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Three-Dimensional Adaptive Grid Generation for Body-Fitted Coordinate System

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THREE-DIMENSIONAL ADAPTIVE GRID GENERATION FOR BODY-FITTED COORDINATE SYSTEM

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1. SUMMARY

This report describes a numerical method for generating three-dimensional grids for general configurations. The basic method involves the solution of a set of quasi-linear elliptic partial differential equations via pointwise relaxation with a local relaxation factor. It allows specification of the grid spacing off the boundary surfaces and the grid orthogonality at the boundary surfaces. It includes adaptive mechanisms to improve smoothness, orthogonality, and flow resolution in the grid interior.

2. INTRODUCTION

Three-dimensional computational fluid dynamics codes require computational grids with suitable resolution, smoothness, and orthogonality. High grid resolution allows complex flow physics to be modelled near shocks and in shear layers. Smoothness of the metric data prevents the flow solution from being dominated by truncation error in the metric coefficients. Grid orthogonality at the boundaries simplifies and improves the accuracy of any boundary condition involving normal gradients.

This report describes a numerical method for generating three-dimensional grids for computational fluid dynamics codes with the potential and the ability to satisfy the foregoing requirements. The basic method is general and involves the solution of a quasi- linear elliptic partial differential equation via pointwise successive relaxation with a local relaxation factor. The governing equation contains forcing functions that depend upon the boundary point distribution and the boundary surface gradient. The method allows specification of the grid point distribution on the boundary surfaces, the grid spacing off the boundary surfaces, and the grid orthogonality at the boundary surfaces. It includes adaptive mechanisms to improve smoothness, orthogonality, and flow resolution in the grid interior.

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The mechanism of adaptation contains two branches: cumulative movement and non-cumulative movement each with its own characteristic.

In non-cumulative action, the amount of adjustment made on the nodal point distribution is governed by a penalty type of behavior, with fixed upper and lower limits, on the forcing functions.

The system appears to be very stable numerically. The overall accuracy of the system, in terms of residues of the physical aspects under adaptation, is limited by the strength of the penalty function.

In cumulative action, the adjustments are guided by physical constraints. Movement of the grid will not stop unless a homogeneous condition is reached. There are no directly predetermined upper or lower limits on the forcing functions with this technique.

The strength of the adaptive mechanism is determined by a scaling factor, which in turn controls the magnification of disturbances at each iteration. The behavior of this technique resembles an explicit scheme, the scaling factor should be small enough to assure stability of the system, and yet large enough to achieve better accuracy.

This paper includes a discussion of the mathematical formulation of the method and some representative results.

3. MATHEMATICAL FORMULATION

The quasi-linear elliptic governing equation is taken from reference 1 as

$$\sum_{i=1}^{3}\sum_{j=1}^{3} g^{ij}\,\frac{\partial^2 \tilde{x}}{\partial\xi_i \partial\xi_j} + \sum_{k=1}^{3} g^{kk}\, P_k \frac{\partial \tilde{x}}{\partial \xi_k} = 0. \tag{1}$$

The metric tensor components g^{ij} and g^{kk} in equation (1) are defined as

$$g^{ij} = \tilde{a}^i \cdot \tilde{a}^j$$

where

$$\begin{aligned}
\tilde{a}^i &= \tilde{a}_j \times \tilde{a}_k/\sqrt{g} \quad i,j,k \text{ cyclic} \\
\sqrt{g} &= \tilde{a}_1 \cdot (\tilde{a}_2 \times \tilde{a}_3) \\
\tilde{a}_i &= \frac{\partial \tilde{x}}{\partial \xi_i}.
\end{aligned}$$

The P_k in equation (1) are the forcing functions specified by the user. They represent one-dimensional stretching in each index coordinate direction.

Equation (1) can be rewritten using matrix notation as

$$\mathrm{A} + \mathrm{B}\,\mathrm{P} = 0 \tag{2}$$

where

$$A = \begin{pmatrix} g^{11}\frac{\partial^2 \tilde{x}}{\partial \xi_1^2} & g^{12}\frac{\partial^2 \tilde{x}}{\partial \xi_1 \partial \xi_2} & g^{13}\frac{\partial^2 \tilde{x}}{\partial \xi_1 \partial \xi_3} \\ g^{21}\frac{\partial^2 \tilde{x}}{\partial \xi_2 \partial \xi_1} & g^{22}\frac{\partial^2 \tilde{x}}{\partial \xi_2^2} & g^{23}\frac{\partial^2 \tilde{x}}{\partial \xi_2 \partial \xi_3} \\ g^{31}\frac{\partial^2 \tilde{x}}{\partial \xi_3 \partial \xi_1} & g^{32}\frac{\partial^2 \tilde{x}}{\partial \xi_3 \partial \xi_2} & g^{33}\frac{\partial^2 \tilde{x}}{\partial \xi_3^2} \end{pmatrix}$$

$$B = \begin{pmatrix} g^{11}\frac{\partial^2 \tilde{x}}{\partial \xi_1^2} & 0 & 0 \\ 0 & g^{22}\frac{\partial^2 \tilde{x}}{\partial \xi_2^2} & 0 \\ 0 & 0 & g^{33}\frac{\partial^2 \tilde{x}}{\partial \xi_3^2} \end{pmatrix}$$

$$P = \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix}$$

Equation (2) can be solved for P_k on the boundaries as

$$\mathrm{P}_0 = -\mathrm{B}_0^{-1}\,\mathrm{A}_0 \tag{3}$$

where the subscript "0" indicates values on the boundary. Tangential derivatives for terms on the right hand side of equations (3) are determined by applying standard difference formulas to the prescribed boundary point distribution on the surface. Normal derivatives are determined by specifying the first normal derivative equal to the desired spacing off the boundary (for detail, see reference 2) and using the approximation

$$\frac{\partial^2 \tilde{x}_0}{\partial n^2} = \frac{2(\tilde{x}_1 - \tilde{x}_0)}{(\Delta n)^2} - \frac{2}{\Delta n}\frac{\partial \tilde{x}_0}{\partial n}$$

where n indicates the normal direction, the subscript "0" indicates values on the boundary, and the subscript "1" indicates values one point away from the boundary.

Once the boundary values are known, the interior values of P_k can be determined using

$$P_k(\xi_1, \xi_2, \xi_3) = \sum_{l=1}^{3} P_{0_{k,l,1}}(1 + \alpha_{k,l,1})\gamma_{k,l,1} + \sum_{l=1}^{3} P_{0_{k,l,2}}(1 + \alpha_{k,l,2})\gamma_{k,l,2}$$

where

$$
\begin{aligned}
\alpha_{k,l,1} &= C_{\alpha_{k,l,1}}((\xi_l - \xi_{l_{min}})/(\xi_{l_{max}} - \xi_{l_{min}}))^{D_{k,l,1}} \\
\alpha_{k,l,2} &= C_{\alpha_{k,l,2}}((\xi_{l_{max}} - \xi_1)/(\xi_{l_{max}} - \xi_{l_{min}}))^{D_{k,l,2}} \text{when } C_\alpha \geq 0 \\
\alpha_{k,l,1} &= \tanh(\alpha_{k,l,1}) \\
\alpha_{k,l,2} &= \tanh(\alpha_{k,l,2}) \qquad \text{when } C_\alpha < 0 \\
\beta_{k,l,1} &= [(\xi_{l_{max}} - \xi_{l_{min}})/(\xi_l - \xi_{l_{min}})]^{C_{\beta_{l,1}}} \\
\beta_{k,l,2} &= [(\xi_{l_{max}} - \xi_{l_{min}})/(\xi_{l_{max}} - \xi_l)^{C_{\beta_{l,2}}} \\
\gamma_{k,l,1} &= \beta_{k,l,1}/\Big(\sum_{m=1}^{3} \beta_{k,l,1} + \sum_{m=1}^{3} \beta_{k,l,2}\Big) \\
\gamma_{k,l,2} &= \beta_{k,l,2}/\Big(\sum_{m=1}^{3} \beta_{k,l,1} + \sum_{m=1}^{3} \beta_{k,l,2}\Big).
\end{aligned}
$$

The value of $P_{0_{k,l,1}}$ represents the k-th component of the $\mathbf{P}_0$ vector on the minimum boundary surface in the l-th direction. The value of $P_{0_{k,l,1}}$ represents the k-th component of the $\mathbf{P}_0$ vector on the maximum boundary surface in the l-th direction. The functions α, β, and γ have subscript notation similar to that of P_0. The α function represents polynomial extrapolation from a controlled boundary using a constant factor C_α. The γ function represents the combined effect of the β functions, which represent power-law factorization with constant exponent C_β to control the depth of influence away from a controlled boundary.

The values of the forcing functions can be modified for improved smoothness by using

$$P'_k = \theta \; P_k$$

where

$$\theta = 1 - \tanh(C_{\theta_1}(1 - \sigma^{C_{\theta_2}}))$$

$$\sigma = J/(\| l_1 \| \cdot \| l_2 \| \cdot \| l_3 \|).$$

The constants C_{θ_1} and C_{θ_2} define the rate and order of the adaptation. The variable σ is a measure of the shear of the grid, with J representing the Jacobian and l_1, l_2, l_3 representing the grid cell lengths in each direction.

A measure of the local orthogonality of the grid can be defined as

$$\phi = \sigma^{C_\theta}/J. \tag{4}$$

The constant C_θ defines the order of the adaptation. A one-dimensional variational form of equation (4) can then be written as

$$\phi'_k = \frac{\delta_k(J)}{J} - C_\theta \frac{\delta_k(\sigma)}{\sigma}.$$

If a flow error index E is computed by the flow solver such that $E \geq 0$, a measure of the local flow resolution can be defined as

$$\psi = J(1 + f(E)). \tag{5}$$

where f represents a suitable functional of E.

A one-dimensional variational form of equation (5) can then be written as

$$\psi'_k = \frac{\delta_k(J)}{J} - \frac{\delta_k f(E)}{1 + f(E)}$$

The values of the forcing functions can be modified for improved local orthogonality and flow resolution using

$$P'_k = (\theta + \lambda_k \cdot \theta_\lambda)P_k \qquad F_k \cdot P_k > 0$$

$$\tag{6}$$

$$P'_k = (\theta - \lambda_k \cdot \theta_\lambda)P_k \qquad F_k \cdot P_k > 0$$

where θ_λ, similar to θ, is the skewness index for the adaptive constributions, and

$$\lambda_k = C_{\lambda_1} \tanh(C_{\lambda_2} |F_k|^{C_{\lambda_3}})$$
$$F_k = (W_\phi \phi'_k + W_\psi \psi'_k)/(W_\phi + W_\psi)$$

with the constants C_{λ_1}, C_{λ_2}, and C_{λ_3} determining the range, rate, and order of the adaptation for the non-cumulative mechanism. The variable F represents a weighted combination of the skewness and flow error variations.

When cumulative adjustment is desired, the focus must be on the disturbances of F_k instead of F_k itself. Thus let

$$\begin{aligned}
\Delta F_k &= F_k^{(n-1)} - F_k^{(n)} & \\
\lambda_k^* &= (\Delta F_k \cdot F_k^{(n)} / |F_k^{(n)}|)(e^{|F_k^{(n)}|C_{\lambda_3}} - 1) & \\
\lambda_k^{**} &= \tanh(C_{\lambda_2}\, \lambda_k^*) & \text{when } \lambda_k^* \geq 0 \\
\lambda_k^{**} &= 0 & \text{when } \lambda_k^* < 0
\end{aligned}$$

and then

$$\lambda_k = \lambda_k^{(n-1)} + \lambda_k^{**} \tag{7}$$

Equation (7) is used in conjuction with equation (6) for evaluating the proper value of the forcing functions at the N_{th} step.

4. RESULTS AND DISCUSSION

Two groups of test cases using a turbine blade geometry are attached. Each of these cases represents a particular capability of the technique.

Within each group, the nodal point distribution on the boundary lines and the basic values of the forcing functions are constructed in exactly the same fashion.

Figure 1 illustrates the effect of adaptation according the the skewness of the grid only. Highly sheared grids (around the trailing edge) are sufficiently improved compared to results without this adaptation (not shown).

Figure 2 shows the effect of the non-cumulative adaptation according to the skewness and the flow property with constant error index everywhere (equivalent to requiring a zero variation in the Jacobian). The result has indicated that the nearly collapsed grid lines (from Figure 1) at the inlet corner are greatly improved whereas the grid elsewhere is well preserved.

Figure 3 illustrates the effect of the non-cumulative adaptation according to the skewness and the general orthogonality of the grids. The result shows the tendency of the expansion of a good grid and the suppression of a sheared grid.

Two representatives of the three-dimensional capability of the current technique are illustrated in Figure 4 and 5. Figure 4 shows a C-grid configuration with adaptation according to the skewness of the grid only. Figure 5 shows an H-grid configuration with the adaptation according to the skewness and the non-cumulative orthogonality of the grid.

Figure 6, 7, and 8 illustrate the effects of the solution adaptive mechanisms. The error index used here is a fixed artivicial pattern with

$$E = ((J-1)/JHalf)^4 \qquad \text{when } J \leq JHalf + 1$$

$$= ((Jmax - J)/JHalf)^4 \qquad \text{when } J > JHalf + 1$$

where $JHalf = (Jmax - 1)/2 + 1$.

The error functional f used is

$$1 + f(E) = e^{A \cdot E}$$

with a constant A.

Figure 6 represents the optional grid with a skewness adaptation only. Figure 7 shows the effect of the non-cumulative solution adaptive mechanism with A = 10. Figure 8 illustrates the effect of the cumulative solution adaptive technique with A = 2. The results have indicated a positive response on the redistribution of the grids. The adjustments achieved by the cumulative mechanism are more profound than that indicated by the non-cumulative mechanism.

The stability of the adaptive technique is satisfactory. Failure to reach convergence is possible, however, particularly when the spacing requirements of the grids are extremely small and when the physical configuration forces the appearances of highly sheared grids in the interior. These difficulties can be relaxed, only to a degree, by the specification of a suitable strength for the adaptive schemes.

The solution adaptive technique introduced here is still at its early developing stage. The adaptation according to a real slow property has not yet been tested. Although current test results have shown positive response on the grid movements to the adaptive mechanisms, they also appear to indicate a major weakness of this technique, that is, the adjustments on the grid are ultimately limited by the basic requirements, which are specified by the user. The residue of the property under adaptation can only reach a relative minimum, not an absolute minimum, in order to maintain the numerical stability.

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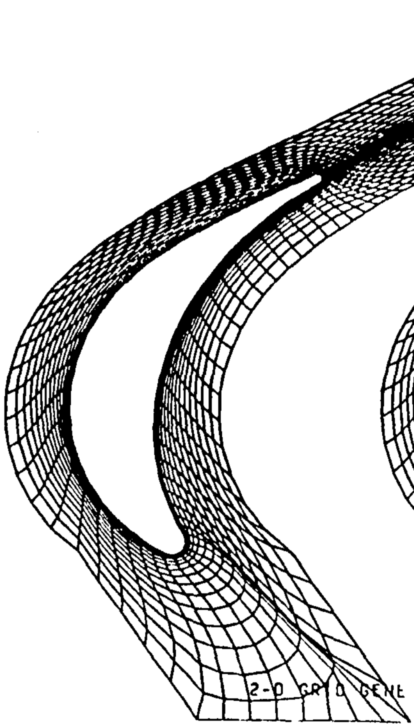


Fig. 1 Skewness adaptation only. 101*16 grids.

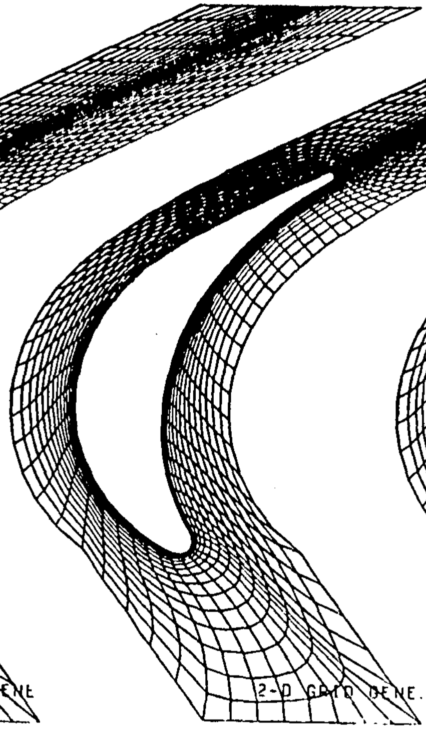


Fig. 2 Skewness and zero variation in the Jacobian. Non-cumulative.

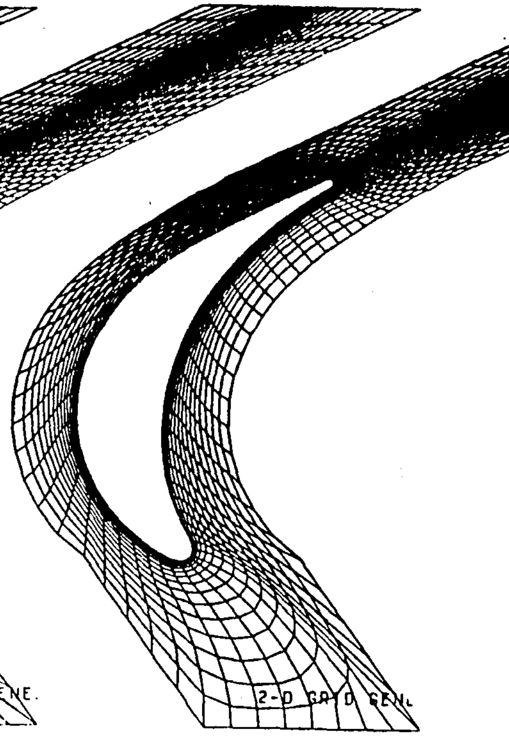
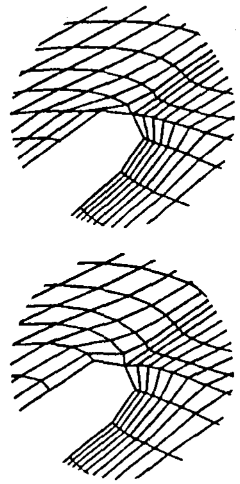


Fig. 3 Skewness and orthogonality control. Non-cumulative.



Magnified views for Fig.2 and Fig.3 respectively.

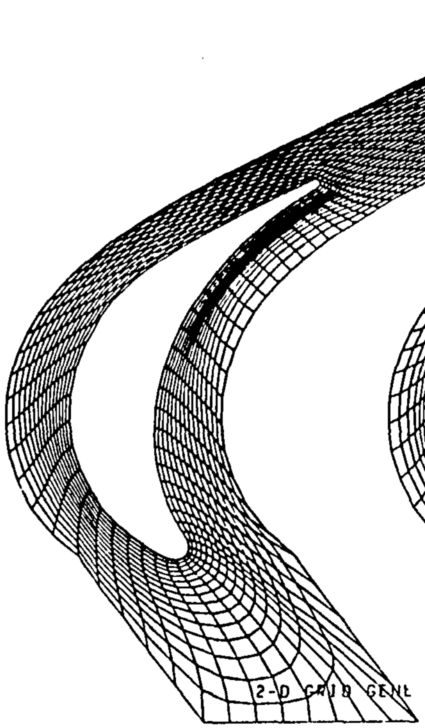


Fig. 6 Original grids with the skewness control only.

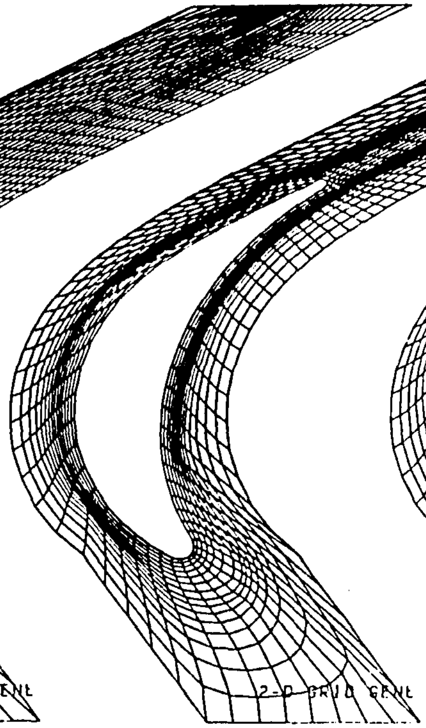


Fig. 7 Solution adaptation. Non-cumulative.

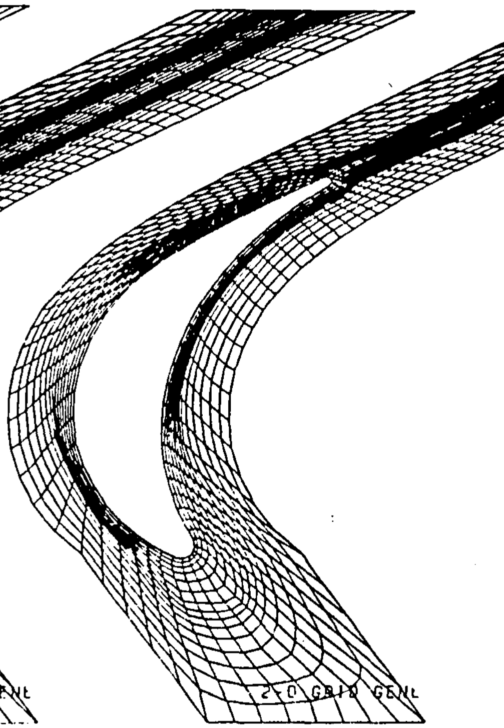
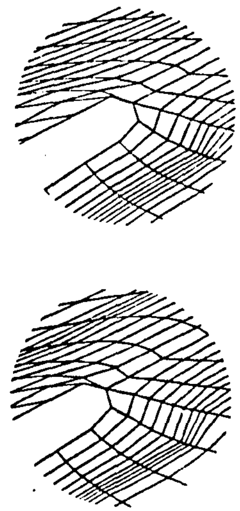


Fig. 8 Solution adaptation. Cumulative.



Magnified views for Fig.7 and Fig.8 respectively.

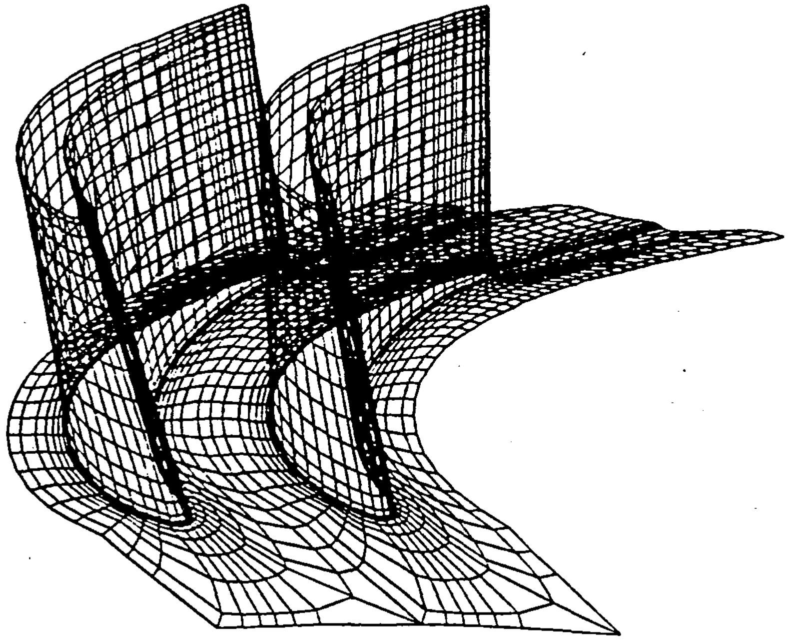


Fig. 4 Three dimensional C-grid configuration.

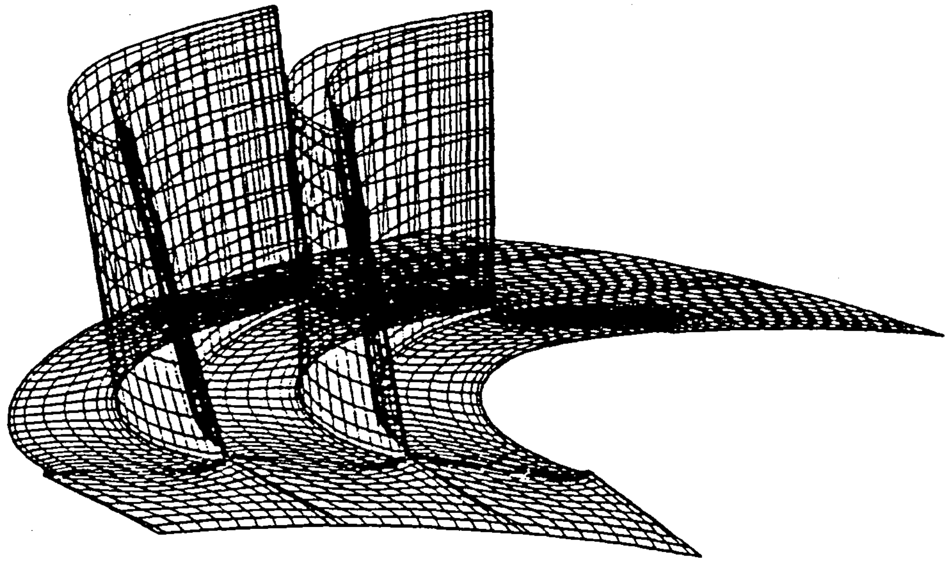


Fig. 5 Three dimensional H-grid configuration.

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16. Abstract

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