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September 1988

NASA-TP-2847 19880018793

More on Exact State Reconstruction in Deterministic Digital Control Systems

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More on Exact State Reconstruction in Deterministic Digital Control Systems

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National Aeronautics
and Space Administration

Scientific and Technical
Information Division

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I. INTRODUCTION

Books on modern digital control systems usually address the problem of controlling a continuous-time plant driven by a zero-order-hold with a sampled output as shown in Figure 1. (For example, see Reference 1.) A common solution to this problem is to reconstruct the state of the system at the sampling instants using a state observer and then feed back the reconstructed states [2]. However, the state observer has two undesirable characteristics. First of all, it is a dynamical system in itself and, hence, adds additional states and eigenvalues to the system, which can affect system stability. Second, as a consequence, the reconstructed state is normally an approximation to the true state and is usually not a good one early in the state reconstruction process unless the initial state of the system is well known. Recently, Polites developed a new approach to state reconstruction which has neither of these problems [3]. Subsequently, he extended this work and developed what he called the Ideal State Reconstructor [4,5]. It was so named because: if the plant parameters are known exactly, its output will exactly equal, not just approximate, the true state of the plant and accomplish this without any knowledge of the plant's initial state. Besides this, it adds no new states or eigenvalues to the system. Nor does it affect the plant equation for the system in any way; it affects the measurement equation only. It is characterized by the fact that discrete measurements are generated every T/N seconds and input into a multi-input/multi-output moving-average (MA) process [6]. The output of this process is sampled every T seconds and utilized in reconstructing the state of the system. In this paper, a special form of the Ideal State Reconstructor is presented which is simpler to implement than the most general form. Before presenting this simpler form, some pertinent results to date for continuous-time plants driven by a zero-order hold are reviewed in Section II. Then, the Ideal State Reconstructor, in its most general form, is summarized in Section III. Finally, the special form of it is presented in Section IV. An example of this special form is given in Section V. The conclusions and some recommendations for future study are presented in Section VI.

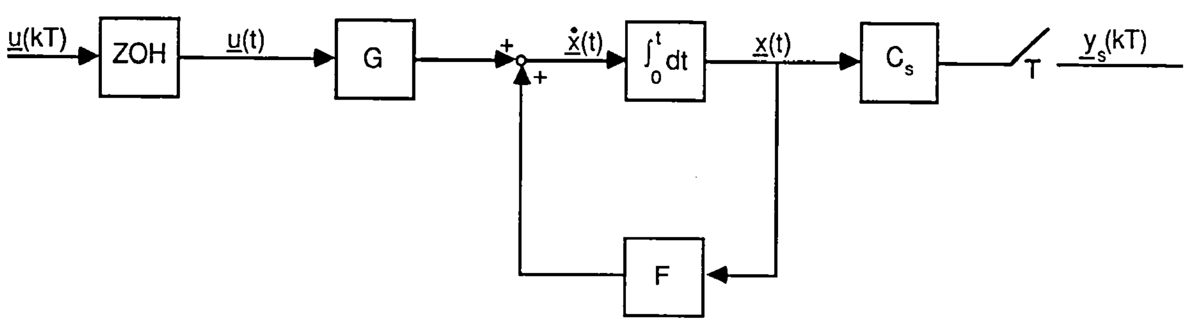

Figure 1. Continuous-time plant driven by a zero-order-hold with standard measurements.

II. PRELIMINARY

For the plant in Figure 1, $\underline{x}(t) \in R^n$ is the state vector, $\underline{u}(kT) \in R^r$ is the control input vector, $\underline{y}_S(kT) \in R^m$ is the standard output or measurement vector, $F \in R^{nxn}$ is the system matrix, $G \in R^{nxr}$ is the control matrix, and $C_S \in R^{mxn}$ is the standard output matrix. It is well known that this system can be modeled at the sampling instants by the discrete state equations [1]

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k) \tag{1}$$

$$\underline{y}_S(k) = C_S\, \underline{x}(k) \quad , \tag{2}$$

where

$$\phi(t) = \mathscr{L}^{-1}\, [(sI-F)^{-1}] \quad , \tag{3}$$

$$A = \phi(T) \quad , \tag{4}$$

and

$$B = \left[\int_0^T \phi(\lambda)\, d\lambda \right] G \quad . \tag{5}$$

$\phi(t) \in R^{nxn}$ is the state transition matrix. $A \in R^{nxn}$ is the system matrix and $B \in R^{nxr}$ is the control matrix for the discrete state equations (1) and (2).

A and B can be determined analytically using equations (3) to (5). An alternative approach, which is also quite suitable for numerical computation, is as follows [7]: $\phi(t)$ and $\int_0^t \phi(\lambda)d\lambda$ can be expressed in the form of matrix exponential series as

$$\phi(t) = \sum_{i=0}^{\infty} \frac{F^i\, t^i}{i!} \tag{6}$$

and

$$\int_0^t \phi(\lambda)d\lambda = \sum_{i=0}^{\infty} \frac{F^i \, t^{i+1}}{(i+1)!} \quad , \tag{7}$$

respectively. From equations (6) and (7),

$$\phi(t) = I + F\left[\int_0^t \phi(\lambda)d\lambda\right] \quad , \tag{8}$$

where $I \in R^{nxn}$ is the identity matrix. Hence, $\int_0^T \phi(\lambda)d\lambda$ can be determined using equation (7) with $t = T$ and this result substituted into equation (8) to get $\phi(T)$. With these results, A and B can be found using equations (4) and (5).

Now consider the plant in Figure 2, which is a generalization of the one in Figure 1. In addition to the standard output $\underline{y}_S(kT)$, the plant in Figure 2 has the output $\underline{y}_F'(kT)$ generated as follows. First, the continuous-time output $\underline{z}(t) \in R^p$ is sampled every T/N seconds. Every N samples are multiplied by the weighting matrices $H_j \in R^{qxp}$, $j = 0,1,\ldots,N-1$, and then summed to generate the output $\underline{y}_F(kT) \in R^q$, every T seconds. Functionally, this is equivalent to passing the discrete measurements generated every T/N seconds through a multi-input/multi-output MA process with coefficient matrices H_j, $j = 0,1,\ldots,N-1$, and then sampling the output of this process every T seconds to generate $\underline{y}_F(kT)$. Then $\underline{y}_F(kT)$ has $E_- \underline{u}[(k-1)T]$ subtracted from it, where $E_- \in R^{qxr}$, to produce the modified MA-prefiltered measurement vector $\underline{y}_F'(kT) \in R^q$. This is catenated with $\underline{y}_S(kT)$ to form the total measurement vector $\underline{y}_T(kT) \in R^{m+q}$. In Figure 2, $C_F \in R^{pxn}$ since $\underline{z}(t) \in R^p$ and $\underline{x}(t) \in R^n$.

Previously, Polites [8] showed that when

$$E_- = H\beta \tag{9}$$

where $H \in R^{qx(Np)}$ is given by

$$H = [H_0 \mid H_1 \mid \ldots \mid H_{N-1}] \tag{10}$$

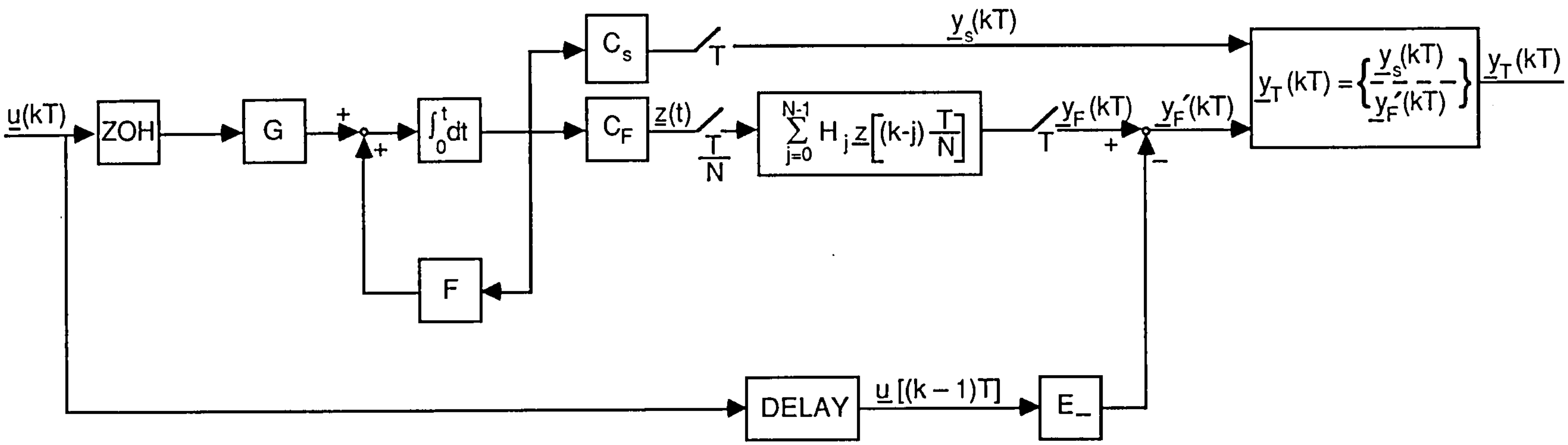

Figure 2. Continuous-time plant driven by a zero-order-hold with standard and modified MA-prefiltered measurements.

and $\beta \in R^{(Np)xr}$ is

$$\beta = \begin{bmatrix} C_F \left[\int_0^{0} \phi(\lambda) d\lambda \right] G \\ C_F \left[\int_0^{-(T/N)} \phi(\lambda) d\lambda \right] G \\ \vdots \\ C_F \left[\int_0^{-(N-1)(T/N)} \phi(\lambda) d\lambda \right] G \end{bmatrix} , \tag{11}$$

the discrete state equations for the plant in Figure 2 become

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k) \tag{12}$$

$$\underline{y}_T(k) = \begin{bmatrix} \underline{y}_S(k) \\ ---- \\ \underline{y}_F'(k) \end{bmatrix} = \begin{bmatrix} C_S \\ ---- \\ D_- \end{bmatrix} \underline{x}(k) = C_T \, \underline{x}(k) , \tag{13}$$

where $D_- \in R^{qxn}$ is given by

$$D_- = H\alpha \tag{14}$$

and $\alpha \in R^{(Np)xn}$ is

$$\alpha = \begin{bmatrix} C_F \, \phi(0) \\ C_F \, \phi(-T/N) \\ \vdots \\ C_F \, \phi[-(N-1)T/N] \end{bmatrix} . \tag{15}$$

From equation (13),

$$C_T = \begin{bmatrix} C_S \\ ---- \\ D_- \end{bmatrix} \tag{16}$$

where $C_T \in R^{(m-q)xn}$.

E_- and D_- can be evaluated analytically using equations (3), (9) to (11), (14), and (15). An alternative approach, which can be either analytical or numerical, is as follows. Let $t = -j(T/N)$, where $j = 0,1,\ldots,N-1$, and use equation (7) to determine $\int_0^{-j(T/N)} \phi(\lambda)d\lambda$, $j = 0,1,\ldots,N-1$. Substitute these results into equation (8) to get $\phi[-j(T/N)]$, $j = 0,1,\ldots,N-1$. At this point, E_- and D_- can be found using equations (9) to (11), (14), and (15).

III. THE IDEAL STATE RECONSTRUCTOR

A general block diagram of the plant and the Ideal State Reconstructor, in its most general form, is shown in Figure 3. Notice the similarity between Figures 2 and 3. By virtue of this, if E_- is given by equation (9), then equations (12) to (16) define the discrete state equations for the system in Figure 3 up to the output $\underline{y}_T(k)$. Proceeding further, $\underline{y}_T'(k)$ is related to $\underline{y}_T(k)$ by the expression

$$\underline{y}_T'(k) = (C_T^T\, C_T)^{-1}\, C_T^T\, \underline{y}_T(k) \quad . \tag{17}$$

However, for equation (17) to be meaningful, $(C_T^T\, C_T)^{-1}$ must exist, and this occurs only when $(C_T^T\, C_T)$ is nonsingular. Recall that $C_T \in R^{(m+q)xn}$. If $(m+q) \geqslant n$ and C_T has maximal rank (i.e., rank n), then $(C_T^T\, C_T)$ is positive definite and therefore nonsingular [9]. Hence, equation (17) requires that $(m+q) \geqslant n$ and rank $(C_T) = n$ for it and the Ideal State Reconstructor to be meaningful. Assuming this is the case, it follows from equations (12), (13), and (17) that the discrete state equations for the system in Figure 3 are

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k) \tag{18}$$

$$\underline{y}_T'(k) = \underline{x}(k) \quad . \tag{19}$$

Hence, the output of the Ideal State Reconstructor, $\underline{y}_T'(kT)$, exactly equals the true state of the plant, $\underline{x}(kT)$. Consequently, if one is given the plant in Figure 1 and modifies it to conform to Figure 3, he can exactly reconstruct the state of the plant without adding any new states, eigenvalues, or dynamics to it, since the plant equation (18) for the system in Figure 3 is identical to the plant equation (1) for the plant in

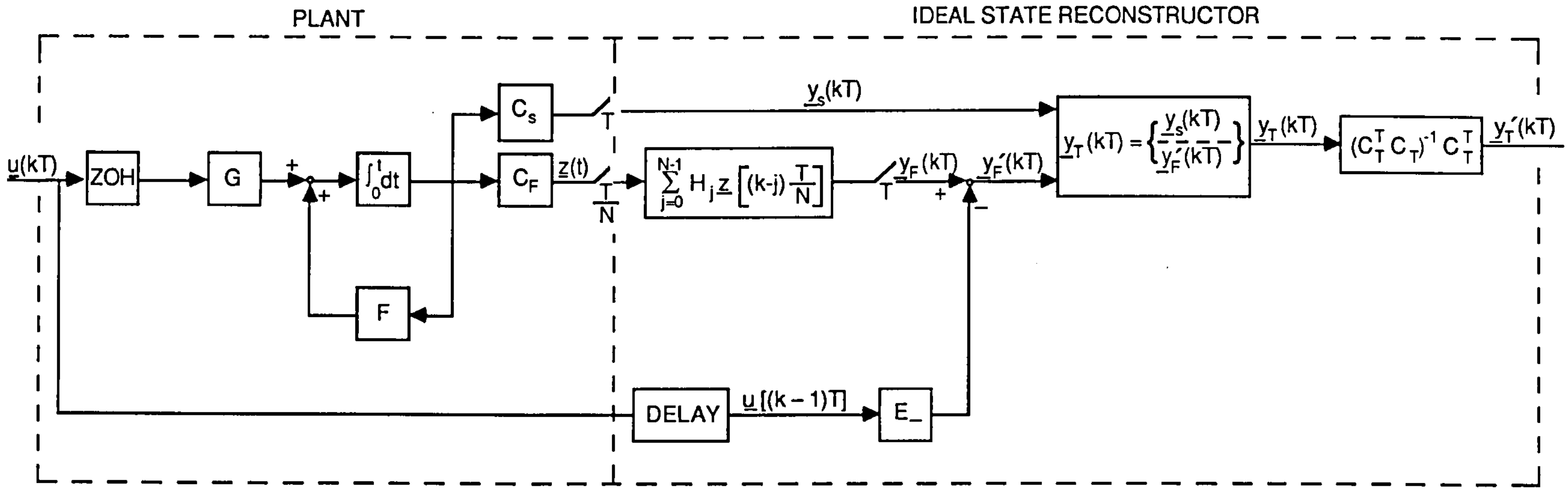

Figure 3. Block diagram of the plant and the Ideal State Reconstructor in its most general form.

Figure 1. In Figure 3, exact state reconstruction is achieved when E_- is given by equation (9) and C_T is given by equation (16) where D_- is given by equation (14). In addition, C_T must satisfy the requirements just imposed on it.

One of these is that the dimensions of C_T, namely $(m+q)xn$, satisfy the relationship $(m+q) \geqslant n$. However, this can be rewritten as $q \geqslant (n-m)$. Hence, the number of rows, q, in the weighting matrices H_j, $j = 0,1,\ldots,N-1$, must equal or exceed the number of states, n, in the plant state vector, $\underline{x}(t)$ or $\underline{x}(kT)$, minus the number of elements, m, in the standard measurement vector, $\underline{y}_S(kT)$. Since q can be chosen arbitrarily, this requirement can be easily satisfied. The other requirement on C_T is that it have rank n. One approach to satisfying this requirement is as follows. Recall that C_T is defined by equation (16) where D_- is defined by equation (14). Assuming C_S is given, then one can choose D_- so that C_T has rank n and then find H to give the desired D_-. One solution to the problem of finding H to give the desired D_- is to let

$$H = D_- (\alpha^T \alpha)^{-1} \alpha^T \quad . \tag{20}$$

This follows from equation (14). However, like before, this requires that $(\alpha^T \alpha)$ be nonsingular. Recall that $\alpha \in R^{(Np)xn}$. If $(Np) \geqslant n$, or equivalently $N \geqslant n/p$, and α has maximal rank (i.e., rank n), then $(\alpha^T \alpha)$ is nonsingular. The first requirement can be easily satisfied because the number of weighting matrices N, where the weighting matrices are H_j, $j = 0,1,\ldots,N-1$, can be arbitrarily chosen so that $N \geqslant n/p$. Recall that n is the number of states in the plant state vector, $\underline{x}(t)$, and p the number of elements in the output vector $\underline{z}(t)$.

In summary, the procedure to achieve exact state reconstruction with the Ideal State Reconstructor is as follows. Given the plant in Figure 1, modify it to conform to Figure 3. Choose the number of rows, q, in the weighting matrices H_j, $j = 0,1,\ldots,N-1$, so that $q \geqslant (n-m)$ where n is the number of states in the plant state vector, $\underline{x}(t)$ or $\underline{x}(kT)$, and m is the number of elements in the standard measurement vector, $\underline{y}_S(kT)$. Choose the number of weighting matrices, N, so that $N \geqslant n/p$ where p is the number of elements in the output vector $\underline{z}(t)$. Assuming the $(Np)xn$ matrix α, defined by equation (15), has maximal rank (i.e., rank n), let H be given by equation (20) where D_- is chosen so that the $(m+q)xn$ matrix C_T, defined by equation (16), has maximal rank (i.e., rank n). The weighting matrices H_j, $j = 0,1,\ldots,N-1$, are found by partitioning H as in equation (10). Finally, let E_- in Figure 3 be given by equation (9) where β is defined by equation (11). The discrete state equations for the system in Figure 3 are now given by equations (18) and (19). Hence, the output of the Ideal State Reconstructor, $\underline{y}_T'(kT)$, exactly equals the true state of the system, $\underline{x}(kT)$.

In the event no standard measurements are used in the state reconstruction process, then $\underline{y}_S(kT)$ is a null vector, C_S is a null matrix, and the Ideal State Reconstructor in Figure 3 degenerates to the one presented in Reference 4. In this case, the methods in Reference 4 for choosing the parameters in the Ideal State Reconstructor to achieve exact state reconstruction, as well as the one described here, are applicable.

IV. A SPECIAL FORM OF THE IDEAL STATE RECONSTRUCTOR

As indicated in Section III, one requirement of the Ideal State Reconstructor is: the number of rows, q, in the weighting matrices H_j, $j = 0,1,\ldots,N-1$, must be chosen so that $q \geq (n-m)$ where n is the number of states in the plant state vector, $\underline{x}(t)$ or $\underline{x}(kT)$, and m is the number of elements in the standard measurement vector $\underline{y}_S(kT)$. A special form of the Ideal State Reconstructor, which is simpler to implement than the most general form, can be obtained by letting $q = n\text{-}m$. In general, $C_T \in R^{(m+q)xn}$. Hence, for this special case, $C_T \in R^{nxn}$, which is a square matrix. Now, if D_- can be chosen so that C_T is nonsingular, then

$$\underline{y}_T'(k) = C_T^{-1}\, \underline{y}_T(k) \quad , \tag{21}$$

which follows from equation (17), and the Ideal State Reconstructor assumes the special form shown in Figure 4. From equations (12), (13), and (21), the discrete state equations for the system in Figure 4 become equations (18) and (19), like before. Again, the output of the Ideal State Reconstructor, $\underline{y}_T'(kT)$, exactly equals the true state of the plant, $\underline{x}(kT)$, but with a minimum number of rows in the weighting matrices, in this case.

Observe, from equations (12) and (13), that when C_T is square and nonsingular, $\underline{y}_T(k)$ becomes a legitimate state vector for the system. Solving for $\underline{x}(k)$ in equation (13) and substituting this result into equation (12) yields the plant equation for the system in terms of the state vector $\underline{y}_T(k)$. The result is

$$\underline{y}_T(k+1) = A^*\, \underline{y}_T(k) + B^*\, \underline{u}(k)$$

where $A^* = C_T\, A\, C_T^{-1}$ and $B^* = C_T\, B$. Consequently, if it is sufficient to construct the state vector $\underline{y}(kT)$ in place of the state vector $\underline{x}(kT)$, then the special form of the Ideal State Reconstructor in Figure 4 can be simplified, because there is no reason to calculate $\underline{y}_T'(kT)$.

V. AN EXAMPLE

Consider the double integrator plant and the special form of the Ideal State Reconstructor shown in Figure 4. Manipulating this plant into the format of Figure 3 yields

$$F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad , \tag{22}$$

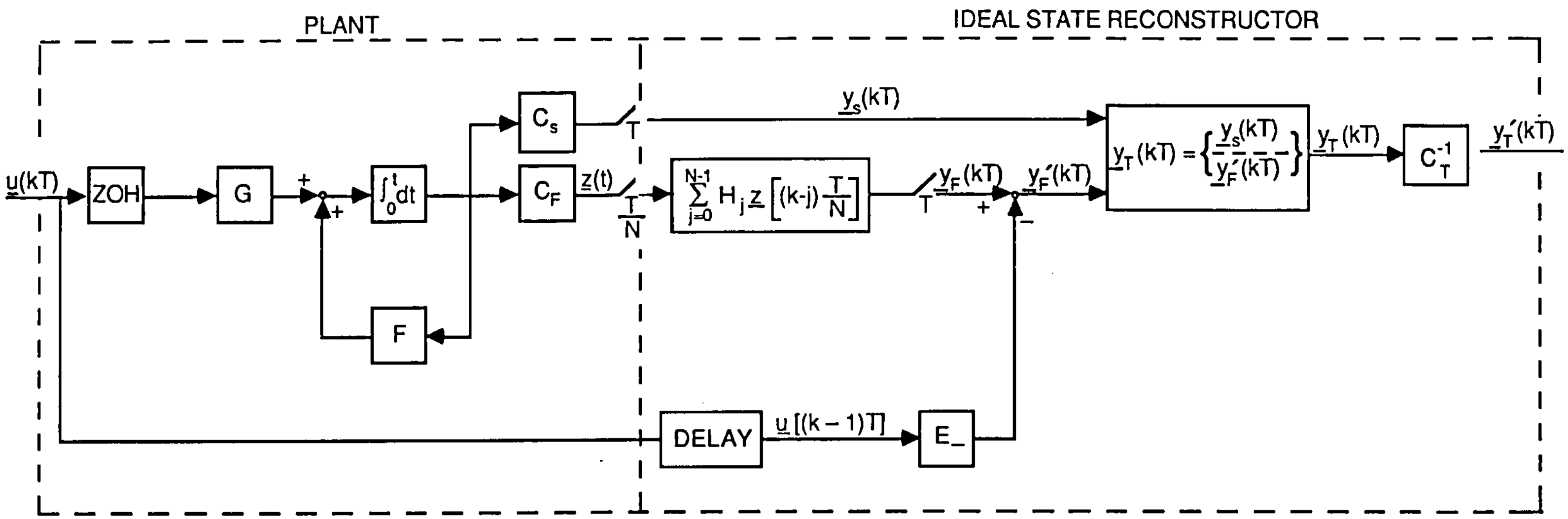

Figure 4. Block diagram of the plant and the special form of the Ideal State Reconstructor.

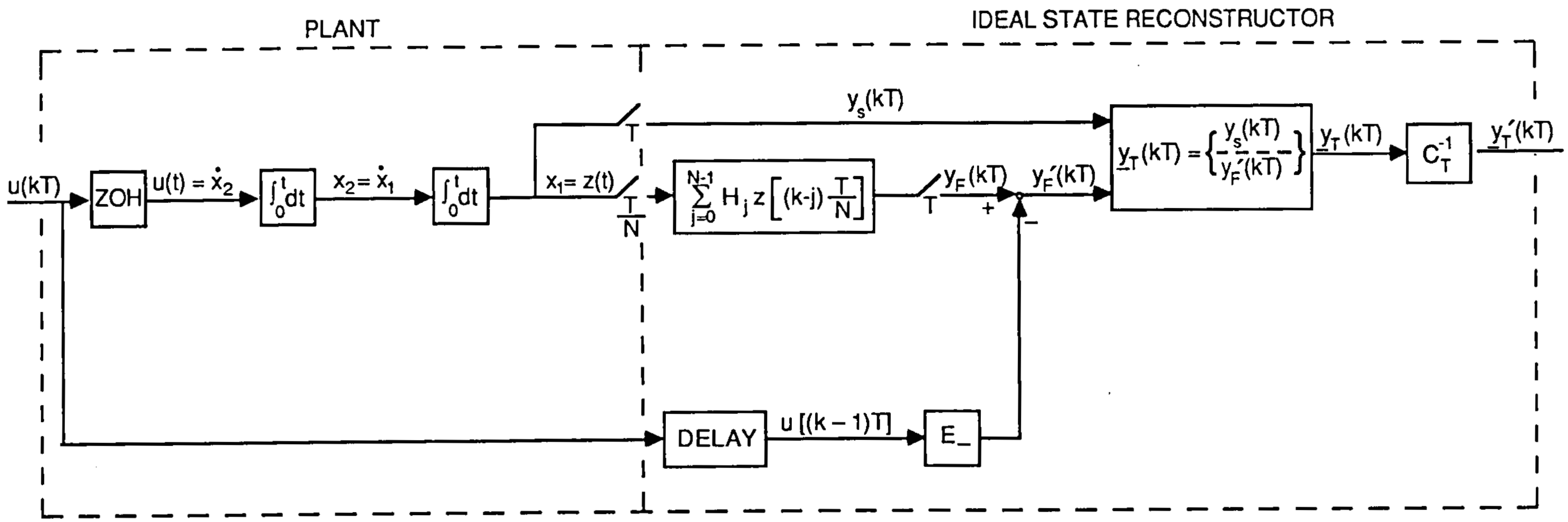

Figure 5. Double integrator plant and the special form of the Ideal State Reconstructor.

$$G = \begin{bmatrix} 0 \\ 1 \end{bmatrix} , \tag{23}$$

and

$$C_S = C_F = [1 \quad 0] \ . \tag{24}$$

Since $F \epsilon R^{nxn}$, $G \epsilon R^{nxr}$, $C_S \epsilon R^{mxn}$, and $C_F \epsilon R^{pxn}$, it follows from equations (22) to (24) that $n = 2$ and $r = m = p = 1$. Using equations (22) and (23) and the formulas presented in Section II,

$$\phi(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} , \tag{25}$$

$$\int_0^t \phi(\lambda) d\lambda = \begin{bmatrix} t & t^2/2 \\ 0 & t \end{bmatrix} , \tag{26}$$

$$A = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} ,$$

and

$$B = \begin{bmatrix} T^2/2 \\ T \end{bmatrix} .$$

For the special form of the Ideal State Reconstructor, $q = n\text{-}m = 1$. The requirement $N \geqslant n/p$ can be satisfied by letting $N = 4$. Now α and β can be evaluated using equations (11), (15), (23) to (25), and (26), and are found to be

$$\alpha = \begin{bmatrix} 1 & 0 \\ 1 & -\frac{T}{4} \\ 1 & -\frac{2T}{4} \\ 1 & -\frac{3T}{4} \end{bmatrix} \tag{27}$$

and

$$\beta = \begin{bmatrix} 0 \\ \frac{T^2}{32} \\ \frac{4T^2}{32} \\ \frac{9T^2}{32} \end{bmatrix} , \tag{28}$$

respectively. In equation (27), eliminating any two rows forms a 2x2 matrix with nonzero determinant, assuming of course $T > 0$. Hence, rank $(\alpha) = 2 = n$ and so $(\alpha^T \alpha)$ is nonsingular. Consequently, $(\alpha^T\alpha)^{-1} \alpha^T$ exists and is found to be

$$(\alpha^T \alpha)^{-1} \alpha^T = \begin{bmatrix} \left(\frac{7}{10}\right) & \left(\frac{4}{10}\right) & \left(\frac{1}{10}\right) & \left(-\frac{2}{10}\right) \\ \left(\frac{6}{5T}\right) & \left(\frac{2}{5T}\right) & \left(-\frac{2}{5T}\right) & \left(-\frac{6}{5T}\right) \end{bmatrix} \tag{29}$$

using equation (27). Since $(\alpha^T \alpha)^{-1} \alpha^T$ exists, H can be given by equation (20) where D_- needs to be chosen so C_T is square and nonsingular. Since $D_- \in R^{q \times n}$, then D_- becomes a 1x2 matrix. Now, if D_- is chosen to be

$$D_- = [0 \quad 1] \quad , \tag{30}$$

it follows from equations (16), (24), and (30) that

$$C_T = C_T^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} .$$

Obviously, C_T is square and nonsingular and there is no need to calculate $\underline{y}_T'(kT)$ in Figure 5. From equations (10), (20), (29), and (30),

$$H = [H_0 \mid H_1 \mid H_2 \mid H_3] = \left[\left(\frac{6}{5T}\right) \mid \left(\frac{2}{5T}\right) \mid \left(-\frac{2}{5T}\right) \mid \left(-\frac{6}{5T}\right) \right] , \tag{31}$$

which reveals the weighting matrices H_j, $j = 0,1,2,3$. From equations (9), (28), and (31),

$$E_- = -3T/8 \quad .$$

The special form of the Ideal State Reconstructor is now completely defined for this example.

VI. CONCLUSIONS AND RECOMMENDATIONS

This paper has presented a special form of the Ideal State Reconstructor for deterministic digital control systems which is simpler to implement than the most general form. The Ideal State Reconstructor is so named because: if the plant parameters are known exactly, its output will exactly equal, not just approximate, the true state of the plant and accomplish this without any knowledge of the plant's initial state. Besides this, it adds no new states or eigenvalues to the system. Nor does it affect the plant equation for the system in any way; it affects the measurement equation only. It is characterized by the fact that discrete measurements are generated every T/N seconds and input into a multi-input/multi-output MA process. The output of this process is sampled every T seconds and utilized in reconstructing the state of the system.

The Ideal State Reconstructor is ideally suited for systems where measurements are available at a faster rate than the control law equations need to be solved. A good implementation of it would be to have a microprocessor dedicated to solving the MA-prefilter calculations recursively as the measurements become available every T/N seconds. Every T seconds or N calculations, the result could be transferred to a central processor where the remaining calculations in the reconstructor are made and the control law equations are solved.

Since it is an open loop type state reconstructor, the Ideal State Reconstructor may be less robust than the state observer when parameter uncertainties and measurement and process noise are considered. This is an area for further study. However, intuitively it would seem: the more measurements used in reconstructing a given state, the more robust the reconstructor. If so, then making N as large as possible would help. Perhaps, N could be as large as 100, or even 1000, with the suggested implementation, in many practical problems. If it turns out that robustness is still a problem, then the future of the Ideal State Reconstructor may be that of a prefilter to the state observer, or better yet the Kalman filter. In a system where measurements are available at a much faster rate than the Kalman filter algorithms can be solved, the Ideal State Reconstructor might provide a descent estimate of the state which could then be further improved by the Kalman filter. This also is an area for future study.

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1. REPORT NO.	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.
NASA TP-2847		

4. TITLE AND SUBTITLE	5. REPORT DATE
More on Exact State Reconstruction in Deterministic Digital Control Systems	September 1988
	6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S) Michael E. Polites	8. PERFORMING ORGANIZATION REPORT #
9. PERFORMING ORGANIZATION NAME AND ADDRESS George C. Marshall Space Flight Center, Marshall Space Flight Center, Alabama 35812	10. WORK UNIT NO. M-597
	11. CONTRACT OR GRANT NO.
12. SPONSORING AGENCY NAME AND ADDRESS National Aeronautics and Space Administration, Washington, D.C. 20546	13. TYPE OF REPORT & PERIOD COVERED Technical Paper
	14. SPONSORING AGENCY CODE

15. SUPPLEMENTARY NOTES

Prepared by Structures and Dynamics Laboratory, Science and Engineering Directorate.

16. ABSTRACT

This paper presents a special form of the Ideal State Reconstructor for deterministic digital control systems which is simpler to implement than the most general form. The Ideal State Reconstructor is so named because: if the plant parameters are known exactly, its output will exactly equal, not just approximate, the true state of the plant and accomplish this without any knowledge of the plant's initial state. Besides this, it adds no new states or eigenvalues to the system. Nor does it affect the plant equation for the system in any way; it affects the measurement equation only. It is characterized by the fact that discrete measurements are generated every T/N seconds and input into a multi-input/multi-output moving-average (MA) process. The output of this process is sampled every T seconds and utilized in reconstructing the state of the system.

17. KEY WORDS	18. DISTRIBUTION STATEMENT
Digital Control Systems Multirate Sampling State Estimation State Reconstruction	Unclassified – Unlimited Subject Category: 31

19. SECURITY CLASSIF. (of this report)	20. SECURITY CLASSIF. (of this page)	21. NO. OF PAGES	22. PRICE
Unclassified	Unclassified	24	A02

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