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Final Technical Report

COMPUTATIONAL METHODS FOR THE PROBLEMS OF THE TIP VORTEX

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The grant has been devoted to the study of the tip vortex problem for helicopter rotors in transonic flow. The first model to be studied is that of potential flow past a single blade in parallel transonic flow. The incidence on the wing of a vortex sheet trailing from a blade in front has been studied by modeling the incident sheet by a single vortex filament. David Ross has written a 3D potential flow code to incorporate the usual boundary conditions at infinity together with the model of such a vortex filament and to analyze the resulting flow over the wing. This model runs rapidly on the CRAY computer and gives results comparable to those of the standard code FLO22 written by Antony Jameson.

The code has been used to study the effect of the vortex filament on the production or shocks at the tip of the wing. We start from shock flow past a virtually shockfree wing at free stream Mach numbers which would otherwise produce an enclosed supersonic zone with a shock wave. Then the vortex sheet modeled by the filament is allowed to approach the wing. This produces a shock of moderate strength at the tip. We have analyzed the effect of twisting the wing by a moderate amount at its tip. It has been shown that a twist of not more than 10 degrees will eliminate the shock at the tip of the wing caused by the incident vortex sheet. This suggests the possibility of controlling such shocks at the tip of a rotor blade through an appropriate twist of the tip section in a hovering helicoptor.

Another problem studied within the framework of potential flow theory has been the rollup of the vortex sheet behind the wing. Robert Krasny has developed a two-dimensional time-dependent model for the rollup of such a vortex sheet in two dimensions. He studies the time-dependent effect of the rollup of the vortex sheet when it has been initialized with a standard distribution of vorticity. His code is based on a classical integral equation of Birkhoff. The Birkhoff integral equation has a singular kernel that has been smoothed by Krasny in a fashion analogous to filtering of a Fourier series. This procedure allows him to study rolling of the vortex through as many as several dozen turns. The resolution of the method far exceeds that of earlier attempts. His code provides a benchmark for further modeling of the tip vortex through potential flow simulation.

A principal issue investigated in the grant has been a comparison of flow models described by the potential equation, by the Euler equations for steady flow, and by the Navier-Stokes equations. Octavio Betancourt has written a new spectral code that solves the partial differential equations for steady rotational flow without viscosity. This problem has a direct analogy with the system of nonlinear equations for magnetohydrodynamic equilibrium. The new spectral code allows one to study the amount of artificial viscosity required to model weak solutions in which many vortex sheets may occur simultaneously. On the basis of calculations with the new spectral method, it is conjectured that fourth-order terms introduce an artificial viscosity that scales like the square of the mesh size that will be effective in simulating flows of this kind. Such procedures also provide an interesting model of coherent structures in turbulent flow.

The question has been studied whether there is an advantage in solving the Navier-Stokes equations rather than the Euler equations for steady flow. Straightforward analysis suggests that in problems of aerodynamics such as occur in the study of helicoptor blades the Reynolds number of the physical flow will be so large that the physical viscosity has in effect orders of magnitude less than that of the numerical viscosity. This comment applies even when the artificial viscosity scales like the square of the mesh size as described above. Thus the effect of the Navier-Stokes terms will appear to be negligible in computational methods of this kind. Boundary layer effects are of course an additional issue and should be subject to a separate analysis.

The conclusion of the grant is that major additional efforts must be made to model adequately the tip vortex problem for helicoptor rotor blades. The potential flow equations appear to be the most accurate and efficient approach. However, details of coding to track the vortex sheet in potential flow are unattractive and tedious so that it would also be desirable to consider the Euler equations to capture the vortex sheet. Numerical experiments thus far indicate, however, that the vortex sheet is appreciably smeared by numerical viscosity. More specifically, its evolvement with time is presumably misleading due to the small value of the numerical Reynolds number. Krasny's benchmark calculations of the rollup of a vortex sheet in two dimensions provide examples with which to calibrate the various Euler codes. Unfortunately, the spectral method has not yet been refined so as to model the interaction of the vortex sheet with a blade or wing. Surely this remains an open problem on which much research has yet to be done.

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