# Dual Adaptive Control:

# Design Principles and Applications

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## NASA

# DUAL ADAPTIVE CONTROL: DESIGN PRINCIPLES AND APPLICATIONS

Purusottam Mookerjee, Ph.D.
The University of Connecticut, 1986

The design of an actively adaptive "dual" controller based on an approximation of the stochastic dynamic programming equation for a multi-step horizon is presented. A dual controller that can enhance identification of the system while controlling it at the same time is derived for multidimensional problems. This dual controller uses sensitivity functions of the expected future cost with respect to the parameter uncertainties. A passively adaptive "cautious" controller and the actively adaptive "dual" controller are examined. In many instances, the cautious controller is seen to turn off while the latter avoids the turn-off of the control and the slow convergence of the parameter estimates, characteristic of the cautious controller. The algorithms have been applied to:
1) a multivariable static model which represents a simplified linear version of the relationship between the vibration output and the higher harmonic control input for a helicopter and
2) a dynamic model that has similarity with an ore-crushing plant or a heat exchanger model.

Monte Carlo comparisons based on parametric and nonparametric statistical analysis indicate the superiority of the dual controller over the baseline controller.

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## ACKNOWLEDGEMENTS

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## LIST OF SYMBOLS

| | |
|---|---|
| $A$ | memory parameter matrix |
| $a_i'$ | ith row of $A$ matrix |
| $a_{ij}$ | ijth element of $A$ matrix |
| $B$ | matrix of unknown gain parameters |
| $b$ | unknown gain parameter (scalar) |
| $b_i'$ | ith row of $B$ matrix |
| $b_{ij}$ | ijth element of $B$ matrix |
| $c$ | uncontrolled vibrations |
| $C(k)$ | cost from step k to N |
| $C_k$ | evaluation cost at time step k |
| $\bar{C}_k$ | Monte Carlo average $C_k$ at time k |
| $c_{ik}^{(j)}$ | cost in ith Monte Carlo run at time step k using algorithm j |
| $a, b, c, d, n$ | observed frequencies in $\chi^2$ table |
| $E\{\bullet\}$ | expected value |
| $E\{\ |I^k\}$ | conditional expected value based on information I at time k |
| $EI_k$ | percentage estimated improvement |
| $e_i$ | basis vector in i coordinate |
| $e(k)$ | measurement error |

| | |
|---|---|
| $F$ | correction matrix |
| $f$ | correction vector |
| $H(k)$ | measurement matrix at time k |
| $I^k$ | information at time k |
| $\bar{J}_1$ | zeroth order term in an expansion |
| $J(k)$ | expected value of the cost from step k to N |
| $J_k^{(i)}$ | ith population mean at time k used in Appendix |
| $J^*(k)$ | optimal expected cost to go from time step k to N |
| $J_\theta(k)$ | first partials of cost with respect to parameters at time k |
| $J_{\theta\theta}(k)$ | second partials of the cost with respect to parameters at time k |
| $J_p(k)$ | first partials of the cost with respect to parameter covariance matrix elements at time k |
| $J^*_{k,k+1}$ | optimal expected cost to go from time step k to k+1 |

x

| | |
|---|---|
| $J_y(k)$ | first partial derivatives of cost with respect to measurement |
| $J_{yy}(k)$ | second partial derivatives of cost with respect to measurements |
| $k$ | time index |
| $K(k)$ | Kalman gain at time $k$ |
| $K_{90}$ | 90th percentile point |
| $l$ | number of unknown parameters |
| $m$ | dimension of control vector |
| $n$ | dimension of measurement and state vectors |
| $N$ | N step horizon |
| $P(k)$ | covariance matrix of estimated parameter vector at time $k$ |
| $P_b(k)$ | associated variance of $\hat{b}(k)$ |
| $\bar{P}(k)$ | nominal covariance at time $k$ |
| $P_u(k+1)$ | first partials of the covariance matrix elements at time $k+1$ with |

|  | respect to control inputs at time k |
| $P_{uu}(k+1)$ | second partials of the covariance matrix elements at time k+1 with respect to the control inputs at time k |
| $P^{ij}(k)$ | ijth element of the covariance matrix at time k |
| $P^{ij}_u(k)$ | first partials of $P^{ij}(k)$ with respect to the control |
| $P^{ij}_{uu}(k)$ | second partials of $P^{ij}(k)$ with respect to the control |
| Q | state weights |
| R | control weights |
| S | sample size |
| u(k) | higher harmonic control input at time k |
| $u^*(k)$ | optimal control at time k |
| $u^*_i(k)$ | ith component of $u^*(k)$ |
| $u_D(k)$ | dual controller at time k |
| $u_C(k)$ | cautious controller with one-step ahead horizon at time k |

| | |
|---|---|
| $u^I(k)$ | improved nominal control about which the future covariance is expanded |
| $u_i(k)$ | ith control component |
| $\bar{u}(k)$ | nominal control |
| $\bar{u}_i(k)$ | ith nominal control component |
| $V$ | covariance used in (66) |
| $w(k)$ | measurement error at time k |
| $w_i(k)$ | ith component of $w(k)$ |
| $W$ | measurement noise covariancve |
| $x(k)$ | vibration state at time k |
| $x_i(k)$ | ith component of vibration state at time k |
| $y(k)$ | measurement at time k |
| $y_i(k)$ | ith component of measurement at time k |
| $y_r$ | reference measurement in ARMA model |
| $\bar{y}(k)$ | nominal measurement |
| $z_k$ | test statistic |
| $()'$ | transpose |
| $()^{-1}$ | inverse |

| | |
|---|---|
| $\hat{\phantom{x}}$ | estimate |
| $\alpha$ | probability of error |
| $\chi^2$ | chi-square distribution |
| $\Delta_k$ | population difference at time k |
| $\Delta_{ik}$ | sample difference at time k |
| $\bar{\Delta}_k$ | average sample difference |
| $\delta_{kj}$ | Kronecker delta |
| $\gamma$ | threshold from Normal distribution |
| $\mu$ | mean used in (65) |
| $\nu(k)$ | innovation at time k |
| $\theta(k)$ | parameter vector at time k |
| $\theta_i(k)$ | ith component of parameter vector at time k |
| $\bar{\theta}(k)$ | nominal parameter estimate at time k |
| $\sigma^2_{\bar{\Delta}_k}$ | variance of sample difference |

Chapter 1


INTRODUCTION


Research on adaptive control started in the
early fifties [A2].   The design of autopilots for
aircraft for a wide range of speeds and altitudes
motivated the research on adaptive control.   For this
wide range of operating conditions the use of adaptive
control was deemed necessary.   However, progress in
this field has been quite slow because of the lack of
understanding of the inherently nonlinear adaptive
systems and the first results began to appear only in
the sixties.   During that period, pioneering research
toward understanding the theory of adaptive control
was conducted [B6, F1] and this laid down the
foundations of adaptive control research of today.   At
the present time [A1] this research has gained a lot
of momentum because of :   (1) the advent of digital
computers, and, in particular, microprocessors, and


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(2) the successful applications of adaptive control in the aircraft industry.

Most application areas of adaptive control can be mathematically modeled by multi-variable systems with some or all parameters unknown. The control of such systems can not be handled by deterministic control theory. The unknown parameters are modeled by random variables and the unknown disturbances in the system are modeled as stochastic processes and their control constitutes the framework of stochastic control theory. The use of the Proportional-Integral-Derivative (PID) regulator for the control of such industrial processes is appealing for its simplicity. For an industrial process, however, it is a colossal task to tune a large number of control gains involved. Under these circumstances, adaptive control is needed. The adaptive control techniques handle the industrial processes with uncertain parameters by combining system identification and control design. In the Bayesian framework these controllers assume that the parameters have prior probability density functions and large

uncertainty associated with their initial estimates.
In the process of simultaneous system identification
and control, these controllers reduce the uncertainty
associated with the parameter estimates, i.e.,
*learn* and *control* the system.   This is the
basic philosophy of adaptive control.

The design of a controller is a result of an
optimization algorithm on a performance index or cost
function.   This index is generally defined as a
function of system's actual output and its desired
output.   For systems with uncertain parameters, the
control solution which optimizes over a multistage
horizon is obtained by solving the stochastic dynamic
programming equation [B5, and eq(10) of this report].
However, it is not possible to achieve an optimal
solution because of the dimensionality involved in the
stochastic dynamic programming .   In such situations,
emphasis is put on obtaining a suboptimal solution

that incorporates the intrinsic properties of the optimal solution. For stochastic systems, the control has in general a dual effect [B1,F1]: it affects the system's state as well as the future state and/or parameter uncertainty. This property is shared by all control policies, whether, or not, it has the property incorporated in its design. Thus a control law, which explicitly utilizes this property in its design, called a *dual controller*, offers significant improvement potential for the control of uncertain linear plants. In multistage problems it *probes* the system to enhance real-time identification of the system's parameters in order to increase the accuracy of the subsequent control decisions and regulates the system at the same time [B3,D1]. Thus the controller has two different tasks and the dual controller *compromises between good control and good identification of the system.*

Simpler controllers which do not account for any dual effect are also investigated here. One of them estimates the system's parameters based upon all available information and uses those estimates as though they were true. This is called the Heuristic

Certainty Equivalence (HCE) controller [B1]. It is similar in form to the deterministic controller except it uses the parameters' estimates in the derivation of the control input. The other one, called the cautious controller, uses the parameter estimates as well as their associated current covariances. In an uncertain situation, the latter can be overly 'cautious' because of the parameter uncertainty. Another problem of this controller is the *turn-off* phenomenon, when the control almost vanishes over significant lengths of time. Thus the controller cannot estimate the system's parameters and loses control over the system [A1].

Two classes of dual controllers exist presently. In the first class [E1, G1, M1, M5, W1], the control minimizes a one-step-ahead criterion augmented by a second term which penalizes for poor identification. The approach is simple but does not fully exploit the dual property and often requires tuning of some parameters. Padilla and Cruz [P1] give a dual control solution for a plant by minimizing the

control objective function subject to an upper bound in the total estimation cost. Their objective function includes a standard cost function and also a constraint term which reflects the sensitivity of the parameters to the state of the system. Thus the solution adjusts itself to exercise better estimation for such sensitive parameters within the upper bound. The second class [B2, B4, S1, S2, T1] uses the stochastic dynamic programming equation and expands the future cost about a nominal trajectory. The approach of this second class is different from that discussed in [A1]. The method proposed in [A1] formulates the Stochastic Dynamic Programming Equation but suggests no expansion of the expected future cost about any nominal trajectory. Thus no minimization is possible explicitly except at the last step and the expected cost is minimized for two steps by numerical integration.

The recently developed linear feedback dual controller of [B4] is based upon a first order Taylor series expansion of the expected future cost and is called the first order dual solution, D1. This solution, D1, although simple, does not capture all

the dual effect available from the future expected cost [M4]. A second order Taylor series expansion handles it better and yields the second order dual solution, D2, in [M2]. The D2 solution modifies the cautious controller with a numerator "probing" term and a denominator correction term. Performance comparisons are available in [M2] among the cautious, D1 and D2 solutions for a *scalar* model. Both the cautious and the D1 solutions turn off but the D2 solution avoids turn-off, indicating that D1 is not a satisfactory dual solution. In this dissertation, the D2 solution is developed for multi-variable input-output system in Chapter 2 and both the cautious and the D2 solutions are applied to a multi-variable input-output system. Monte Carlo simulations are made which indicate that the D2 solution prevents the turn-off phenomenon prevalent with a cautious solution. However, there are few occasions where it demonstrates excessive probing; this is handled by a control limiter. A second order Taylor series expansion of the future expected cost is performed

about a nominal trajectory and a dual controller is developed and applied to a MIMO dynamic (ARMA of lag one) model in Chapter 3. Monte Carlo simulations and parametric and nonparametric statistical tests of significance indicate the superiority of the dual over the cautious and the heuristic certainty equivalence controllers.

# Chapter 2

## Dual Control and Prevention of the Turn-Off
## Phenomenon in a Class of MIMO Systems

### 2.1 INTRODUCTION

In this chapter, a dual solution is developed
based on a second order expansion of the expected
future cost and both the cautious and the D2 solutions
are applied to a multi-variable input-output system.
Monte Carlo simulations are made which indicate that
the D2 solution prevents the turn-off phenomenon
prevalent with a cautious solution. However, there
are few occasions where it demonstrates excessive
probing; this is handled by a control limiter. Monte
Carlo simulations and statistical tests of
significance indicate the superiority of the dual over
the cautious and the heuristic certainty equivalence
controllers.

Section 2 gives the problem formulation. Section 3 discusses the turn-off phenomenon observed in a stochastic environment. The approximate dual controller for the multi-variable input-output system is provided in Section 4. Section 5 describes the simulation of the plant and compares the performances of the cautious, dual (D2) and the HCE solutions. Section 6 concludes the chapter.

## 2.2 PROBLEM FORMULATION

The multivariable plant considered is

$$x(k+1) = c + B\,u(k) \qquad (1)$$

where  c  is an unknown vector and  B  is a matrix of unknown parameters.  This static model with constant parameters represents a simplified helicopter vibration control problem under steady flight conditions [M4, W2] and defines a relationship between the *higher harmonic control* input vector  u  and the vector  x  of vibration output amplitudes.  These controls can cancel some of the unsteady air loads on the blades.  The unknown elements of  c  and  B comprise the parameter vector  $\theta(k)$  whose estimate at time  k  is  $\hat{\theta}(k)$  with covariance matrix  $P(k)$  .  Assuming the parameters are time-invariant, we have

$$\theta(k+1) = \theta(k) \qquad (2)$$

The measurement vector is given by.

$$y(k) = x(k) + w(k) \tag{3}$$

where

$$E[w(k)] = 0; \quad E[w(k)w'(j)] = W\delta_{kj} \tag{4}$$

with $x(k)$ , $y(k)$ being $n$ dimensional vectors. The performance criterion to be minimized is the expected value of the cost from step $0$ to $N$ ,

$$J(0) = E\{C(0)\} = E\{\sum_{k=1}^{N} x'(k)Qx(k) + u'(k-1)Ru(k-1)II^k\} \tag{5}$$

where $N=2$ for the two-step horizon.

## 2.3 CAUTIOUS CONTROL AND THE TURN-OFF PHENOMENON

For the sake of illustration let us consider a scalar plant with one unknown gain parameter as

$$x(k+1) = c + b\, u(k) \qquad (6)$$

and obeying (2) - (5).

The cautious controller, designed with a one step horizon (N=1) , is obtained by minimizing (5) for the plant (6) with Q=1 and R=0 i.e.,

$$\min_{u(0)} E\{x^2(1)\} \qquad (7)$$

This is given by

$$u_c(0) = -\frac{\hat{b}(0)\, c}{\hat{b}^2(0) + P_b(0)} \qquad (8)$$

where $P_b(0)$ is the associated variance of the

parameter estimate $\hat{b}(0)$ .

The covariance update equation is

$$P_b(1) = \frac{P_b(0)W}{P_b(0)u_c^2(0) + W} \qquad (9)$$

In the case of constant but unknown parameter, the controller assumes initially that the parameter has a prior probability density function with a large uncertainty. The parameter uncertainty will evolve as (9) and, the controller tends to adapt itself to the system and gradually *learn* the system with time. From (8) it is clear that if $\hat{b}^2(0)$ is very small compared to $P_b(0)$ , the control $u_c(0)$ will also be very small. Moreover, if $u_c(0)$ is small, there is no learning and the covariance stays practically unchanged. When this situation occurs, it stays so until there is a large measurement noise which alters the parameter estimate and brings the

system out of *turn-off*. This leads often to a *burst* phenomenon. The dual controller presented here and in [M2] have sensitivity correction terms which are usually large in such situations and avoid the *turn-off* phenomenon. The occurrence of the *turn-off* phenomenon is well understood in the context of a scalar model. This is further discussed later for a multidimensional system in Section 5.

## 2.4 DUAL CONTROL WITH A TWO-STEP HORIZON

A dual control solution with a two-step horizon is obtained by minimizing (5) with respect to the control u(0) for the multidimensional plant (1)-(4). This is obtained by solving the general equation of Stochastic Dynamic Programming [B6, B7]

$$J^*(k) = \min_{u(k)} E\{C(k) + J^*(k+1)|I^k\} \quad k=N-1,....,1,0 \qquad (10)$$

where $J^*(k)$ is the *cost to go* from k to N and $I^k$ is the cumulated information at time k when the control u(k) is to be applied.

For N=1 , Eq. (10) becomes

$$J^*(0) = \min_{u(0)} E\{x'(1)Qx(1) + u'(0)Ru(0) + J^*(1) \mid I^0\} \qquad (11)$$

where $J^*(1)$ is the optimal cost at the last step and is obtained by minimization of $J(N-1)$ for $N=2$.

The last control is easily obtained by minimizing $J(1)$ and is given by

$$u^*(1) = -[ R + E\{B'(1)QB(1)|I^1\}]^{-1} E\{B'(1)Qc(1)|I^1\} \qquad (12)$$

Thus inserting $u^*(1)$ into $J(1)$ we obtain

$$J^*(1) = E\{c'(1)Qc(1)|I^1\}$$

$$- E\{c'(1)QB(1)|I^1\}[R + E\{B'(1)QB(1)|I^1\}]^{-1}E\{B'(1)Qc(1)|I^1\} \qquad (13)$$

where $E\{\cdot|I^1\}$ is the conditional expectation given the available information $I^1$.

The parameter vector estimate $\hat{\theta}(1)$ and the associated covariance matrix $P(1)$ are obtained from a Kalman filter according to

$$\hat{\theta}(1) = \hat{\theta}(0) + K(1)[y(1) - H(1)\hat{\theta}(0)] = \hat{\theta}(0) + K(1) \; v(1) \qquad (14)$$

$$K(1) = P(0) \; H'(1) \; [H(1)P(0)H'(1) + W]^{-1} \qquad (15)$$

$$P(1) = P(0) - K(1) \; H(1) \; P(0) \qquad (16)$$

where

$$H(1) = \text{diag} \; [\tilde{H}(1) \; , \; \tilde{H}(1)] \qquad (17)$$

$$\tilde{H}(1) = [1 \; u'(0)] \qquad (18)$$

From (13) it is clear that $J^*(1)$ is a nonlinear function of the estimated parameter vector $\hat{\theta}(1)$ and covariance $P(1)$ . But the estimated vector $\hat{\theta}(1)$ and the covariance $P(1)$ are not known until the control $u(0)$ is applied.

A control $u(0)$ with a two-step horizon can be obtained from (11) if a Taylor series expansion of $J^*(1)$ is performed about a suitable nominal

trajectory.   Here a second order expansion of

$J^*(1)$   is proposed about a nominal parameter

estimate   $\bar{\theta}(1)$   and a nominal covariance   $\bar{P}(1)$.

Expansion of (13) about   $\bar{\theta}(1) = \hat{\theta}(0)$

and   $\bar{P}(1)$   results in,

$$J^*(1) = J^*[1,\bar{\theta}(1),\bar{P}(1)] + [J_\theta(1)]'[\hat{\theta}(1) - \hat{\theta}(0)]$$

$$+ \frac{1}{2}[\hat{\theta}(1) - \hat{\theta}(0)]'J_{\theta\theta}(1)[\hat{\theta}(1) - \hat{\theta}(0)] + tr[J_P(1)\{P(1) - \bar{P}(1)\}] \quad (19)$$

where the sensitivities defined by

$$J_\theta(1) \triangleq \left[ \frac{\partial J^*(1)}{\partial \hat{\theta}_i(1)} \right] \quad (20)$$

$$J_{\theta\theta}(1) \triangleq \left[ \frac{\partial^2 J^*(1)}{\partial \hat{\theta}_i(1)\partial \hat{\theta}_j(1)} \right] \quad (21)$$

$$J_P(1) \triangleq \left[ \frac{\partial J^*(1)}{\partial P^{ij}(1)} \right] \quad (22)$$

are evaluated at $\bar{\theta}(1) = \hat{\theta}(0)$ and at $\bar{P}(1)$ ;

and $P^{ij}(1)$ is the ij-th element of the covariance matrix associated with the parameter estimates $\hat{\theta}_i(1)$ and $\hat{\theta}_j(1)$ . With this particular choice of $\bar{\theta}(1)$ and using (14) the conditional expected value of (19) is

$$E[J^*(1)|I^0] = J^*[1, \hat{\theta}(0), \bar{P}(1)]$$

$$+ \tfrac{1}{2}tr[J_{\theta\theta}(1)K(1) \, E\{v(1)v'(1)|I^0\}K'(1)] + tr[J_P(1)\{P(1) - \bar{P}(1)\}] \quad (23)$$

Making use of (15), (16) and the innovation covariance it is clear that (23) yields,

$$E[J^*(1)|I^0] = J^*[1, \hat{\theta}(0), \bar{P}(1)] + \tfrac{1}{2}tr[J_{\theta\theta}(1)(P(0) - P(1))]$$

$$+ tr[J_P(1)\{P(1) - \bar{P}(1)\}] \quad (24)$$

The expected future cost (24) is a function of the

covariances multiplied by appropriate sensitivity functions $J_{\theta\theta}(1)$ and $J_P(1)$ . These sensitivities introduce the dual effect into (11). For the first order dual solution D1 of [84] the sensitivity $J_{\theta\theta}(1)$ is not present and thus the second order dual solution D2 is expected to exploit better the dual effect in the problem. Again, it must be noted now that the covariance $P(1)$ is nonlinear in $u(0)$ and is not yet known. Hence a second order expansion of $P(1)$ is proposed about a nominal control $\bar{u}(0)$ and a nominal covariance $\bar{P}(1)$ in order to obtain a (suboptimal) dual solution $u_D(0)$ in a closed form from (11). Two choices of $\bar{u}(0)$ will be discussed later on when the implementation of the algorithm is described.

This expansion is performed as follows

$$P(1) = \bar{P}(1) + \sum_{i,j} e_i e'_j \{ P_u^{ij}(1)(u(0) - \bar{u}(0))$$

$$+ \frac{1}{2}[u(0) - \bar{u}(0)]'P_{uu}^{ij}(1)[u(0) - \bar{u}(0)]\} \qquad (25)$$

with superscript here denoting matrix element and

$$P_u^{ij}(1) \triangleq \left[ \frac{\partial P^{ij}(1)}{\partial u(0)} \right] \qquad (26)$$

$$P_{uu}^{ij}(1) \triangleq \left[ \frac{\partial^2 P^{ij}(1)}{\partial u^2(0)} \right] \qquad i,j = 1,....,n \qquad (27)$$

evaluated at $\bar{P}(1)$ and $\bar{u}(0)$ .

Now a (suboptimal) dual solution $u_D(0)$ can

be obtained from (11) using (24) - (27) and is given

by

$$u_D(0) = - [R + E\{B'(0)QB(0)|I^0\} + F]^{-1}[E\{B'(0)Qc(0)|I^0\} + f](28)$$

where the elements of the matrix  F  and those of the

vector  f  are given by

$$F_{i,j} = tr\left[ \frac{1}{2}(J_P(1) - \frac{1}{2}J_{\theta\theta}(1))\frac{\partial^2 P(1)}{\partial u_i(0)\partial u_j(0)} \right]$$

$$i,j = 1,....,m \qquad\qquad (29)$$

and

$$f_i = \sum_{j=1}^{m} tr\left[ \frac{1}{2}(J_P(1) - \frac{1}{2}J_{\theta\theta}(1))\left( \frac{\partial P(1)}{\partial u_i(0)} - \frac{\partial^2 P(1)}{\partial u_i(0)\partial u_j(0)}\bar{u}_j(0) \right) \right]$$

$$i=1,......,m \qquad\qquad (30)$$

m being the dimension of the control vector.

It is clear from (28) that this approximate dual solution $u_D(0)$ is a modification of the cautious solution by the sensitivity terms $J_P(1)$, $J_{\theta\theta}(1)$, $P_u(1)$, $P_{uu}(1)$. These account for the dual effect.

The implementation of this second order dual solution (D2) (28)-(30) can be performed in three ways:

(D2a) direct or explicit method,

(D2b) multidimensional grid search method, and

(D2c) adaptive grid search method.

These are summarized next:

Algorithm D2a

1. Choose a nominal control $\bar{u}(0)$.

2. Using this nominal control $\bar{u}(0)$ evaluate $\bar{P}(1)$ according to (15) - (18).

3. Using the $\bar{u}(0)$ , $\bar{\theta}(1) = \hat{\theta}(0)$ , $\bar{P}(1)$ , compute the sensitivities required in (29), (30) and obtain $u_0(0)$ from (28).

## Algorithms D2b and D2c

1. Choose a nominal control $\bar{u}(0)$ .

2. Using this nominal control $\bar{u}(0)$ evaluate $\bar{P}(1)$ according to (15) - (18). This is the first nominal control $\bar{u}(0)$ and covariance $\bar{P}(1)$ .

3. Compute the sensitivity functions $J_{\theta\theta}(1)$ , $J_P(1)$ for (24) with $\bar{\theta}(1) = \hat{\theta}(0)$ and the first nominal values $\bar{u}(0)$ , $\bar{P}(1)$ .

4. Search on (11) with (24) (with the sensitivity functions computed above) starting with the first nominal values $\bar{u}(0)$ , $\bar{P}(1)$ over $u(0)$ to obtain an improved nominal $u^1(0)$ for which $J^*(0)$ is lower than that with the first nominal $\bar{u}(0)$ . $P(1)$ is expanded about this $u^1(0)$ in

(25). This search is a fine multidimensional grid search in D2b. It is quite time consuming in terms of computation and may not be justified as a practical implementation. It is improved by the adaptive grid search in D2c. Instead of a fine multidimensional grid in D2b, a coarse grid is selected for D2c and an improved nominal control is obtained. Then another coarse grid is chosen about the latter nominal control over a narrower interval and a refined $u^I(0)$ is obtained. This reduces the computational burden considerably, especially for multidimensional systems.

5. Using this $u^I(0)$ compute $P_u(1)$, $P_{uu}(1)$; together with the previously computed $J_{\theta\theta}(1)$, $J_P(1)$ obtain $F$, $f$ from (29), (30) and get a $u_D(0)$ from (28).

## 2.5 SIMULATION RESULTS

Performance was evaluated from Monte Carlo runs for the following controllers:

1) Heuristic Certainty Equivalence,

2) One step ahead cautious controller, and

3) Dual solution (D2) based upon sensitivity correction (with two-step horizon).

This is implemented in three ways:

(D2a) direct or explicit method,

(D2b) multidimensional grid search method, and

(D2c) adaptive grid search method.

The plant equations are [M4, W2]

$$x_1(k+1) = \theta_1 + \theta_2 u_1(k) + \theta_3 u_2(k) \qquad (31)$$

$$x_2(k+1) = \theta_4 + \theta_5 u_1(k) + \theta_6 u_2(k) \qquad (32)$$

This model represents a simplified helicopter vibration control problem where the first state $x_1$ is the rotor hub force amplitude and the second state $x_2$ is the rotor blade bending moment amplitude at a given frequency (i.e., one of the harmonics of the rotor r.p.m.). The two controls are the *higher harmonic controls* and they cancel some of the unsteady air loads on the rotor.

The measurements are

$$y_1(k) = x_1(k) + w_1(k) \tag{33}$$

$$y_2(k) = x_2(k) + w_2(k) \tag{34}$$

where

$$E\{w(k)w'(j)\} = W\delta_{kj} = \text{diag } (W_1, W_2);$$

$$W_1 = 7.52^2, \ W_2 = 43^2 \tag{35}$$

Only the gain parameters were unknown and their

initial estimates were generated as $N(\theta_i, \theta_i^2)$, i = 2,3,5,6 where the true values are

$$\theta_1 = 23.8 \qquad \theta_4 = -135.87$$

$$\theta_2 = -74.84 \qquad \theta_5 = 53.31 \qquad (36)$$

$$\theta_3 = -51.04 \qquad \theta_6 = -82.56$$

A large uncertainty is chosen in the initial parameter estimates in order to test the learning capabilities of the various adaptive algorithms. The cost weighting matrices are

$$Q = diag(q_1, q_2): q_1 = 1.0, q_2 = 1.0$$

$$R = diag (r_1, r_2): r_1 = 0. , r_2 = 0. \qquad (37)$$

For the model chosen (31)-(36) the optimal control solution is

$$u_1^* = 1.0, u_2^* = -1.0 \qquad (38)$$

In terms of the notation of Section 2

$$c = \begin{bmatrix} \theta_1 \\ \theta_4 \end{bmatrix} \tag{39}$$

$$B = \begin{bmatrix} \theta_2 & \theta_3 \\ \theta_5 & \theta_6 \end{bmatrix} \tag{40}$$

$$u(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} \tag{41}$$

The controllers are implemented with a *sliding horizon* for a total of 20 time steps. The evaluation criterion is

$$C_k = q_1 x_1^2(k) + q_2 x_2^2(k) \tag{42}$$

## Analysis of the Monte Carlo Average Costs

Comparisons are made between the performances of the cautious and the various dual algorithms (D2a-D2c) on the system and a conventional statistical significance analysis is done using the normal theory approach (i.e., it is assumed that the central limit

theorem holds for the sample mean from a large number of runs). This is given in Appendix A. Tables 1-6 contain the results of the simulation runs. Table 1 compares the average cost $\bar{C}_k$ over 100 Monte Carlo runs for the first 10 time steps for HCE, Cautious and the dual algorithms, with an active control limiter $|u_1| \leq 1.5$, $1-1$, 2.

Clearly it is seen that the cumulative average cost is the lowest for the dual controller. The HCE increases the vibrations in time step 1 by using too large control magnitudes because of lack of caution. This however helps to learn the parameters faster and reduces the vibration earlier than the others. The dual controller sometimes demands large control magnitudes but less often than the HCE. In a realistic situation large control magnitudes are not permitted because of the active control limiters discussed above. Tables 2-4 provide a statistical significance test for the run with the limiter and

show that the dual solutions improve upon the cautious

solution on the average by 60% with at least 95%

confidence. Table 5 shows the percentile test

comparing the cautious and the dual (D2c) solutions

(Appendix A). It clearly indicates that from time

steps 3 onwards the tail of the dual is lighter than

that of the cautious solution. This test was carried

out for 500 Monte Carlo runs. Also a sample

distribution function plot was made for the vibration

cost at each time step comparing the two algorithms;

figures 6, 7 are typical examples. From the plots a

threshold value of 5000 was chosen for the cost and

Table VI indicates the percentage of runs the

vibration cost exceeds 5000 for the two algorithms.

This also indicates the light tailed nature of the

distribution obtained by the dual algorithm.


Individual Time History Runs


Analysis of the Monte Carlo Average Cost

indicates the improvement offered by the dual

solution: but provides no information about the

cautious controls turning off. Hence a careful investigation of the individual runs is required to discover these occurrences. Turn-off phenomenon is observed in many runs among the 100 Monte Carlo runs while using the cautious controller; runs 11, 60, 94, 98 are typical examples of it. In run 11 control 2 turns off between the time steps 0 and 16. In run 60, the control 2 turns off between the time steps 0 and 8. In run 94 also the control 2 turns off between the time steps 0 and 6. The worst case of turn-off occurs in run 98. Both the controls are off between the time steps 0 and 12. Here at time step 13 another interesting phenomenon called *burst* occurs. The cautious control exceeds the limits and this reflects in a small hump in the cost curve at time step 13. In all these cases the dual does better and avoids the *turn-off* and the *burst* phenomena. As explained in Section 3, the control for a constant parameter plant revokes from the turn-off situation by the burst phenomenon. A large measurement noise helps

the plant to come back to life by causing a
*burst*.   Run 89 (Figure 5) is a case where the
cautious controller exercises excess of caution and is
slow in convergence.   This is avoided by the dual
solution.   These cases are portrayed in Figures 3-7.
For the latter case, the controller goes to the right
direction of control by utilizing the dual effect from
the very outset.  Analysis of the simulation runs has
shown that this new dual control solution applied to a
multi-variable input-output model improves the cost on
the average by 60%.   The key improvement is in the
avoiding of situations like turn-off, burst and slow
convergence, typical of the cautious solution.

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OF POOR QUALITY

| Time Step $k$ | HCE | | Cautious | | D2a | | D2b | | D2c | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\bar{c}_k$ | $\sum_{i=1}^{k}\bar{c}_i$ | $\bar{c}_k$ | $\sum_{i=1}^{k}\bar{c}_i$ | $\bar{c}_k$ | $\sum_{i=1}^{k}\bar{c}_i$ | $\bar{c}_k$ | $\sum_{i=1}^{k}\bar{c}_i$ | $\bar{c}_k$ | $\sum_{i=1}^{k}\bar{c}_i$ |
| 1 | 21931 | 21931 | 11223 | 11223 | 12527 | 12527 | 10869 | 10869 | 11117 | 11117 |
| 2 | 4900 | 26831 | 7202 | 18425 | 9055 | 21582 | 7430 | 18299 | 5932 | 17049 |
| 3 | 1778 | 28609 | 3984 | 22409 | 3943 | 25525 | 1886 | 20185 | 1949 | 18998 |
| 4 | 651 | 29260 | 2788 | 25197 | 1656 | 27181 | 1486 | 21671 | 1673 | 20671 |
| 5 | 399 | 29659 | 2092 | 27289 | 1037 | 28218 | 720 | 22391 | 1173 | 21844 |
| 6 | 331 | 29990 | 1690 | 28979 | 1137 | 29355 | 687 | 23078 | 626 | 22470 |
| 7 | 225 | 30215 | 1530 | 30509 | 479 | 29834 | 432 | 23510 | 435 | 22905 |
| 8 | 203 | 30418 | 1027 | 31536 | 422 | 30256 | 357 | 23867 | 372 | 23277 |
| 9 | 216 | 30634 | 989 | 32525 | 435 | 30691 | 374 | 24241 | 387 | 23664 |
| 10 | 187 | 30821 | 770 | 33295 | 385 | 31076 | 294 | 24535 | 304 | 23968 |

Table 1: Average costs for the three algorithms in the
static model with a limiter (100 Monte Carlo
Runs; $|u_1| \leq 1.5$; $|u_2| \leq 1.5$)

| Time Step | Test Statistic | Estimated Improvement |
|:---:|:---:|:---:|
| k | $z_k$ | $EI_k$ % |
| 1 | -1.0 | -11. |
| 2 | -1.2 | -26. |
| 3 | .03 | 1.0 |
| 4 | 2.4 | 40. |
| 5 | 2.1 | 50. |
| 6 | 1.0 | 32. |
| 7 | 2.5 | 69. |
| 8 | 2.1 | 59. |
| 9 | 1.9 | 56. |
| 10 | 1.5 | 50. |

Table 2 :   Statistical significance test for comparisons of cautious and the dual algorithms (D2a) in the static model with a limiter ( $|u_1| \le 1.5$, $|u_2| \le 1.5$ ) (100 Monte Carlo Runs).

| Time Step | Test Statistic | Estimated Improvement |
|:---:|:---:|:---:|
| k | $z_k$ | $EI_k$ % |
| 1 | 1.2 | 3. |
| 2 | -.17 | -3. |
| 3 | 4.6 | 53. |
| 4 | 2.4 | 47. |
| 5 | 3.5 | 66. |
| 6 | 2.6 | 59. |
| 7 | 2.8 | 72. |
| 8 | 2.8 | 65. |
| 9 | 2.5 | 62. |
| 10 | 2.3 | 62. |

Table 3 : Statistical significance test for comparisons of cautious and the dual algorithms (D2b) in the static model with a limiter ( $|u_1| \le 1.5$, $|u_2| \le 1.5$ ) (100 Monte Carlo Runs).

| Time Step | Test Statistic | Estimated Improvement |
|-----------|----------------|------------------------|
| k | $z_k$ | $EI_k$ % |
| 1 | .29 | .9 |
| 2 | 2.2 | 17. |
| 3 | 4.4 | 51. |
| 4 | 2.0 | 40. |
| 5 | 1.7 | 44. |
| 6 | 2.7 | 63. |
| 7 | 2.8 | 71. |
| 8 | 2.7 | 64. |
| 9 | 2.4 | 61. |
| 10 | 2.2 | 60. |

Table 4 :  Statistical significance test for comparisons of cautious and the dual algorithms (D2c) in the static model with a limiter ( $| u_1 | \le 1.5$, $| u_2 | \le 1.5$ ) (100 Monte Carlo Runs).

| Time Step | $\chi^2$ test statistics at $K_{90}$ |
|:---------:|:------------------------------------:|
| 1 | 1.6 |
| 2 | 0.4 |
| 3 | 30 |
| 4 | 48 |
| 5 | 33 |
| 6 | 25 |
| 7 | 27 |
| 8 | 27 |
| 9 | 23 |
| 10 | 19 |

Table 5: Percentile test for comparisons of cautious and the dual algorithms (D2c) in the static model with a limiter (500 Monte Carlo Runs $|u_1| \le 1.5$, $|u_2| \le 1.5$)

| Time | Percentage of runs the vibration exceeds 5000 | |
| --- | --- | --- |
| k | Cautious | Dual |
| 1 | 94 | 86 |
| 2 | 41 | 29 |
| 3 | 23 | 8 |
| 4 | 16 | 2.4 |
| 5 | 11 | 1.2 |
| 6 | 9 | 0.8 |
| 7 | 7.4 | 0.8 |
| 8 | 6 | 0.6 |
| 9 | 6 | 0.6 |
| 10 | 5.2 | 0.4 |

Table 6: Comparison of the tails using the cautious and the dual algorithm (D2c) in the static model with a limiter (500 Monte Carlo Runs; $|u_1| \le 1.5$, $|u_2| \le 1.5$)

SAMPLE DISTRIBUTION TIME STEP 4
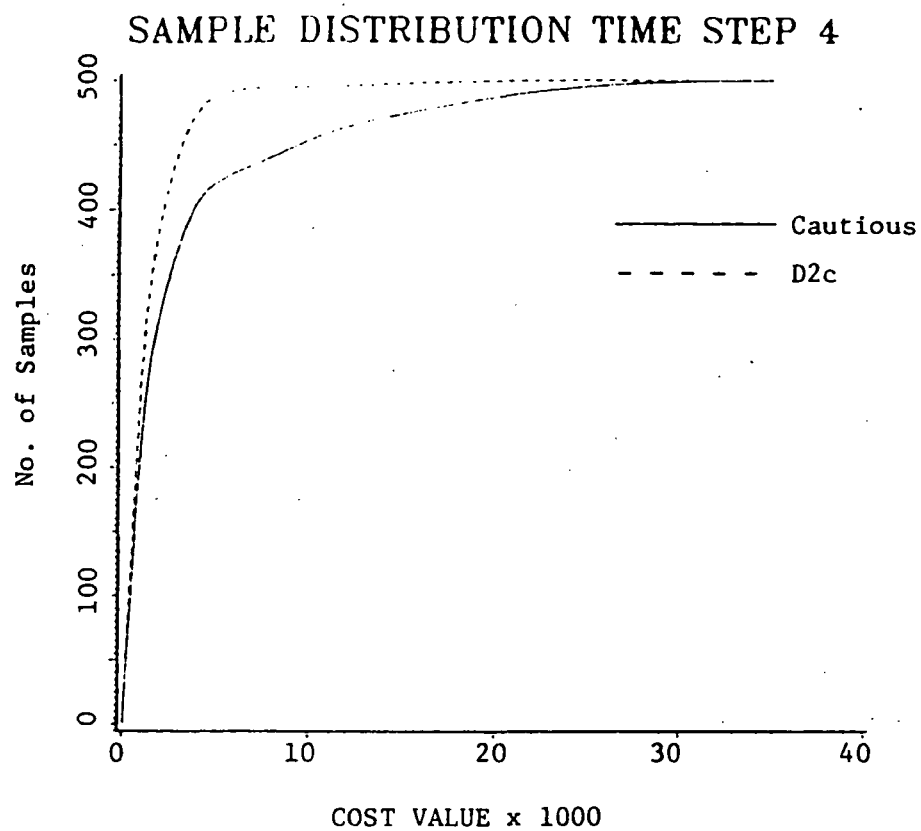


Fig. 1   Sample distribution of vibration cost using
         cautious and dual (D2c) algorithms (500 Monte
         Carlo Runs); ($|u_1| \leq 1.5$; $|u_2| \leq 1.5$)
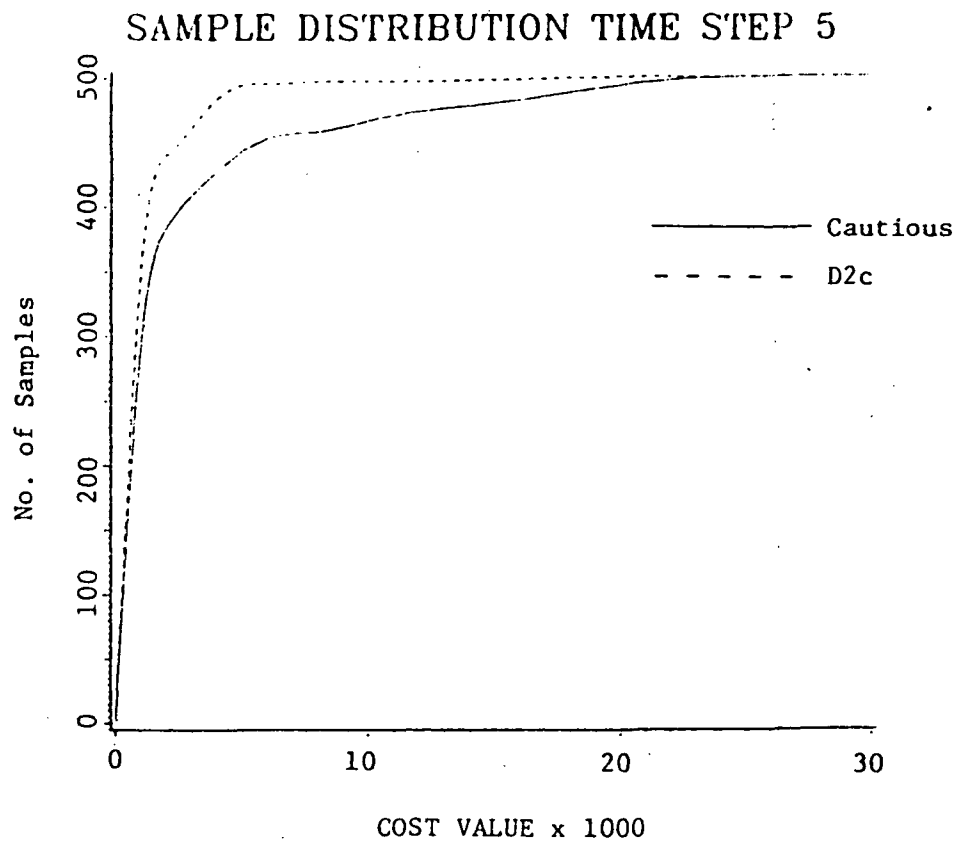
Fig. 2 Sample distribution of vibration cost using cautious and dual (D2c) algorithms (500 Monte Carlo Runs); ($|u_1| \leq 1.5$; $|u_2| \leq 1.5$)

PLOT 11



Fig. 3a   Time history of cost and controls using the
cautious and the dual algorithms for run 11
(100 Monte Carlo Runs: $|u_1| \leq 1.5$; $|u_2| \leq 1.5$)
(see pages 44, 45)
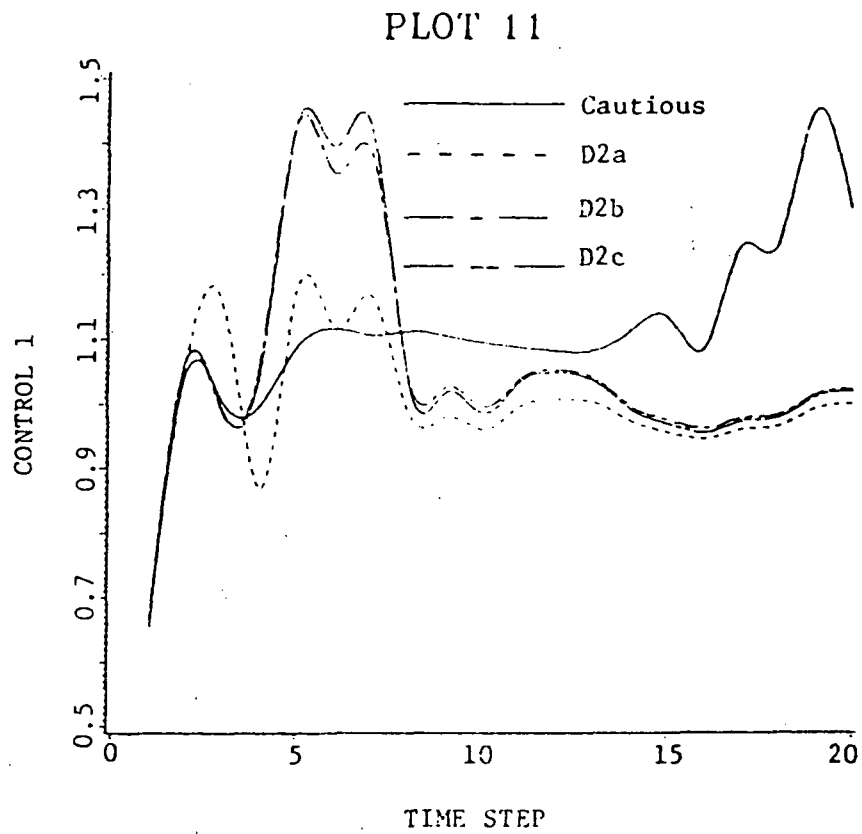
# PLOT 11



Fig. 3b  Control 1 (see pages 43, 45)

# PLOT 11
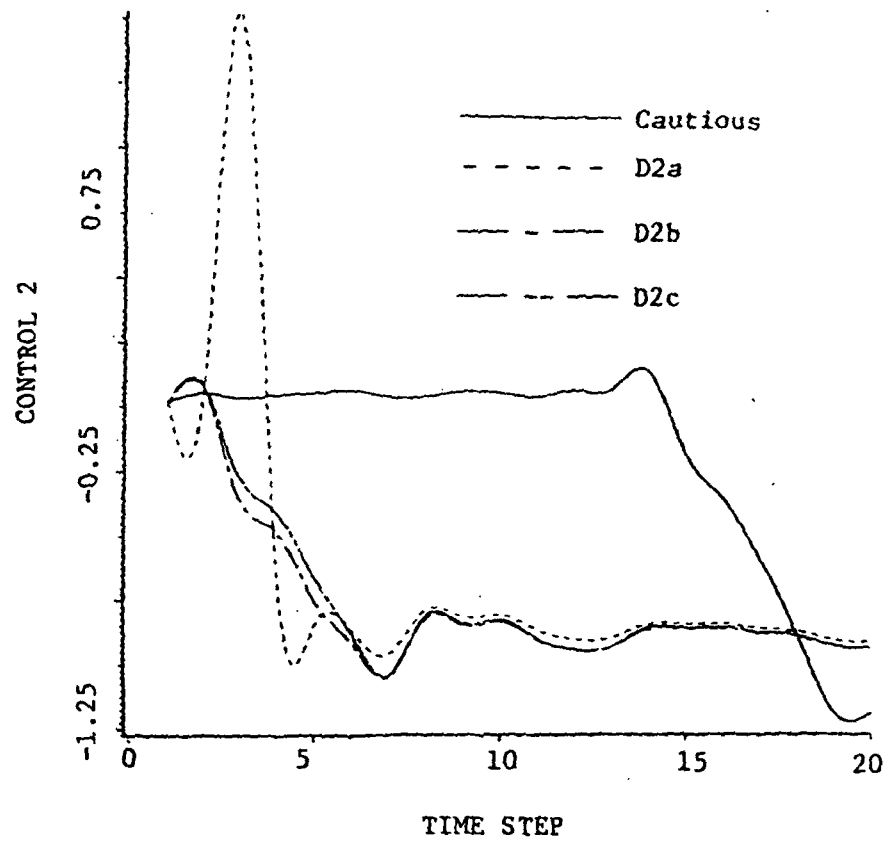


Fig. 3c  Control 2 (see pages 43, 44)

# PLOT 60
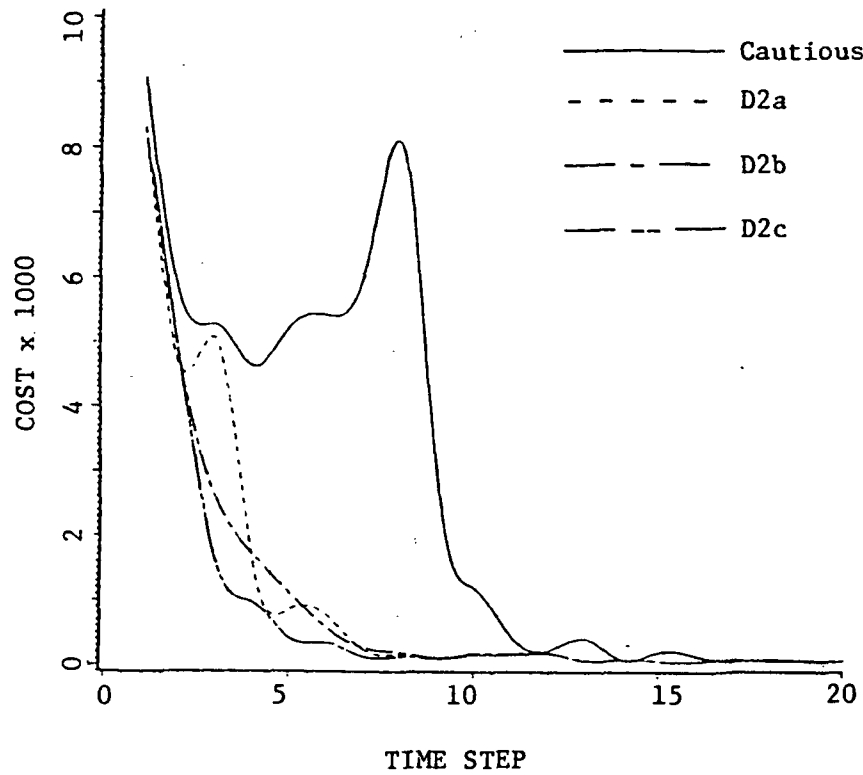


Fig. 4a   Time history of cost and controls using the
cautious and the dual algorithms for run 60
(100 Monte Carlo Runs: $|u_1| \leq 1.5$; $|u_2| \leq 1.5$)
(see pages 47, 48)

## PLOT 60
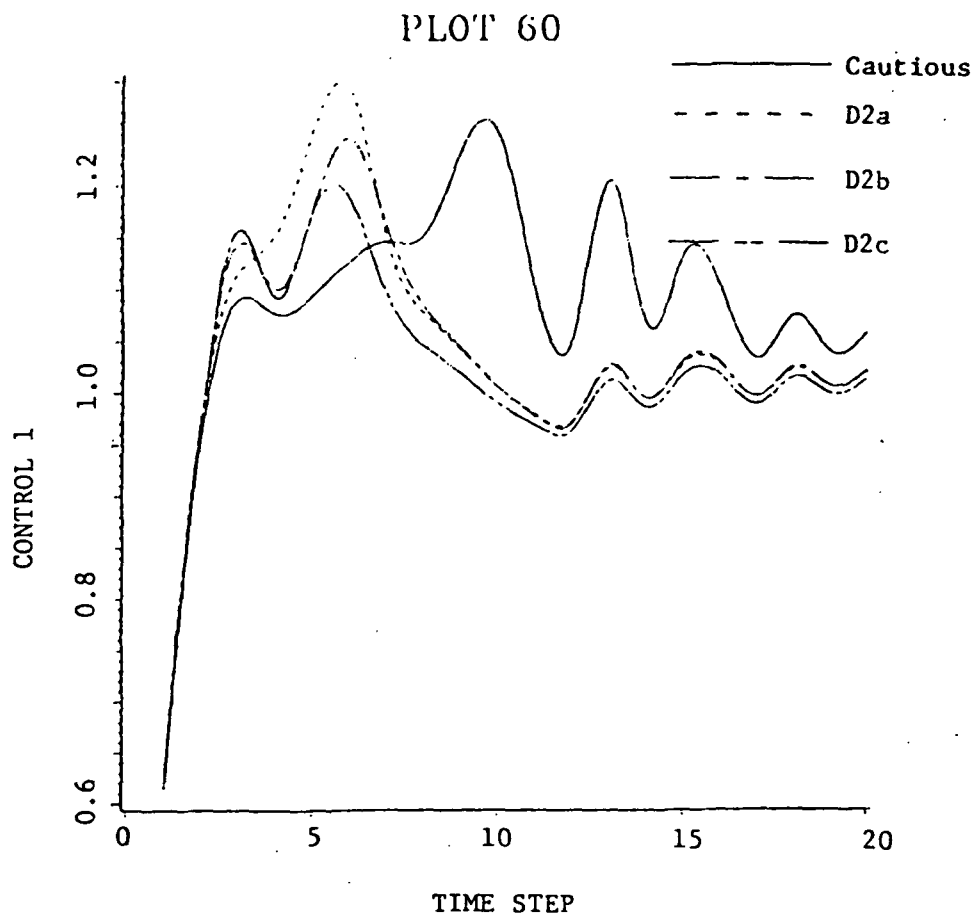
Fig. 4b    Control 1 (see pages 46, 48)

## PLOT 60
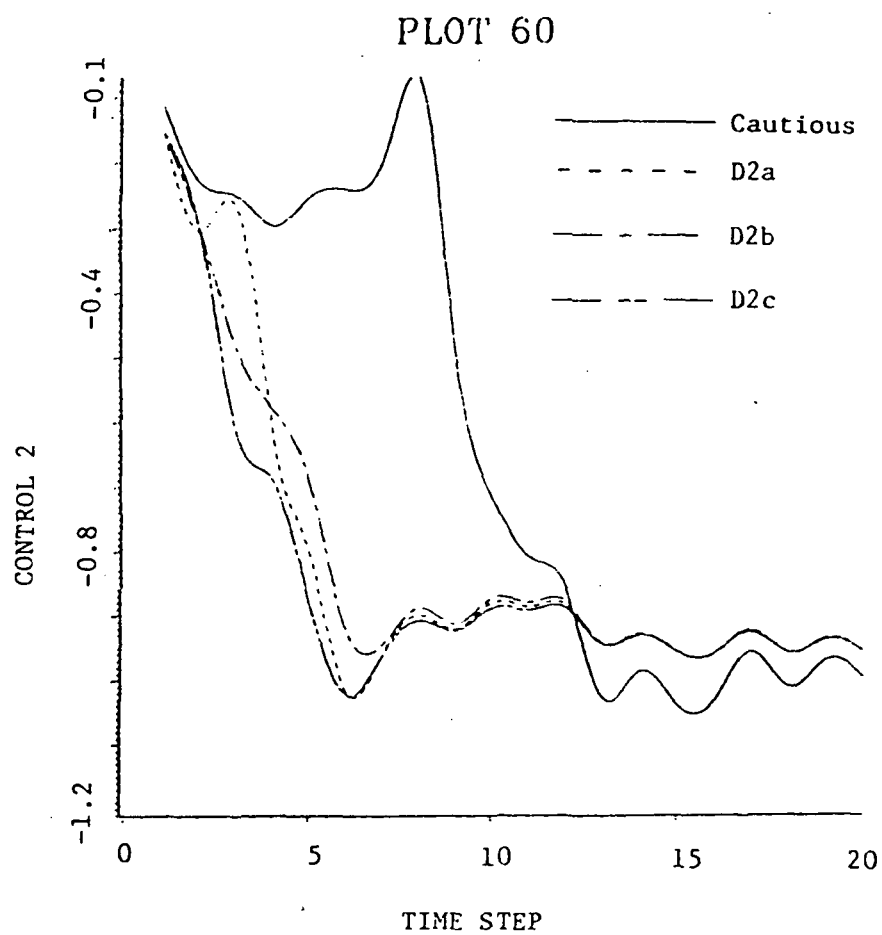
Fig. 4c   Control 2 (see pages 46, 47)

# PLOT 89
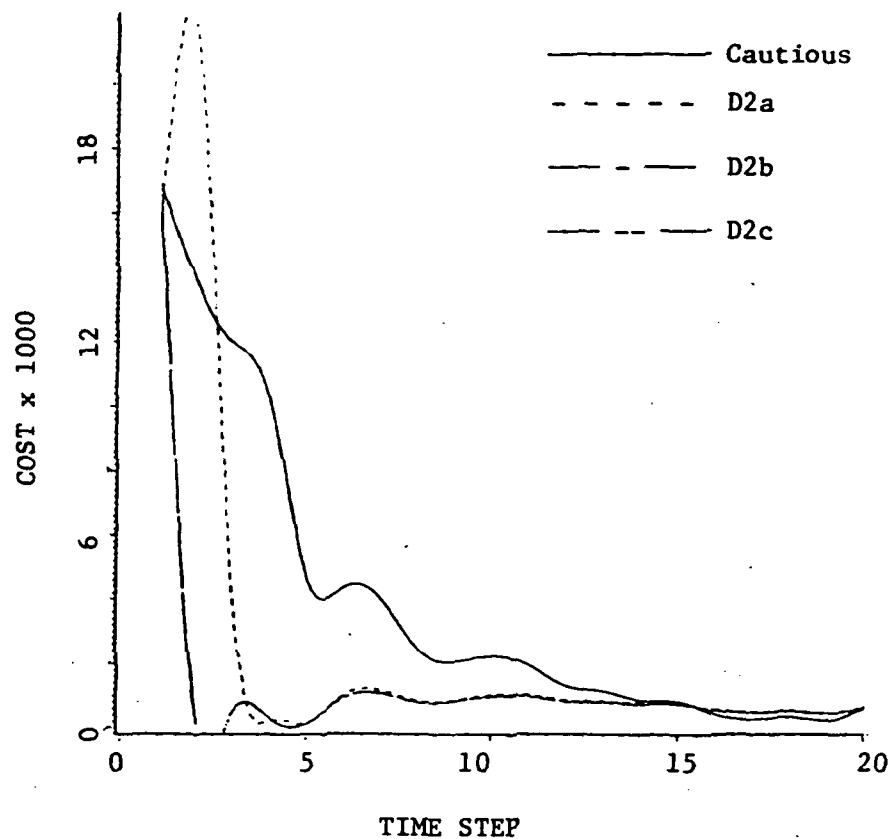


Fig. 5a  Time history of cost and controls using the cautious and the dual algorithms for run 89 (100 Monte Carlo Runs: $|u_1| \leq 1.5$; $|u_2| \leq 1.5$) (see pages 50, 51)

PLOT 89
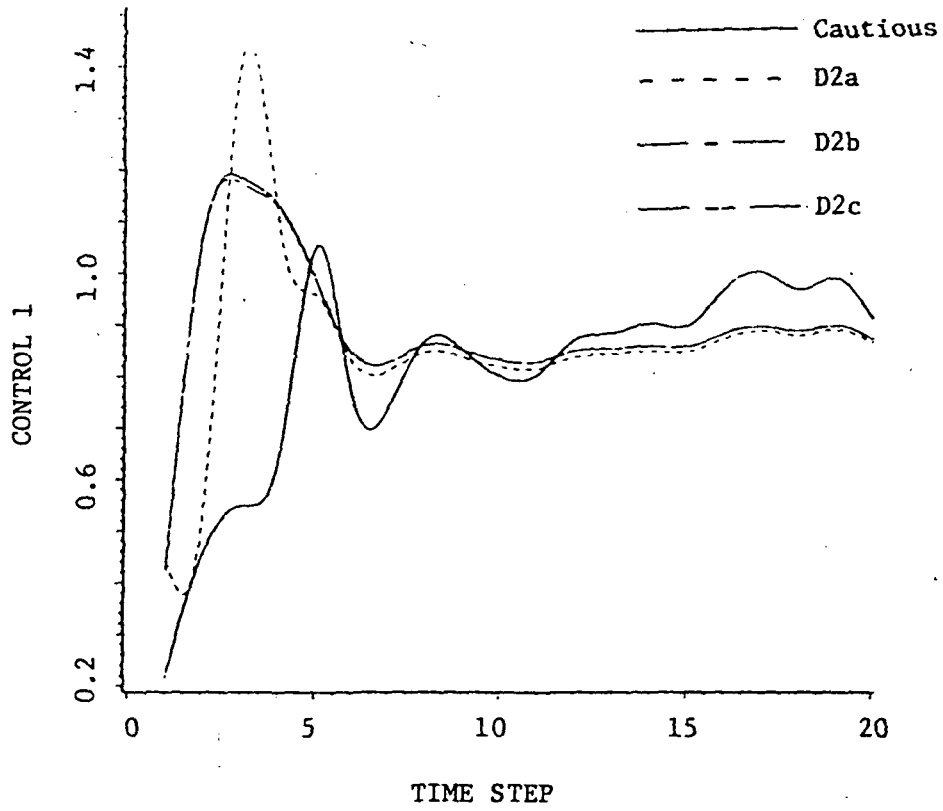
Fig. 5b   Control 1 (see pages 49, 51)

# PLOT 89
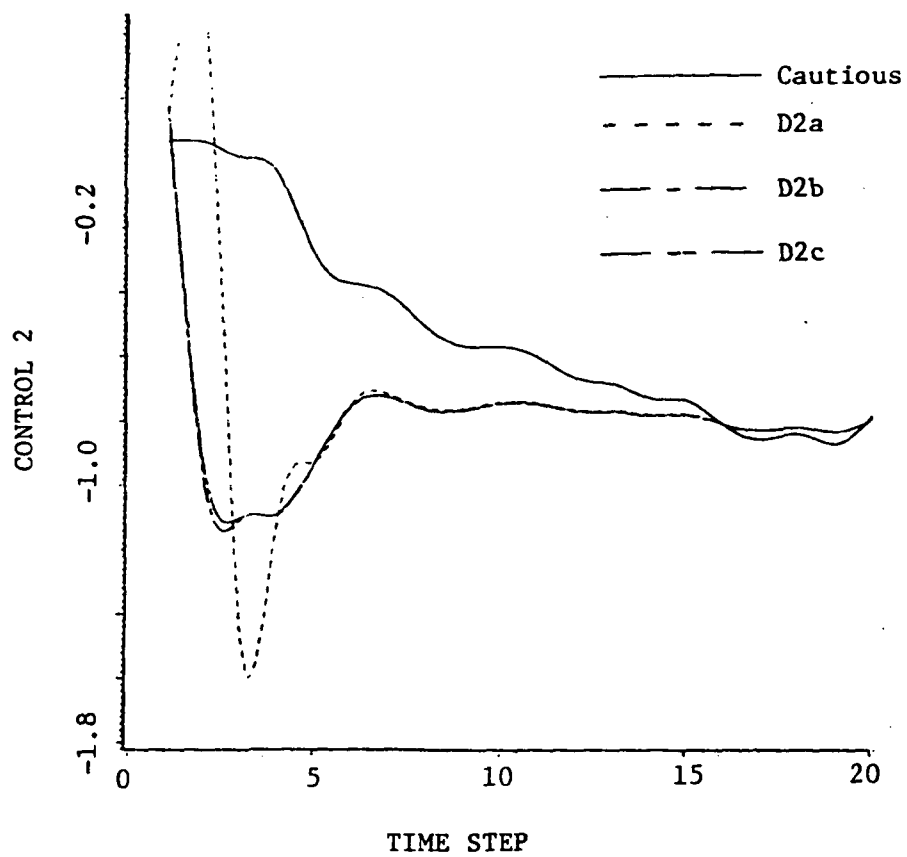


Fig. 5c  Control 2 (see pages 49, 50)

# PLOT 94
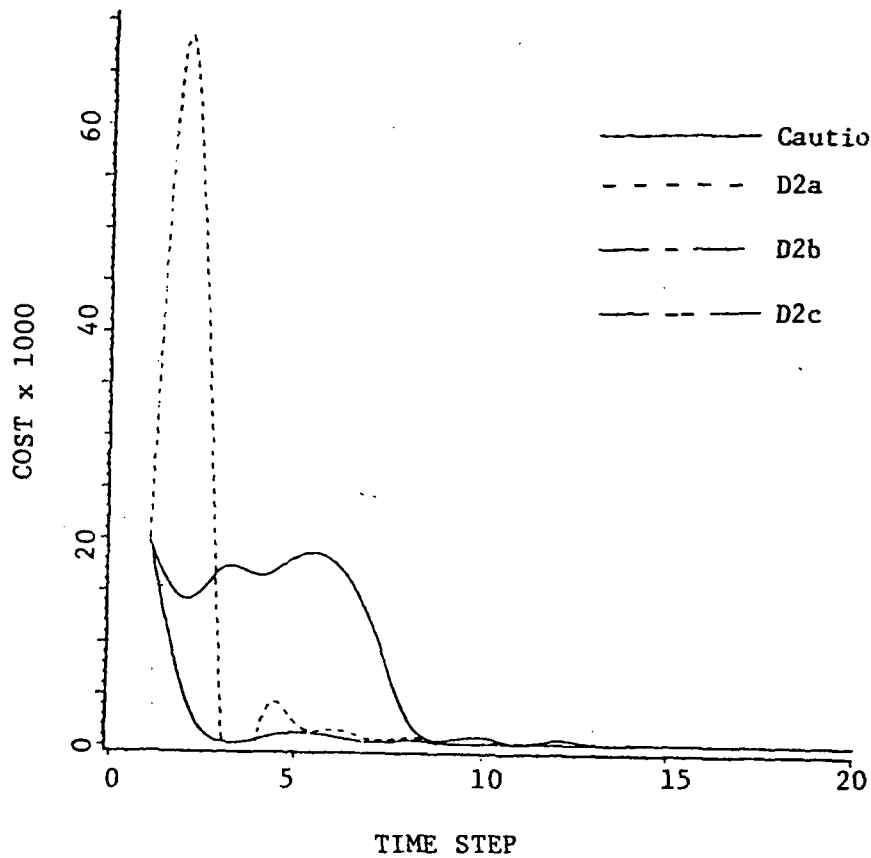


Fig. 6a  Time history of cost and controls using the
cautious and the dual algorithms for run 94
(100 Monte Carlo Runs: $|u_1| \leq 1.5$; $|u_2| \leq 1.5$)
(see pages 53, 54)
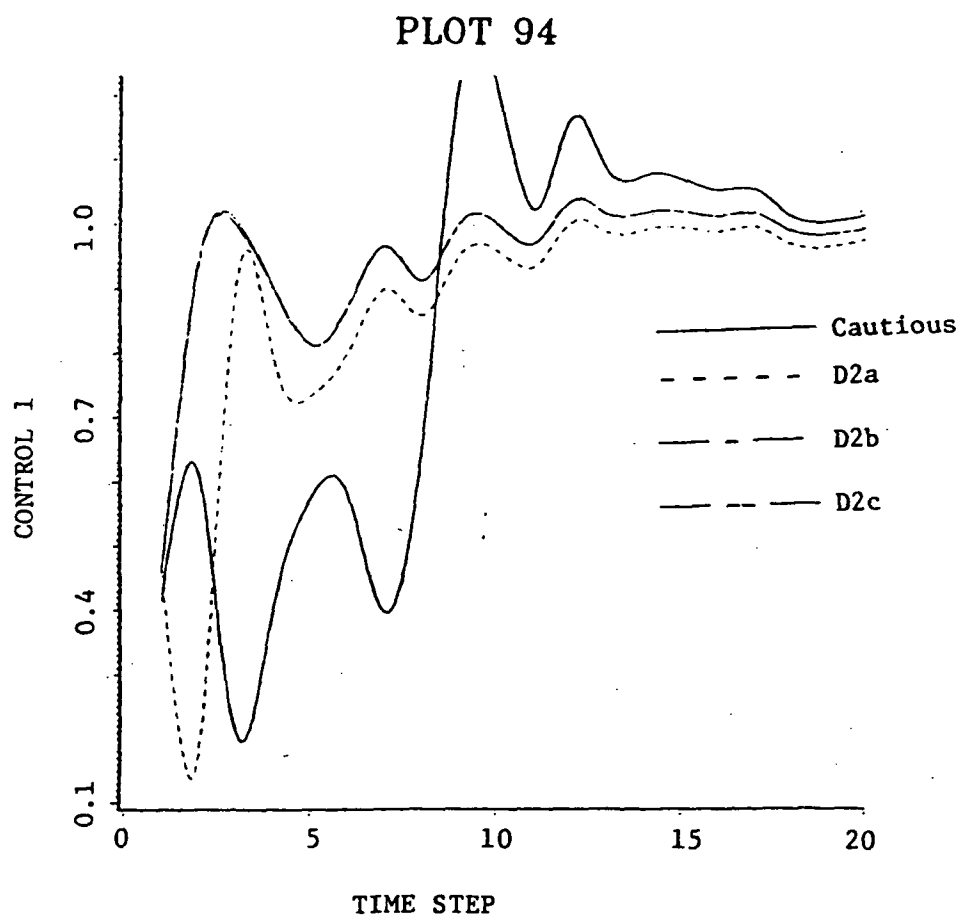
Fig. 6b   Control 1 (see pages 52, 54)

# PLOT 94
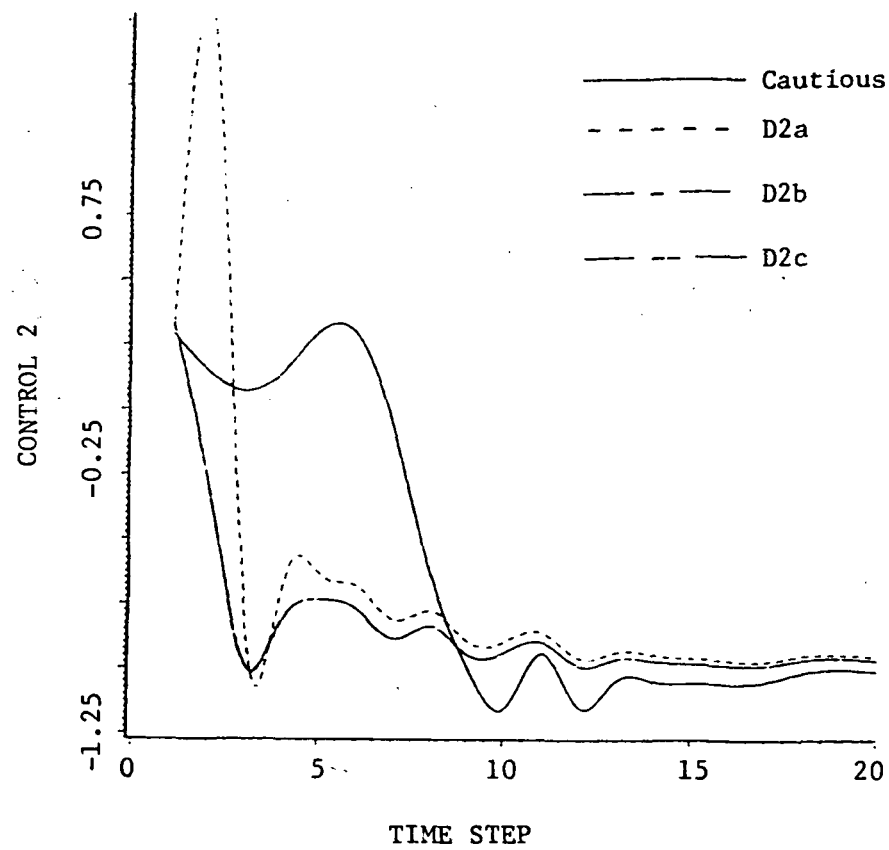


Fig. 6c   Control 2 (see pages 52, 53)

PLOT 98
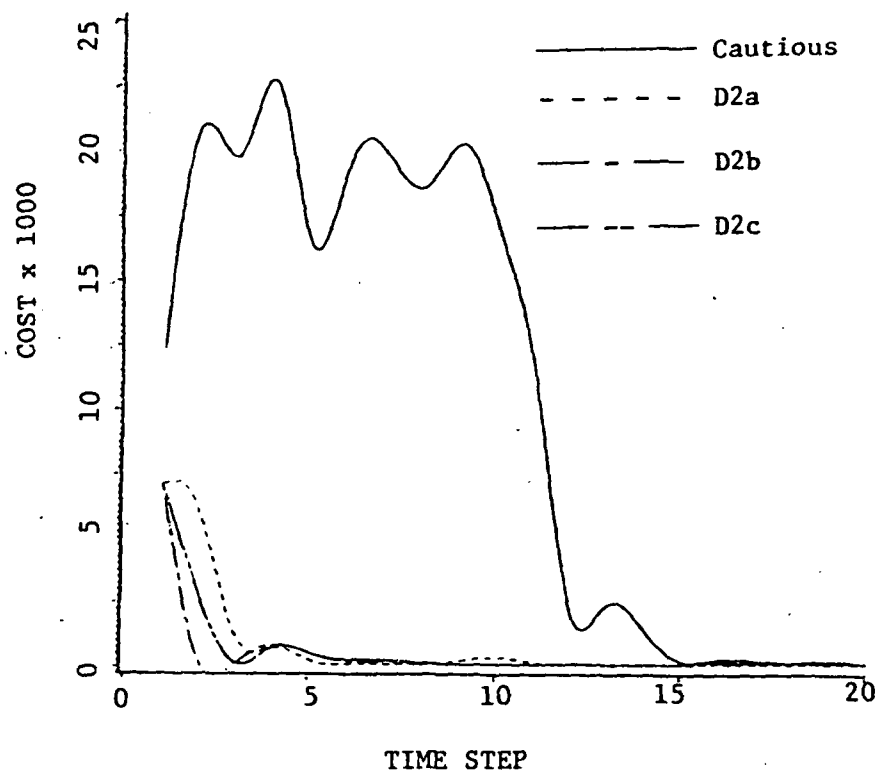


Fig. 7a  Time history of cost and controls using the
cautious and the dual algorithms for run 98
(100 Monte Carlo Runs: $|u_1| \leq 1.5$; $|u_2| \leq 1.5$)
(see pages 56, 57)

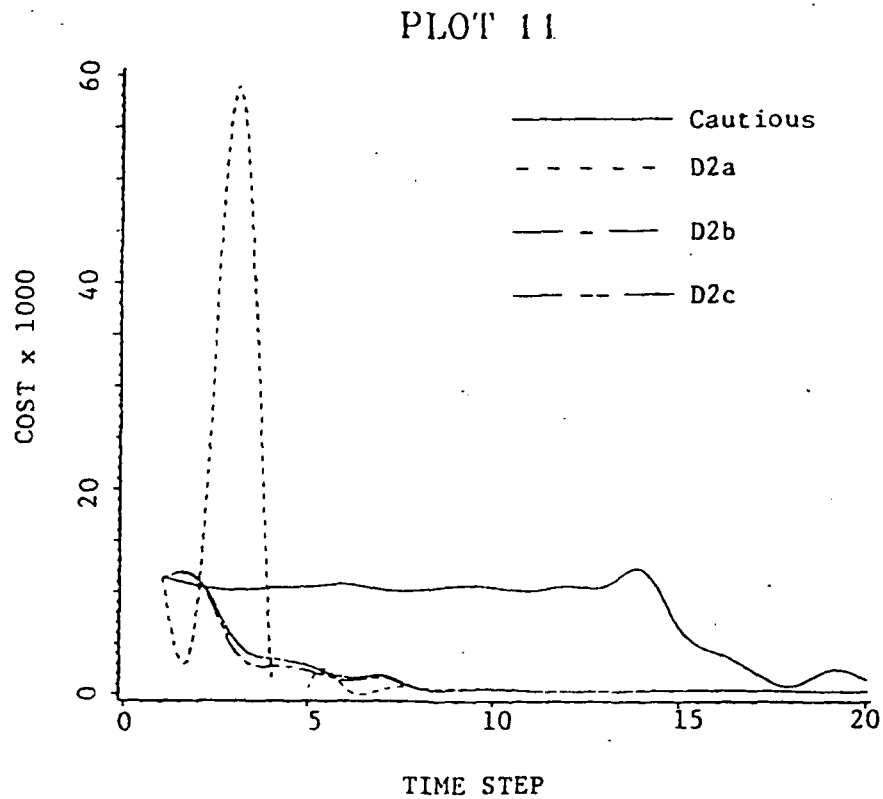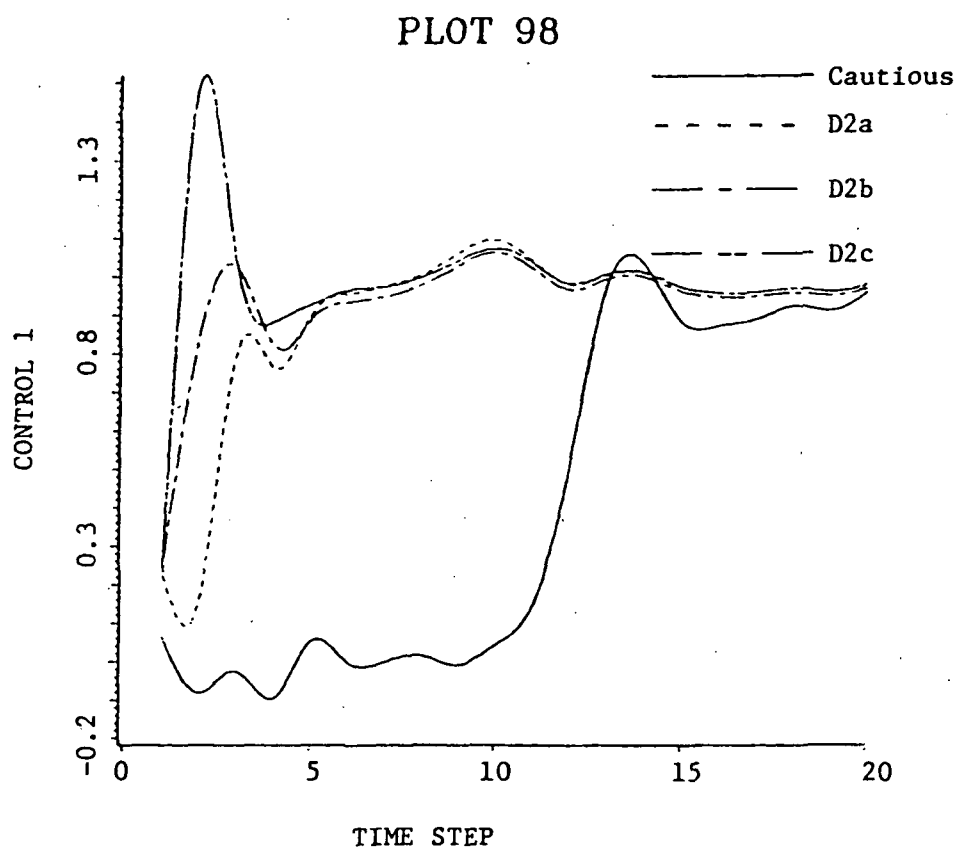Fig. 7b   Control 1 (see pages 55, 57)

PLOT 98
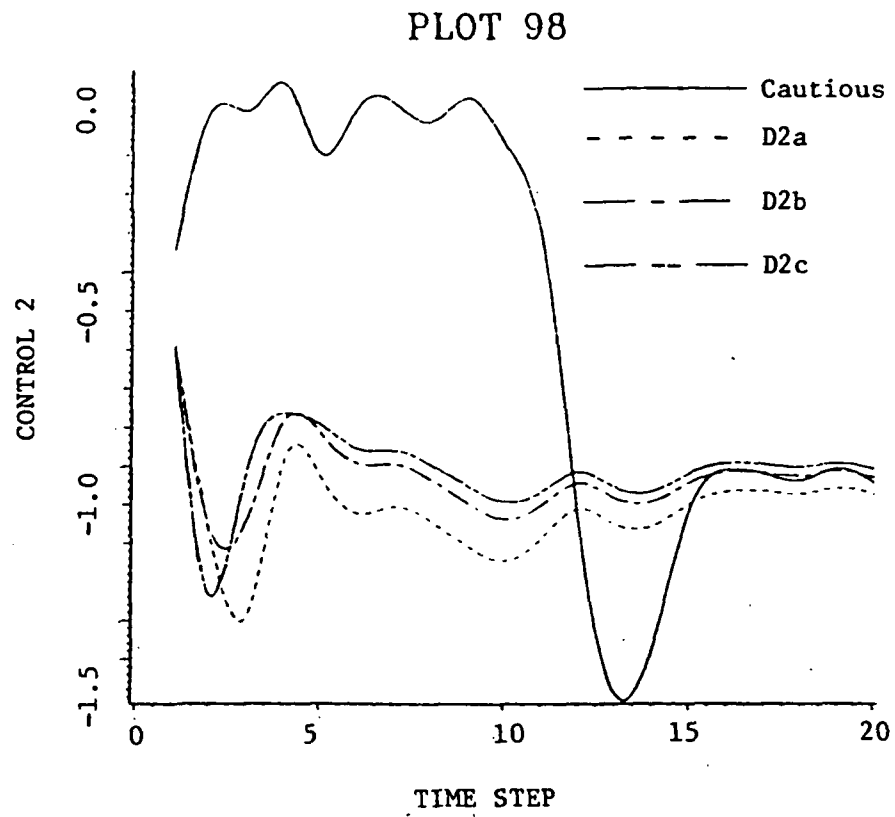


Fig. 7c  Control 2 (see pages 55, 56)

## 2.6 CONCLUSIONS

A new adaptive dual control solution is applied here to a multi-variable input-output system. This solution captures the dual effect by performing a second order Taylor series expansion of the expected future cost. It modifies the cautious solution by numerator and denominator correction terms. It also avoids problems of *turn-off, burst and slow convergence*, typical of the cautious solution.

Chapter 3

An Adaptive Dual Controller for

a Dynamic MIMO System.

## 3.1 INTRODUCTION

In this chapter a second order Taylor series expansion of the future expected cost is performed about a nominal trajectory and a dual controller is developed for a <u>MIMO</u> dynamic (ARMA of lag one) model. The cautious [W1, S1, M3] and the new dual controller are applied to a MIMO ARMA system. Monte Carlo simulations, based on parametric and nonparametric statistical analysis, indicate that the dual controller prevents the turn-off phenomenon and slow convergence prevalent with a cautious solution.

Section 2 gives the problem formulation. The approximate dual controller with a two-step horizon for the MIMO system is derived in Section 3. The

control solution is obtained by approximating the solution of the stochastic dynamic programming equation. A second order Taylor series expansion of the expected future cost is performed about a nominal trajectory and this leads to a dual control solution in a closed form. Following the derivations of the controller, a summary of the algorithm is given. Section 4 describes the simulation of the plant and compares the performances of the cautious and the dual solutions. Section 5 concludes the chapter.

## 3.2 PROBLEM FORMULATION

The MIMO system to be controlled is described by

$$y(k) = -A\ y(k-1) + B\ u(k-1) + e(k) \qquad (43)$$

where

$$E[e(k)] = 0 \quad ; \quad E[e(k)\ e'(j)] = W\delta_{kj} \qquad (44)$$

The parameter matrices A and B are unknown. This model describes some industrial processes like an ore crushing plant, or a heat exchanger [A2]. The unknown elements of A and B comprise the parameter vector $\theta(k)$ whose estimate at time k is $\hat{\theta}(k)$ with covariance matrix $P(k)$. The parameter vector is designated as

$$\theta(k) \triangleq [\ a'_1\ |\ b'_1\ |\ a'_2\ |\ b'_2\ |\ \dots\ |\ a'_n\ |\ b'_n\ ]' \qquad (45)$$

where  n  is the dimension of the output vector  $y(k)$

and  $a'_i$ ,  $b'_i$  are the ith row of the matrices

A  and  B , respectively.  Assuming the parameters are

time-invariant we have

$$\theta(k+1) = \theta(k) \qquad (46)$$

A measurement matrix  H(k)  is defined as

$$H(k) \triangleq \text{diag} \left[ -y'(k) \mid u'(k) , -y'(k) \mid u'(k) , \ldots \right] \qquad (47)$$

where  H(k)  has  n  rows.  For a better understanding

of the form of this matrix please refer to (81)

With these definitions the measurement model is

$$y(k) = H(k-1) \theta(k-1) + e(k) \qquad (48)$$

The performance criterion to be minimized is the expected value of the cost from step 0 to N ,

$$J(0) = E\{C(0)\} = E\left[\sum_{k=0}^{N-1}\{y(k+1) - y_r\}'Q(k)\{y(k+1) - y_r\}|I^k\right] \quad (49)$$

where $Q(k)$ is diagonal and $I^k$ is the cumulated information at time $k$ .

## 3.3 DUAL CONTROL WITH A TWO-STEP HORIZON

First the controller is derived and then a summary of the algorithm is provided.

A dual control solution with a two-step horizon is obtained by minimizing (49) with respect to the control u(0) for the multidimensional plant (43)-(46). This is obtained by solving the general equation of Stochastic Dynamic Programming [B2, B6, B7]

$$J^*(k) = \min_{u(k)} E\{C(k) + J^*(k+1)|I^k\} \qquad k=N-1,...,1,0 \qquad (50)$$

where $J^*(k)$ is the *cost to go* from $k$ to $N$ and $I^k$ is the cumulated information at time $k$ when the control $u(k)$ is to be applied.

Thus for a two-step horizon we have

$$J^*_{k,k+2} = \min_{u(k)} E\{C(k) + J^*_{k+1,k+2}||I^k\}$$

$$= \min_{u(k)} E[\{y(k+1) - y_r\}' Q(k) \{y(k+1) - y_r\} + J^*_{k+1,k+2}||I^k] \quad (51)$$

where $J^*_{k+1,k+2}$ is the optimal cost at the last step and is obtained by minimization of $J_{k+1,k+2}$ .

The cautious control with a one step sliding horizon at $k+1$ is given by

$$u(k+1) = [E\{B'Q(k+1)B||I^{k+1}\}]^{-1}E[B'Q(k+1)\{Ay(k+1) + y_r\}||I^{k+1}] \quad (52)$$

This helps us in obtaining the optimal cost to go at the penultimate stage.

The cost from step   k+1   to   k+2   is,

$$J_{k+1,k+2} = \text{tr } Q(k+1)W + E[\{Ay(k+1) + y_r\}'Q(k+1)\{Ay(k+1) + y_r\}$$

$$+ u'(k+1)B'Q(k+1)Bu(k+1) - 2\{Ay(k+1) + y_r\}'Q(k+1)Bu(k+1)|I^{k+1}] \quad (53)$$

and inserting (52) into (53) the optimal cost at the last step is,

$$J^*_{k+1,k+2} = \text{tr } Q(k+1)W + E[\{Ay(k+1) + y_r\}'Q(k+1)\{Ay(k+1) + y_r\}|I^{k+1}]$$

$$- E[\{A\,y(k+1) + y_r\}'Q(k+1)B|I^{k+1}][E\{B'Q(k+1)B|I^{k+1}\}]^{-1}.$$

$$E[B'Q(k+1)\{A\,y(k+1) + y_r\}|I^{k+1}] \quad (54)$$

where  $E\{\cdot\,|\,I^{k+1}\}$  is  the  conditional expectation given the available information $I^{k+1}$  .

The parameter vector estimate  $\hat{\theta}(k+1)$   and the

associated covariance matrix $P(k+1)$ are obtained from a Kalman filter according to

$$K(k+1) = P(k) H'(k) [H(k) P(k) H'(k) + W]^{-1} \qquad (55)$$

$$\hat{\theta}(k+1) = \hat{\theta}(k) + K(k+1) [y(k+1)) - H(k) \hat{\theta}(k)]$$

$$= \hat{\theta}(k) + K(k+1) v(k+1) \qquad (56)$$

$$P(k+1) = P(k) - P(k) H'(k) [H(k) P(k) H'(k) + W]^{-1} H(k) P(k) \qquad (57)$$

From (54) it is clear that $J^*_{k+1,k+2}$ is a nonlinear function of the estimated parameter vector $\hat{\theta}(k+1)$ and covariance $P(k+1)$ . But the estimated vector $\hat{\theta}(k+1)$ and the covariance $P(k+1)$ are not known until the control $u(k)$ is applied.

A control   u(k)   with a two-step horizon can be

obtained from (51) if a second order Taylor series

expansion of   $J^{*}_{k+1,k+2}$   is performed about a

suitable nominal trajectory.     Here the nominal

trajectory is defined by


1) a nominal parameter estimate   $\bar{\theta}(k+1)=\hat{\theta}(k)$

2) a nominal control   $\bar{u}(k)$

3) a nominal covariance   $\bar{P}(k+1)$   obtained by

   using   $\bar{u}(k)$

4) a nominal measurement   $\bar{y}(k+1)$   obtained by

   using   $\bar{u}(k)$   and   $\hat{\theta}(k)$   .

Expansion of (54) about this nominal trajectory results in

$$J^*_{k+1,k+2} = \bar{J}_1 + J'_y(k+1)[y(k+1) - \bar{y}(k+1)]$$

$$+ \frac{1}{2}[y(k+1) - \bar{y}(k+1)]'J_{yy}(k+1)[y(k+1) - \bar{y}(k+1)]$$

$$+ J'_\theta(k+1)[\hat{\theta}(k+1) - \hat{\theta}(k)] + tr[J_p(k+1)\{P(k+1) - \bar{P}(k+1)\}]$$

$$+ \frac{1}{2}[\hat{\theta}(k+1) - \hat{\theta}(k)]'J_{\theta\theta}(k+1)[\hat{\theta}(k+1)-\hat{\theta}(k)] \qquad (58)$$

where $\bar{J}_1$ is the zeroth order term and the cost sensitivities are

$$J_y(k+1) \triangleq \left[ \frac{\partial J^*_{k+1,k+2}}{\partial y_i(k+1)} \right] \qquad (59)$$

$$J_{yy}(k+1) \triangleq \left[ \frac{\partial^2 J^*_{k+1,k+2}}{\partial y_i(k+1)\partial y_j(k+1)} \right] \qquad (60)$$

$$J_\theta(k+1) \triangleq \left[ \frac{\partial J^*_{k+1,k+2}}{\partial \hat{\theta}_i(k+1)} \right] \qquad (61)$$

$$J_{\theta\theta}(k+1) \triangleq \left[ \frac{\partial^2 J_{k+1,k+2}}{\partial \hat{\theta}_i(k+1) \partial \hat{\theta}_j(k+1)} \right] \qquad (62)$$

$$J_p(k+1) \triangleq \left[ \frac{\partial J^*_{k+1,k+2}}{\partial P^{ij}(k+1)} \right] \qquad (63)$$

The above sensitivities are evaluated at $\hat{\theta}(k)$ , $\bar{P}(k+1)$ and $\bar{y}(k+1)$ ; and $P^{ij}(k+1)$ is the ij-th element of the covariance matrix associated with the parameter estimates $\hat{\theta}_i(k+1)$ and $\hat{\theta}_j(k+1)$ .

Under Gaussian assumption for the noise,

$$y(k+1) - \bar{y}(k+1) \sim \mathcal{N}[\mu, V] \qquad (64)$$

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where the mean is

$$\mu = E\{H(k)\theta(k) + e(k+1) - \bar{H}(k)\hat{\theta}(k) || I^k\}$$

$$= [H(k) - \bar{H}(k)]\hat{\theta}(k) \tag{65}$$

and the covariance is

$$V = E[\{y(k+1) - \bar{y}(k+1) - \mu\}\{y(k+1) - \bar{y}(k+1) - \mu\}' || I^k\}$$

$$= H(k)\, P(k)\, H'(k) + W \tag{66}$$

With the choice of the nominal path as defined

earlier and using (55), (65) and (66) the conditional

expected value of (54) is

$$E\{J^*_{k+1,k+2}|I^k\} = \bar{J}_1 + J'_y(k+1)[H(k) - \bar{H}(k)]\hat{\theta}(k) +$$

$$\frac{1}{2} \mu'J_{yy}(k+1)\mu + \frac{1}{2} tr[J_{yy}(k+1) V] + \frac{1}{2}tr[J_{\theta\theta}(k+1)\{P(k) - P(k+1)\}]$$

$$+ tr [J_p(k+1)\{P(k+1) - \bar{P}(k+1)\}] \qquad (67)$$

The above expected future cost (67) is a function of the nominal parameters multiplied by appropriate sensitivity functions $J_y(k+1)$ , $J_{yy}(k+1)$ , $J_{\theta\theta}(k+1)$ and $J_p(k+1)$ . These sensitivities introduce the dual effect into (51) which is then used to yield $u(k)$ . It must also be noted that the covariance $P(k+1)$ is nonlinear in $u(k)$ and is not yet known. Hence a second order expansion of $P(k+1)$ is proposed about a nominal control $\bar{u}(k)$ and a nominal covariance $\bar{P}(k+1)$

in order to obtain a (suboptimal) dual solution

$u_D(k)$ in a <u>closed form</u> from (51).

This expansion is performed as follows

$$P(k+1) = \bar{P}(k+1) + \sum_{i,j} e_i e'_j \{P_u^{ij}(k+1)[u(k) - \bar{u}(k)]$$

$$+ \frac{1}{2} [u(k) - \bar{u}(k)]'P_{uu}^{ij}(k+1)[u(k) - \bar{u}(k)]\} \qquad (68)$$

with superscript here denoting matrix element, $e_i$

the i-th cartesian basis vector and

$$P_u^{ij}(k+1) \triangleq \frac{\partial P^{ij}(k+1)}{\partial u(k)} \; ; \; P_{uu}^{ij}(k+1) \triangleq \frac{\partial^2 P^{ij}(k+1)}{\partial u^2(k)} \qquad i,j = 1,...,l \quad (69)$$

evaluated at $\bar{P}(k+1)$ and $\bar{u}(k)$ and $l$ the

number of unknown parameters.

Now a (suboptimal) dual solution $u_D(k)$ can be

obtained from (51) using (67)-(69) and is given in

closed form by

$$u_D(k) = [E\{B'Q(k)B||^k\} + F]^{-1} [E\{B'Q(k)(Ay(k) + y_r)||^k\} + f](70)$$

where the elements of the matrix $F$ and those of the

vector $f$ are given by

$$F_{i,j} = \frac{1}{2} tr\left[ \{J_P(k+1) - \frac{1}{2} J_{\theta\theta}(k+1)\} \frac{\partial^2 P(k+1)}{\partial u_i(k)\partial u_j(k)} \right]$$

$$+ \frac{1}{2} tr\left[ J_{yy}(k+1)\frac{\partial H(k)}{\partial u_i(k)} P(k)\left( \frac{\partial H(k)}{\partial u_j(k)} \right)' \right]$$

$$+ \frac{1}{2} tr\left[ J_{yy}(k+1)\left( \frac{\partial H(k)}{\partial u_i(k)} \hat{\theta}(k) \right)\left( \frac{\partial H(k)}{\partial u_j(k)} \hat{\theta}(k) \right)' \right] \qquad (71)$$

$$i,j=1,...,m$$

and

$$f_i = -\frac{1}{2}\left(\frac{\partial H(k)}{\partial u_i(k)}\hat{\theta}(k)\right)' J_y(k+1)$$

$$-\frac{1}{2}\,\mathrm{tr}\left[\left\{J_p(k+1) - \frac{1}{2}J_{\theta\theta}(k+1)\right\}\frac{\partial P(k+1)}{\partial u_i(k)}\right]$$

$$+\frac{1}{2}\sum_{j=1}^{m}\mathrm{tr}\left[\{J_p(k+1) - \frac{1}{2}J_{\theta\theta}(k+1)\}\frac{\partial^2 P(k+1)}{\partial u_i(k)\partial u_j(k)}\right]\bar{u}_j(k)$$

$$+\frac{1}{2}\,\mathrm{tr}\sum_{j=1}^{m}\mathrm{tr}\left[J_{yy}(k+1)\left(\frac{\partial H(k)}{\partial u_i(k)}\hat{\theta}(k)\right)\left(\frac{\partial H(k)}{\partial u_j(k)}\hat{\theta}k\right)'\right]\bar{u}_j(k)\quad(72)$$

$$i=1,\dots,m$$

and   m   is the dimension of the control vector.

It is clear from (70) that this approximate dual

solution $u_D(k)$ is a modification of the cautious solution by the cost sensitivity terms. The cautious solution is (70) with $F=0$ and $f=0$. These account for the dual effect. The implementation of this second order dual solution is performed by the method described below.

## Algorithm Summary

1) Compute the sensitivity functions $J_{\theta\theta}(k+1)$ , $J_p(k+1)$ , $J_y(k+1)$ , $J_{yy}(k+1)$ for (67) with $\bar{\theta}(k+1)=\hat{\theta}(k)$ and the nominal values $\bar{u}(k)$ , $\bar{P}(k+1)$ , $\bar{y}(k+1)$ defining the nominal path.

2) Search on (51) with (67) [with the sensitivity functions computed above, starting with first nominal values $\bar{u}(k)$ , $\bar{P}(k+1)$ ] over $u(k)$ to obtain an improved nominal for which

$J^*_{k,k+2}$  is  lower.  This  search  is  done  by
selecting  a  first coarse  grid.  A  grid  search  is
necessary  to  avoid  locking  in  on  a  local  minimum.
Then  another  grid  is  chosen  about  the  latter
control  over  a  narrower  interval  and  from  a  second
search  $u^I(k)$  is  obtained.  This  is  the  control
about  which  the  covariances  are  expanded.  It  is
not  the  control  law  applied.

3)  Using    $u^I(k)$    compute  the  covariance
sensitivities    $P_u(k+1)$  ,    $P_{uu}(k+1)$;
together  with  the  previously  computed  cost
sensitivities    $J_{\theta\theta}(k+1)$    ,    $J_p(k+1)$  ,
$J_{yy}(k+1)$  ,  $J_y(k+1)$  obtain  F  ,  f
defined  in  (71),  (72).  Finally  the  control  to  be
applied,    $u_D(k)$  ,  is  calculated  from  its
expression  (70)  in  terms  of  the  various
expectations  and  sensitivities.

The  iteration  described  in  step  2  above  is  carried
out  to  obtain  better  covariance  sensitivities.  The
control    $u_D(k)$    could  have  been  obtained  directly

from (70) by skipping step 2 above; however, as indicated in [M2, M3] this results in unsatisfactory performance. With this iteration of step 2, the "improved" sensitivities yield good performance as shown in the next section.

## 3.4 SIMULATION RESULTS

Performance is evaluated from 500 Monte Carlo runs

for the following controllers:

1) Heuristic Certainty Equivalence [B2] (with a

    one step horizon),

2)   One-step-ahead cautious controller, and

3) Dual controller based upon sensitivity

    functions (with a two-step horizon) derived

    in Sec. 3.

The plant equations for a 2-input 2-output system

are

$$y_1(k+1) = -a_{11}y_1(k) - a_{12}y_2(k) + b_{11}u_1(k) + b_{12}u_2(k) + e_1(k+1) \quad (73)$$

$$y_2(k+1) = -a_{21}y_1(k) - a_{22}y_2(k) + b_{21}u_1(k) + b_{22}u_1(k) + e_2(k+1) \quad (74)$$

where

$$E\{e(k)e'(j)\} = W\delta_{kj} = \text{diag}(W_1, W_2):$$

$$W_1 = 7.52^2 \; ; \; W_2 = 43^2 \tag{75}$$

The true values of the parameters are

$$
\begin{array}{llll}
a_{11} = .8 & & b_{11} = -74.84 \\
a_{12} = .1 & & b_{12} = -51.04 \\
a_{21} = .2 & & b_{21} = 53.31 & \qquad (76) \\
a_{22} = .75 & & b_{22} = -82.56
\end{array}
$$

Only the gain parameters (B matrix) are considered unknown for testing the dual effect and their initial estimates where generated as $\mathcal{N}(b_{ij}, b_{ij}^2)$, i, j=1, 2. This choice of system was motivated by the helicopter vibration study [M2].

A large initial uncertainty is chosen in the parameter estimates in order to test the learning capabilities of the various adaptive algorithms. The cost weighting matrices are

$$Q(k) = \text{diag } (q_1, q_2) : q_1 = 1.0 , q_2 = 1.0 \tag{77}$$

The desired response is

$$y_r = [-18 \quad 80]' \tag{78}$$

For the model chosen (73)-(78) the optimal control solution is

$$u_1^* = 1.0 , u_2^* = -1.0 \tag{79}$$

In terms of the notation of (45) and (47)

$$\hat{\theta}(k) \triangleq [a_{11} \quad a_{12} \quad \hat{b}_{11}(k) \quad \hat{b}_{21}(k) \quad a_{21} \quad a_{22} \quad \hat{b}_{21}(k) \quad \hat{b}_{22}(k)]' \tag{80}$$

and

$$H(k) \triangleq \begin{bmatrix} -y_1(k) & -y_2(k) & u_1(k) & u_2(k) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -y_1(k) & -y_2(k) & u_1(k) & u_2(k) \end{bmatrix} \quad (81)$$

The controllers are implemented with a *sliding horizon* for a total of 40 time steps. The evaluation criterion is

$$C_k = (y(k+1) - y_r)' \, Q(k) \, (y(k+1) - y_r) \quad (82)$$

## Analysis of the Monte Carlo Average Costs

Comparisons are made between the performances of the cautious and the dual algorithm on the system and a statistical significance analysis is done using the normal theory approach (i.e., it is assumed that the central limit theorem holds for the sample mean from

C-2

a large number of runs) [M3]. Tables 7-10 contains the results of the simulation runs. Table 7 compares the average cost $\bar{C}_k$ over 500 Monte Carlo runs for the first 40 time steps for HCE, cautious and the dual algorithms, with a control limiter $|u_l| \leq 2$, $l = 1, 2$.

Clearly it is seen that the cumulative average cost is the lowest for the dual controller. The HCE incurs an excessive penalty in time step 1 because of lack of caution. The cautious controller is overly cautious and exhibits slow convergence. However, the dual controller incurs less penalty in step 1 than the HCE and makes a judicious choice of caution and probing to learn the parameters fast. Figure 8 compares the performances of the three algorithms for 500 Monte Carlo runs.

Table 8 provides a statistical significance test and shows the improved performances of the dual

solution from time step 4 onwards with at least 98% confidence.

Table 9 indicates the percentage of runs the cost exceeds 2000 for the two algorithms. This threshold of 2000 is selected from a sample distribution study of the cost at each time step. Table 10 shows the percentile test [M3, N1] comparing the cautious and the dual solution. They clearly indicate from time step 4 onwards the light tailed nature of the distribution of the cost yielded by the new dual control algorithm.

## Individual Time History Runs

Analysis of the Monte Carlo Average Cost indicates the improvement offered by the dual solution; it provides no information about the cautious control's turning off phenomenon [S1, W1]. Hence a careful investigation of the individual runs is required to examine these occurrences.

The turn-off phenomenon is observed in many runs among the 500 Monte Carlo simulation while using the cautious controller; run 90 is a typical example of it. Both components are almost off betwen time steps 0 and 20 during which the dual controller already identified the parameters and reached the desired trajectory. Figures 9-12 portray this result.

| Time Step k | HCE | | Cautious | | Dual | |
|---|---|---|---|---|---|---|
| | $\bar{C}_k$ | $\sum\limits_{i=1}^{k}\bar{C}_i$ | $\bar{C}_k$ | $\sum\limits_{i=1}^{k}\bar{C}_i$ | $\bar{C}_k$ | $\sum\limits_{i=1}^{k}\bar{C}_i$ |
| 1 | 14851 | 14851 | 3623 | 3623 | 6944 | 6944 |
| 2 | 6241 | 21092 | 3961 | 7584 | 6722 | 13666 |
| 3 | 3578 | 24670 | 3246 | 10830 | 4230 | 17896 |
| 4 | 1616 | 26286 | 2836 | 13666 | 1866 | 19762 |
| 5 | 1354 | 27640 | 2505 | 16171 | 1492 | 21254 |
| 6 | 807 | 28447 | 2154 | 18325 | 953 | 22207 |
| 7 | 593 | 29040 | 1921 | 20246 | 700 | 22907 |
| 8 | 462 | 29502 | 1670 | 21916 | 582 | 23489 |
| 9 | 397 | 29899 | 1623 | 23539 | 535 | 24024 |
| 10 | 347 | 30246 | 1327 | 24866 | 385 | 24409 |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| 40 | 77 | 34444 | 281 | 43810 | 89 | 29178 |

Table 7. Average Costs for the three algorithms in the dynamic model with a limiter (500 Monte Carlo Runs; $|u_1| \leq 2.0$, $|u_2| \leq 2.0$)

| Time Step | Test Statistic | Estimated Improvement |
|:---------:|:--------------:|:---------------------:|
| k | $z_k$ | $EI_k\%$ |
| 1 | -8.1 | -91 |
| 2 | -5.3 | -69 |
| 3 | -2.2 | -30 |
| 4 | 3.5 | 34 |
| 5 | 3.3 | 40 |
| 6 | 6.0 | 56 |
| 7 | 6.3 | 64 |
| 8 | 6.5 | 65 |
| 9 | 6.5 | 67 |
| 10 | 5.7 | 71 |
| 11 | 6.3 | 76 |
| 12 | 5.6 | 70 |
| 13 | 5.9 | 82 |
| 14 | 5.2 | 62 |
| 15 | 5.5 | 79 |
| 16 | 4.9 | 70 |
| 17 | 4.5 | 78 |
| 18 | 4.4 | 74 |
| 19 | 4.4 | 76 |
| 20 | 4.3 | 76 |

Table 8.  Statistical significance test for comparison of cautious and the dual algorithm in the dynamic model with a limiter (500 Monte Carlo Runs; $|u_1| \leq 2.0$, $|u_2| \leq 2.0$)

| Time | Percentage of runs which exceed 2000 | |
| --- | --- | --- |
| k | Cautious | Dual |
| 1 | 86 | 76 |
| 2 | 60 | 52 |
| 3 | 43 | 40 |
| 4 | 33 | 25 |
| 5 | 31 | 17 |
| 6 | 22 | 10 |
| 7 | 22 | 8 |
| 8 | 19 | 7 |
| 9 | 16 | 3 |
| 10 | 12 | 2 |
| 11 | 12 | 1.2 |
| 12 | 10 | 1.4 |
| 13 | 11 | 1.4 |
| 14 | 7 | 1 |
| 15 | 8 | 0.4 |
| 16 | 6 | 0.4 |
| 17 | 6 | 0.2 |
| 18 | 6 | 0.4 |
| 19 | 5 | 0.4 |
| 20 | 5 | 0.2 |

Table 9: Comparison of the tails using the cautious and the dual algorithm in the dynamic model with a limiter (500 Monte Carlo Runs; $| u_1 | \leq 2.0$, $| u_2 | \leq 2.0$)

| Time Step | $\chi^2$ test statistics at $K_{90}$ |
|:---:|:---:|
| 1 | -- |
| 2 | -- |
| 3 | -- |
| 4 | 10 |
| 5 | 19 |
| 6 | 23 |
| 7 | 32 |
| 8 | 35 |
| 9 | 57 |
| 10 | 37 |
| 11 | 40 |
| 12 | 40 |
| 13 | 40 |
| 14 | 16 |
| 15 | 32 |
| 16 | 11 |
| 17 | 16 |
| 18 | 16 |
| 19 | 18 |
| 20 | 25 |

Table 10.    Percentile test for comparison of cautious and the dual algorithm in the dynamic model with a limiter (500 Monte Carlo Runs; $|u_1| \leq 2.0$, $|u_2| \leq 2.0$)

## CAUTIOUS, DUAL AND HCE
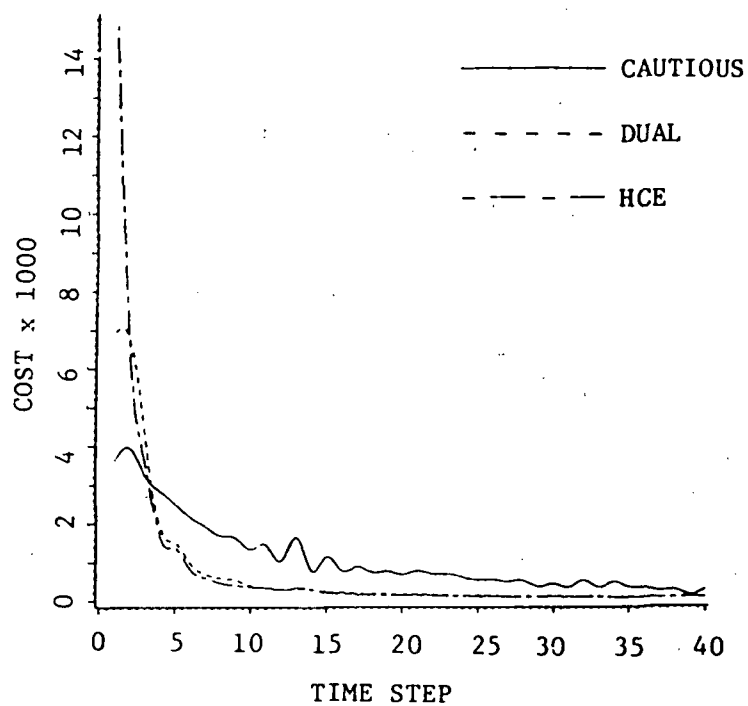


Fig. 8 Time history of the average cost using the heuristic certainty equivalence, cautious and the dual controllers. (500 Monte Carlo runs; $|u_1| \leq 2.0$, $|u_2| \leq 2.0$)
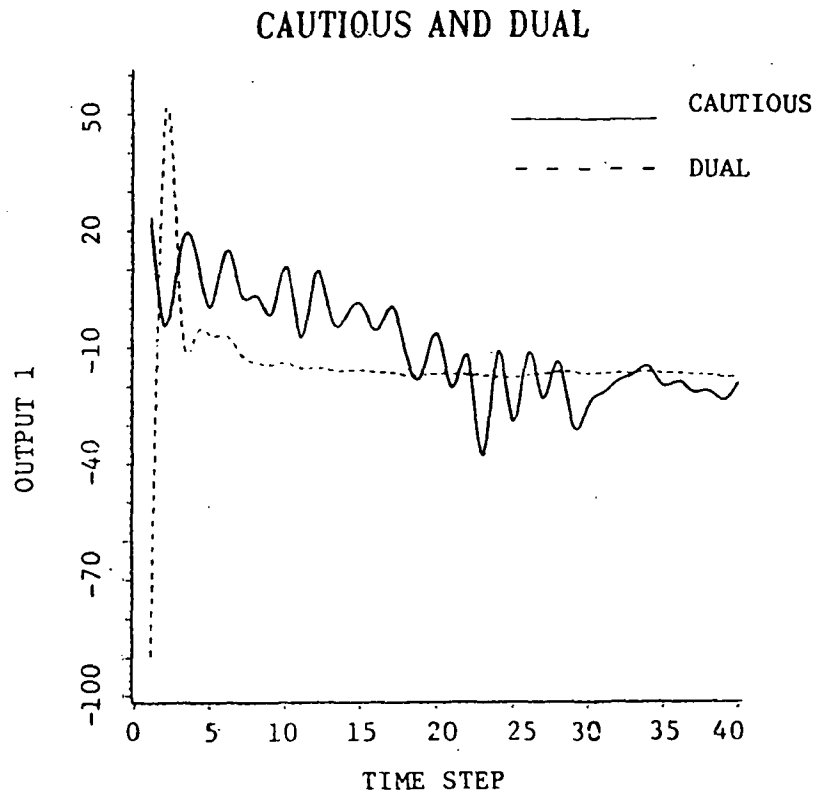
CAUTIOUS AND DUAL

Fig. 9  Time history of output 1 using the cautious
and the dual algorithms for run 90 (500 Monte
Carlo runs; $|u_1| \leq 2.0$; $|u_2| \leq 2.0$)
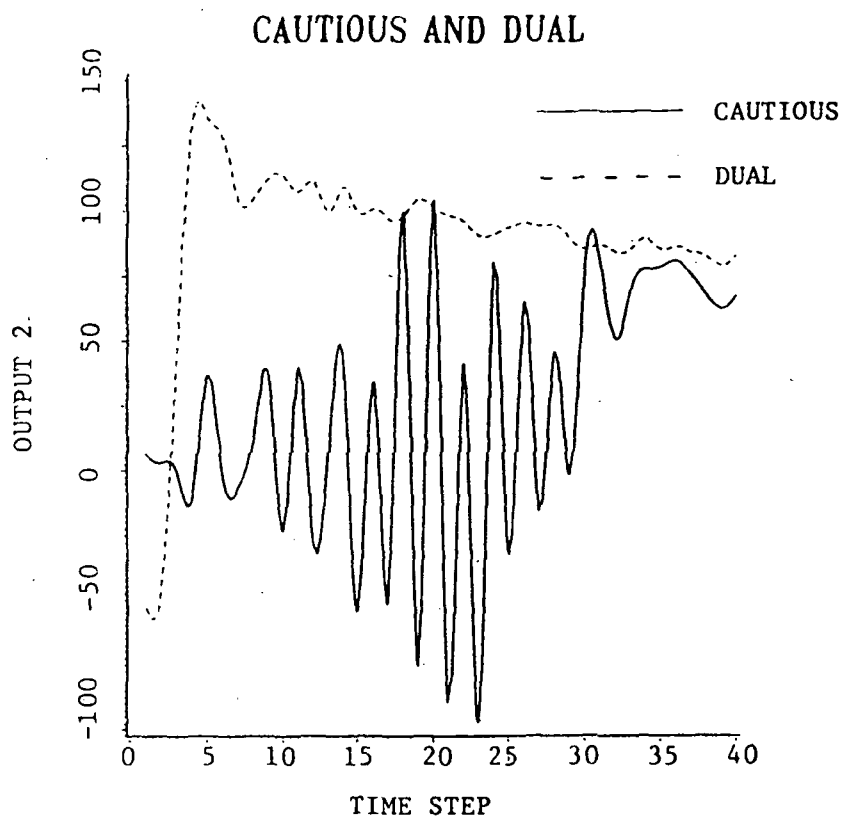
Fig. 10 Time history of output 2 using the cautious
and the dual algorithms for run 90 (500 Monte
Carlo runs; $|u_1| \leq 2.0$; $|u_2| \leq 2.0$)
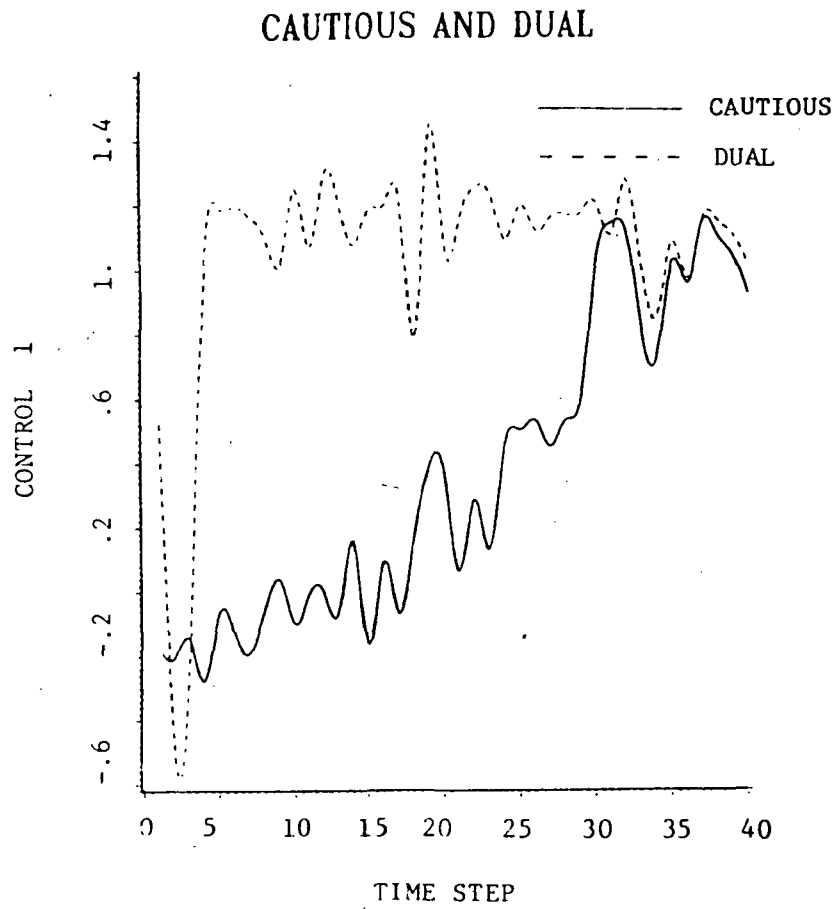
## CAUTIOUS AND DUAL



Fig. 11   Time history of control 1 using the cautious
and the dual algorithms for run 90 (500 Monte
Carlo runs; $|u_1| \leq 2.0$; $|u_2| \leq 2.0$)
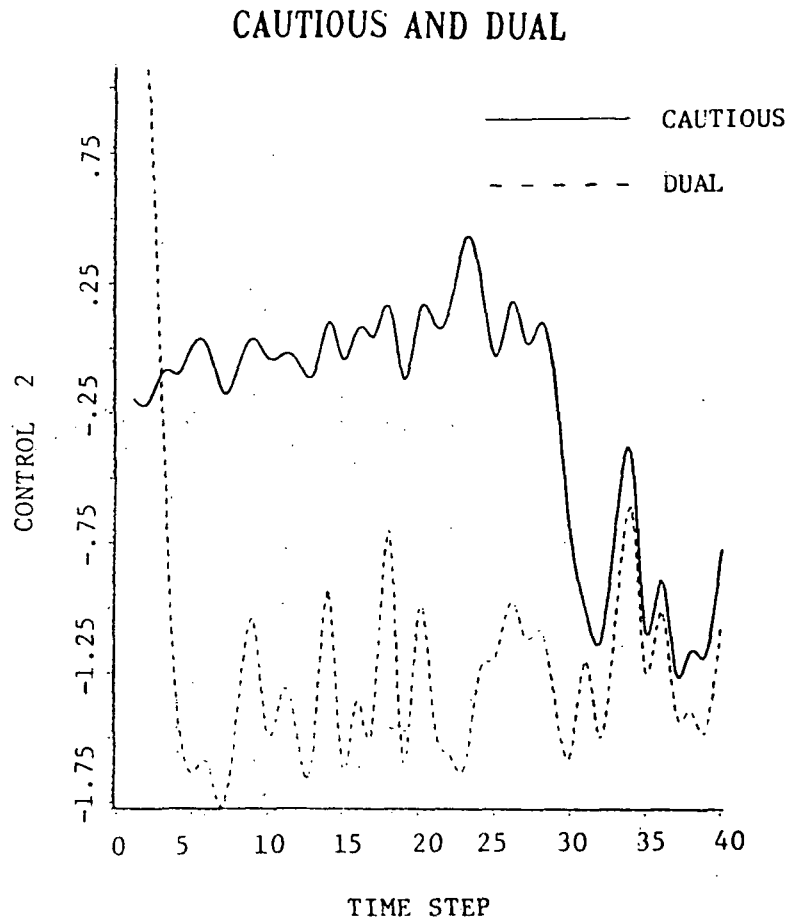
Fig. 12 Time history of control 2 using the cautious and the dual algorithms for run 90 (500 Monte Carlo runs; $|u_1| \leq 2.0$; $|u_2| \leq 2.0$)

## 3.5 CONCLUSIONS

A new adaptive dual control solution has been developed for an ARMA MIMO system. This solution utilizes the dual effect by performing a second order Taylor series expansion of the expected future cost. It modifies the cautious solution by numerator and denominator correction terms. Analysis of the simulation runs has shown that this new dual control solution applied to a multi-input multi-output model improves over the cautious controller. The key improvement is in the avoiding of situations like turn-off and slow convergences, typical of the cautious solution.

# APPENDIX A

## Statistical Significance in the
## Comparison of Controller Performance

Two control algorithms are compared by performing a Monte Carlo simulation. S independent runs with the two algorithms, under the same homogeneous conditions, yield a set of i.i.d. samples $C_{ik}^{(1)}$, $C_{ik}^{(2)}$, $i = 1, 2, \ldots, S$ from two distributions with true but unknown means $J_k^{(1)}$ and $J_k^{(2)}$, respectively, for each time step k.

The sample means

$$\bar{C}_k^{(j)} = \frac{1}{S} \sum_{i=1}^{S} C_{ik}^{(j)} \qquad j=1,2 \qquad (A.1)$$

are point estimates of the respective true means.

A statement that

$$\bar{C}_k^{(1)} < \bar{C}_k^{(2)} \qquad (A.2)$$

indicating that algorithm 1 is better than 2 for time

step   k   has to be accompanied by a level of

significance   $\alpha$   of type I error.

Thus we test the hypothesis

$$H_0: \Delta = J_k^{(2)} - J_k^{(1)} \leq 0 \qquad \text{(algorithm 1 not better)} \qquad \text{(A.3)}$$

against the one sided alternative

$$H_1: \Delta = J_k^{(2)} - J_k^{(1)} > 0 \qquad \text{(algorithm 1 better)} \qquad \text{(A.4)}$$

for a particular   $\alpha$   level at each time step   k.

This probability of error   $\alpha$   is defined as

$$\alpha \triangleq P\{\text{accept } H_1 | H_0 \text{ true}\} \qquad \text{(A.5)}$$

Since we get a set of data of the performances of

the two algorithms on the plant under similar

conditions we regard it as a set of naturally paired

observations.

We consider the sample differences

$$\Delta_{ik} = C_{ik}^{(2)} - C_{ik}^{(1)} \qquad \text{(A.6)}$$

and this set of differences $\Delta_{ik}$ represents a sample with mean

$$\Delta_k = J_k^{(2)} - J_k^{(1)} \tag{A.7}$$

Thus we have reduced the two-sample problem to a one-sample problem. The hypothesis is tested by examining whether $\Delta_k$ can be accepted as being positive with high confidence. The test statistic is

$$z_k \triangleq \frac{\bar{\Delta}_k}{\sigma_{\bar{\Delta}_k}} \tag{A.8}$$

where

$$\bar{\Delta}_k = \frac{1}{S} \sum_{i=1}^{S} \Delta_{ik} \tag{A.9}$$

$$\sigma_{\bar{\Delta}_k}^2 = \frac{1}{S(S-1)} \sum_{i=1}^{S} (\Delta_{ik} - \bar{\Delta}_k)^2 \tag{A.10}$$

The test statistic $z_k$ has a $t$ - distribution with $(S-1)$ degrees of freedom. For S large $(>50)$ $z_k$ has a normal distribution. Then we have

$$\sigma_{\bar{\Delta}_k}^2 \cong \frac{1}{S^2} \sum_{i=1}^{S} (\Delta_{ik} - \bar{\Delta}_k)^2 \tag{A.11}$$

and the hypothesis $H_1$ is accepted if

$$z_k > \gamma \qquad (A.12)$$

where $\gamma$ is taken from the normal distribution tables. For a one-sided test with $\alpha = 0.05$, one has $\gamma = 1.645$.

The estimated improvement for each time step k is defined as

$$EI_k \triangleq \frac{\bar{C}_k^{(2)} - \bar{C}_k^{(1)}}{\bar{C}_k^{(2)}} \times 100\% \qquad (A.13)$$

For our problem the costs have a probability density function which is not symmetric and also not normal. For this class of problems nonparametric tests for two samples are applicable [N1]. A percentile test is recommended here to further substantiate that the vibrations obtained by using the dual and the cautious algorithms come from two different distributions and the tail of the distribution obtained from the dual is lighter than the tail of that obtained by the cautious algorithm. This test is described next. The two samples are

pooled together and a 90 percentile point denoted as $K_{90}$ is chosen. Then a 2x2 contingency table is computed as follows

|  | $\leq K_{90}$ | $> K_{90}$ | Totals |
|---|---|---|---|
| Dual | $a$ | $b$ | $a + b$ |
| Cautious | $c$ | $d$ | $c + d$ |
| Totals | $a + c$ | $b + d$ | $n = a+b+c+d$ |

where $a$, $b$, $c$, $d$ are the observed frequencies for the four cells.

The $\chi^2$ (chi-square) value is obtained by

$$\chi^2 = \frac{n(ad-bc)^2}{(a+b)(d+b)(c+d)(a+c)} \quad \text{at 1 degree of freedom} \quad (A.14)$$

This should be greater than 3.8 at $\alpha = 0.05$ to prove that the tail of dual is lighter than that of the cautious.

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**16. Abstract**

The design of an actively adaptive "dual" controller based on an approximation of the stochastic dynamic programming equation for a multi-step horizon is presented. A dual controller that can enhance identification of the system while controlling it at the same time is derived for multi-dimensional problems. This dual controller uses sensitivity functions of the expected future cost with respect to the parameter uncertainties. A passively adaptive "cautious" controller and the actively adaptive "dual" controller are examined. In many instances, the cautious controller is seen to turn off while the latter avoids the turn-off of the control and the slow convergence of the parameter estimates, characteristic of the cautious controller. The algorithms have been applied to a multi-variable static model which represents a simplified linear version of the relationship between the vibration output and the higher harmonic control input for a helicopter. Monte Carlo comparisons based on parametric and nonparametric statistical analysis indicate the superiority of the dual controller over the baseline controller.

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