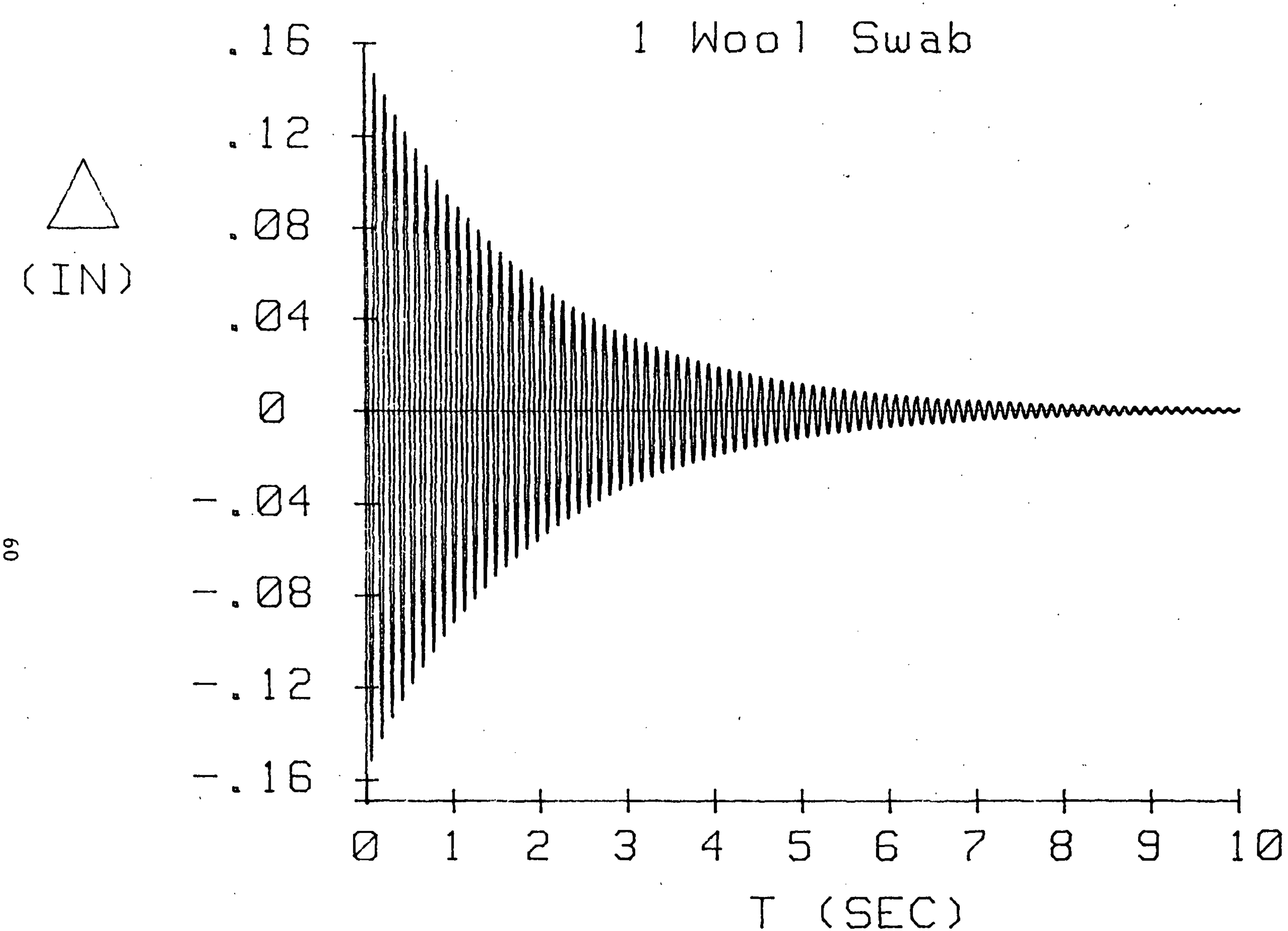


Figure 22. Average Δ-t plots for members with 1 wool swab damper.

most eff

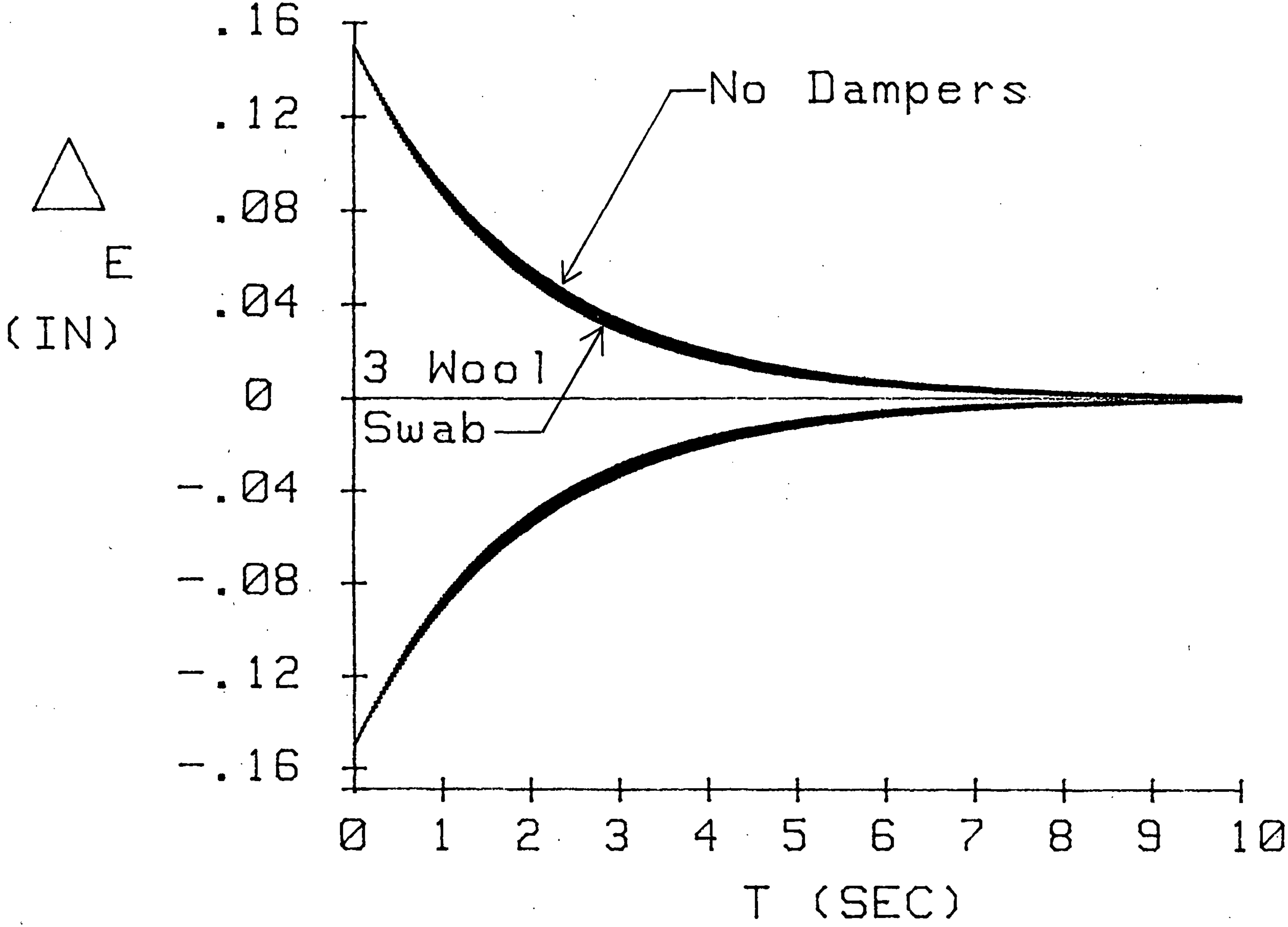


Figure 23. Effect of 3 wool swab dampers on Δ-t envelope.

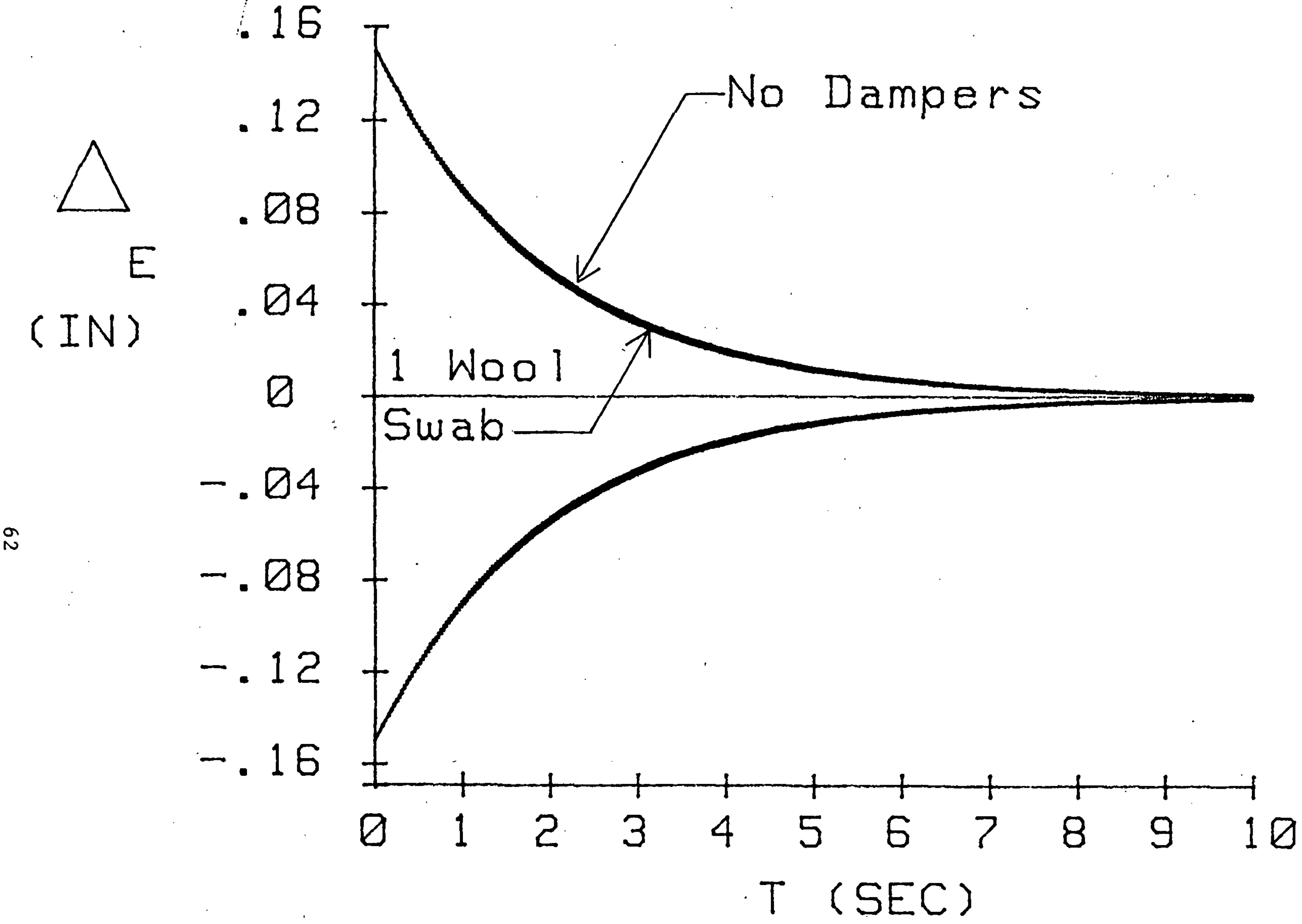


Figure 24. Effect of 1 wool swab damper on Δ-t envelope.

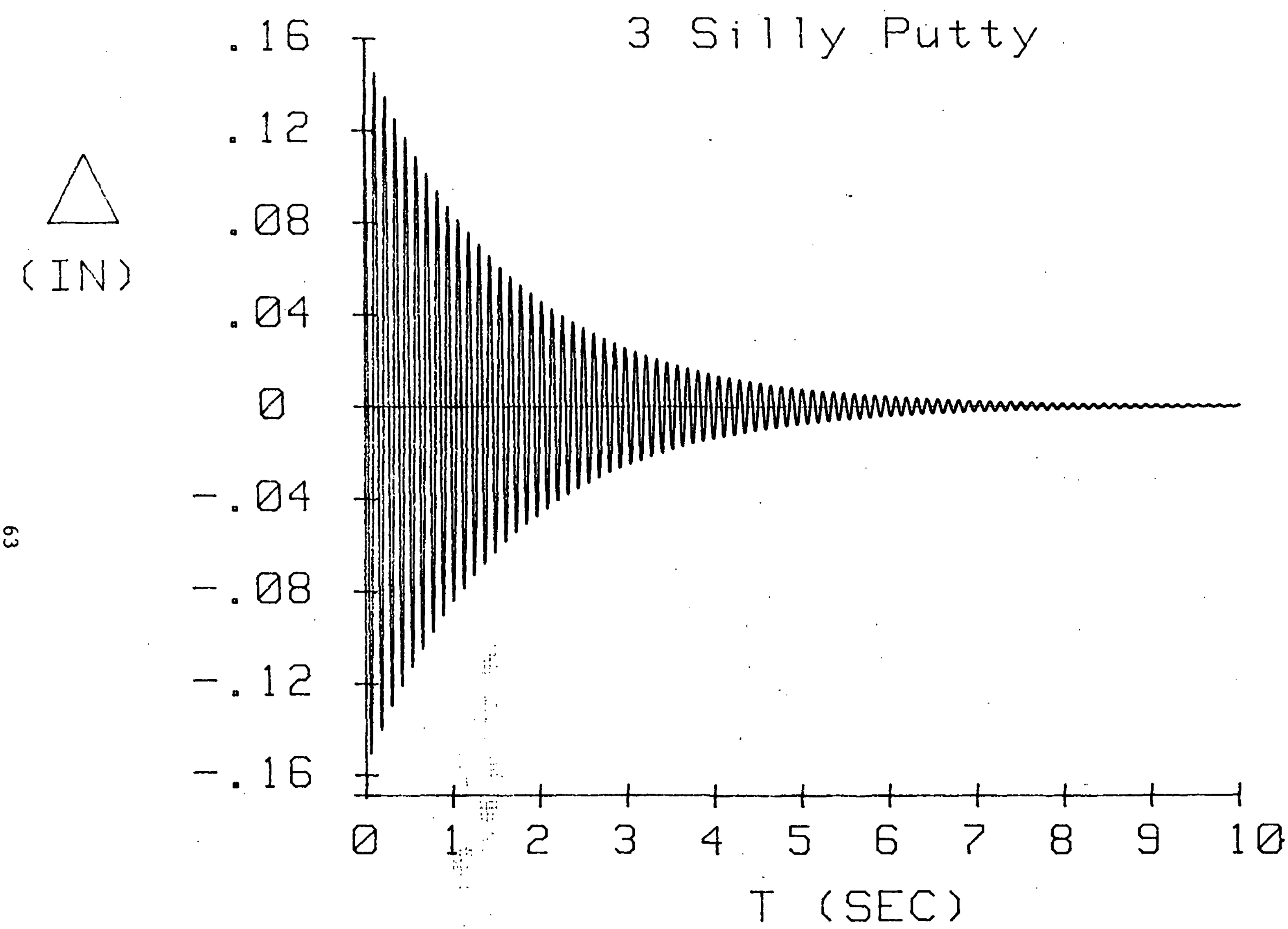


Figure 25. Average Δ-t plot for member with 3 silly putty in chamber dampers.

Best 3

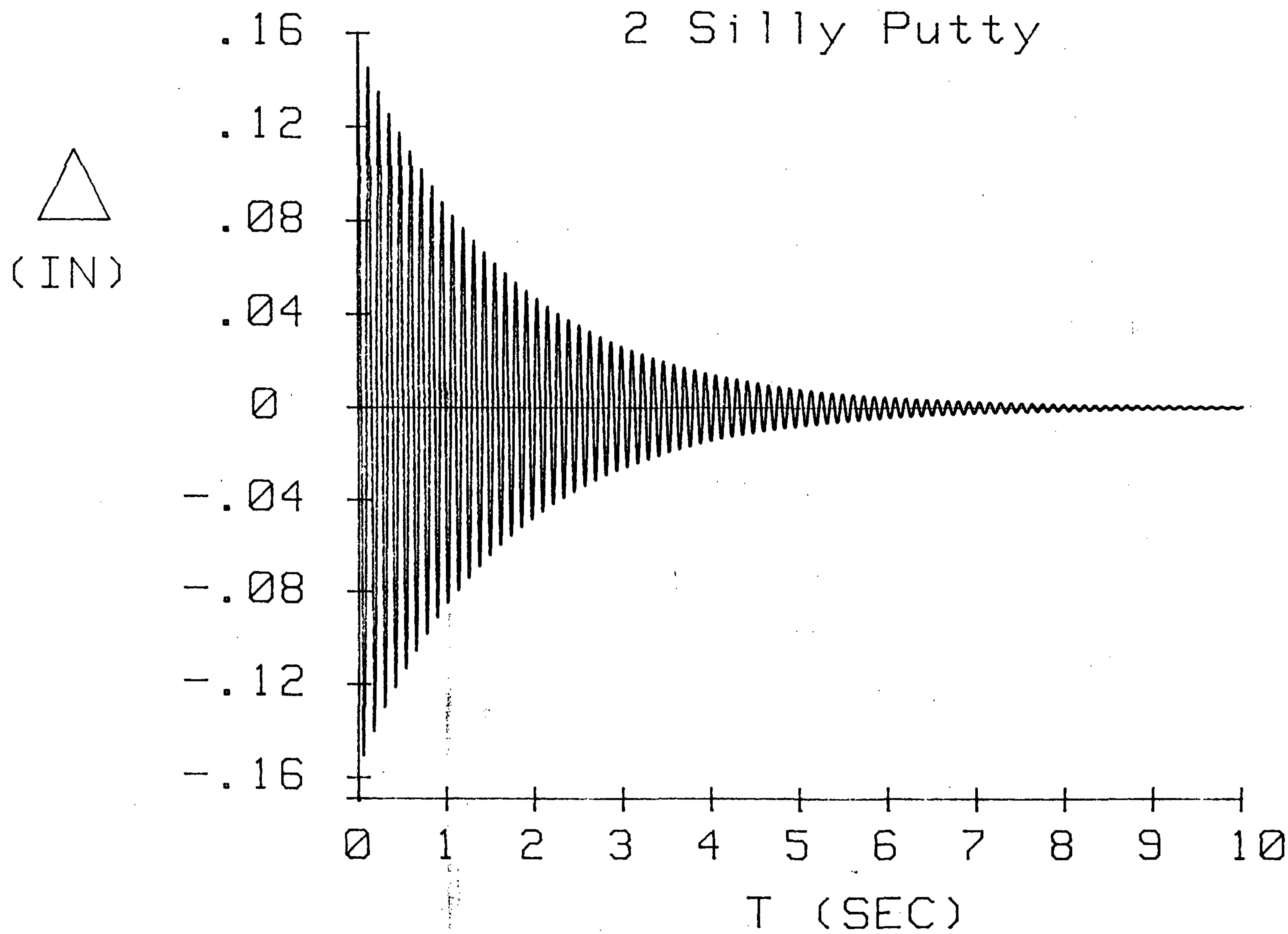


Figure 26. Average Δ-t plot for member with 2 silly putty in chamber dampers.

MOST eff

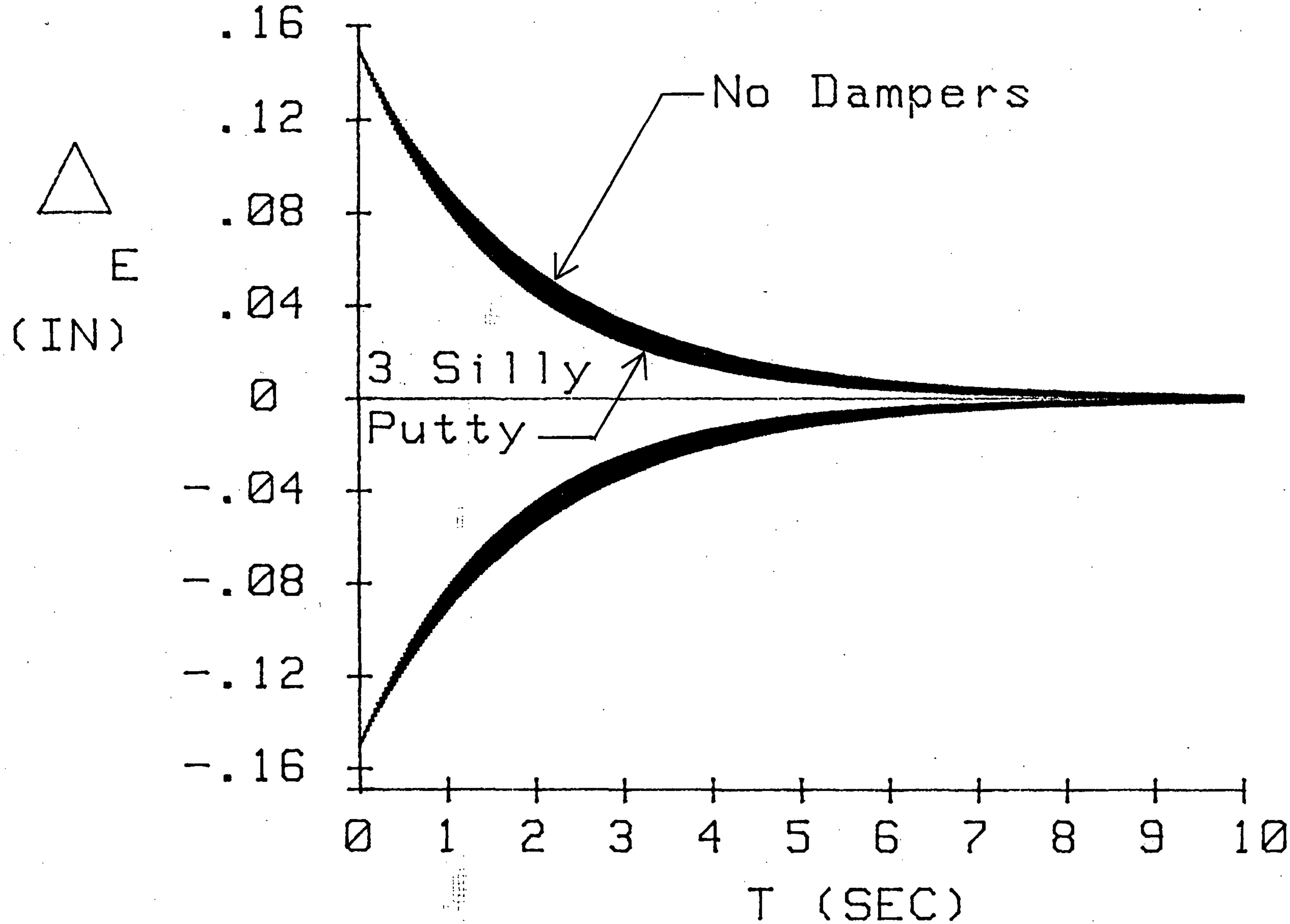


Figure 27. Effect of 3 silly putty in chamber dampers on Δ-t.

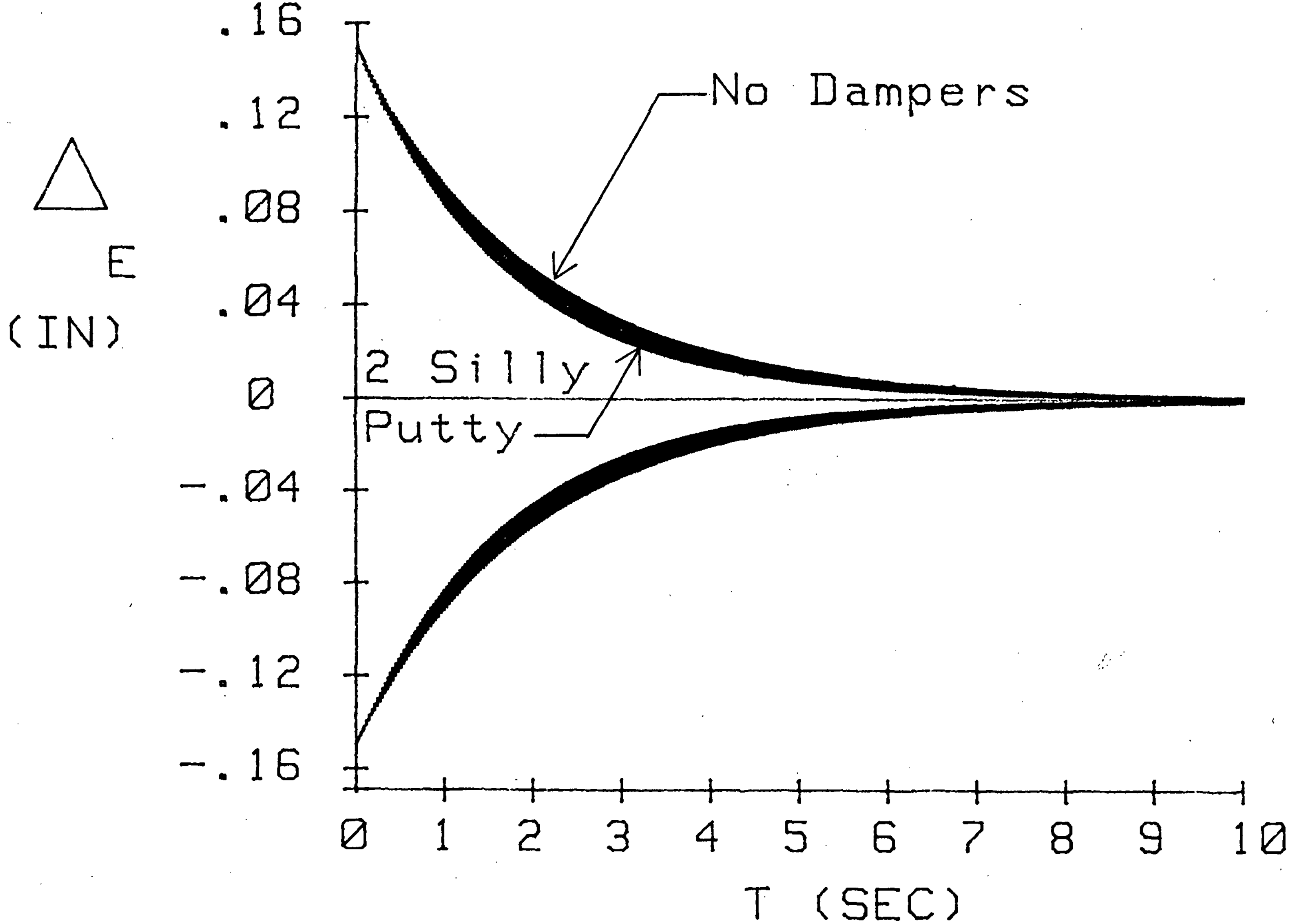


Figure 28. Effect of 2 silly putty in chamber dampers on Δ-t envelope.

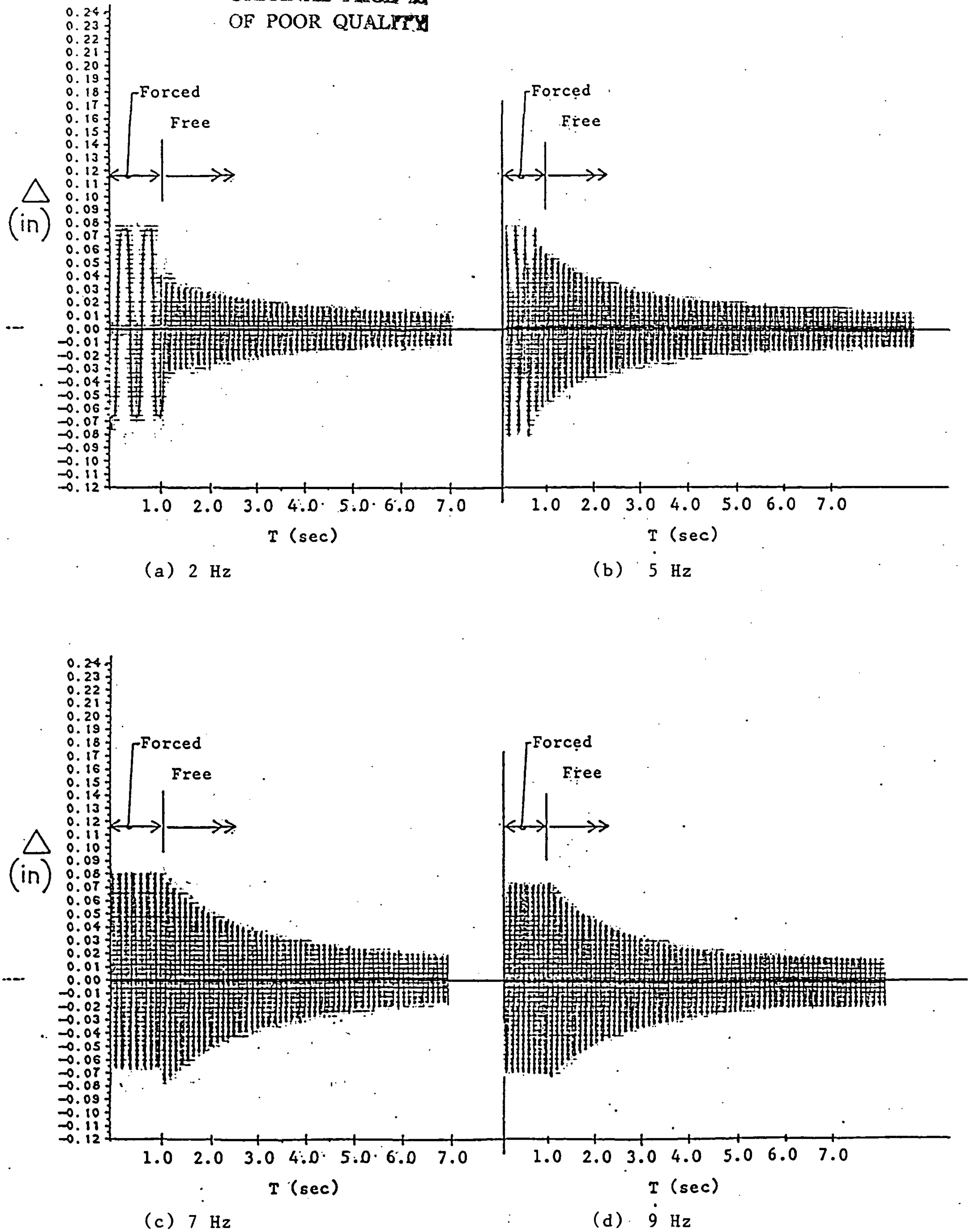


Figure 29. Experimental Δ-t plots for member "constrained" forced then free vibration with no dampers and frequencies of 2, 5, 7 and 9 Hz.

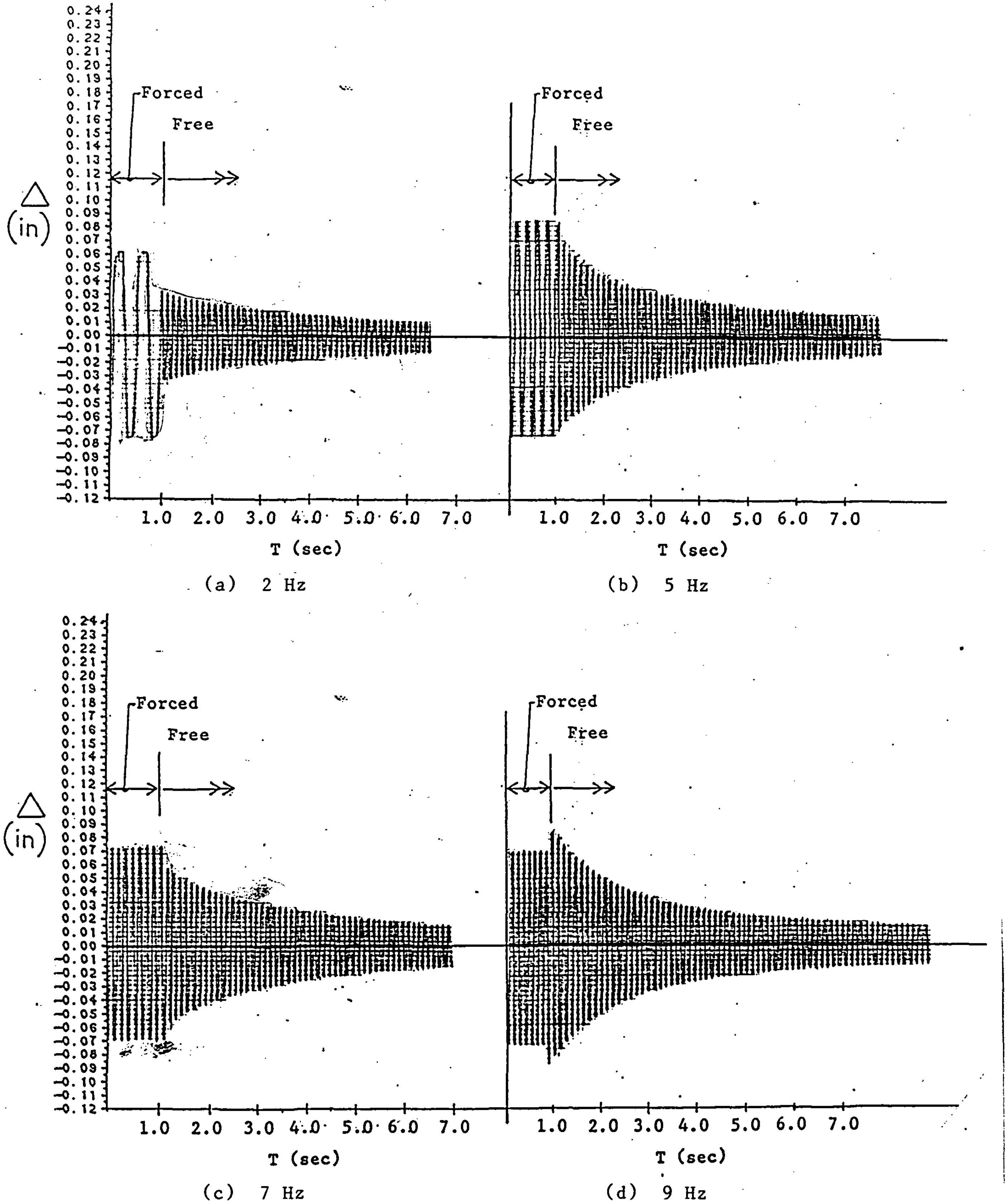


Figure 30. Experimental Δ-t plot for member "constrained" forced then free vibration with 2 silly putty in chamber damper and forcing function frequencies of 2, 5, 7 and 9 Hz.

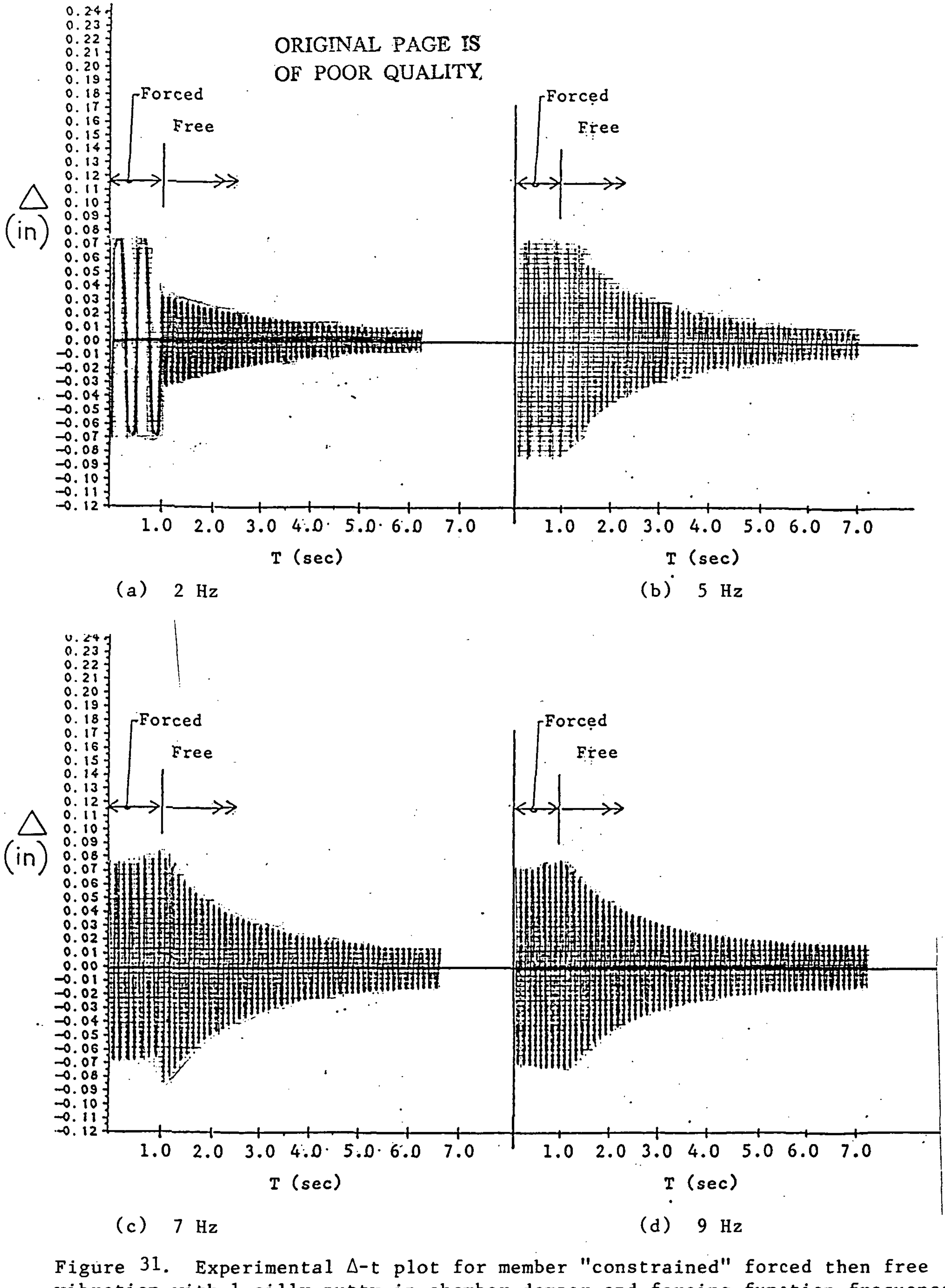


Figure 31. Experimental Δ-t plot for member "constrained" forced then free vibration with 1 silly putty in chamber damper and forcing function frequencies of 2, 5, 7 and 9 Hz.

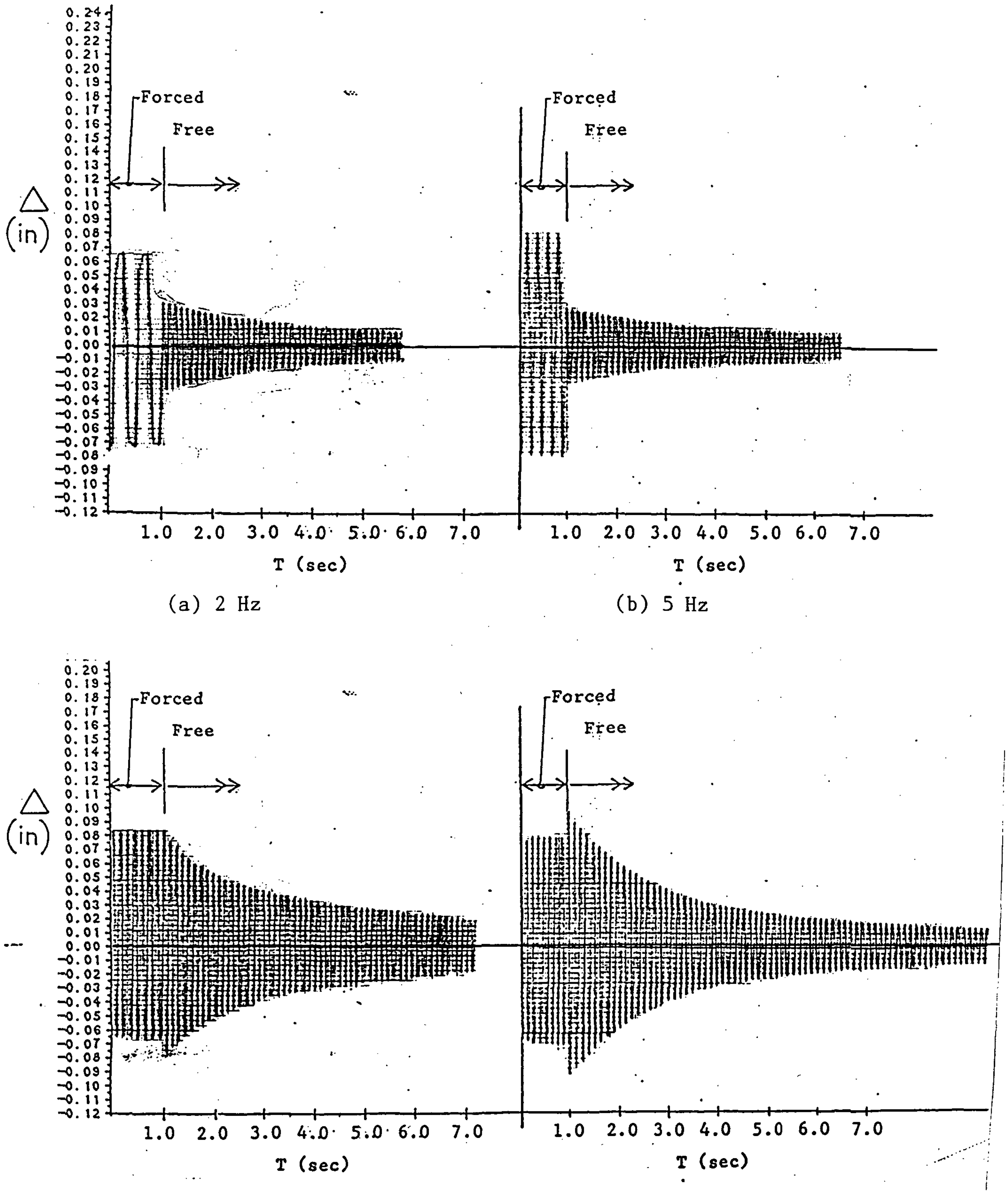


Figure 32. Experimental Δ -t plots for member "constrained" forced then free vibration with 3 silly putty in chamber dampers and forcing function frequencies of 2, 5, 7, and 9 Hz.

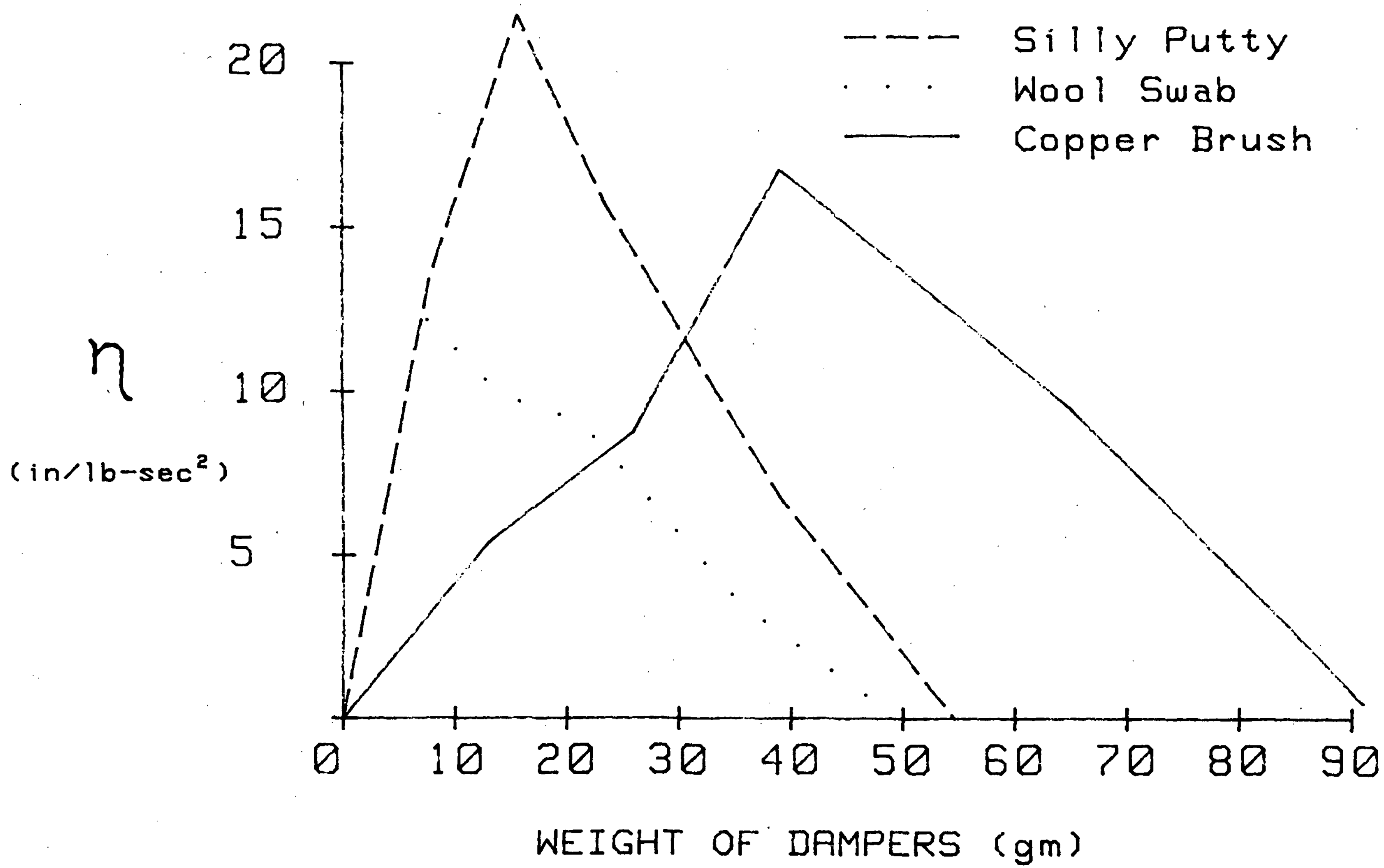


Figure 33. Damping efficiency index versus weight curves for natural vibration tests.

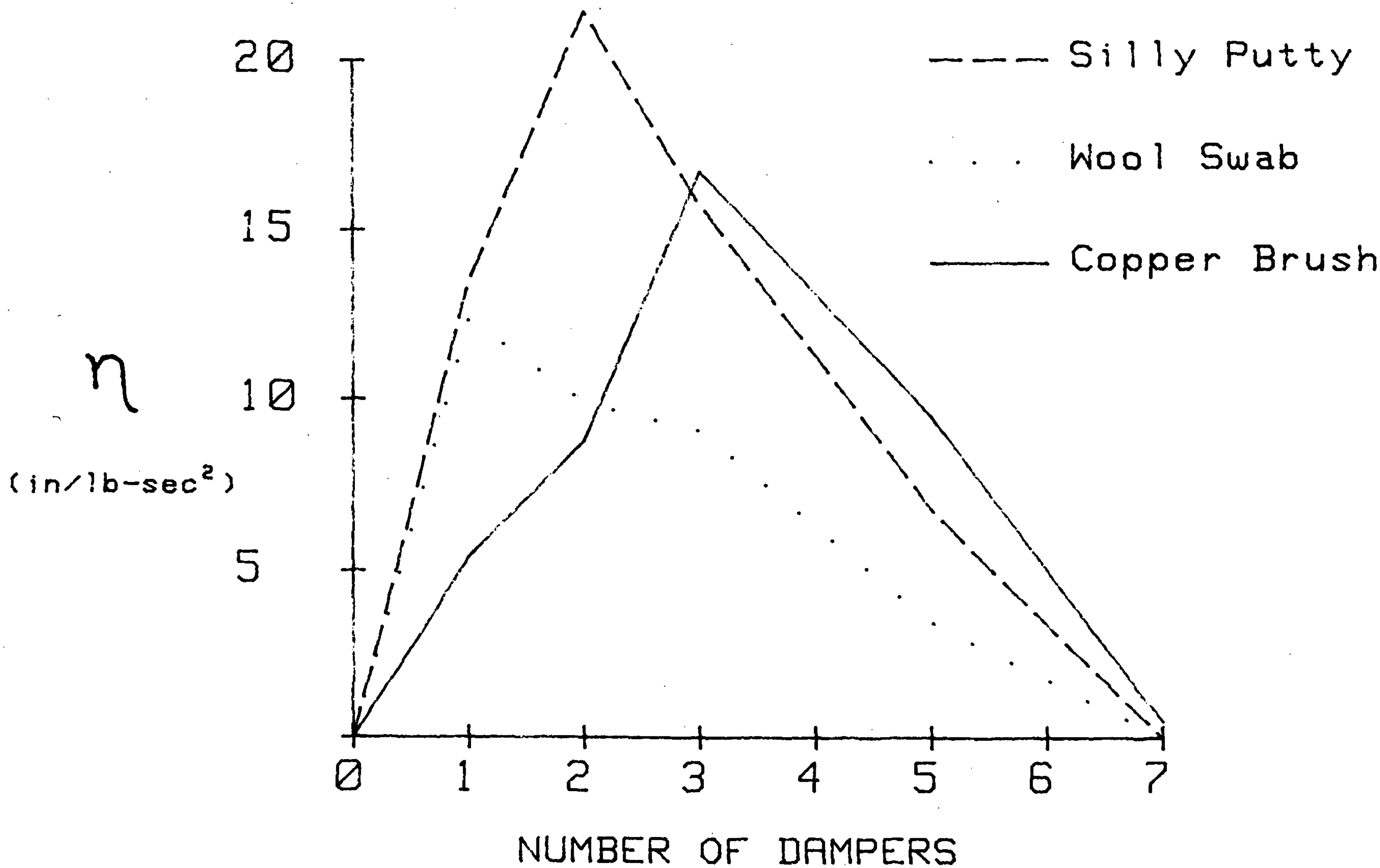


Figure 34. Damping efficiency index versus number of dampers for natural vibration tests.

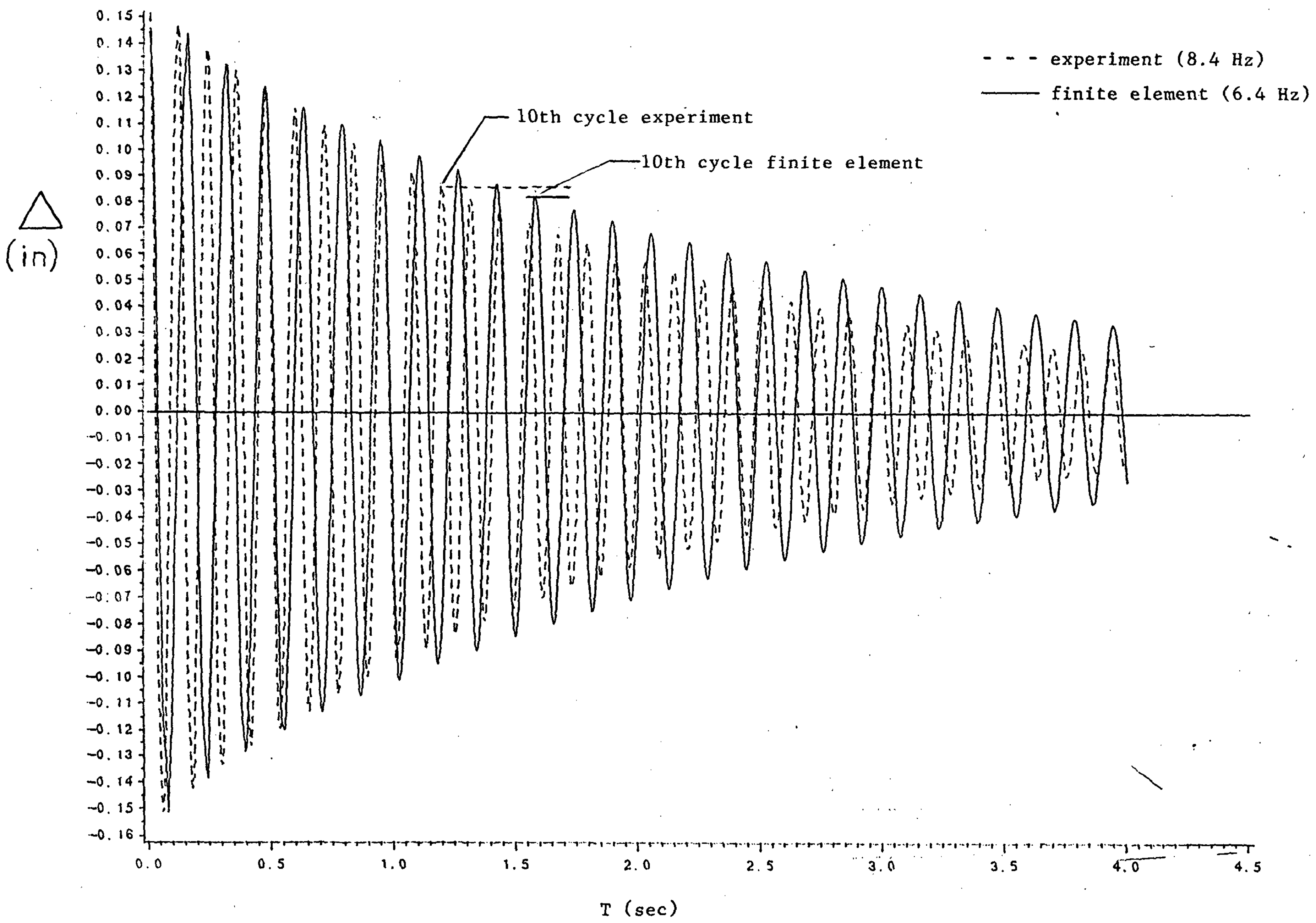


Figure 35. Finite Element versus average experimental Δ-t plots for member with no dampers.

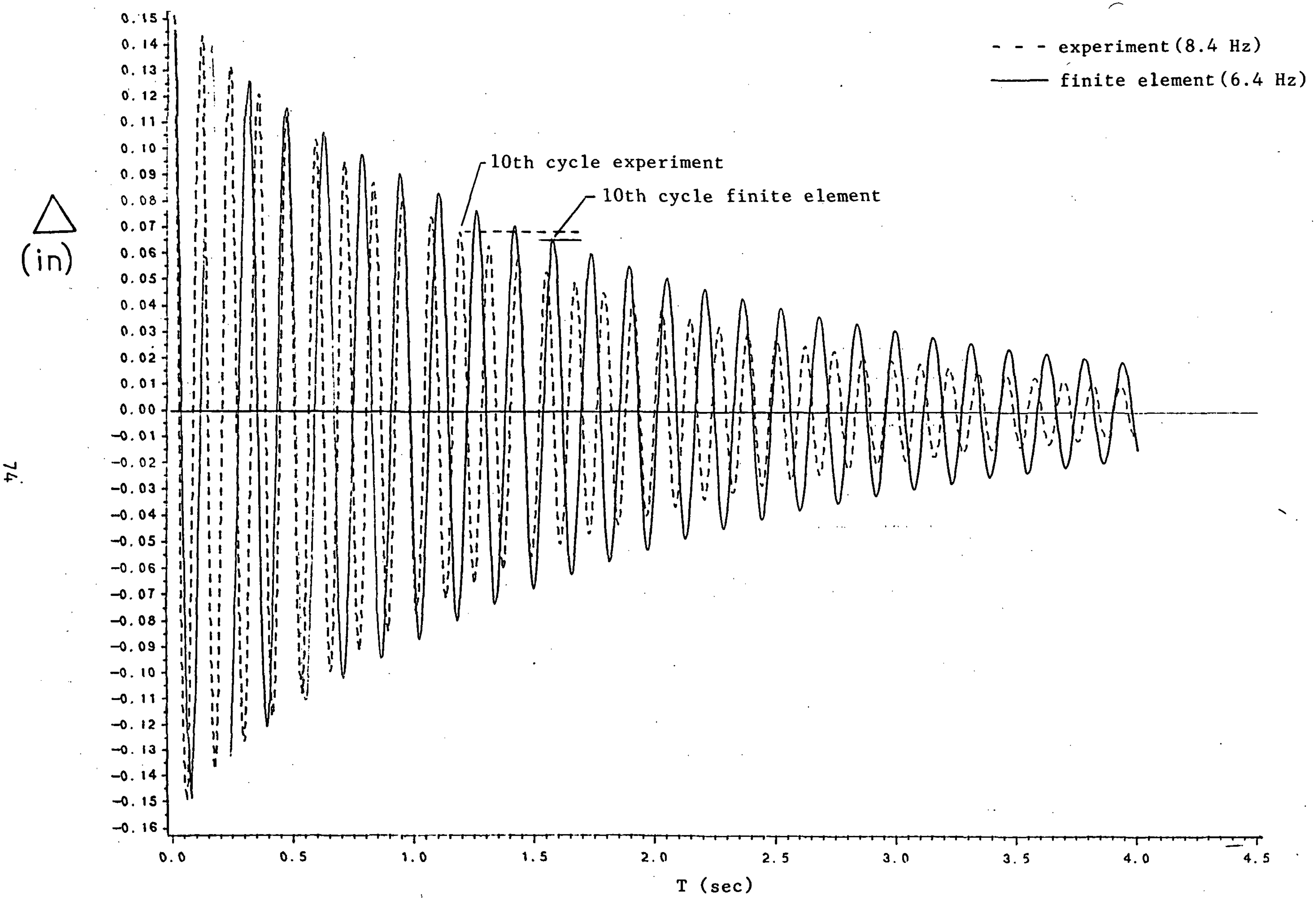


Figure 36. Finite element versus average experimental Δ-t plots for member with 3 copper brush dampers.

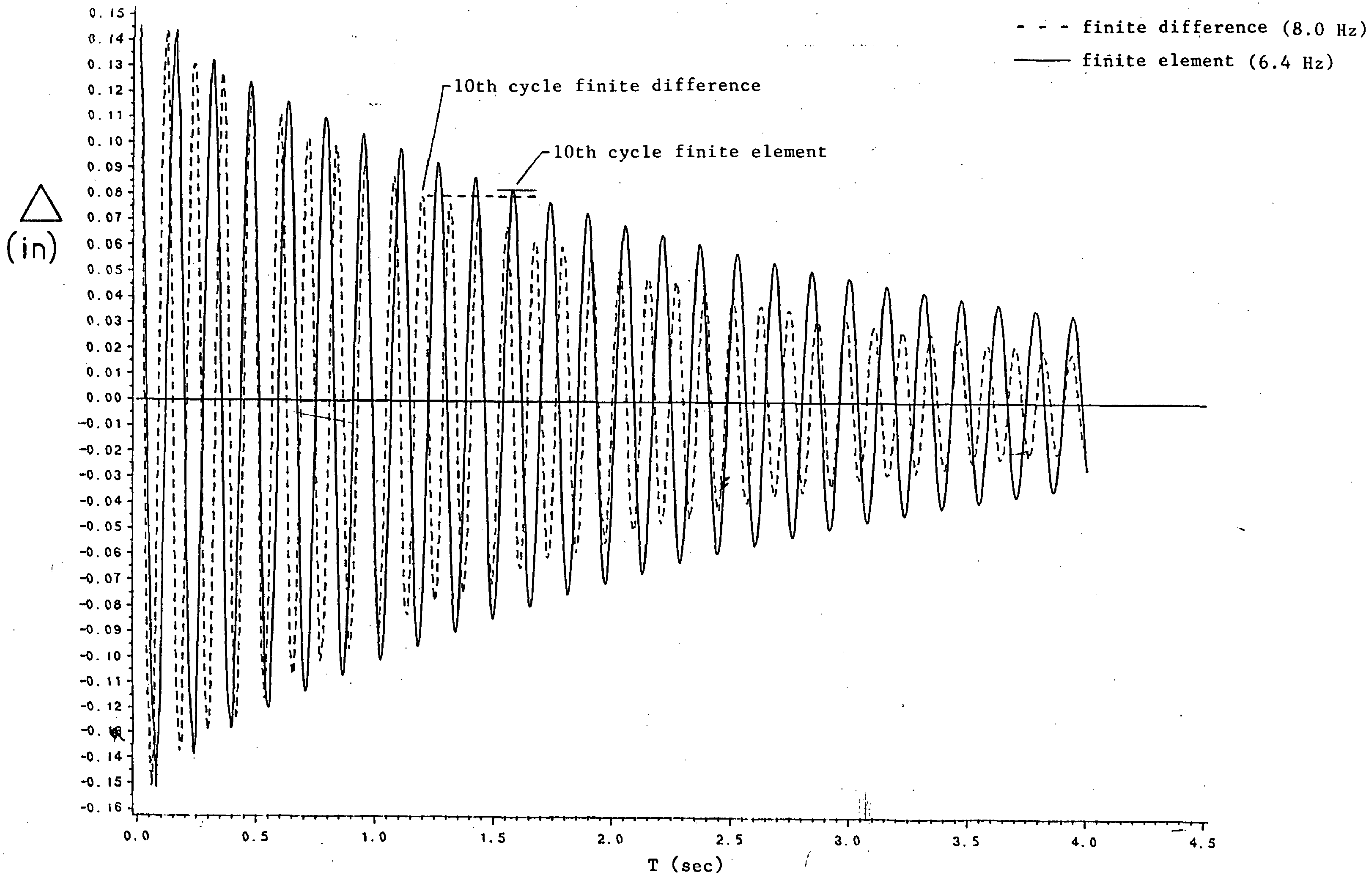


Figure 37. Finite element versus finite difference Δ-t plots for member with no dampers.

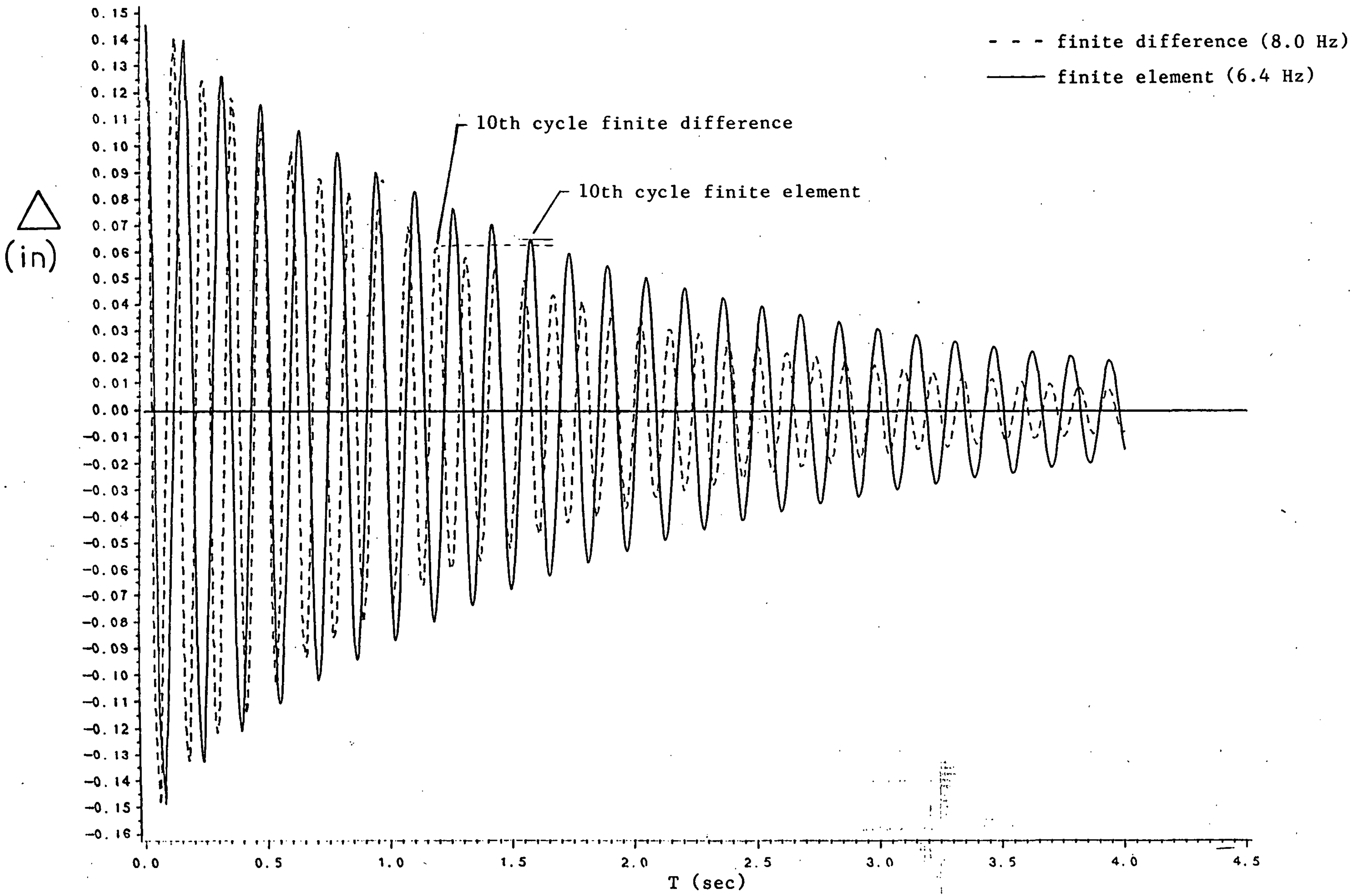


Figure 38. Finite element versus finite difference Δ-t plots for member with 3 copper brush dampers.

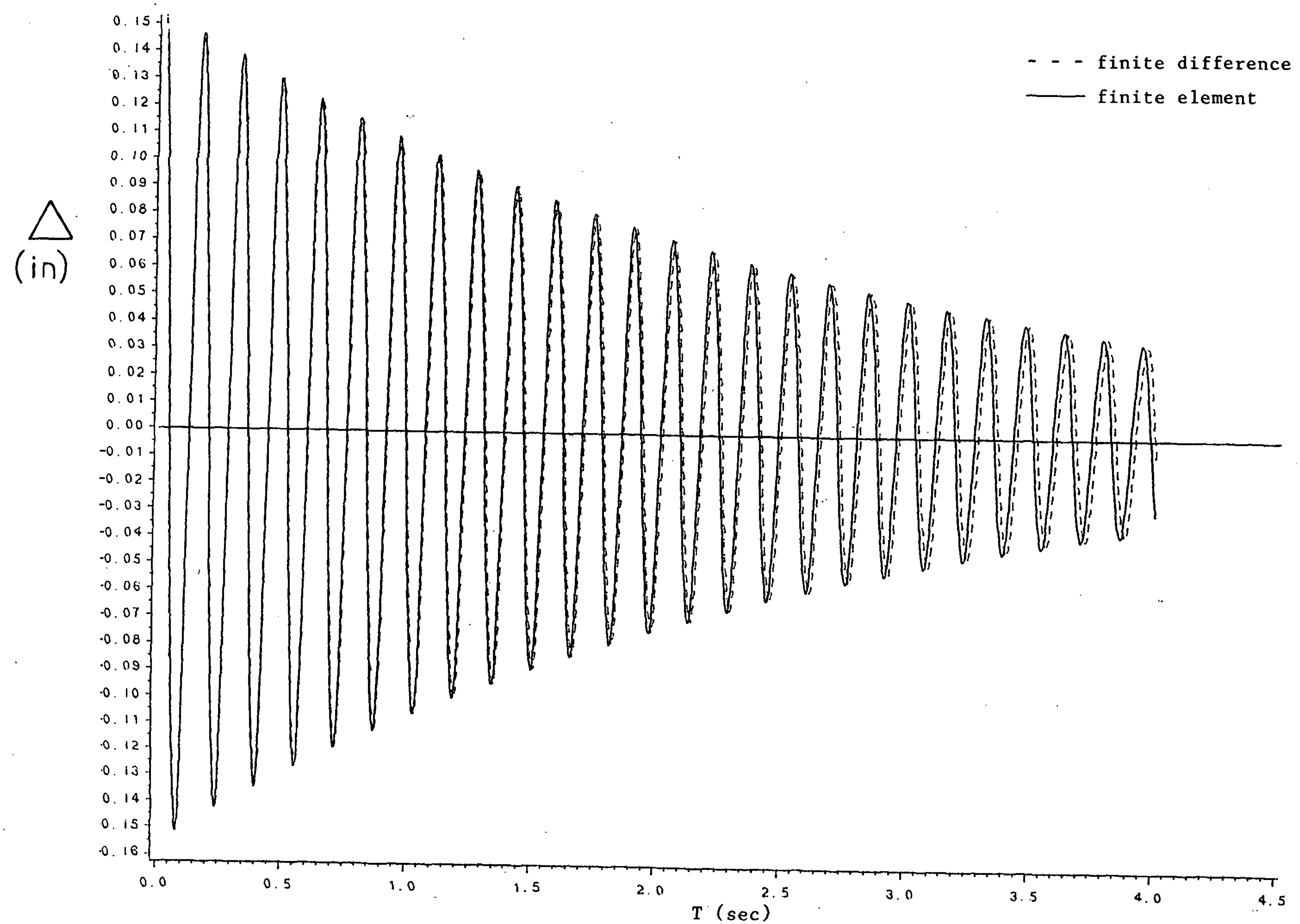


Figure 39. Finite element versus finite difference Δ-t plots for simply supported beam with no dampers.

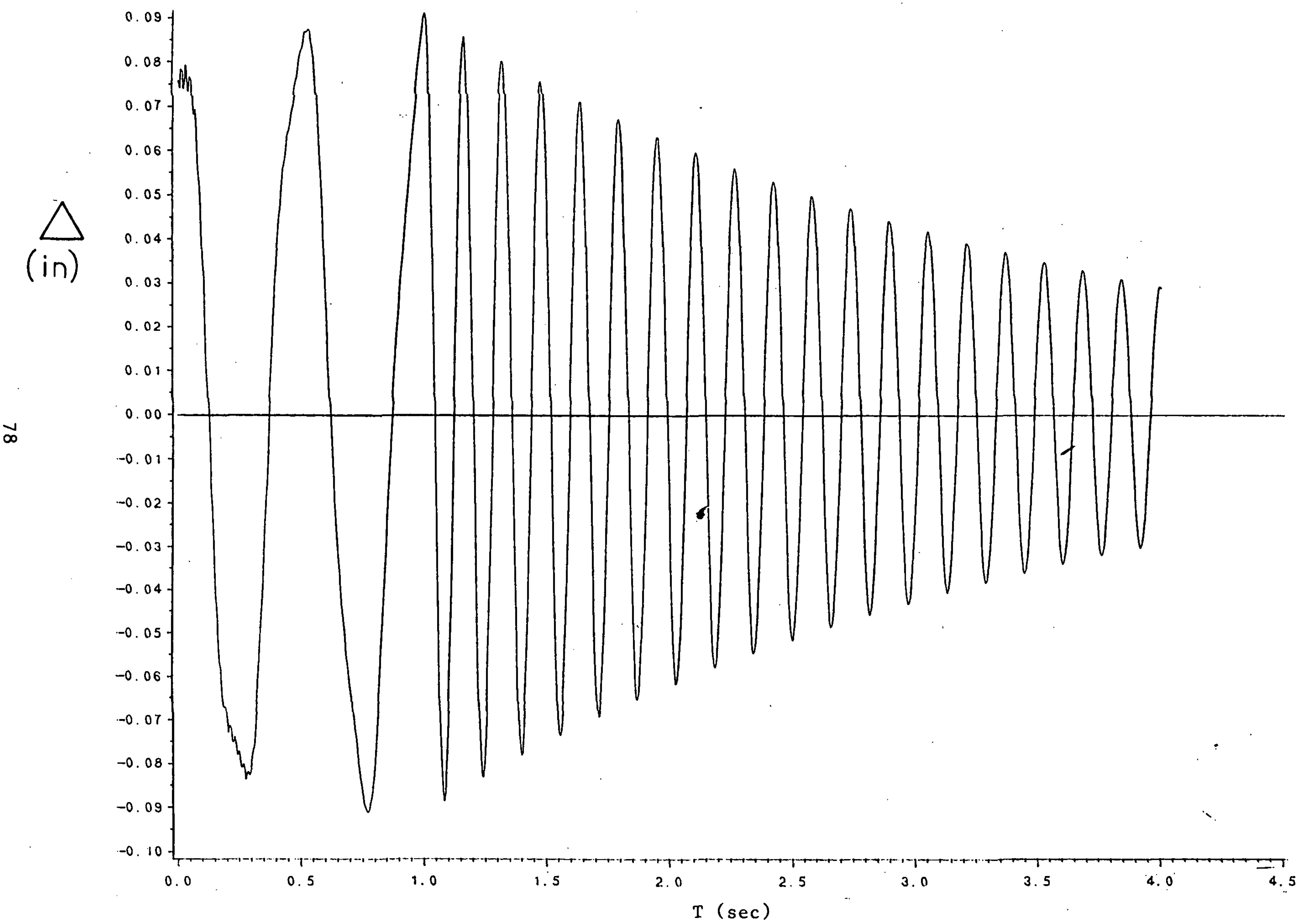


Figure 40. Finite element Δ-t plot for member with no dampers and subjected to a 4.0 lb. force at 2 Hz for 1 second.

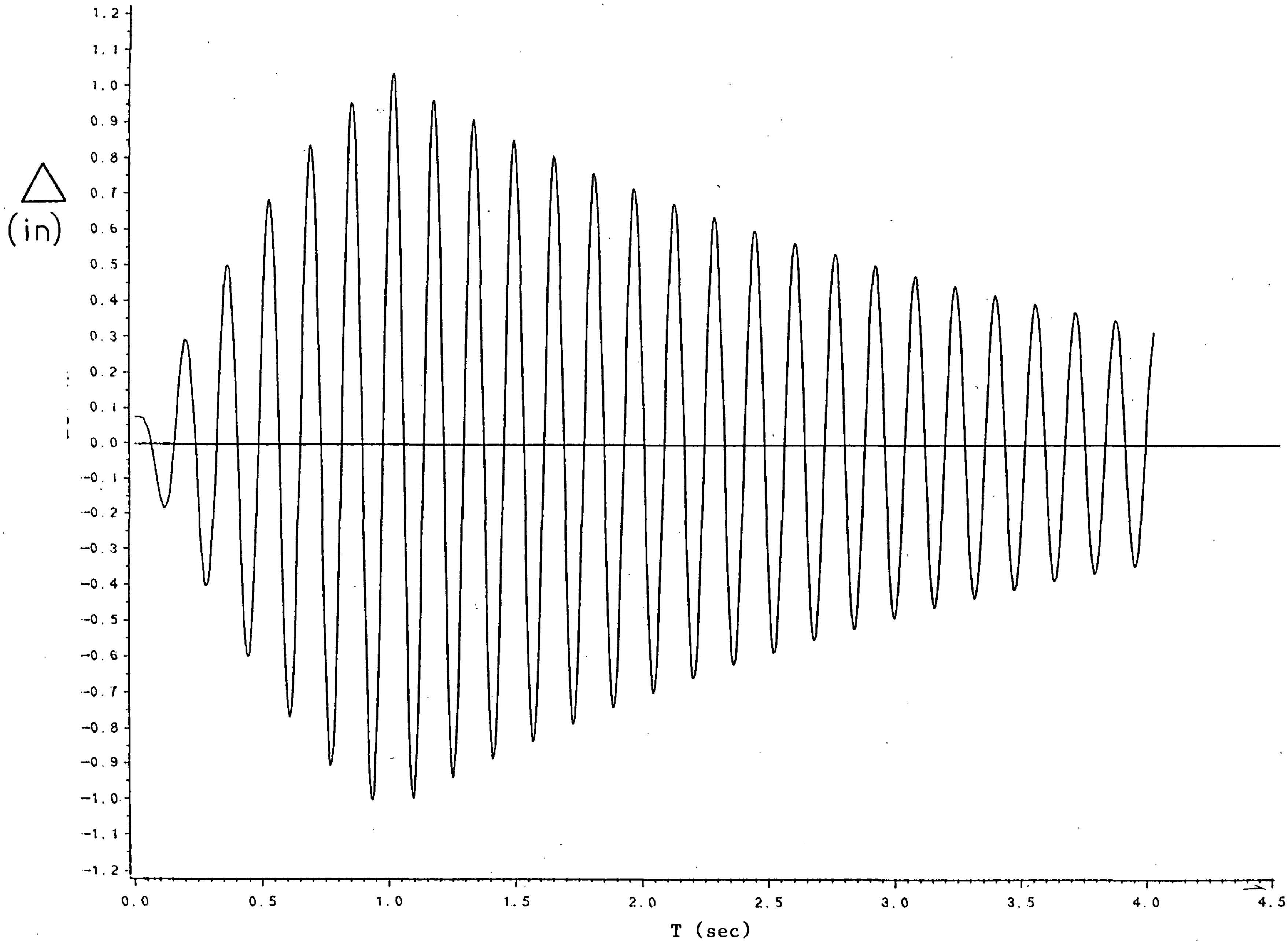
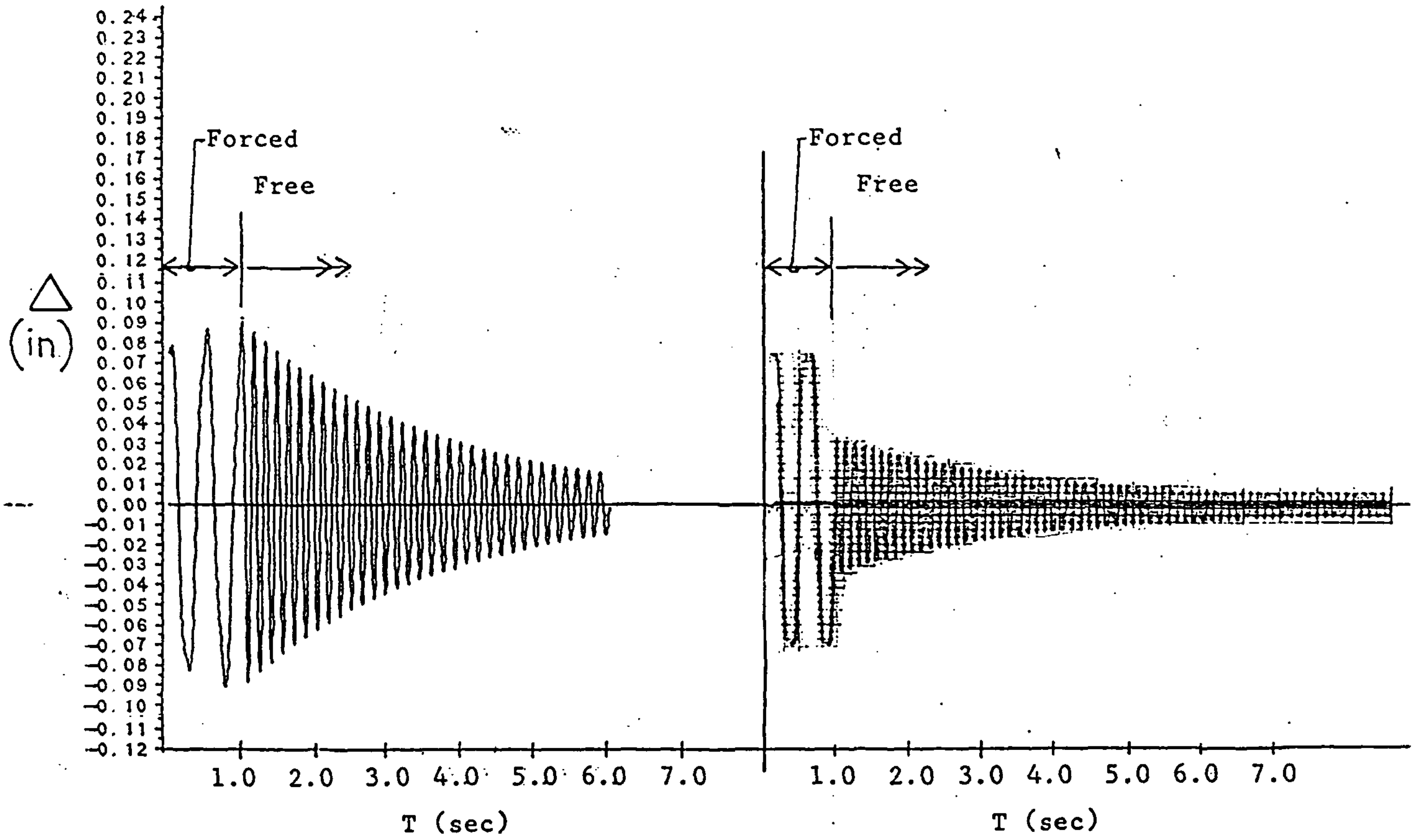


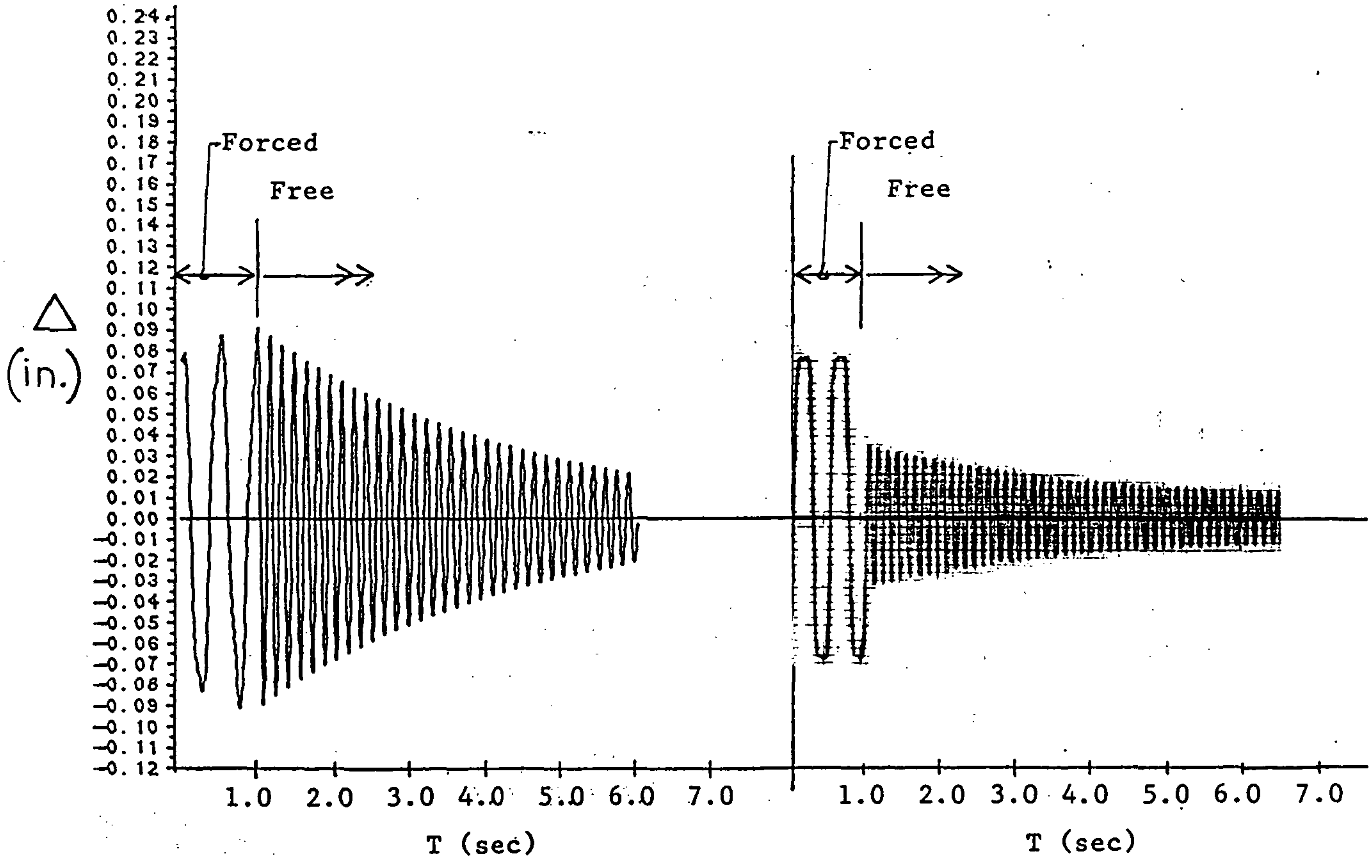
Figure 41. Finite element forced then free Δ-t plot for member with no dampers and subjected to a 4.0 lb. force at 6 Hz for 1 second.



Theoretical curve with ideal disengagement.

Experimental curve with mechanical disengagement.

Figure 42. Theoretical and experimental forced then free Δ-t plots for a 4.0 lb. force at 2 Hz for 1 second on member with one silly putty in chamber damper.



Theoretical curve with ideal disengagement.

Experimental curve with mechanical disengagement.

Figure 43. Theoretical and experimental forced then free Δ-t plots for a 4.0 lb. force at 2 Hz for 1 second on member with no dampers.

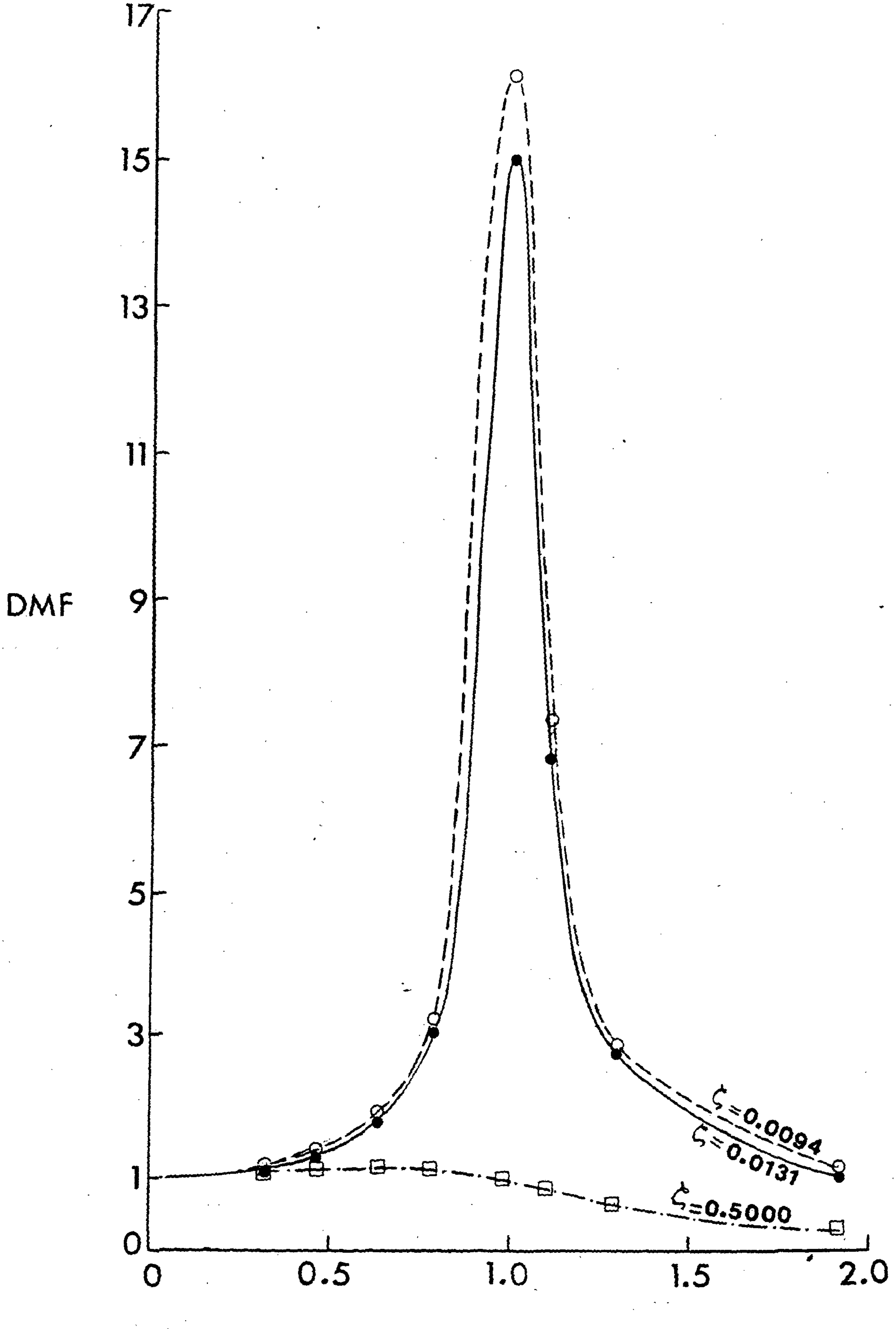


Figure 44. Theoretical dynamic magnification factor versus frequency ratio for damping ratios of 0.0094, 0.0131, and 0.50.

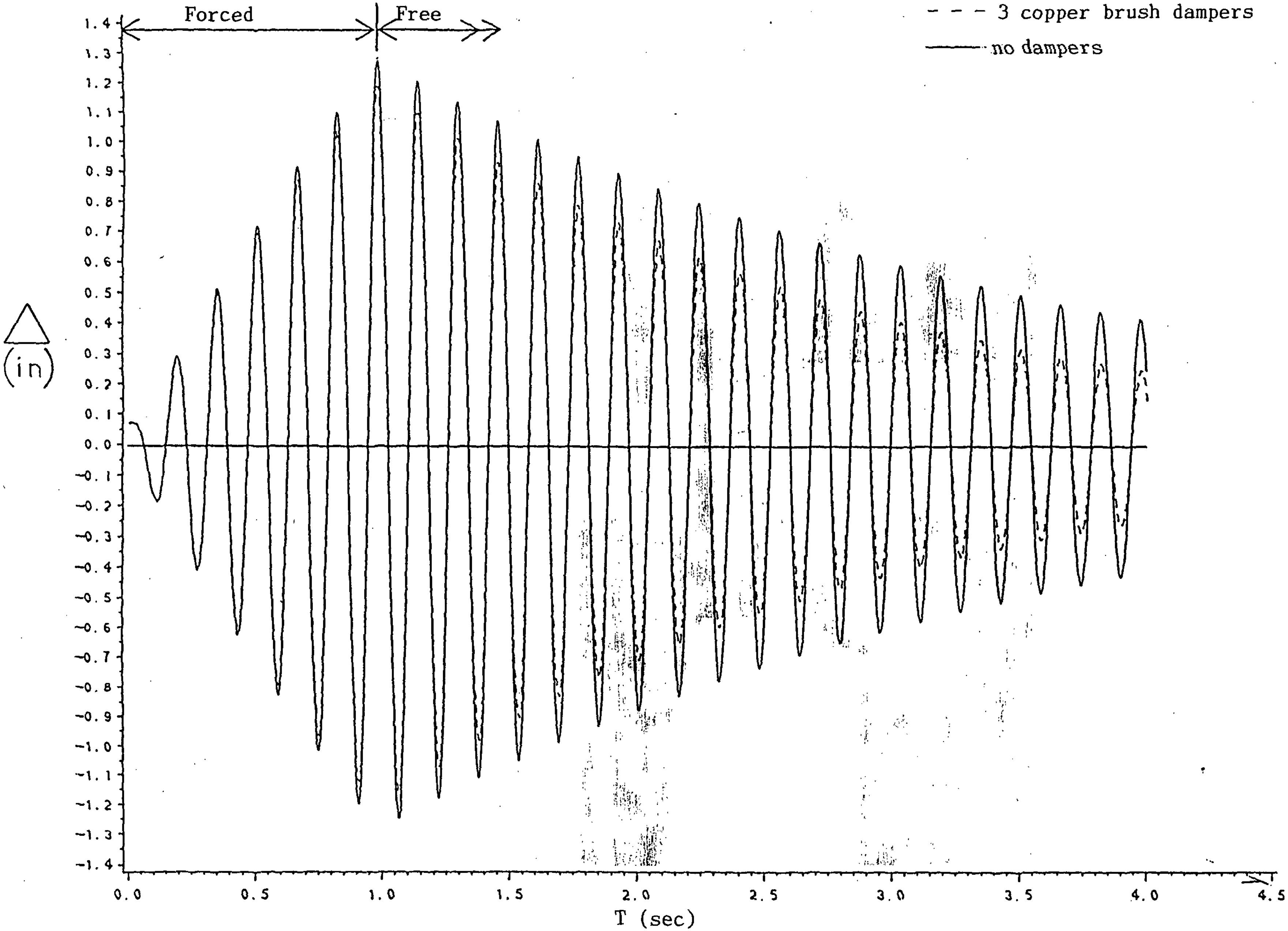


Figure 45. Theoretical Δ-t plots for member with no dampers and with 3 copper brush dampers with a forcing function of 6.35 Hz for one second.

APPENDICES

APPENDIX A

EXAMPLE OF FOUR ELEMENT BEAM STIFFNESS MATRIX

In this appendix the procedure used to assemble the beam stiffness matrix using a beam composed of four elements is presented.

The typical element stiffness matrices for Elements b and c as shown in Figure 2 are given as (Reference 6):

$$[K]_{b,c} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ & \frac{4EI}{L} & \frac{-6EI}{L^2} & \frac{2EI}{L^2} \\ & & \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ \text{Symmetric} & & & \frac{4EI}{L} \end{bmatrix} \qquad (A.1)$$

Since only planar motion is considered, axial effects are negligible and, therefore, not included in the element stiffness matrix.

Derivation of the stiffness matrix for Element a as shown in Figure 2 is as follows. The flexibility matrix for the element is given by:

$$[F] = [H]^t[F]_{c1}[H] + [F]_m + [F]_{c2} \qquad (A.2)$$

in which [H] is the equilibrium matrix given by:

$$[H] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & L \\ 0 & 0 & 1 \end{bmatrix} \qquad (A.3)$$

and $[F]_{c1}$ represents the flexibility of the connection at end one, $[F]_m$ is the flexibility of the element itself and $[F]_{c2}$ is the flexibility of the connection at end two. These are defined as follows:

$$[F]_{c1} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{L}{4EIk} \end{bmatrix} \qquad (A.4)$$

$$[F]_m = \begin{bmatrix} \frac{L^3}{3EI} & \frac{L^2}{2EI} \\ \frac{L^2}{2EI} & \frac{L}{EI} \end{bmatrix} \qquad (A.5)$$

$$[F]_{c2} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{L}{4EI} \end{bmatrix} \qquad (A.6)$$

therefore the flexibility matrix in full can be written as

$$[F] = \begin{bmatrix} \frac{L^3}{EI}\left(\frac{1}{3} + \frac{1}{4k}\right) & \frac{L^2}{EI}\left(\frac{1}{2} + \frac{1}{4k}\right) \\ \frac{L^2}{EI}\left(\frac{1}{2} + \frac{1}{4k}\right) & \frac{L}{EI}\left(1 + \frac{1}{4k}\right) \end{bmatrix} \qquad (A.7)$$

The inverse of [F] is given by:

$$[K_{22}]_a = [F]^{-1} = \begin{bmatrix} \frac{3EI}{L^3}\,\frac{(4k + 1)}{(k + 1)} & \frac{-3EI}{L^2}\,\frac{(2k + 1)}{(k + 1)} \\ \frac{-3EI}{L^2}\,\frac{(2k + 1)}{(k + 1)} & \frac{EI}{L}\,\frac{(4k + 3)}{(k + 1)} \end{bmatrix} \qquad (A.8)$$

The other stiffness matrices now follow from:

$$[K_{11}]_a = [H]\ [K_{22}]\ [H]^t = \begin{bmatrix} \frac{3EI}{L^3}\,\frac{(4k + 1)}{(k + 1)} & \frac{3EI}{L^2}\,\frac{(2k)}{(k + 1)} \\ \frac{3EI}{L^2}\,\frac{(2k)}{(k + 1)} & \frac{EI}{L}\,\frac{(4k)}{(k + 1)} \end{bmatrix} \qquad (A.9)$$

$$[K_{12}]_a = [K_{21}]^t = -[H]\ [K_{22}] = \begin{bmatrix} \frac{-3EI}{L^3}\frac{(4k+1)}{(k+1)} & \frac{3EI}{L^2}\frac{(2k+1)}{(k+1)} \\ \frac{-3EI}{L^2}\frac{(2k)}{(k+1)} & \frac{EI}{L}\frac{(2k)}{(k+1)} \end{bmatrix} \qquad \text{(A.10)}$$

Therefore, the total stiffness matrix for Element a is:

$$[K]_a = \begin{bmatrix} \frac{3EI}{L^3}\frac{(4k+1)}{(k+1)} & \frac{3EI}{L^2}\frac{(2k)}{(k+1)} & \frac{-3EI}{L^3}\frac{(4k+1)}{(k+1)} & \frac{3EI}{L^2}\frac{(2k+1)}{(k+1)} \\ & \frac{EI}{L}\frac{(4k)}{(k+1)} & \frac{-3EI}{L^2}\frac{(2k)}{(k+1)} & \frac{EI}{L}\frac{(2k)}{(k+1)} \\ & \text{Symmetric} & \frac{3EI}{L^3}\frac{(4k+1)}{(k+1)} & \frac{-3EI}{L^2}\frac{(2k+1)}{(k+1)} \\ & & & \frac{EI}{L}\frac{(4k+3)}{(k+1)} \end{bmatrix} \qquad \text{(A.11)}$$

Similarly, Element d shown in Figure 2:

$$[K_{22}]_d = \begin{bmatrix} \frac{3EI}{L^3}\frac{(4k+1)}{(1+k)} & \frac{-6EI}{L^2}\frac{(k)}{(1+k)} \\ \frac{-6EI}{L^2}\frac{(k)}{(1+k)} & \frac{4EI}{L}\frac{(k)}{(1+k)} \end{bmatrix} \qquad \text{(A.12)}$$

$$[K_{11}]_d = [H]\ [K_{22}]_d\ [H^t] = \begin{bmatrix} \frac{3EI}{L^3}\frac{(4k+1)}{(1+k)} & \frac{3EI}{L^2}\frac{(2k+1)}{(1+k)} \\ \frac{3EI}{L^2}\frac{(2k+1)}{(1+k)} & \frac{EI}{L}\frac{(4k+3)}{(1+k)} \end{bmatrix} \qquad \text{(A.13)}$$

C-2

$$[K_{12}]_d = [K_{21}]^t{}_d = -[H]\,[K] = \begin{bmatrix} \frac{3EI}{L^3}\frac{(4k+1)}{(1+k)} & \frac{6EI}{L^2}\frac{k}{(1+k)} \\ \frac{-3EI}{L^2}\frac{(2k+1)}{(1+k)} & \frac{2EI}{L}\frac{(k)}{(k+1)} \end{bmatrix} \tag{A.14}$$

$$[K]_d = \begin{bmatrix} \frac{3EI}{L^3}\frac{(4k+1)}{(1+k)} & \frac{3EI}{L^2}\frac{(2k+1)}{(1+k)} & \frac{-3EI}{L^3}\frac{(4k+1)}{(k+1)} & \frac{6EI}{L^2}\frac{(k)}{(k+1)} \\ & \frac{EI}{L}\frac{(4k+3)}{(1+k)} & \frac{-3EI}{L^2}\frac{(2k+1)}{(1+k)} & \frac{2EI}{L}\frac{(k)}{(1+k)} \\ & & \frac{3EI}{L^3}\frac{(4+1)}{(1+k)} & \frac{-6EI}{L^2}\frac{(k)}{(1+k)} \\ \text{Symmetric} & & & \frac{4EI}{L}\frac{(k)}{(1+k)} \end{bmatrix} \tag{A.15}$$

Using the above element matrices, the following global matrix is assembled:

$$[K] = \begin{bmatrix} [K_{11}]_a & [K_{22}]_a & [0] & [0] & [0] \\ [K_{21}]_a & [K_{22}]_a+[K_{11}]_b & [K_{12}]_b & [0] & [0] \\ [0] & [K_{21}]_b & [K_{22}]_b+[K_{11}]_c & [K_{12}]_c & [0] \\ [0] & [0] & [K_{21}]_c & [K_{22}]_c+[K_{11}]_d & [K_{12}]_d \\ [0] & [0] & [0] & [K_{21}]_d & [K_{22}]_d \end{bmatrix} \tag{A.16}$$

This is an n x n matrix where n = 2N + 2, and N = the number of elements. The first two boundary conditions are enforced by putting 1.0 in the diagonal corresponding to the translational degrees of freedom at the supports and setting all other entries in that row and column equal to zero. The last two boundary conditions are accounted for in the derivation of the individual stiffness matrices.

Note that an adjustment to the stiffness matrix must be made when the stiffness of the rotational restraint at each end approaches zero. For this case the diagonal term corresponding to the rotational stiffness at the support should be set equal to unity.

APPENDIX B

COMPUTER PROGRAMS

As a part of this study, two computer programs were developed to solve the dynamic equilibrium matrix equation given in Chapter 2. A brief description of these programs along with their listings and sample outputs are given in this appendix.

B.1 NEWMARK

This program is based on the analysis described in Section 2.2. A description of the required input data is given at the beginning of the program listing. Data is input by means of the data statements in lines 48 to 53 of the program listing. The output consists of the time in seconds and corresponding midspan deflection in inches.

B.2 CENDIF

Program CENDIF is based on the analysis described in Section 2.3. Data input and output are the same on NEWMARK.

```
      IMPLICIT REAL*8(A-H,O-Z)

      DOUBLE PRECISION L,K1,K2,K(70,70),M(70,70),C(70,70),U(70),UDT(70),
     &RT(70),C1(70,70),C2(70,70),C9(70,70),C4(70),KEL(4,4),F(70),C5(70),
     &DUM(70),UT(70),MINV(70,70),C8(70,70),UDTP(70),UDDT(70),UDDTP(70),U
     &TP(70),UTN(70),C3(70),C6(70),FREQ(5),KINV(70,70),X(70,70),A(70,70)
     &,B(70,70),D(70),EIGV(70),DAMRAT(70),ANS(70,70),RES(70,70),XT(70,70
     &)
      IFPR=0
C********************  INPUT  DATA  ******************************
C**                                                             **
C**  IFPR = PRINT REQUEST VARIABLE FOR JACOBI                   **
C**         0 = DO NOT PRINT INTERMEDIATE VALUES                **
C**         1 = PRINT INTERMEDIATE VALUES                       **
C**                                                             **
C**  L = LENGTH  (IN)                                           **
C**                                                             **
C**  NUMEL = NUMBER OF ELEMENTS (MUST BE AN EVEN NUMBER)        **
C**                                                             **
C**  TS = TIME STEP; DELTA 'T'  (SEC)                           **
C**                                                             **
C**  ROW = MASS PER UNIT LENGTH  (KIP*SEC**2/IN**2)             **
C**                                                             **
C**  E = MODULUS OF ELASTICITY  (KSI)                           **
C**                                                             **
C**  XI = MOMENT OF INERTIA  (IN**4)                            **
C**                                                             **
C**  AR = AREA  (IN**2)                                         **
C**                                                             **
C**  K1 = ROTATIONAL STIFFNESS AT END 1  (K*IN/RAD)             **
C**                                                             **
C**  K2 = ROTATIONAL STIFFNESS AT END 2  (K*IN/RAD)             **
C**                                                             **
C**  ZETA = DAMPING RATIO                                       **
C**                                                             **
C**  TT = TOTAL TIME FOR PROGRAM EXECUTION  (SEC)               **
C**                                                             **
C**  PO = MAGNITUDE OF THE FORCING FUNCTION  (KIPS)             **
C**                                                             **
C**  OMEGA = FREQUENCY OF THE FORCING FUNCTION (RAD)            **
C**                                                             **
C**  DELO = PRESCRIBED INITIAL DEFLECTION AT MEMBER MIDSPAN    **
C**                                                      (IN)   **
C**                                                             **
C*****************************************************************


      DATA L,NUMEL,TS,ROW/ 177,10,0.000500,202.1454E-09/
      DATA E,XI,AR/10000.,0.32500000,0.7363/
      DATA K1,K2,TT/53.1100,53.1100,6.00/
      DATA PO,OMEGA,ZETA/0.002226367,12.56637062,0.007200/
      DATA GAMA,BETA,DELO/0.50,0.25,0.007914/

      ICOUNT=0
```

```
      TIME=0.0
        WRITE(2,1)
      H=L/NUMEL
 1    FORMAT(/1X,'THIS IS NEWMARKS SOLUTION')
      WRITE(2,1059)K1
 1059 FORMAT(/1X,'STIFFNESS -- ',D16.9)
      WRITE(2,1060)ZETA
 1060 FORMAT(/1X,'DAMPING ---- ',D16.9)
      WRITE(2,1061)PO
 1061 FORMAT(/1X,'FORCE ------ ',D16.9)
      WRITE(2,1062)OMEGA
 1062 FORMAT(/1X,'FREQUENCY -- ',D16.9)

      N=2*NUMEL+2

C     WRITE(2,176)
C     WRITE(2,177)

      DO 10 I=1,N
      DO 10 J=1,N
 10   K(I,J)=0.0


      K(1,1)=1000
      K(N-1,N-1)=1000


      K(2,2)=E*XI*4.*K1/(H*(K1+1.))
      K(2,3)=(-1.)*3.*E*XI*2.*K1/((H**2)*(K1+1.))
      K(2,4)=E*XI*2.*K1/(H*(K1+1.))
      K(3,2)=K(2,3)
      K(3,3)=3.*E*XI*(4.*K1+1.)/((H**3)*(K1+1.))
      K(3,4)=(-3.)*E*XI*(2.*K1+1.)/((H**2)*(K1+1.))
      K(4,2)=K(2,4)
      K(4,3)=K(3,4)
      K(4,4)=E*XI*(4.*K1+3.)/(H*(K1+1.))


      K(N-3,N-3)=3.*E*XI*(4.*K2+1.)/((H**3)*(K2+1.))
      K(N-3,N-2)=(3.)*E*XI*(2.*K2+1.)/((H**2)*(K2+1.))
      K(N-3,N)=6.*E*XI*K2/((H**2)*(K2+1.))
      K(N-2,N-3)=K(N-3,N-2)
      K(N-2,N-2)=E*XI*(4.*K2+3.)/(H*(K2+1.))
      K(N-2,N)=2.*E*XI*K2/(H*(K2+1.))
      K(N,N-3)=K(N-3,N)
      K(N,N-2)=K(N-2,N)
      K(N,N)=E*XI*4.*K2/(H*(K2+1.))



      KEL(1,1)  =  12.*E*XI/(H**3)
      KEL(1,2)  =  6.*E*XI/(H**2)
      KEL(1,3)  =  (-1.)*KEL(1,1)
      KEL(1,4)  =  KEL(1,2)
```

```
      KEL(2,2)  =  4.*E*XI/(H)
      KEL(2,3)  =  (-1.)*KEL(1,2)
      KEL(2,4)  =  KEL(2,2)/2.
      KEL(3,3)  =  KEL(1,1)
      KEL(3,4)  =  KEL(2,3)
      KEL(4,4)  =  KEL(2,2)

      IF(K(2,2).LE.0.00001)  K(2,2)=1.0
      IF(K(N,N).LE.0.00001)  K(N,N)=1.0
      DO 30 I=1,4
      DO 30 J=1,4
 30   IF(J.GT.I)KEL(J,I)=KEL(I,J)

      DO 50 JK =2,NUMEL-1
      II=JK*2-2
      JJ=II
      DO 45 I=1,4
      DO 40 J=1,4

      K(II+I,JJ+J)  =  K(II+I,JJ+J)+KEL(I,J)

 40   CONTINUE
 45   CONTINUE
 50   CONTINUE

 66   DO 75 I=1,N
      DO 70 J=1,N
      M(I,J)=0.0
      C(I,J)=0.0
 70   CONTINUE
 75   CONTINUE

      M(1,1)=39.
      M(2,2)=H**2
      M(N-1,N-1)=39.
      M(N,N)=H**2

      DO 80 I=3,N-3,2
      J=I+1
      M(I,I)=78.
      M(J,J)=2.*(H**2)
 80   CONTINUE

      DO 90 I=1,N
 90   M(I,I)=M(I,I)*(ROW*H/78.)
      CALL JACOBI(K,M,N,IFPR,X,EIGV)
      DO 95 I=1,N
 95   DAMRAT(I)=ZETA

      CALL DAMP(N,EIGV,X,M,DAMRAT,C)


      PRINT*,'IN START'

      DO 300 I=1,N
```

```
 300  RT(I)=0.0
      PI=ACOS(-1.0)
      RT(N/2)=PO*(DCOS(OMEGA*TIME))

      CALL INVERT(M,MINV,N)

      DO 333 I=1,NUMEL+1
      DUM1=K1*L/(4*PI*E*XI)
      Z=PI*H*(I-1)/L
      UT(2*I)=(DELO/(1.+DUM1*2))*((PI/L*DCOS(Z))+(DUM1*2*PI/L*DSIN(2*Z)
     &))
      UT(2*I-1)=(DELO/(1.+DUM1*2))*(DSIN(Z)+DUM1*(1.-DCOS(2*Z)))
 333   CONTINUE
      DO 302 I=1,N
      SUM=0.0
      DO 301 J=1,N
 301  SUM=SUM+K(I,J)*UT(J)*(-1.)
 302  DUM(I)=SUM

      DO 306 I=1,N
      SUM=0.0
      DO 305 J=1,N
 305  SUM=SUM+MINV(I,J)*DUM(J)
 306  UDDT(I)=SUM

      DO 310 I=1,N
      UDT(I)=0.0
 310  CONTINUE

      PRINT*,'OUT START'

      DO 320 I=1,N
      DO 320 J=1,N
 320  C1(I,J)= K(I,J)+GAMA*C(I,J)/(BETA*TS)+M(I,J)/(BETA*(TS**2))
      CALL INVERT(C1,C2,N)
 120  DO 122 I=1,N
      C3(I)=(GAMA/(BETA*TS))*UT(I)+(GAMA/BETA-1.0)*UDT(I)+TS*((GAMA/(BE
     &TA*2.))-1.0)*UDDT(I)

      C4(I)=UT(I)/(BETA*(TS**2))+UDT(I)/(BETA*TS)+((1./(2.*BETA))-1.0)*U
     &DDT(I)
 122  CONTINUE

      DO 130 I=1,N
      SUM=0.0
      DO 125 J=1,N
 125  SUM=SUM+C(I,J)*C3(J)
 130  C5(I)=SUM

      DO 410 I=1,N
      SUM=0.0
      DO 400 J=1,N
 400  SUM=SUM+M(I,J)*C4(J)
 410  C6(I)=SUM
```

```
      DO 420 I=1,N
 420  F(I)=RT(I)+C5(I)+C6(I)

      DO 430 I=1,N
      SUM=0.0
      DO 425 J=1,N
 425  SUM=SUM+C2(I,J)*F(J)
 430  UTP(I)=SUM

       ICOUNT=ICOUNT+1
        TIME=TIME+TS
       IF(TIME.GT.1.00)GO TO 199
       RT(N/2)=PO*(DCOS(OMEGA*TIME))
       GO TO 200
 199   RT(N/2)=0.0
 200   EXACT=0.0
       JEST=1
       IF(ICOUNT.EQ.10)GO TO 141
       GO TO 143
 141   WRITE(2,175)TIME,UTP(N/2),JEST
       ICOUNT=0

 143   DO 150 I=1,N
       UDDTP(I)=(UTP(I)-UT(I)-(TS*UDT(I))-((TS**2)*(0.5-BETA)*UDDT(I)))*
     &(1./((TS**2)*BETA))
 150   CONTINUE

      DO 161 I=1,N
      UDTP(I)=UDT(I)+TS*(((1.0-GAMA)*UDDT(I))+(GAMA*UDDTP(I)))
      UT(I)=UTP(I)
      UDT(I)=UDTP(I)
      UDDT(I)=UDDTP(I)
 161  CONTINUE
       IF(TIME.GT.TT)GO TO 500

        GO TO 120
 175    FORMAT(F10.8,1X,F10.8,1X,I1)
 176    FORMAT(/1X,'  TIME           DEFLECTION AT L/2')
 177    FORMAT('--------------------------------------------------------------
     &-----------')
 500    STOP
        END


       SUBROUTINE INVERT(AO,A,N)
       DOUBLE PRECISION A(70,70),AO(70,70)

       DO 1 I=1,N
       DO 1 J=1,N
 1     A(I,J)=AO(I,J)

       NP=N+1
       A(1,NP)=1.0
       DO 10 I=2,N
 10    A(I,NP)=0.0
```

```
       DO 40 J=1,N
       DO 20 LX=1,N
 20    A(NP,LX)=A(1,LX+1)/A(1,1)
       DO 30 KX=2,N
       DO 30 LX=1,N
 30    A(KX-1,LX)=A(KX,LX+1)-A(KX,1)*A(NP,LX)
       DO 40 LX=1,N
 40    A(N,LX)=A(NP,LX)

       RETURN
      END
      SUBROUTINE JACOBI(K,M,N,IFPR,X,EIGV)
C     SUBROUTINE JACOBI
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION A(70,70),B(70,70),X(70,70),EIGV(70),D(70),
     &K(70,70),M(70,70)
       IFPR=0
C     COMMON/K,M/
C      WRITE(2,1051)
C1051 FORMAT(/1X,'  INPUT DATA ')
C     READ(1,*)N,IFPR
C     WRITE(2,1001)N,IFPR
C     DO 1010 I=1,N
C     READ(1,*) (A(I,J),J=1,N)
C     WRITE(2,1110) (A(I,J),J=1,N)
C1010   CONTINUE
C     DO 1020 I=1,N
C     READ(1,*) (B(I,J),J=1,N)
C     WRITE(2,1110) (B(I,J),J=1,N)
C1020   CONTINUE
C1001  FORMAT(2I10)
C1110  FORMAT(8F10.4)
      DO 2 I=1,N
      DO 1 J=1,N
      A(I,J)=K(I,J)
      B(I,J)=M(I,J)
 1    CONTINUE
 2    CONTINUE
      NSMAX=15
C     WRITE(2,1980)
 1980 FORMAT(/1X,'   EIGENVALUES  ')
      RTOL=1.D-12
      IOUT=2
      DO 10 I=1,N
      IF(A(I,I).GT.O.AND.B(I,I).GT.O.)GO TO 4
      WRITE(IOUT,2020)
      STOP
 4    D(I)=A(I,I)/B(I,I)
 10   EIGV(I)=D(I)
      DO 30 I=1,N
      DO 20 J=1,N
 20   X(I,J)=0.
 30   X(I,I)=1.0
      IF(N.EQ.1)RETURN
```

```
C
C  INITIALIZE SWEEP COUNTER AND EIGEN ITERATION
C
      NSWEEP=0
      NR=N-1
 40   NSWEEP=NSWEEP+1
      IF(IFPR.EQ.1)WRITE(IOUT,2000)NSWEEP
      PRINT*,' SWEEP NUMBER... ',NSWEEP
C
C   CHECK IF PRESENT OFF DIAGONAL ELEMENT IS TOO LARGE
C
      EPS=(0.01**NSWEEP)**2
      DO 210 J=1,NR
      JJ=J+1
      DO 210 K1=JJ,N
      IF(DABS(A(J,K1)).LT.1.D-20)GO TO 211
      EPTOLA=(A(J,K1)*A(J,K1))/(A(J,J)*A(K1,K1))
      GO TO 212
 211  EPTOLA=0.0
 212  EPTOLB=(B(J,K1)*B(J,K1))/(B(J,J)*B(K1,K1))
      IF((EPTOLA.LT.EPS).AND.(EPTOLB.LT.EPS))GO TO 210
      AKK=A(K1,K1)*B(J,K1)-B(K1,K1)*A(J,K1)
      AJJ=A(J,J)*B(J,K1)-B(J,J)*A(J,K1)
      AB=A(J,J)*B(K1,K1)-A(K1,K1)*B(J,J)
      CHECK=(AB*AB+4.*AKK*AJJ)/4.
      IF(CHECK)50,60,60
 50   WRITE(IOUT,2020)
      STOP
 60   SQCH=DSQRT(CHECK)
      D1=AB/2.+SQCH
      D2=AB/2.-SQCH
      DEN=D1
      IF(DABS(D2).GT.DABS(D1))DEN=D2
      IF(DEN)80,70,80
 70   CA=0.
      CG=(-1.)*A(J,K1)/A(K1,K1)
      GO TO 90
 80   CA=AKK/DEN
      CG=(-1.)*AJJ/DEN
 90   IF(N-2)100,190,100
 100  JP1=J+1
      JM1=J-1
      KP1=K1+1
      KM1=K1-1
      IF(JM1-1)130,110,110
 110  DO 120 I=1,JM1
      AJ=A(I,J)
      BJ=B(I,J)
      AK=A(I,K1)
      BK=B(I,K1)
      A(I,J)=AJ+CG*AK
      B(I,J)=BJ+CG*BK
      A(I,K1)=AK+CA*AJ
 120  B(I,K1)=BK+CA*BJ
 130  IF(KP1-N)140,140,160
```

```
 140  DO 150 I=KP1,N
      AJ=A(J,I)
      BJ=B(J,I)
      AK=A(K1,I)
      BK=B(K1,I)
      A(J,I)=AJ+CG*AK
      B(J,I)=BJ+CG*BK
      A(K1,I)=AK+CA*AJ
 150  B(K1,I)=BK+CA*BJ
 160  IF(JP1-KM1)170,170,190
 170  DO 180 I=JP1,KM1
      AJ=A(J,I)
      BJ=B(J,I)
      AK=A(I,K1)
      BK=B(I,K1)
      A(J,I)=AJ+CG*AK
      B(J,I)=BJ+CG*BK
      A(I,K1)=AK+CA*AJ
 180  B(I,K1)=BK+CA*BJ
 190  AK=A(K1,K1)
      BK=B(K1,K1)
      A(K1,K1)=AK+2.*CA*A(J,K1)+CA*CA*A(J,J)
      B(K1,K1)=BK+2.*CA*B(J,K1)+CA*CA*B(J,J)
      A(J,J)=A(J,J)+2.*CG*A(J,K1)+CG*CG*AK
      B(J,J)=B(J,J)+2.*CG*B(J,K1)+CG*CG*BK
      A(J,K1)=0.
      B(J,K1)=0.
C
C UPDATE EIGENVECTOR MATRIX
C
      DO 200 I=1,N
      XJ=X(I,J)
      XK=X(I,K1)
      X(I,J)=XJ+CG*XK
 200  X(I,K1)=XK+CA*XJ
 210  CONTINUE
C
C  UPDATE EIGENVALUES
C
      DO 220 I=1,N
      IF(A(I,I).GT.0.AND.B(I,I).GT.0)GO TO 220
      WRITE(IOUT,2020)
      STOP
 220  EIGV(I)=A(I,I)/B(I,I)
      IF(IFPR.EQ.0)GO TO 230
      WRITE(IOUT,2030)
      WRITE(IOUT,2010) (EIGV(I),I=1,N)
C
C  CHECK FOR CONVERGENCE
C
 230  DO 240 I=1,N
      TOL=RTOL*D(I)
      DIF=DABS(EIGV(I)-D(I))
      IF(DIF.GT.TOL)GO TO 280
 240  CONTINUE
```

```
C
C  CHECK ALL OFF DIAG ELEMENTS TO SEE IF ANOTHER SWEEP IS REQ'D
C
      EPS=RTOL**2
      DO 250 J=1,NR
      JJ=J+1
      DO 250 K1=JJ,N
      IF(DABS(A(J,K1)).LT.1.D-30)GO TO 251
      EPSA=(A(J,K1)*A(J,K1))/(A(J,J)*A(K1,K1))
      GO TO 252
 251  EPSA=0.0
 252  EPSB=(B(J,K1)*B(J,K1))/(B(J,J)*B(K1,K1))
      IF((EPSA.LT.EPS).AND.(EPSB.LT.EPS))GO TO 250
      GO TO 280
 250   CONTINUE
C
C  FILL OUT BOTTOM TRIANGLE OF RESULTANT MATRICES & SCALE EIGENVECTORS
C
 255  DO 260 I=1,N
      DO 260 J=1,N
      A(J,I)=A(I,J)
 260  B(J,I)=B(I,J)
      DO 270 J=1,N
      BB=DSQRT(B(J,J))
      DO 270 K1=1,N
 270  X(K1,J)=X(K1,J)/BB
C     WRITE(IOUT,310)
C     DO 300 I=1,N
C300  WRITE(IOUT,2010) (X(I,J),J=1,N)
 310  FORMAT(/1X,' THE EIGENVECTORS ARE ')




      RETURN
C
C   UPDATE THE 'D' MATRIX AND START NEW SWEEP IF ALLOWED
C
 280  DO 290 I=1,N
 290  D(I)=EIGV(I)
      IF(NSWEEP.LT.NSMAX)GO TO 40
      GO TO 255
 2000 FORMAT(/1X,'  SWEEP NUMBER IN JACOBI = ',I4)
 2010 FORMAT(/1X,6E20.12)
 2020 FORMAT(/1X,'  ****  ERROR SOLUTION STOP / MATRICES NOT POSITIVE
     &DEFINITE')
 2030 FORMAT(/1X,' CURRENT EIGENVALUES IN JACOBI ARE ')
      END

      SUBROUTINE DAMP(N,EIGV,X,M,DAMRAT,C)
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION X(70,70),T(70,70),M(70,70),C(70,70),EIGV(70),DAMR
     &AT(70)

      DO 10 I=1,N
```

```
      EIGV(I)=DSQRT(EIGV(I))
      DO 10 J=1,N
 10   C(I,J)=0.0

      DO 20 II=1,N
      DA=2.*DAMRAT(II)*EIGV(II)
      DO 20 I=1,N
      DO 20 J=1,N
 20   C(I,J)=C(I,J)+X(I,II)*X(J,II)*DA

      DO 30 I=1,N
      DO 30 J=1,N
      T(I,J)=0.0
      DO 30 K1=1,N
 30   T(I,J)=T(I,J)+M(I,K1)*C(K1,J)

      DO 40 I=1,N
      DO 40 J=1,N
      C(I,J)=0.0
      DO 40 K1=1,N
 40   C(I,J)=C(I,J)+T(I,K1)*M(K1,J)

C     DO 50 I=1,N
C50    WRITE(2,120) (C(I,J),J=1,N)
 120   FORMAT(6D14.4)
      RETURN

      END
```

FILE: DAVE OUT A OLD DOMINION UNIVERSITY -- CMSL 4.0 8706

THIS IS NEWMARKS SOLUTION

STIFFNESS -- 0.531100006D+02

DAMPING ---- 0.130999982D-01

FORCE ------ 0.400000066D-02

FREQUENCY -- 0.251300049D+02

TIME	DEFLECTION AT L/2	
0.00500000	0.07560742	1
0.01000000	0.07411487	1
0.01500000	0.07822479	1
0.02000000	0.07755833	1
0.02500000	0.07321689	1
0.03000000	0.07439081	1
0.03500001	0.07596175	1
0.04000001	0.07030083	1
0.04500001	0.06480695	1
0.05000001	0.06403831	1
0.05500001	0.05838368	1
0.06000001	0.04628549	1
0.06500001	0.03770042	1
0.07000001	0.03044851	1
0.07500001	0.01549482	1
0.08000001	-0.00155219	1
0.08500001	-0.01381828	1
0.09000001	-0.02866951	1
0.09500002	-0.04917751	1
0.10000002	-0.06667677	1
0.10500002	-0.08058453	1
0.11000002	-0.09807491	1
0.11500002	-0.11601054	1
0.12000002	-0.12788046	1
0.12500002	-0.13797468	1
0.13000002	-0.14964225	1
0.13500002	-0.15657816	1
0.14000002	-0.15730855	1
0.14500002	-0.15770900	1
0.15000002	-0.15621028	1
0.15500003	-0.14753221	1
0.16000003	-0.13531903	1
0.16500003	-0.12327482	1
0.17000003	-0.10672159	1
0.17500003	-0.08461688	1
0.18000003	-0.06275240	1
0.18500003	-0.04085973	1
0.19000003	-0.01462197	1
0.19500003	0.01269540	1
0.20000003	0.03673648	1
0.20500003	0.06085743	1
0.21000003	0.08652612	1

```
      IMPLICIT REAL*8(A-H,O-Z)


C     THIS PROGRAM SOLVES FOR THE DEFLECTIONS OF A BEAM

C     SUBJECT TO A FORCING FUNCTION AT THE MIDPOINT

C     USING CENTRAL DIFFERENCE METHOD



      DOUBLE PRECISION L,K1,K2,K(70,70),M(70,70),C(70,70),U(70),UTN(70),
     &RT(70),C1(70,70),C2(70,70),C3(70,70),C4(70),KEL(4,4),B(70),C5(70),
     &DUM(70),UT(70),MINV(70,70),C6(70,70),X(70,70),EIGV(70),DAMRAT(70),
     &UDDT(70)
      IFPR=0
C*********************  SET CONSTANT DATA  ***********************
C**                                                             **
C**  L = LENGTH  (IN)                                           **
C**                                                             **
C**  NUMEL = NUMBER OF ELEMENTS (MUST BE AN EVEN NUMBER)        **
C**                                                             **
C**  TS = TIME STEP; DELTA 'T'  (SEC)                           **
C**                                                             **
C**  ROW = MASS PER UNIT LENGTH  (KIP*SEC**2/IN**2)             **
C**                                                             **
C**  E = MODULUS OF ELASTICITY  (KSI)                           **
C**                                                             **
C**  XI = MOMENT OF INERTIA  (IN**4)                            **
C**                                                             **
C**  AR = AREA  (IN**2)                                         **
C**                                                             **
C**  K1 = ROTATIONAL STIFFNESS AT END 1  (K*IN/RAD)             **
C**                                                             **
C**  K2 = ROTATIONAL STIFFNESS AT END 2  (K*IN/RAD)             **
C**                                                             **
C**  ZETA = DAMPING RATIO                                       **
C**                                                             **
C**  TT = TOTAL TIME FOR PROGRAM EXECUTION  (SEC)               **
C**                                                             **
C**  PO = MAGNITUDE OF THE FORCING FUNCTION  (KIPS)             **
C**                                                             **
C**  OMEGA = FREQUENCY OF THE FORCING FUNCTION (HZ)             **
C**                                                             **
C**                                                             **
C*****************************************************************



      DATA L,NUMEL,TS,ROW/177.,12,0.00010,181.9527D-09/
      DATA E,XI,AR/10000.,.325,.7363/
      DATA K1,K2,TT/00.000,00.000,4.00/
      DATA PO,OMEGA,ZETA/0.00,0.000000000,0.0000/
      ICOUNT=0
```

```
      TIME=0.0

      H=L/NUMEL

      N=2*NUMEL-2

      DO 10 I=1,N
      DO 10 J=1,N
 10   K(I,J)=0.0

      K(1,1)=3.*E*XI*(4.*K1+1.)/((H**3)*(K1+1.))
      K(1,2)=(-1.)*3.*E*XI*(2.*K1+1.)/((H**2)*(K1+1.))
      K(2,1)=K(1,2)
      K(2,2)=E*XI*(4.*K1+3.)/(H*(K1+1.))

      K(N-1,N-1)=3.*E*XI*(4.*K2+1.)/((H**3)*(K2+1.))
      K(N-1,N)=3.*E*XI*(2.*K2+1.)/((H**2)*(K2+1.))
      K(N,N-1)=K(N-1,N)
      K(N,N)=E*XI*(4.*K2+3.)/(H*(K2+1.))

      KEL(1,1)  =  12.*E*XI/(H**3)
      KEL(1,2)  =  6.*E*XI/(H**2)
      KEL(1,3)  =  (-1.)*KEL(1,1)
      KEL(1,4)  =  KEL(1,2)
      KEL(2,2)  =  4.*E*XI/(H)
      KEL(2,3)  =  (-1.)*KEL(1,2)
      KEL(2,4)  =  KEL(2,2)/2.
      KEL(3,3)  =  KEL(1,1)
      KEL(3,4)  =  KEL(2,3)
      KEL(4,4)  =  KEL(2,2)

      DO 30 I=1,4
      DO 30 J=1,4
 30   IF(J.GT.I)KEL(J,I)=KEL(I,J)

      DO 50 JK =1,NUMEL-2
      II=JK*2-2
      JJ=II
      DO 45 I=1,4
      DO 40 J=1,4

      K(II+I,JJ+J)  =  K(II+I,JJ+J)+KEL(I,J)

 40   CONTINUE
 45   CONTINUE
 50   CONTINUE

C***************************************************************************

C   THIS IS A TEST OF THE STIFFNESS MATRIX FOR THE STATIC LOAD CASE

C     DO 61 I=1,N
C61   RT(I)=0.0
C     RT(N/2)=.10
C     CALL INVERT(K,C6,N)
```

```
C      DO 63 I=1,N
C      SUM=0.0
C      DO 62 J=1,N
C62    SUM=SUM+C6(I,J)*RT(J)
C63    U(I)=SUM
C      DO 65 I=1,N
C      PRINT*,'DEFLECTION AT NODE ',I,' IS ',U(I)
C65    WRITE(2,64) I,U(I)
C64    FORMAT(/1X,'STATIC SOLUTION... U(',I2,')  = ',D23.16)
C      GO TO 500
C
C**********************************************************************

 66    DO 75 I=1,N
       DO 70 J=1,N
       M(I,J)=0.0
       C(I,J)=0.0
 70    CONTINUE
 75    CONTINUE


       DO 80 I=1,N-1,2
       J=I+1
       M(I,I)=78.
       M(J,J)=2.*(H**2)
 80    CONTINUE

       DO 90 I=1,N
 90    M(I,I)=M(I,I)*(ROW*H/78.)

C      DO 100 I=1,N
       IFPR=0
        CALL JACOBI(K,M,N,IFPR,X,EIGV)

        DO 100 I=1,N
 100    DAMRAT(I)=ZETA

        CALL DAMP(N,EIGV,X,M,DAMRAT,C)



C100   C(I,I)=ZETA

C*****  PRINT STIFFNESS, MASS, AND DAMPING MATRICES  ***********
C
C      WRITE(2,220)N/2
C      DO 210 I=1,N
C210   WRITE(2,215) (K(I,J),J=1,N/2)
C      WRITE(2,221)N/2
C      DO 211 I=1,N
C211   WRITE(2,215) (K(I,J),J=N/2+1,N)
C      WRITE(2,222)N/2
C      DO 212 I=1,N
C212   WRITE(2,215) (M(I,J),J=1,N/2)
C      WRITE(2,223)N/2
```

```
C       DO 213 I=1,N
C213    WRITE(2,215) (M(I,J),J=N/2+1,N)
C       WRITE(2,224)N/2
C       DO 214 I=1,N
C214    WRITE(2,215) (C(I,J),J=1,N/2)
C       WRITE(2,225)N/2
C       DO 216 I=1,N
C216    WRITE(2,215) (C(I,J),J=N/2+1,N)

 215    FORMAT(/1X,5D14.7)

 220    FORMAT(/1X,'THESE ARE THE FIRST ',I3,' COLUMNS OF K ')
 221    FORMAT(/1X,'THESE ARE THE LAST ',I3,' COLUMNS OF K ')
 222    FORMAT(/1X,'THESE ARE THE FIRST ',I3,' COLUMNS OF M ')
 223    FORMAT(/1X,'THESE ARE THE LAST ',I3,' COLUMNS OF M ')
 224    FORMAT(/1X,'THESE ARE THE FIRST ',I3,' COLUMNS OF C ')
 225    FORMAT(/1X,'THESE ARE THE LAST  ',I3,' COLUMNS OF C ')


        PRINT*,'IN START'

        DO 300 I=1,N
 300    RT(I)=0.0

C       RT(N/2)=PO*(DSIN(OMEGA*TIME))

        CALL INVERT(M,MINV,N)


        PI=ACOS(-1.0)
        DO 333 I=1,NUMEL
        Z=PI*H*I/L
        UT(2*I)=(.1563/(1.+DUM1*2))*((PI/L*DCOS(Z))+(DUM1*2*PI/L*DSIN(2*Z)
       &))
        UT(2*I-1)=(.1563/(1.+DUM1*2))*(DSIN(Z)+DUM1*(1.-DCOS(2*Z)))
 333    CONTINUE
C       DO 334 I=1,N
C334    PRINT*,'UT(',I,') = ',UT(I)



         DO 302 I=1,N
         SUM=0.0
         DO 301 J=1,N
 301     SUM=SUM+K(I,J)*UT(J)*(-1.0)
 302     DUM(I)=SUM

         DO 306 I=1,N
         SUM=0.0
         DO 305 J=1,N
 305     SUM=SUM+MINV(I,J)*DUM(J)
 306     UDDT(I)=SUM

         DO 303 I=1,N
 303     UTN(I)=UT(I)+UDDT(I)*(TS**2)/2.
```

```
C       DO 302 I=1,N
C       SUM=0.0
C       DO 301 J=1,N
C301    SUM=SUM+MINV(I,J)*RT(J)
C302    UTN(I)=SUM

C       DO 303 I=1,N
C303    UTN(I)=UTN(I)*(TS**2)/(2.0)

        DO 310 I=1,N
        U(I)=0.0
C       UT(I)=0.0
 310    CONTINUE

        PRINT*,'OUT START'

         DO 320 I=1,N
         DO 315 J=1,N
         C1(I,J) = M(I,J)/(TS**2) + C(I,J)/(2.0*TS)
         C2(I,J) = K(I,J) - (2.*M(I,J)/(TS**2))
         C3(I,J) = (M(I,J)/(TS**2)) - (C(I,J)/(2.*TS))
 315     CONTINUE
 320     CONTINUE

         CALL INVERT(C1,C6,N)

C        DO 122 I=1,N
C122     WRITE(2,123)I,C6(I,I)
 123     FORMAT(/1X,'THIS IS C6(',I2,')   ',D23.16)
C       WRITE(2,176)
C       WRITE(2,177)
 130     DO 440 I=1,N
         SUM=0.0
         DO 410 J=1,N
C        WRITE(2,131)TIME,C2(I,J),UT(I)
 410     SUM=SUM+C2(I,J)*UT(J)
 440     C4(I)=SUM
 131     FORMAT(/1X,'TIME ',F5.3,' C2 ',D18.10,' UT ',D18.10)
         DO 442 I=1,N
         SUM=0.0
         DO 411 J=1,N
 411     SUM=SUM+C3(I,J)*UTN(J)
 442     C5(I)=SUM

          RT(N/2)=PO*(DSIN(OMEGA*TIME))

          DO 140 I=1,N
 140     B(I)=RT(I)-C4(I)-C5(I)

C       DO 142 I=1,N
C142    WRITE(2,149)I,B(I)
 149    FORMAT(/1X,'THIS IS B(',I2,')  =  ',D23.16)
```

```
C      WRITE(2,176)
C      WRITE(2,177)
        DO 542 I=1,N
        SUM=0.0
        DO 511 J=1,N
 511    SUM=SUM+C6(I,J)*B(J)
 542    U(I)=SUM

        ICOUNT=ICOUNT+1
         TIME=TIME+TS

C      SUM=0.0
C      DO 199 I=1,7,2
C      X=I
C      D1=((E*XI/(ROW*(L**4)))**0.5)*(X**2)*(9.869604404)
C      D2=((DSIN((X)*3.14592654/2.0))**2)
C      D3=1.0-(DCOS(D1*TIME))
C199   SUM=SUM+D2*D3/(D1**2)
C199   SUM=SUM+D3/(D1**2)
C      EXACT=(2.*PO/(ROW*L))*SUM

C      DI=2.*PO*(DSIN(OMEGA*TIME))*(L**3)/((3.14592654**4)*E*XI)
C      SUM=0.0
C      DO 199 I=1,NUMEL-1,2
C      X=I
C199   SUM=SUM+(1./((X**4)-0.25))
C      EXACT=SUM*DI

        LINE=3
        IF(ICOUNT.EQ.50)GO TO 141
        GO TO 143
 141    WRITE(2,175)TIME,U(N/2),LINE
C       DIF=(U(N/2)-EXACT)/EXACT
C       IF(DABS(DIF).LE.0.15)WRITE(2,177)
        ICOUNT=0

 143    DO 150 I=1,N
        UTN(I)=UT(I)
 150    UT(I)=U(I)

        IF(TIME.GT.TT)GO TO 500

         GO TO 130
 175     FORMAT(F10.8,1X,F10.8,1X,I1)
 176     FORMAT(/1X,'  TIME           DEFLECTION AT L/2',15X,'EXACT')
 177     FORMAT('-------------------------------------------------------
      &-----------')
 500     STOP
         END


        SUBROUTINE INVERT(AO,A,N)
        DOUBLE PRECISION A(70,70),AO(70,70)

        DO 1 I=1,N
```

```
      DO 1 J=1,N
 1    A(I,J)=AO(I,J)

      NP=N+1
      A(1,NP)=1.0
      DO 10 I=2,N
 10   A(I,NP)=0.0

      DO 40 J=1,N
      DO 20 LX=1,N
 20   A(NP,LX)=A(1,LX+1)/A(1,1)
      DO 30 KX=2,N
      DO 30 LX=1,N
 30   A(KX-1,LX)=A(KX,LX+1)-A(KX,1)*A(NP,LX)
      DO 40 LX=1,N
 40   A(N,LX)=A(NP,LX)

      RETURN
      END

      SUBROUTINE JACOBI(K,M,N,IFPR,X,EIGV)
C     SUBROUTINE JACOBI
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION A(70,70),B(70,70),X(70,70),EIGV(70),D(70),
     &K(70,70),M(70,70)
      IFPR=0
C     COMMON/K,M/
C     WRITE(2,1051)
C1051 FORMAT(/1X,'  INPUT DATA ')
C     READ(1,*)N,IFPR
C     WRITE(2,1001)N,IFPR
C     DO 1010 I=1,N
C     READ(1,*) (A(I,J),J=1,N)
C     WRITE(2,1110) (A(I,J),J=1,N)
C1010   CONTINUE
C     DO 1020 I=1,N
C     READ(1,*) (B(I,J),J=1,N)
C     WRITE(2,1110) (B(I,J),J=1,N)
C1020   CONTINUE
C1001  FORMAT(2I10)
C1110  FORMAT(8F10.4)
      DO 2 I=1,N
      DO 1 J=1,N
      A(I,J)=K(I,J)
      B(I,J)=M(I,J)
 1    CONTINUE
 2    CONTINUE
      NSMAX=15
C     WRITE(2,1980)
 1980 FORMAT(/1X,'   EIGENVALUES  ')
      RTOL=1.D-12
      IOUT=2
      DO 10 I=1,N
C     PRINT*,'FLAG  ',I,' A = ',A(I,I),' B = ',B(I,I)
      IF(A(I,I).GT.0.AND.B(I,I).GT.0.)GO TO 4
```

```
      WRITE(IOUT,2020)                                                  CEN03860
      STOP                                                              CEN03870
 4    D(I)=A(I,I)/B(I,I)                                                CEN03880
 10   EIGV(I)=D(I)                                                      CEN03890
      DO 30 I=1,N                                                       CEN03900
      DO 20 J=1,N                                                       CEN03910
 20   X(I,J)=0.                                                         CEN03920
 30   X(I,I)=1.0                                                        CEN03930
      IF(N.EQ.1)RETURN                                                  CEN03940
C                                                                       CEN03950
C  INITIALIZE SWEEP COUNTER AND EIGEN ITERATION                         CEN03960
C                                                                       CEN03970
      NSWEEP=0                                                          CEN03980
      NR=N-1                                                            CEN03990
 40   NSWEEP=NSWEEP+1                                                   CEN04000
      IF(IFPR.EQ.1)WRITE(IOUT,2000)NSWEEP                               CEN04010
      PRINT*,' SWEEP NUMBER... ',NSWEEP                                 CEN04020
C                                                                       CEN04030
C   CHECK IF PRESENT OFF DIAGONAL ELEMENT IS TOO LARGE                  CEN04040
C                                                                       CEN04050
      EPS=(0.01**NSWEEP)**2                                             CEN04060
      DO 210 J=1,NR                                                     CEN04070
      JJ=J+1                                                            CEN04080
      DO 210 K1=JJ,N                                                    CEN04090
      IF(DABS(A(J,K1)).LT.1.D-20)GO TO 211                              CEN04100
      EPTOLA=(A(J,K1)*A(J,K1))/(A(J,J)*A(K1,K1))                        CEN04110
      GO TO 212                                                         CEN04120
 211  EPTOLA=0.0                                                        CEN04130
 212  EPTOLB=(B(J,K1)*B(J,K1))/(B(J,J)*B(K1,K1))                        CEN04140
      IF((EPTOLA.LT.EPS).AND.(EPTOLB.LT.EPS))GO TO 210                  CEN04150
      AKK=A(K1,K1)*B(J,K1)-B(K1,K1)*A(J,K1)                             CEN04160
      AJJ=A(J,J)*B(J,K1)-B(J,J)*A(J,K1)                                 CEN04170
      AB=A(J,J)*B(K1,K1)-A(K1,K1)*B(J,J)                                CEN04180
      CHECK=(AB*AB+4.*AKK*AJJ)/4.                                       CEN04190
C     PRINT*,'THIS IS CHECK... ',CHECK                                  CEN04200
      IF(CHECK)50,60,60                                                 CEN04210
 50   WRITE(IOUT,2020)                                                  CEN04220
      STOP                                                              CEN04230
 60   SQCH=DSQRT(CHECK)                                                 CEN04240
      D1=AB/2.+SQCH                                                     CEN04250
      D2=AB/2.-SQCH                                                     CEN04260
      DEN=D1                                                            CEN04270
      IF(DABS(D2).GT.DABS(D1))DEN=D2                                    CEN04280
      IF(DEN)80,70,80                                                   CEN04290
 70   CA=0.                                                             CEN04300
      CG=(-1.)*A(J,K1)/A(K1,K1)                                         CEN04310
      GO TO 90                                                          CEN04320
 80   CA=AKK/DEN                                                        CEN04330
      CG=(-1.)*AJJ/DEN                                                  CEN04340
 90   IF(N-2)100,190,100                                                CEN04350
 100  JP1=J+1                                                           CEN04360
      JM1=J-1                                                           CEN04370
      KP1=K1+1                                                          CEN04380
      KM1=K1-1                                                          CEN04390
      IF(JM1-1)130,110,110                                              CEN04400
```

```
  110 DO 120 I=1,JM1
      AJ=A(I,J)
      BJ=B(I,J)
      AK=A(I,K1)
      BK=B(I,K1)
      A(I,J)=AJ+CG*AK
      B(I,J)=BJ+CG*BK
      A(I,K1)=AK+CA*AJ
  120 B(I,K1)=BK+CA*BJ
  130 IF(KP1-N)140,140,160
  140 DO 150 I=KP1,N
      AJ=A(J,I)
      BJ=B(J,I)
      AK=A(K1,I)
      BK=B(K1,I)
      A(J,I)=AJ+CG*AK
      B(J,I)=BJ+CG*BK
      A(K1,I)=AK+CA*AJ
  150 B(K1,I)=BK+CA*BJ
  160 IF(JP1-KM1)170,170,190
  170 DO 180 I=JP1,KM1
      AJ=A(J,I)
      BJ=B(J,I)
      AK=A(I,K1)
      BK=B(I,K1)
      A(J,I)=AJ+CG*AK
      B(J,I)=BJ+CG*BK
      A(I,K1)=AK+CA*AJ
  180 B(I,K1)=BK+CA*BJ
  190 AK=A(K1,K1)
      BK=B(K1,K1)
      A(K1,K1)=AK+2.*CA*A(J,K1)+CA*CA*A(J,J)
      B(K1,K1)=BK+2.*CA*B(J,K1)+CA*CA*B(J,J)
      A(J,J)=A(J,J)+2.*CG*A(J,K1)+CG*CG*AK
      B(J,J)=B(J,J)+2.*CG*B(J,K1)+CG*CG*BK
      A(J,K1)=0.
      B(J,K1)=0.
C
C UPDATE EIGENVECTOR MATRIX
C
      DO 200 I=1,N
      XJ=X(I,J)
      XK=X(I,K1)
      X(I,J)=XJ+CG*XK
  200 X(I,K1)=XK+CA*XJ
  210 CONTINUE
C
C  UPDATE EIGENVALUES
C
      DO 220 I=1,N
      IF(A(I,I).GT.0.AND.B(I,I).GT.0)GO TO 220
      WRITE(IOUT,2020)
      STOP
  220 EIGV(I)=A(I,I)/B(I,I)
      IF(IFPR.EQ.0)GO TO 230
```

```
      WRITE(IOUT,2030)
      WRITE(IOUT,2010) (EIGV(I),I=1,N)
C
C  CHECK FOR CONVERGENCE
C
 230  DO 240 I=1,N
      TOL=RTOL*D(I)
      DIF=DABS(EIGV(I)-D(I))
      IF(DIF.GT.TOL)GO TO 280
 240  CONTINUE
C
C  CHECK ALL OFF DIAG ELEMENTS TO SEE IF ANOTHER SWEEP IS REQ'D
C     PRINT*,'  RTOL ',RTOL
      EPS=RTOL**2
      DO 250 J=1,NR
      JJ=J+1
      DO 250 K1=JJ,N
      IF(DABS(A(J,K1)).LT.1.D-30)GO TO 251
      EPSA=(A(J,K1)*A(J,K1))/(A(J,J)*A(K1,K1))
      GO TO 252
 251  EPSA=0.0
C     PRINT*,' EPSA ',EPSA,' EPS ',EPS
 252  EPSB=(B(J,K1)*B(J,K1))/(B(J,J)*B(K1,K1))
C     PRINT*,' EPSB ',EPSB,' EPS ',EPS
      IF((EPSA.LT.EPS).AND.(EPSB.LT.EPS))GO TO 250
      GO TO 280
 250   CONTINUE
C
C  FILL OUT BOTTOM TRIANGLE OF RESULTANT MATRICES & SCALE EIGENVECTORS
C
 255  DO 260 I=1,N
      DO 260 J=1,N
      A(J,I)=A(I,J)
 260  B(J,I)=B(I,J)
      DO 270 J=1,N
      BB=DSQRT(B(J,J))
      DO 270 K1=1,N
 270  X(K1,J)=X(K1,J)/BB
C     WRITE(IOUT,310)
C     DO 300 I=1,N
C300  WRITE(IOUT,2010) (X(I,J),J=1,N)
 310  FORMAT(/1X,' THE EIGENVECTORS ARE ')




      RETURN
C
C   UPDATE THE 'D' MATRIX AND START NEW SWEEP IF ALLOWED
C
 280  DO 290 I=1,N
 290  D(I)=EIGV(I)
      IF(NSWEEP.LT.NSMAX)GO TO 40
      GO TO 255
 2000 FORMAT(/1X,'  SWEEP NUMBER IN JACOBI = ',I4)
```

```
 2010 FORMAT(/1X,6E20.12)
 2020 FORMAT(/1X,'  ****  ERROR SOLUTION STOP / MATRICES NOT POSITIVE
     &DEFINITE')
 2030 FORMAT(/1X,' CURRENT EIGENVALUES IN JACOBI ARE ')
      END

      SUBROUTINE DAMP(N,EIGV,X,M,DAMRAT,C)
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION X(70,70),T(70,70),M(70,70),C(70,70),EIGV(70),DAMR
     &AT(70)

      DO 10 I=1,N
      EIGV(I)=DSQRT(EIGV(I))
      DO 10 J=1,N
 10   C(I,J)=0.0

      DO 20 II=1,N
      DA=2.*DAMRAT(II)*EIGV(II)
      DO 20 I=1,N
      DO 20 J=1,N
 20   C(I,J)=C(I,J)+X(I,II)*X(J,II)*DA

      DO 30 I=1,N
      DO 30 J=1,N
      T(I,J)=0.0
      DO 30 K1=1,N
 30   T(I,J)=T(I,J)+M(I,K1)*C(K1,J)

      DO 40 I=1,N
      DO 40 J=1,N
      C(I,J)=0.0
      DO 40 K1=1,N
 40   C(I,J)=C(I,J)+T(I,K1)*M(K1,J)

C     DO 50 I=1,N
C50    WRITE(2,120) (C(I,J),J=1,N)
 120   FORMAT(6D14.4)
      RETURN

      END
```

THIS IS CENTRAL DIFFERENCE SOLUTION

STIFFNESS -- 0.531100006D+02

DAMPING ---- 0.130999982D-01

FORCE ------ 0.400000066D-02

FREQUENCY -- 0.251300049D+02

TIME	DEFLECTION AT L/2	
0.00500000	0.07560742	1
0.01000000	0.07411487	1
0.01500000	0.07822479	1
0.02000000	0.07755833	1
0.02500000	0.07321689	1
0.03000000	0.07439081	1
0.03500001	0.07596175	1
0.04000001	0.07030083	1
0.04500001	0.06480695	1
0.05000001	0.06403831	1
0.05500001	0.05838368	1
0.06000001	0.04628549	1
0.06500001	0.03770042	1
0.07000001	0.03044851	1
0.07500001	0.01549482	1
0.08000001	-0.00155219	1
0.08500001	-0.01381828	1
0.09000001	-0.02866951	1
0.09500002	-0.04917751	1
0.10000002	-0.06667677	1
0.10500002	-0.08058453	1
0.11000002	-0.09807491	1
0.11500002	-0.11601054	1
0.12000002	-0.12788046	1
0.12500002	-0.13797468	1
0.13000002	-0.14964225	1
0.13500002	-0.15657816	1
0.14000002	-0.15730855	1
0.14500002	-0.15770900	1
0.15000002	-0.15621028	1
0.15500003	-0.14753221	1
0.16000003	-0.13531903	1
0.16500003	-0.12327482	1
0.17000003	-0.10672159	1
0.17500003	-0.08461688	1
0.18000003	-0.06275240	1
0.18500003	-0.04085973	1
0.19000003	-0.01462197	1
0.19500003	0.01269540	1
0.20000003	0.03673648	1
0.20500003	0.06085743	1
0.21000003	0.08652612	1