Phasing the Antennas of the Very Large Array for Reception of Telemetry From Voyager 2 at Neptune Encounter

J. S. Ulvestad
Tracking Systems and Applications Section

The Very Large Array (VLA) radio telescope is being instrumented at 8.4 GHz to receive telemetry from Voyager 2 during its encounter with Neptune in 1989. This article examines the procedure in which the 27 antennas have their phases adjusted in near real time so that the signals from the individual elements of the array can be added coherently. Calculations of the expected signal-to-noise ratio, tests of the autophasing (or self-calibration) process at the VLA, and off-line simulations of that process are all presented in this article. Various possible procedures for adjusting the phases are considered. It is shown that the signal-to-noise ratio at the VLA is adequate for summing the signals from the individual antennas with less than 0.1 dB of loss caused by imperfect coherence among the antennas. Tropospheric variations during the summer of 1989 could cause enough loss of coherence to make the losses higher than 0.1 dB. Experiments show that the losses caused by the troposphere can probably be kept below 0.2 dB if the time delay inherent in the phase adjustment process is no longer than ~5 seconds. This relatively small combining loss meets the goal established in order to minimize the bit error rate in the Voyager telemetry and implies that the autophasing of the VLA should be adequate for reception of telemetry from Voyager 2 at its Neptune encounter.

I. Introduction

The Very Large Array (VLA) will be used in 1989 to receive telemetry from Voyager 2 when it encounters Neptune. The VLA, an array of 27 telescopes located on the Plains of San Agustin in New Mexico, is described in detail in [1]. Several years ago, it was identified as a potentially important contributor to the reception of telemetry from the Voyager 2 spacecraft at Neptune [2]. Work is currently proceeding to equip all 28 VLA antennas (27 in operation plus one in the maintenance cycle) for signal reception at 8.4 GHz and to design the optimal system for telemetry reception from Voyager. One of the crucial issues is the best procedure for keeping all 27 operational antennas in phase with one another so that the signals received by those antennas can be added coherently. The individual antenna phases will be adjusted in near real time so as to keep the summed signals coherent by maximizing the interferometric fringe amplitude of the Voyager signal. Preliminary tests of this real-time self-calibration procedure (hereafter referred to as "autophasing") and a discussion of some of the issues involved can be found in [3]. Several key questions remain to be answered more completely. Specifically, those questions are as follows:

(1) Is the signal-to-noise ratio on the Voyager signal sufficient to allow the autophasing process to work?

(2) How well will the "global" autophasing procedure improve the signal-to-noise ratio in comparison with the previous method for determining antenna phases? (The difference between the two methods is described in Section IIA.)

(3) To what extent can the summer troposphere corrupt the phases?

(4) Given the constraints of both the troposphere and the expected signal-to-noise ratio, what is the optimal algorithm for phase correction that can be easily implemented to work in near real time?

For Voyager, the goal of the autophasing process has been to keep the combining loss for 27 VLA antennas below 0.2 dB. Such a loss would have minimal effect on the bit error rate of the Voyager telemetry. The purpose of the work described in this article is twofold: to address the above questions by means of real and simulated autophasing and to determine the best way to meet the goal of less than 0.2 dB of combining loss.

II. Autophasing Procedure

A. Description

The autophasing process is conceptually fairly simple. It will be described briefly here, with details about the implementation of models and corrections in VLA fringe rotators being largely ignored. A point-like celestial radio source, either a natural source or an artificial source such as the Voyager spacecraft, is observed by the individual VLA antennas. During the observations, standard phase corrections are implemented at each antenna based on a geometric model. For each pair of antennas, observed fringe phases after the application of that model are found from the cross-correlated data. The on-line computers then compute residual phases for each individual antenna. The residuals may be caused by a variety of problems, such as errors in the radio source position, imperfect refraction models, and instabilities in electronics. Most such problems give rise to slowly varying phases that are easily corrected, so the most stringent autophasing "problem" is that of trying to correct for a rapidly varying troposphere along each antenna's line of sight. The residual phases are used as error signals to derive phase corrections. These corrections are applied to a local oscillator at each antenna in order to keep the phase of that antenna near zero relative to the other antennas. If the corrections are successful, the VLA will be "phased up," and the individual antenna signals will sum coherently.

In the past, the VLA phasing algorithm has used only one baseline in computing the residual phase for each antenna. The phases on the baselines from each antenna to a single reference antenna have been used to compute the individual antenna phases in an attempt to bring the phases together with that of the reference antenna. Since not all possible baselines are used in the phase computations, the highest signal-to-noise ratio (SNR) that might be available is not achieved. With the VLA being controlled by a new computer system that was brought on line in January 1988, the individual phases are computed "globally" via a least-squares algorithm that uses all available baselines to solve for the individual antenna phases. This improves the determination of the individual antenna phases, as described more fully below.

The determination of residual antenna phases is only part of the autophasing process. The other important aspect of the procedure is the correction algorithm by which these residual phases are "fed back" to the local oscillators of each antenna. It is important to recognize a key limitation of this feedback loop: namely, it takes a period of time for the residual phases to be computed and for phase corrections to be communicated to the individual antennas, *and the VLA must continue observing during this time*. Therefore, there is a delay before any correction can be applied to the antennas, and data taken during the delay interval will have a somewhat outdated phase correction applied. In previous implementations of autophasing at the VLA, a 20-second delay has existed between the end of one 10-second integration and the beginning of the 10-second integration in which the phase information from the earlier integration is used. Taking an average time at the middle of each integration period, the net delay in this procedure is approximately 30 seconds. (Also see Section IIC below.) Any correction or feedback loop acts as a filter that corrects for low-frequency tropospheric fluctuations but misses the highest-frequency fluctuations. Minimizing both the integration time and delays in the feedback loop should help correct more of the higher-frequency fluctuations and reduce the combining loss when a large number of antenna signals are added.

B. Signal-to-Noise Calculations

1. General considerations. One can calculate the SNR of a single interferometer pair in a variety of ways. An extensive description of such a computation for the VLA is given in [4], and the calculation made here largely follows that reference. The application to Voyager is fairly straightforward, with one major exception. The applicable case for almost all of radio astronomy is that of a source with no significant circular polarization, whereas the signal from Voyager is 100 percent circularly polarized. Therefore, the antenna temperature (or signal strength) contributed by Voyager in a single circularly polarized channel is a factor of 2 higher than that for an unpolarized source having the same total power.

The noise power of a single antenna at the input to the correlator is

$$P_N = k_B T_{sys} \Delta\nu G \tag{1}$$

where k_B is Boltzmann's constant, T_{sys} is the system temperature of the antenna, $\Delta\nu$ is the bandwidth, and G is the power gain of the antenna. When two such antennas (whose noises are the results of independent, random processes) are correlated, the variance of the noise power coming out of the correlator is 1/2 the product of the powers divided by the number of independent samples in the correlator:

$$P^2_{rms} = \frac{P^2_N}{2\Delta\nu\Delta t} \tag{2}$$

where Δt is the integration time. Therefore,

$$P_{rms} = \frac{k_B G T_{sys}\sqrt{\Delta\nu}}{\sqrt{2\Delta t}} \tag{3}$$

The correlated signal power from a source contributing an antenna temperature T_a is $P_a = \eta_c k_B G T_a \Delta\nu$, and the antenna temperature contributed by Voyager, a 100 percent circularly polarized source, is $T_a = \eta_a A P_V/(k_B \Delta\nu)$. Here, η_c is an efficiency factor including the effect of the correlator sampling and the duty cycle of the observations; η_a is the antenna aperture efficiency; A is the physical area of the antenna; and P_V is the spacecraft power per unit area at Earth. Combining all these expressions, we arrive at a formula for the SNR (R) for Voyager:

$$R \equiv \frac{P_a}{P_{rms}} = \frac{\eta_c \eta_a A P_V \sqrt{2\Delta t}}{k_B T_{sys}\sqrt{\Delta\nu}} \tag{4}$$

A small correction for atmospheric attenuation, which is dependent on the weather and on the elevation angle of the spacecraft during the observations, should also be applied but is assumed to be negligible in deriving Eq. (4).

2. Standard parameters and assumptions. We use the following standard parameters for the Voyager observations at the VLA:

$$\eta_c = 0.79$$

$$\eta_a = 0.62 \pm 0.03$$

$$T_{sys} = 35^{+6}_{-4} \text{ K at a 30-degree elevation angle}$$

$$P_V = 5.0 \pm 0.4 \times 10^{-21} \text{ W/m}^2 \text{ at Neptune}$$

$$\Delta\nu = 8 \text{ MHz}$$

$$\Delta t = 10 \text{ seconds}$$

$$\text{zenith opacity} = 0.010^{+0.008}_{-0.0015} \text{ neper}$$

Using the above values in Eq. (4), we find a Voyager SNR of $3.90^{+0.58}_{-0.76}$ for a single VLA baseline. Some of the errors and quantities quoted above are taken from [5], while others were estimated from unpublished data acquired by the author while performing tests at the VLA.

The probability distribution of the phase errors on each baseline has a well-known dependence on the SNR (see [4]). At a moderately high SNR, that distribution approaches a Gaussian distribution whose standard deviation is

$$\sigma_\phi = \frac{1}{R} \tag{5}$$

For $R \approx 3.9$, the actual probability of a given phase error differs from the value given by the Gaussian approximation by $\leqslant 3$ percent for phase errors with a substantial probability of occurrence, so the use of Eq. (5) gives a "typical" phase error within a few percent of the value derived from the exact probability distribution. In the case of a perfect autophasing algorithm (or no tropospheric phase problems), the rms residual phase after autophasing should be equal to the rms phase errors predicted by the limiting SNR. For $R = 3.90^{+0.58}_{-0.76}$, the predicted rms residual phase for each antenna relative to a user-selected reference antenna would be $0.26^{+0.05}_{-0.04}$ radian, or $14.7^{+2.9}_{-2.2}$ degrees. In general, at the VLA, there is no absolute knowledge of the phase of any individual antenna. Therefore, the phase at the user-selected reference antenna is *defined* to be zero, and all other antenna phases are determined relative to that value.

3. Scaling with elevation angle (air mass). Using the results given in [2], a typical X-band atmospheric temperature at the VLA is found to be ~2.73 K per air mass. We will estimate errors of +1.0 K and −0.15 K per air mass (already incorporated in the T_{sys} uncertainty at 30-degree elevation that was given above). These errors are estimates based on the variations seen in the data from tipping curves taken under various weather conditions at the VLA. Large positive excursions in atmospheric temperature can occur during the summer thunderstorm season, but there are not enough data to permit the statistical distribution of the summer atmospheric temperatures to be accurately determined. Relative to the 30-degree elevation value, the additional T_{sys} at 20-degree elevation will be $2.52^{+0.92}_{-0.14}$ K. At 10-degree elevation, the additional temperature will be $10.26^{+3.76}_{-0.56}$ K. The opacity caused by the atmosphere will be assumed to be as quoted above, $0.010^{+0.008}_{-0.0015}$ neper per air mass (again based on VLA tipping data). As with system temperature, the higher positive error in the opacity partially accounts for the possibility of thunderstorms at the VLA. However, the most severe thunderstorms may give greater increases in system temperature and opacity than are assumed here.

Crane[1] recently measured gain curves for VLA antennas at 22 GHz. He found significant changes at low elevations which may be caused by antenna deformations and/or failures of the antennas to track changing foci. From Fig. 2 of Crane,[1] we estimate median gain losses at 22 GHz of 5 percent in going from 30-degree to 20-degree elevation and 11 percent in going from 30-degree to 10-degree elevation. Assuming that the losses are dominated by antenna deformation or pointing errors, these losses should be reduced at X band. We estimate (conservatively) that the gain changes scale inversely with wavelength. Therefore, at X band, the expected gain losses in going to 20-degree and 10-degree elevation will be 2 percent and 4 percent, respectively. Errors of +50 percent and −25 percent are assigned to the degradation at each elevation angle, with the larger positive error caused by the increased spread in the antenna gain losses at the lowest elevation angles.

4. Changing integration times. Because an integral number of phase-switching cycles must be included in each unit VLA integration time, the allowed integration times are quantized in units of a single complete phase-switching cycle. This cycle time is 1 2/3 seconds (32 of the 19.2-Hz VLA waveguide cycles). Therefore, allowed integration times, in seconds, are 1 2/3, 3 1/3, 5, and so on. Integration times longer than 10 seconds will not be considered in this work. The SNR per baseline scales as the square root of the integration time.

5. Changing bandwidths. The SNR scales as $1/\sqrt{\Delta\nu}$. In addition to the nominal value of 8 MHz, a 6-MHz bandwidth will briefly be considered below. It will be assumed that all significant telemetry sideband power can be accommodated in the 6-MHz bandwidth, so the Voyager signal power will not be reduced for the narrower bandwidth. At worst, this may give an error of ~0.1 dB, a value considerably smaller than the 0.3-dB uncertainty quoted for the Voyager power above.

6. Global autophasing. In the case of global autophasing, every baseline in the array is used to compute the phases of the individual antennas. Theoretical work (e.g., [6]) shows that the determination of individual antenna phases via a least-squares calculation will give rms errors that are reduced from the single-baseline values by $\sqrt{N-2}$, where N is the number of antennas. Simulations[2] and actual VLA observations (see below) show that this scaling may overestimate the accuracy of the phase determination by ~25 to 30 percent. It will be assumed that the rms phase improvement given by global autophasing is $(0.7 \pm 0.1)\sqrt{N-2}$.

7. Calculations of expected phase errors. Table 1 summarizes the various parameters and scaling laws described above. Using the numbers quoted in that table and assuming global autophasing, we find the following formulas for the rms phase errors at three different elevation angles:

$$\sigma_\phi(30^\circ) \approx \left(4.38^{\circ\,+1.06}_{\ \ -0.90}\right) \times \sqrt{\frac{10}{\Delta t}}\sqrt{\frac{23}{N-2}} \qquad (6a)$$

$$\sigma_\phi(20^\circ) \approx \left(4.88^{\circ\,+1.23}_{\ \ -0.99}\right) \times \sqrt{\frac{10}{\Delta t}}\sqrt{\frac{23}{N-2}} \qquad (6b)$$

$$\sigma_\phi(10^\circ) \approx \left(6.18^{\circ\,+1.73}_{\ \ -1.23}\right) \times \sqrt{\frac{10}{\Delta t}}\sqrt{\frac{23}{N-2}} \qquad (6c)$$

Table 2 gives the rms phases predicted *only from the SNR* for different elevations and integration times. For the global autophasing, it is assumed that 25 VLA antennas are operational. The right-hand column gives the expected values for each set of parameters. The predicted phase residuals for the highest SNRs in the table are not far above the rms phase noise of ~2 degrees on time scales of ~5 to 10 seconds for the VLA. (This phase noise is determined by factors such as the quantization in the lobe rotators.) All phases in the right-hand column of the table will give combining losses much less than 0.2 dB (see Section V), so the signal-to-noise ratio for short integrations on the Voyager spacecraft is adequate to give phase determinations of very high accuracy in the case of global autophasing. Narrowing the filter bandwidth to 6 MHz would give a slightly higher SNR and predicted rms phases about 15 percent lower than those in the table, but the calcu-

[1] P. C. Crane, "Measurements of Flux Densities and Gain Corrections at 22485.1 MHz," VLA Test Memorandum No. 149 (internal document), Socorro, New Mexico: National Radio Astronomy Observatory, April 1987.

[2] F. R. Schwab, "Robust Solution for Antenna Gains," VLA Scientific Memorandum No. 136 (internal document), Socorro, New Mexico: National Radio Astronomy Observatory, September 1981.

lations show that the decreased observing flexibility of the narrower filters is not necessary to give an adequate SNR.

C. Tropospheric Fluctuations and the Autophasing Feedback Loop

The rms phase computed above is that which would be expected for observations of the Voyager spacecraft through a static troposphere. In fact, the troposphere over each antenna can be quite active. Water vapor cells of various sizes blow over the antennas, changing the effective path length, and hence the observed phase, at each antenna. (See [7] and [8] for more detailed theoretical and empirical treatments of this phenomenon.) The small-scale disturbances blow across an antenna quickly, giving rise to rapid phase fluctuations. These high-frequency fluctuations are the ones that are not adequately corrected by the autophasing feedback loop. Thus, the residual phases after the autophasing process can be much larger than would be predicted just from the signal power and the individual antenna characteristics.

When the VLA is being autophased, the following feedback loop has been implemented for correcting the residual antenna phases:[3]

$$\phi^{t+1}_{\text{corr}} = \phi^{t}_{\text{corr}} + \alpha\phi^{t-k}_{\text{res}} \tag{7}$$

where ϕ^{t}_{corr} is the phase correction applied during the tth integration period, ϕ^{t+1}_{corr} is the correction to be applied in the $(t + 1)$th integration interval, ϕ^{t-k}_{res} is the residual phase found in the $(t - k)$th integration period, and α is the gain of the feedback loop. Strictly speaking, this formula is correct only for the "standard" VLA integration time of 10 seconds, because the exact nature of the time delays in the feedback loop is related to the internal data-handling procedures and to the frequency with which commands updating the local oscillator frequencies are sent to the antennas. For integration times of 10 seconds, the standard VLA phasing algorithm has used $k = 2$ and $\alpha = 0.25$. Note that this feedback loop is different from that described in the Appendix of [3], where the crucial built-in time delay is not taken into account.

Equation (7) can be rewritten in more generality by replacing ϕ^{t}_{corr} with the correction phase at any past integration period $(t - j)$. The index j can take on integer values 0, 1, 2, and so on. A more general expression for the feedback loop is then

$$\phi^{t+1}_{\text{corr}} = \phi^{t-j}_{\text{corr}} + \alpha\phi^{t-k}_{\text{res}} \tag{8}$$

[3]G. Hunt, interoffice memorandum to B. Brundage (internal document), National Radio Astronomy Observatory, Socorro, New Mexico, March 3, 1987.

In this expression, j and k need not take on the same values, although they may be the same if desired.

In [9] (e.g., see Fig. 9 of [9]), it is shown that the autocorrelation function of VLA phase data that are corrupted by the troposphere generally decreases with increasing time lag. As can be seen from the above formulas, there is a considerable time delay built into the feedback algorithm. If the time delay were reduced, more recent phase data from a higher part of the autocorrelation function could be used to generate the corrections of the autophasing process. That should reduce the residual phase substantially in cases where the tropospheric fluctuations dominate the phase errors. One of the major goals of the tests described below has been to determine how much the residual phases could be reduced when compared with the current feedback process.

III. Autophasing Observations of the Voyager Spacecraft

The VLA antennas are gradually being equipped with receivers at X band (8.4 GHz) in order to receive Voyager telemetry. Significant numbers of antennas were not instrumented until late 1986. As part of the monthly JPL tests during 1987, substantial effort was devoted to various tests related to autophasing. This section describes the portion of tests that involved the standard autophasing of the VLA in real time, accomplished by observing Voyager at X band through 8-MHz bandpass filters.

The observations in the spring of 1987 were made when Voyager was transmitting at low power, ~2.0 dB below its high-power mode. However, the spacecraft was also closer to Earth than it will be at Neptune. In March, April, and May, the spacecraft transmitter power should have been ~2.6 dB higher than the low-power values will be at Neptune. Therefore, the SNR on each baseline should have been ~4.4, which would translate into an rms phase residual of 13 degrees. The respective measured values for the rms residual phase in March, April, and May 1987 were 15.0, 16.7, and 15.6 degrees, which would correspond to an average SNR of ~3.6, ~0.8 dB less than the highest possible expected value.

Although the observed phase residuals were slightly higher than those expected based on simple SNR considerations, it is quite likely that they were dominated by the system noise. Several factors could have reduced the average SNR below ~4.4: there is some uncertainty in the Voyager power; the pointing of the Voyager antenna is not updated as frequently during the cruise mode as it is near an encounter; the pointing of the VLA antennas was not completely optimized; several antennas had higher noise receivers than the production receivers which use high-electron-mobility transistors (HEMTs); and two antennas were partially shadowed during some of the observations. Estimated errors in the Voyager power and the power losses due to spacecraft and ground antenna pointing were in the 0.1- to 0.5-dB range. The antennas lacking HEMT amplifiers were ~0.8 dB noisier than those with such amplifiers. The partial shadowing also caused the SNR to be reduced by ~0.2 to 0.3 dB on some baselines. Further, the fact that the autophasing feedback algorithm was not optimal for the SNR-limited case may have added as much as ~1 degree to the phase residuals (see Section IVA). The combination of these effects can easily account for the 0.8-dB (or ~3 degrees of phase) decrement relative to the optimal SNR. The closeness of the measured values to the predicted values, taken together with the factors that would serve to reduce the SNR slightly, implies that the SNR calculation models the actual observations quite well.

IV. Autophasing Simulations

Various off-line simulations have been performed to validate results obtained with the VLA autophasing process and to explore parameter space a bit more thoroughly. These simulations have used a variety of values for the gain and for the number of periods of delay (α, j, and k in Eq. [8]) in the feedback loop. Because of constraints set by the data-handling procedures at the VLA, not all of the feedback loop parameters considered are actually usable at the VLA. However, they serve to indicate the improvement that may be expected by decreasing the integration and delay times. The autophasing simulations have been applied to phase data obtained by the VLA in its "normal" interferometry mode–i.e., with no autophasing applied on line. This section describes the results of the off-line simulations.

A. Observations of Voyager: Single-Antenna-Referenced Phase Computations

The Voyager spacecraft was observed through the 8-MHz filters in normal interferometer mode on several occasions during 1987. The phases relative to a specified reference antenna, which were exactly those that would have gone into the feedback algorithm in real time during autophasing at the VLA, were fed into a simulation program instead. The data in March and May were taken with 10-second integration periods. Simulations were run for three different values of the gain in the feedback loop: 0.125, 0.25, and 0.5. Another set of simulations was run for the same three gains, but with a time delay of only one integration ($k = 1$ in Eq. [8]). All these simulations used the correction phase from the last integration period in the recursive formulation of Eq. (8) (i.e., $j = 0$). The rms residual phases for both sets of simulations are shown in Table 3.

Three important points should be noted from Table 3. First, the residual phases for a gain of 0.5 are very much lower for a time delay of only one integration than for a delay of two integrations. The high residuals for a feedback loop with a delay of two integration periods seem to be caused by the fact that the loop is marginally stable for a gain of 0.5; this is confirmed both by simulations using faked data and by analytical results found by J. W. Armstrong (private communication). Second, the results for a gain of 0.25 and a delay of two integration periods are very close to the values obtained on the same dates using the VLA in its autophasing mode (see Section III). This provides confidence that the simulations are working properly. Third, the simulations with a gain of 0.125 provided slight improvement over those with a gain of 0.25, since the lower gain gives a better performance in the SNR-limited case.

B. Observations of Voyager: Global Phase Computations

It is expected that the global phase computations will increase the SNR in the autophasing, as described in Section II. There were seven antennas used in March 1987 and nine in May 1987. Theoretical analysis implies that global phase computations should have improved the SNR by $\sqrt{5} = 2.24$ in March and by $\sqrt{7} = 2.65$ in May. As mentioned in Section II, these theoretical improvements should probably be reduced by ~25 percent, to ~1.7 and ~2.0. Simulations of global autophasing were run on the same data used in the analysis described in the previous subsection, and with the same choices of parameters in the feedback loop. The VLA program ANTSOL was first run in the off-line computers to compute the phases globally. Then, those phases were fed into the autophasing simulation programs. Results are shown in Table 4.

Several comments are in order regarding Table 4. For the March data, the rms phase residuals for the four cases with gains of 0.25 or less were reduced by factors between 1.71 and 1.80 when the global phase calculations were used. (The simulations with a gain of 0.5 have contributions from algorithm instability that are too large to allow for the reliable estimation of the improvement caused by better SNR.) These improvement factors are close to the predicted value of ~1.7. The actual improvement in global autophasing indicated by the May data was only a factor of ~1.4, substantially less than the predicted improvement. The lack of improvement in the May data is probably attributable to weather that was significantly worse than that in March, with the troposphere-induced phase problems becoming more important as a limiting factor when the SNR was improved by the global phase calculations.

In November 1987, data were again taken on the Voyager spacecraft in the normal interferometer mode. These data were acquired using 14 X-band antennas, all equipped with HEMT receivers. Since the troposphere is relatively quiet during this time of year, these data were expected to be SNR-limited, as the March data had been. The increased number of antennas available meant that the global phase computation could be expected to give a larger improvement than it did with only seven or nine antennas. In fact, the expected improvement factor was ~2.6, whereas the real improvement factors ranged from 1.7 to 2.0. Results are shown in Table 5.

Since the Voyager spacecraft was transmitting in its high-power mode (accounting for the generally lower phase residuals in November when compared to March), the SNR was higher on each baseline. This means that the residual phases were low enough so that the tropospheric "noise" would make a more important contribution and might create a "floor" above that expected from simple SNR considerations. There is some evidence for this effect in simulations using the last previous phase to predict the next phase (not actually possible in the real VLA because of finite computation times). As shown in the bottom line of Table 5, this gives an improvement factor of 2.4, much closer to that predicted. Since only the most recent phase point is used in this case, the effective delay is reduced, and the troposphere should be followed better. However, it is not yet clear whether the troposphere is really at fault or if the actual improvement from the global phase computations will be somewhat less than that expected. Therefore, an improvement factor of $(0.7 \pm 0.1)\sqrt{N-2}$ has been used in Section III and Table 2 rather than the factor of $0.8\sqrt{N-2}$, which is more in line with the results given in VLA Scientific Memorandum No. 136 (see footnote 2).

C. Summertime, Low-Elevation Observations at a High Signal-to-Noise Ratio

The worst tropospheric effects should occur during the summer, during observations of low-elevation sources. The tests run during July 1987 were designed for the observation of strong point radio sources at a variety of elevation angles that are characteristic of the Voyager spacecraft. Strong sources were observed in order to ensure that the phase errors were dominated by the troposphere rather than by limited SNR. For 5-second integrations on sources with several janskys of flux density (1 jansky = 10^{-26} W/m^2/Hz), observing through a 50-MHz bandwidth, the SNR was ~100 to 1 on each baseline. Normal interferometry observations were made of several sources during the 4 1/2-hour test. As stated previously, the fact that the autocorrelation of the phase fluctuations is larger for shorter time lags implies that the shorter integrations should help reduce the residual phases after autophasing corrections are applied.

Two strong radio sources, 0237–233 and 1730–130, were observed at X band at elevations ranging from 9 to 25 degrees. The phase data from these two sources were separated into three bins according to their elevation angle. A third source, 3C345, was observed at an elevation of ~25 degrees. This gave a total of seven different sets of interferometry data. All seven were treated with the autophasing simulation algorithm using various parameters for the integration time and the delay interval. The data were taken in the "A" configuration, in which the three arms of the VLA range from 19 to 21 km in length. There were nine X-band antennas operational.

Using an antenna near the center of the VLA as a reference, the simulations mimicked phase adjustments on the other eight antennas, which were at distances of 0.8 to 20.5 km from the reference antenna. The data were averaged for 10-second intervals, and autophasing was simulated using a feedback algorithm with delays of one or two integration periods. In addition, simulations were performed on the same data with their initial 5-second integration periods. A set of runs was also made with no time delay and with a gain of 1.0 for the 5-second-interval data. This simulation uses the last previous phase as the predictor of the next phase. It is not a physically realizable system, since a finite time is required to calculate the antenna phases before they can be applied to incoming data. However, it does provide a limiting phase residual; all possible systems having the same integration time should have phase residuals greater than those given by this method.

In addition to the feedback loop parameters described above, all of which used $j = 0$ in Eq. (8), simulations were run using a feedback loop in which $j \neq 0$. Specifically, the case where $j = 1$ and $k = 1$ has been considered. This choice of parameters has the effect of using a recent "observed" phase as the predictor of the next observed phase. In this special case, Eq. (8) becomes

$$\phi_{\text{corr}}^{t+1} = \phi_{\text{corr}}^{t-1} + \phi_{\text{res}}^{t-1} \qquad (9)$$

The sum of the phase correction and the residual at time $(t - 1)$ is the phase that would have been observed at time $(t - 1)$ if there had been no autophasing correction at that time.

Table 6 gives the results of the autophasing simulations for the different source observations in July 1987. For each source and elevation angle, results for seven different sets of parameters are shown for the generalized feedback algorithm of Eq. (8). Figure 1 plots the residual phase for two different sources as a function of elevation angle, while Fig. 2 shows results for a single observation as a function of baseline length.

In each case, simulations with different sets of parameters for the autophasing are shown. Table 6 and these figures illustrate several very important points:

(1) When compared with the current autophasing feedback algorithm, the residual phase is reduced by a factor of ~2.5 simply by reducing the integration time to 5 seconds, reducing the time delay in application of the residuals to a single integration period, and increasing the gain to 0.5. A further improvement of ~10 to 15 percent is possible if the same delay is used and if the last available phase for a given antenna is used to predict the next phase at that antenna–i.e., if the algorithm given in Eq. (9) is used.

(2) A further factor of ~2 improvement could be achieved if it were possible to compute antenna phases in an infinitesimally short time and immediately use these phases as predictors for the following integration period. However, this is not a physically realizable system. The delay time could be reduced to some value closer to zero if there were more computing resources, but logistical and financial considerations make this action impractical. Since the integration time must be at least 1 2/3 seconds because of the VLA phase-switching characteristics, and the integration time should be even longer than that to achieve adequate autophasing SNR on the Voyager signal, there must be some delay inherent in the individual integrations (i.e., the data from the beginning of a 5-second integration are 5 seconds old by the end of the integration period).

(3) The residual phase is independent of baseline length for baselines ranging from less than 1 km to over 20 km.

(4) The residual phase depends on elevation angle, with the residuals near 12-degree elevation being ~1.5 to 2 times greater than those for observations of the same source near ~22-degree elevation.

(5) Although results are not displayed, the residuals for phases calculated only with reference to a single antenna are almost identical to those for phases computed via a global phase-determination procedure. Since the SNR on each baseline was well above 50 in 5-second integrations on the strong sources, this result simply confirms the expectation that system noise was not a limiting factor for the strong sources.

The result summarized in point 3 is not surprising given the character of tropospheric fluctuations. Fluctuations over a given antenna can be viewed as being caused by a tropospheric pattern blowing across that antenna (e.g., [7], [8]). For an assumed tropospheric wind speed of 8 m/s, representative of that measured 1 km above Goldstone, California [7], fluctuations move no more than 80 meters in a time of 10 seconds or less. Therefore, phase fluctuations on such short time scales are caused only by atmospheric cells having sizes less than ~80 meters. These fluctuations are completely uncorrelated between antennas separated by more than ~80 meters, and larger (longer time scale) fluctuations are adequately corrected by the feedback loop if the total delay time is no greater than ~10 seconds. Hence the size of the VLA configuration seems to make little difference. Even the smallest possible VLA configuration, with each arm of the array having nine antennas along a line 600 meters long, has the majority of its baselines longer than 100 meters. Since there is considerable mutual antenna shadowing in this "D" configuration, the larger "C" configuration, with arm lengths of ~2 km, is preferred for Neptune encounter.

The apparent elevation dependence of the residual phase is expected because of the increased number of air masses; hence the increased chance of an antenna line of sight intercepting small tropospheric cells at the low elevation angles. It is clear that the shortest possible integration time that still provides adequate SNR is needed to correct the phases near the 8-degree elevation limit of the VLA.

D. High-SNR Observations During Thunderstorms

Several short observations of strong sources were obtained during thunderstorms in August and September 1987. The purpose of these observations was to determine the ability of the autophasing process to correct for the phase fluctuations during the worst weather expected for the Neptune encounter. These observations generally lasted for ~45 minutes and were made when there were thunderstorms and/or rain showers at the control building of the VLA. Since they were made with a small subarray while normal astronomical observing proceeded with the bulk of the VLA, only four antennas were available, and there was no direct control over the integration time. The interferometric data were analyzed via simulations similar to those described above. Table 7 gives the results from the three data sets acquired in August and September 1987.

Table 8 shows results of autophasing simulations on data taken on the radio source 1519–273 during a September thunderstorm (the same results were shown in Table 7) and on a November night. Although the antenna spacings were shorter in November than in September, they were still large enough so that the uncorrected fluctuations over different antennas were uncorrelated. The data during the two months were acquired at identical elevation angles and show a difference of a factor of 5 to 6 in residual phase. This illustrates the magnitude of the possible degradation caused by having the

c - 4

Neptune encounter occur during the summer rather than the winter. Scaling the thunderstorm data to integration times and delay periods of 5 seconds, according to the trend shown in Table 6, yields an expectation for an rms phase residual of less than ~20 degrees in the worst summer weather encountered during these tests. However, it is clear that the previously employed feedback loop and integration time, which can yield residual phases in the 30- to 45-degree range, would have given substantial combining losses for the VLA antennas.

V. Combining Losses

Given a distribution of residual antenna phases, it is possible to compute the power loss expected when the signals from the individual VLA antennas are summed. At the VLA, the signal combination is done in an analog sum device. The power output of this device is

$$P = \sum_{j=1}^{N} E_j e^{i\phi_j} \sum_{k=1}^{N} E_k e^{-i\phi_k} \tag{10}$$

Here, P is the total power, E_j is the real electric field amplitude at antenna j, i is the complex number representing $\sqrt{-1}$, ϕ_j is the residual phase of the jth antenna, and N is the number of antennas.

Assuming that all antennas are identical, E_j and E_k can be set equal to a constant, E, and taken outside the sums. Then, Eq. (10) becomes

$$\begin{aligned} P &= E^2 \sum_{j=1}^{N} \sum_{k=1}^{N} e^{i(\phi_j - \phi_k)} \\ &= E^2 \sum_{j=1}^{N} \sum_{\substack{k=1 \\ k \neq j}}^{N} e^{i(\phi_j - \phi_k)} + NE^2 \end{aligned} \tag{11}$$

In Eq. (11), the terms in which $j = k$ have been evaluated and taken out of the sum. Next, assume that the ensemble average of the phase difference between any two antennas is independent of which antenna pair is chosen. If the phase difference between two antennas is $\Delta\phi$, the sum in Eq. (11) can be evaluated, and the average power coming out of the analog sum device is

$$\langle P \rangle = N(N-1)E^2 \langle e^{i\Delta\phi} \rangle + NE^2 \tag{12}$$

If the residual phases of the identical interferometer pairs are Gaussian distributed with variance σ_ϕ^2, Eq. (12) becomes

$$\begin{aligned} \langle P \rangle &= \frac{N(N-1)E^2}{\sigma_\phi \sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\Delta\phi} \exp\left[\frac{-(\Delta\phi)^2}{2\sigma_\phi^2}\right] d(\Delta\phi) + NE^2 \\ &= N(N-1)E^2 \exp\left[\frac{-\sigma_\phi^2}{2}\right] + NE^2 \end{aligned} \tag{13}$$

The addition of N antennas with zero relative phase gives a total power ($\equiv P_{max}$) of N^2E^2, so the ratio of the summed power to the maximum possible signal power is simply given by

$$\begin{aligned} \frac{\langle P \rangle}{P_{max}} &= \frac{(N-1)}{N} \exp\left[\frac{-\sigma_\phi^2}{2}\right] + \frac{1}{N} \\ &\approx \exp\left[\frac{-\sigma_\phi^2}{2}\right] \end{aligned} \tag{14}$$

The approximation in the second line of Eq. (14) slightly overestimates the combining loss. For 23 to 27 antennas, the overestimate of the loss is only ~0.02 dB for σ_ϕ = 25 degrees and less for lower phase residuals. Therefore, $\exp(-\sigma_\phi^2)/2$ is a slightly conservative estimate of the expected combining loss at the VLA. Identifying σ_ϕ with the rms residual phase after the autophasing procedure, we can immediately see how the summed power will be reduced with increasing phase residual caused either by limited SNR or by tropospheric fluctuations. Figure 3 is a plot of the signal loss vs. rms phase residual. From this figure, note that the desired maximum combining loss of 0.2 dB occurs for a residual phase of ~17.4 degrees.

VI. Trade-offs Between Troposphere and SNR

In August 1989 (prime thunderstorm season), the first part of each Voyager pass will be in the late afternoon, when the atmospheric phases can be expected to fluctuate rapidly. It has been shown that the residual phases expected in the SNR-limited case are no more than ~10 degrees for the VLA with 25 operational antennas as long as the integration period is at least 3 1/3 seconds. Phase errors this small are expected to give combining losses well below 0.2 dB. Since the shortest possible integration time is desired to correct for the high-frequency troposphere delays, this implies that an integration period of 3 1/3 or 5 seconds is desirable during Voyager's Neptune encounter in 1989. An integration time of 5 seconds would provide some signal margin at low elevation angles and should serve to track the troposphere adequately in most cases.

The shortest integration time allowed, 1 2/3 seconds, would be even better for correcting the troposphere. However, there are two arguments against using such a short integration time. First, the SNR on the Voyager signal would be marginal, with predicted phase residuals of ~15 degrees at the lowest elevations. Second, the necessary phase computations, data handling, and communication to the individual antennas will take several seconds, long enough to reduce some of the benefits of the shortest integration period. In fact, the required computation time would probably be increased slightly because of the reduced SNR at the shortest integration time.

VII. Summary

VLA tests combined with computer simulations have been conducted to study the process of autophasing the antennas at the VLA. Investigations of the SNR of the autophasing process have been both theoretical and empirical, while the investigations of the troposphere and of various feedback loops have been largely empirical in nature. A study is needed to determine the statistical properties of the troposphere, the relation of the data to models such as that in [9], and the consequent expectations for various feedback loops.

The analysis reported here has yielded the following major conclusions:

(1) The minimum rms phase achievable by the autophasing procedure can be predicted by considerations of the SNR derived from known properties of the antennas. Both real VLA autophasing and off-line simulations using VLA data taken during observations of Voyager 2 confirm the predictions. Formulas for the rms phase of the phasing process are given as functions of elevation angle, system temperature, and integration time.

(2) Also predicted is the improvement in the autophasing process that occurs when individual antenna phases are computed by means of a least-squares algorithm. The observed improvement for several 1987 tests is somewhat less than that predicted on pure theoretical grounds, but generally consistent with the results found in other simulations. Further tests with more antennas will be performed to see if the real improvement is close to that expected for the full VLA.

(3) It is clear that tropospheric effects can greatly increase the residual phases of the VLA antennas in the autophasing process, especially at the lowest elevations. The residual tropospheric effects can be reduced substantially by decreasing the delay time inherent in the autophasing procedure. If the integration time can be made reasonably short, it appears that simple feedback loops are adequate to correct the VLA phases so that the loss in the process of combining the signals from the individual VLA antennas is less than 0.2 dB.

(4) For short integration times, there is no apparent dependence of the rms residual phase on baseline length. This implies that the best tropospheric corrections can be made without choosing a VLA configuration that is so small that there is significant shadowing of antennas by one another.

(5) Consideration of the trade-offs between tropospheric activity and SNR implies that a VLA integration time of 3 1/3 seconds or 5 seconds will be best for observing the Voyager spacecraft in 1989 and will be adequate to meet the goal of less than 0.2 dB of combining loss.

Acknowledgments

Thanks are owed to J. Armstrong, D. Brown, B. Brundage, J. Davidson, G. Resch, and R. Treuhaft for their comments on earlier versions of this article and for suggestions during various stages of this work.

References

[1] P. J. Napier, A. R. Thompson, and R. D. Ekers, "The Very Large Array: Design and Performance of a Modern Synthesis Radio Telescope," *IEEE Proceedings*, pp. 1295–1320, November 1983.

[2] J. W. Layland et al., "Interagency Array Study Report," *TDA Progress Report 42-74*, vol. April–June 1983, Jet Propulsion Laboratory, Pasadena, California, pp. 117–148, August 15, 1983.

[3] J. W. Layland, P. J. Napier, and A. R. Thompson, "A VLA Experiment—Planning for Voyager at Neptune," *TDA Progress Report 42-82*, vol. April–June 1985, Jet Propulsion Laboratory, Pasadena, California, pp. 136-142, August 15, 1985.

[4] P. C. Crane and P. J. Napier, "Sensitivity," in R. A. Perley, F. R. Schwab, and A. H. Bridle (eds.), *Synthesis Imaging*, Socorro, New Mexico: National Radio Astronomy Observatory, pp. 87–108, 1986.

[5] J. S. Ulvestad, G. M. Resch, and W. D. Brundage, "X-Band System Performance of the Very Large Array," *TDA Progress Report 42-92*, vol. October–December 1987, Jet Propulsion Laboratory, Pasadena, California, pp. 123–137, February 15, 1988.

[6] T. Cornwell, "Self-Calibration," in R. A. Perley, F. R. Schwab, and A. H. Bridle (eds.), *Synthesis Imaging*, Socorro, New Mexico: National Radio Astronomy Observatory, pp. 137–147, 1986.

[7] R. N. Treuhaft and G. E. Lanyi, "The Effect of the Dynamic Wet Troposphere on Radio Interferometric Measurements," *Radio Science*, vol. 22, pp. 251–265, 1987.

[8] J. W. Armstrong and R. A. Sramek, "Observations of Tropospheric Phase Scintillations at 5 GHz on Vertical Paths," *Radio Science*, vol. 17, pp. 1579–1586, 1982.

[9] J. P. Basart and Y. Zheng, "Modeling Very Large Array Phase Data by the Box-Jenkins Method," *Radio Science*, vol. 21, pp. 863–881, 1986.

Table 1. Standard parameters for SNR calculations

Parameter	Value
System temperature (30-degree elevation)	35 ± 3 K
Atmospheric temperature	$2.73^{+1.0}_{-0.15}$ K per air mass
Aperture efficiency (30-degree elevation)	0.62 ± 0.03
Voyager signal strength	$(5.0 \pm 0.4) \times 10^{-21}$ W/m^2
Attenuation	$0.01^{+0.004}_{-0.0015}$ per air mass
Gain loss factor (20-degree elevation)	$0.02^{+0.01}_{-0.005}$
Gain loss factor (10-degree elevation)	$0.04^{+0.02}_{-0.01}$
Global autophase improvement	$(0.7 \pm 0.1)\sqrt{N-2}$

Table 2. Predicted phase residual based only on SNR of Voyager observations with VLA (errors in the rms phase residuals are ~+25 percent and ~−20 percent)

Elevation, degrees	Integration time, seconds	SNR (one baseline)	σ_ϕ(global), degrees
30	10	3.9	4.4
30	5	2.8	6.2
30	3 1/3	2.3	7.6
20	10	3.5	4.9
20	5	2.5	6.9
20	3 1/3	2.0	8.5
10	10	2.8	6.2
10	5	2.0	8.7
10	3 1/3	1.6	10.7

Table 3. RMS phase residuals for Voyager observations after simulations of the old VLA autophasing procedure with 10-second integrations

Gain (α)	Number of periods delayed		RMS phase, degrees	
	j	k	March	May
0.125	0	2	14.7	14.8
0.25	0	2	15.5	15.8
0.5	0	2	24.7	24.1
0.125	0	1	14.8	14.3
0.25	0	1	15.1	14.6
0.5	0	1	17.1	17.4

Table 4. RMS phase residuals for Voyager observations after simulations of the global autophasing procedure

Gain (α)	Number of periods delayed		RMS phase, degrees	
	j	*k*	March	May
0.125	0	2	8.3	10.8
0.25	0	2	8.6	11.5
0.5	0	2	17.8	17.6
0.125	0	1	8.6	10.1
0.25	0	1	8.5	10.1
0.5	0	1	9.6	10.8

Table 5. Phase residuals for autophasing simulations during observations of the Voyager spacecraft in November 1987

Integration time, seconds	Gain (α)	Number of periods delayed		RMS phase, degrees	
		j	*k*	Reference antenna method	Global method
10	0.125	0	2	11.2	6.7
10	0.25	0	2	11.7	6.7
10	0.5	0	2	17.1	10.0
10	0.125	0	1	10.9	6.3
10	0.25	0	1	11.1	6.0
10	0.5	0	1	12.5	6.2
10	1.0	0	0	13.2	5.4

Table 6. RMS phase residuals for autophasing simulations on strong sources on July 22, 1987

Source	Integration time, seconds	Gain (α)	Number of periods delayed		RMS phase, degrees
			j	k	
0237–233, 9- to 14-degree elevation	10	0.25	0	2	24.0
	10	0.25	0	1	21.0
	5	0.25	0	2	15.5
	5	0.25	0	1	13.5
	5	0.5	0	1	10.6
	5	1.0	0	0	5.4
	5	1.0	1	1	9.2
0237–233, 15- to 19-degree elevation	10	0.25	0	2	14.1
	10	0.25	0	1	12.5
	5	0.25	0	2	8.8
	5	0.25	0	1	7.8
	5	0.5	0	1	6.1
	5	1.0	0	0	3.5
	5	1.0	1	1	5.6
0237–233, 20- to 24-degree elevation	10	0.25	0	2	10.2
	10	0.25	0	1	8.8
	5	0.25	0	2	6.6
	5	0.25	0	1	5.9
	5	0.5	0	1	5.0
	5	1.0	0	0	2.9
	5	1.0	1	1	4.5
3C345, 22- to 26-degree elevation	10	0.25	0	2	12.8
	10	0.25	0	1	11.5
	5	0.25	0	2	8.1
	5	0.25	0	1	7.3
	5	0.5	0	1	6.0
	5	1.0	0	0	3.5
	5	1.0	1	1	5.4
1730–130, 9- to 14-degree elevation	10	0.25	0	2	33.0
	10	0.25	0	1	27.2
	5	0.25	0	2	20.0
	5	0.25	0	1	17.1
	5	0.5	0	1	12.2
	5	1.0	0	0	6.1
	5	1.0	1	1	10.8
1730–130, 15- to 19-degree elevation	10	0.25	0	2	29.3
	10	0.25	0	1	23.3
	5	0.25	0	2	17.4
	5	0.25	0	1	15.0
	5	0.5	0	1	11.3
	5	1.0	0	0	5.7
	5	1.0	1	1	9.9
1730–130, 20- to 25-degree elevation	10	0.25	0	2	16.8
	10	0.25	0	1	14.4
	5	0.25	0	2	11.2
	5	0.25	0	1	9.6
	5	0.5	0	1	7.7
	5	1.0	0	0	4.1
	5	1.0	1	1	6.8

Table 7. Residual phases for simulations on data taken during thunderstorms and rain showers

Source	Integration time, seconds	Gain (α)	Number of periods delayed		RMS phase, degrees
			j	k	
1127–145,	8.33	0.25	0	2	25.9
21- to 26-degree	8.33	0.25	0	1	22.1
elevation,	8.33	0.5	0	1	17.7
August 11, 1987	8.33	1.0	1	1	16.3
	8.33	1.0	0	0	8.5
3C279,	8.33	0.25	0	2	17.5
50-degree elevation,	8.33	0.25	0	1	15.3
September 18, 1987	8.33	0.5	0	1	12.0
	8.33	1.0	1	1	10.6
	8.33	1.0	0	0	6.3
1519–273,	10	0.25	0	2	44.6
15- to 22-degree	10	0.25	0	1	37.6
elevation,	10	0.5	0	1	32.2
September 22, 1987	10	1.0	1	1	27.2
	10	1.0	0	0	16.0

Table 8. Comparison of data taken on 1519–273 at 15 to 22 degrees of elevation during a September thunderstorm and during a November night

Integration time, seconds	Gain (α)	Number of periods delayed		RMS phase, degrees	
		j	k	September	November
10	0.25	0	2	44.6	7.6
10	0.25	0	1	37.6	6.3
10	0.5	0	1	32.2	5.7
10	1.0	1	1	27.2	5.1
10	1.0	0	0	16.0	3.3

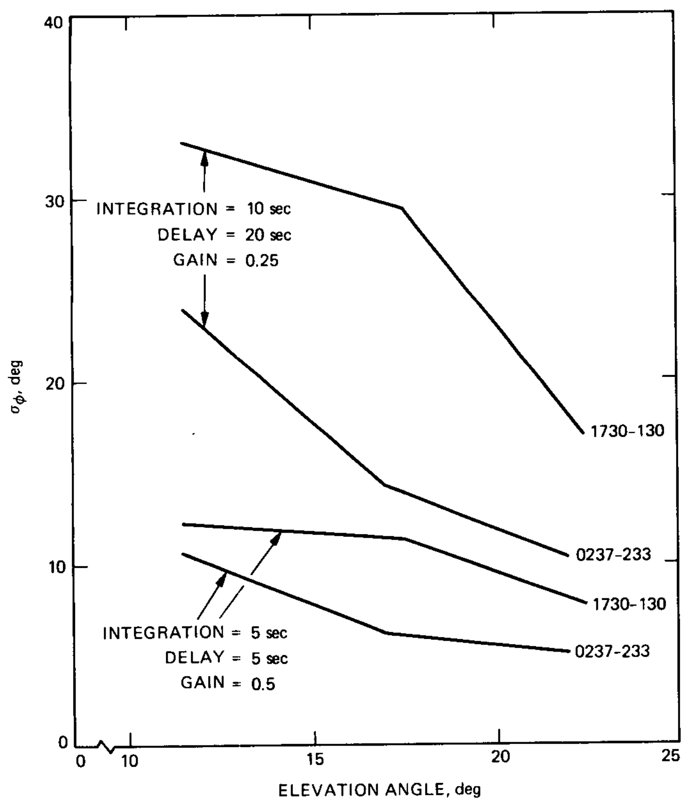

Fig. 1. Residual phase from autophasing simulations for two different sources for different autophasing parameters

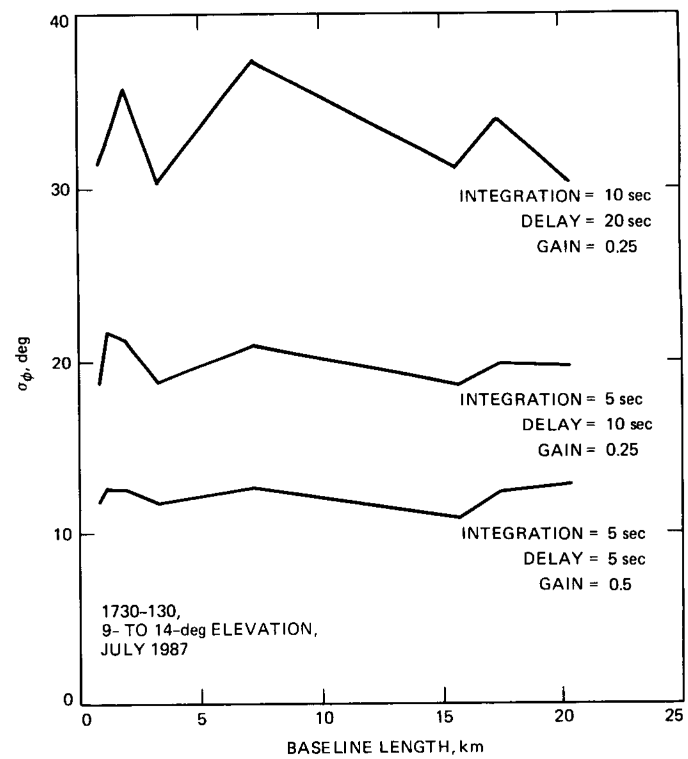

Fig. 2. Plot of residual phase versus baseline length for autophasing simulations on a single set of observations

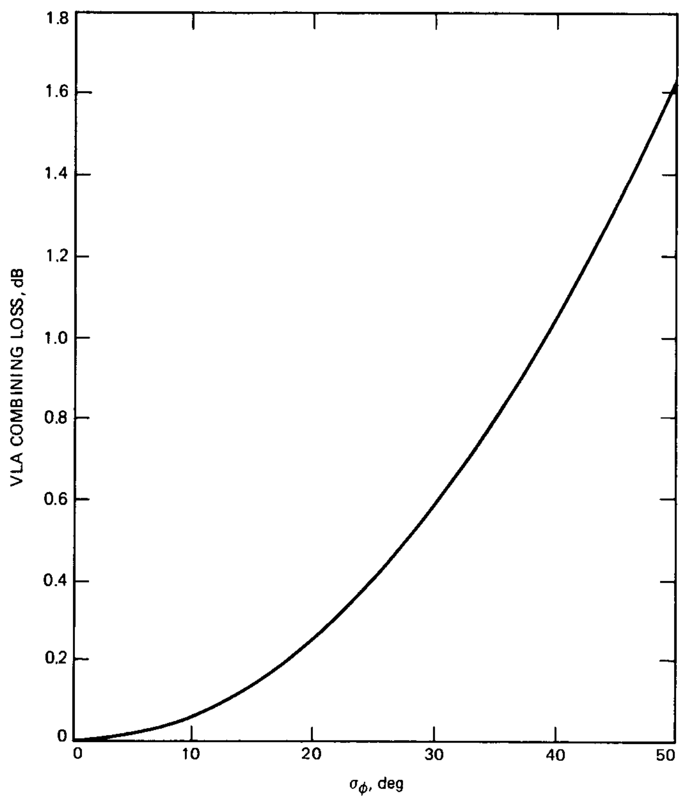

Fig. 3. Combining loss for the sum of many antennas as a function of rms residual phase for each antenna