The Application of Single Particle Hydrodynamics in Continuum Models of Multiphase Flow

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Abstract: A review of the application of single particle hydrodynamics in models for the exchange of interphase momentum in continuum models of multiphase flow is presented. Considered are the equations of motion for a laminar, mechanical two phase flow. Inherent to this theory is a model for the interphase exchange of momentum due to drag between the dispersed particulate and continuous fluid phases. In addition, applications of two phase flow theory to de-mixing flows require the modelling of interphase momentum exchange due to lift forces. The applications of single particle analysis in deriving models for drag and lift are examined.

I. Introduction

Before the advent of numerical approximation techniques, coupled with large computational facilities the study of single particle hydrodynamics was popular. Following the early conceptual efforts on theories of multiphase flow [1],[2] investigations into the form of certain classes of phenomena were undertaken. Specifically, those investigating the form for models of interphase momentum exchange found a wealth of information by evoking the arguments once used in single particle hydrodynamics. As an example, consider the following quote [3], used in a context specific to arguments relating to forms of momentum exchange models in continuum theories of multiphase flows:

"On the other hand, the terms ... and ... have no analogs in single particle calculations and will be neglected."

It would appear that the application of single particle hydrodynamics in continuum models of multiphase flow has received a degree of acceptance.

II. Continuum Theories of Multiphase Flow

Continuum theories of multiphase flow have developed along parallel lines: Mixture Theory and the Theory of Interpenetrating Media with Moving Interfaces. Both approaches are rooted in the same fundamental assumption, namely: That both the dispersed and continuous phases of the flow can be treated as and described within an Eulerian kinematic framework by the conservation equations of a macro-continua. Implicit in this assumption is that the variable fields of each phase are unique and continuous over the flow domain. The limits of this assumption for the case of dilute concentrations of the dispersed phase have been explored [4] and the alternative of a Lagrangian or 'particle tracking' kinematic scheme for the dispersed phase forwarded. In addition, continuum models have been adapted to granular material flows [5] where the dispersed phase concentration approaches a maximum.

Mixture theories arise from the specialization of the classical field theory requirements of internally consistent thermodynamic arguments [6],[7]. In contrast, the theories of interpenetrating media address more directly modifications to the classical transport equations due to discontinuous or 'jump' conditions at moving phase boundaries [8],[9],[10],[11]. To reduce to a local form, the conservation equations resulting from the theories of interpenetrating media must be averaged in either space or time. In fact, the key difference between the mixture theory formulation for multiphase flows and the averaged conservation equations for interpenetrating media is that; while the averaging process is implicit in the mixture theory approach it is an explicit operation in the course of writing the conservation equations from the interpenetrating, moving phase boundary approach.

With few exceptions, it is reassuring to note both approaches result in the same set of conservation equations.

Consider a simple two phase flow. That is one in which the dispersed phase is a dilute, mono-dispersed suspension of non-reacting, smooth, rigid, spherical particles in an incompressible, linearly viscous fluid. Both phases and the surroundings are thermally equilibrated. Laminar flow conditions prevail.

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Regardless of the method of formulation, the conservation equations reduce to the expressions for continuity and balance of momentum [10],[12] and constitute the continuum equations of motion of the mechanical theory of two phase flow.

CONTINUITY: $\dfrac{\partial \phi^\alpha \rho^\alpha}{\partial t} + \underline{\nabla} \cdot \phi^\alpha \rho^\alpha \underline{V}^\alpha$, $\alpha = D$ (DISPERSED), C (CONTINUOUS) (1)

SATURATION: $\phi^D + \phi^C = 1$ (2)

BALANCE OF MOMENTUM: $\phi^\alpha \rho^\alpha \left(\dfrac{\partial \underline{V}^\alpha}{\partial t} + \underline{V}^\alpha \cdot \underline{\nabla}\, \underline{V}^\alpha \right) = -\phi^\alpha \underline{\nabla} p^\alpha + \underline{\nabla} \cdot \underline{T}^\alpha + \phi^\alpha \rho^\alpha \underline{b}^\alpha + \underline{\hat{M}}^\alpha + \left(\lambda - p^\alpha \right) \underline{\nabla} \phi^\alpha$ (3)

WHERE $\phi\,(\underline{x},t)$: CONCENTRATION

ρ : MATERIAL DENSITY

$\underline{V}\,(\underline{x},t)$: VELOCITY

$p\,(\underline{x},t)$: SPHERICAL STRESS (PRESSURE)

$\underline{T}\,(\underline{x},t)$: DEVIATORIC STRESS

$\underline{\hat{M}}\,(\underline{x},t)$: INTERPHASE MOMENTUM EXCHANGE

$\lambda\,(\underline{x},t)$: SATURATION (CONTACT) PRESSURE

The flow has two velocity fields and two concentration fields. Each phase has a unique, constant material density and may be acted upon by a set of external potentials or body forces. In addition, there exists unique expressions for the spherical and deviatoric elements of the dispersed and continuous phase stress tensors. Lastly, under the assumption that the flow is saturated, i.e. no phaseless voids may develop, the phases are coupled by an interface "saturation" pressure and momentum exchange or transfer between the phases.

If the external potentials are specified, the momentum exchange terms modeled and the deviatoric elements of the stress tensors specified by constitutive assumption or neglected via arguments with respect to magnitude, the resulting is the unclosed system of 9 equations in 10 unknown fields; velocity, concentration and spherical stress or pressure, for each phase. Commonly, the assumption of equal phase pressures is used in an effort to create a closed determinant system of fields and conservation equations [1],[11]. However, in should be noted that the wisdom of this assumption has been challenged on both physical and mathematical grounds [12],[13],[14].

III. Models of Momentum Exchange in Two Phase Flow

As a common denominator, all theories of two phase flow embody some model for the exchange of momentum between phases. For simple, mechanical two phase flow it can be demonstrated rigorously that the sum of the interphase momentum transfer must be conservative, i.e. the sum of all momentum transfer terms is zero. Those exceptions to this "summing rule" are more a matter of bookkeeping than conceptual difference [15].

In addition, the degree of coupling between the phases of the flow , both in a physical and mathematical sense is controlled by the momentum transfer model. Two way coupling, i.e. momentum transfer from one phase to the other and vice versa is implicit in the requirement of conservative transfer of momentum. In attempts to simplify the computational complexities associated with applying continuum theories of two phase flow the assumption of one way coupling is often evoked [16]. One way coupling allows for the transfer of momentum from the continuous to the dispersed phase, but not vice versa. In this case the process of momentum exchange is not conservative. Within this context, one way coupling is synonymous with the statement that the velocity field of the continuous phase is unchanged due to the presence of the dispersed phase within the flow. The computational simplifications result from the fact that it is no longer necessary to solve the conservation equations for the continuous phase field variables simultaneously with the conservation equations of the dispersed phase. Given a solution to the continuous phase variable fields, perhaps generated by single phase analysis or experimental techniques, the uncoupled conservation equations can be solved for the dispersed phase variable fields.

The use of a step function to describe the "effectiveness" of momentum transfer has been proposed and applied to the problem of laminar two phase jet flows [1],[17]. This approach allows the degree of coupling to vary, step-wise from uncoupled to one way coupled. The arguments raised are: that in flows where interparticle spacing in large relative to the sum of the particle diameter and twice the fluid boundary layer thickness on the surface of the particles no net momentum transfer between the phases occurs. It is argued that when the interparticle spacings are large and the suspension is dilute, the slip between the phases results only in unrecoverable dissipation in the particle wakes. However, if the multitude of efforts in the analysis of two way coupled, dilute two phase flows can be used as an indication, then it would appear that the "ineffectiveness" of interphase momentum transfer in dilute two phase flows is not generally accepted.

Arguments have been presented for the set of variables which constitute the general class of admissible momentum exchange functions [3],[18].

$$\underline{M}^{\alpha} \cdot \phi^{D} S \left(\underline{V}^{D} - \underline{V}^{C} \right) + \phi^{D} B \, \underline{\nabla} \cdot \underline{\nabla} \, \underline{V}^{C} + C \, \underline{\nabla} \left(\underline{\nabla} \cdot \underline{V}^{C} \right) + L \left(\underline{V}^{D} - \underline{V}^{C} \right) \cdot \underline{D}^{C} + \cdots$$

$$\tag{4}$$

WHERE: $\underline{D}^{C} \left(\underline{x}, t \right) = 1/2 \left(\underline{\nabla} \, \underline{V}^{C} + \underline{V}^{C} \, \underline{\nabla} \right)$, RATE AT DEFORMATION TENSOR

where ... indicates that additional functions, of higher order in gradient, do exist. S,B,C and L are constructed from the scalar invariants of the admissible vector and tensor fields. In addition, owing to the discrete nature of the dispersed phase, the admissibility, as a class of momentum exchange function of gradients of dispersed phase field variables is still debated [11]. If the existence of smooth, continuous first partials derivatives to the dispersed phase variables is at question, consider: Implicit in the assumption of the continuum model of two phase flow was that the dispersed phase could be treated as a macro-continua, i.e. that the dispersed phase field variables are smooth and continuous.

In fact, the use of Divergence (Gauss') theorem in the process of reducing the global conservation equations to the local form requires the existence of continuous first partial derivatives of dispersed phase velocity and concentration fields.

The appropriateness of using phenomenologically based arguments for the purpose of identifying the specific forms of the momentum exchange models from the general admissible class is justified [19]. In fact, more often than not it is the lack of a phenomenological or physical analogue that results in the neglect of a class of the admissible momentum exchange functions.

Depending on the purpose of the analysis, four generic categories of momentum exchange processes and models are identifiable:

-Drag Forces
-Lift Forces
-Inertial Coupling or Virtual Mass Effects
-Inertial History Effects

The arguments for the inclusion of the inertial coupling/virtual mass effects and inertial history effects are raised during the construction of models for momentum exchange in two phase flows [1],[20],[21]. The analogous single particle hydrodynamic forces have been investigated and debated [22],[23],[24],[25]. Inertial coupling stems from the analysis of the forces required to displace a given volume of fluid during the acceleration of a particle through it. Likewise, the inertial history or Basset force is linked with the acceleration history of a particle moving through a quiescent fluid. However, the lack of complete agreement on the single particle hydrodynamic analysis of these effects and the lack of agreement on the forms or even the necessity for retaining these effects [12] in models of momentum exchange in two phase flows precludes them from additional discussion at this point.

Drag Forces

Common to all theories of two phase flow is a model for momentum exchange due to drag between the fluid phase and the particles of the dispersed phase [11],[12],[26],[27],[28]. Intrinsic to each of these models is the existence of a slip velocity between the phases. The result being a net drag force on each phase:

$$\underline{M}^D_{drg} = -\underline{M}^C_{drg} = S(\underline{V}^D - \underline{V}^C) \tag{5}$$

where the factor of proportionality; S is derived from arguments which are rooted in single particle analysis.

The most sophisticated models for S are derived from an analysis of the mean, terminal sedimentation velocities of a dispersion of spheres falling through a quiescent Newtonian fluid under gravity [11],[12],[29]. The analysis is limited to flows, about any one single sphere in the Stokesian regime, where the Reynolds number between the fluid and the sphere is of order unity or smaller. This implies either very small slip velocities and/or a very viscous fluid. The net result being that the inertia of the fluid phase is neglected.

Equilibrium between the force on a single sphere due to the gravitational potential and the terminal or steady state sedimentation velocity drag can be given as:

$$u_o = \frac{2a^2}{9\mu} (\rho^D - \rho^C) \underline{g} \tag{6}$$

WHERE: a: PARTICLE RADIUS

   μ: FLUID VISCOSITY

   $\underline{g}$: GRAVITATIONAL POTENTIAL

This sedimentation velocity potential is modified by the probability of hydrodynamic interaction with the next closest sphere, where the distance separating these spheres is defined probabilistically for a uniform dispersion. The result is the mean sedimentation velocity potential, corrected by the presence of a dispersion of other spheres of a given concentration:

$$\underline{u} = \underline{u}_o (1 - 6.55 \phi^D) \tag{7}$$

The incorporation of this result into a form for the interphase momentum exchange in two phase flow due to drag requires the suppositions that: At a given slip velocity the potentials acting on each phase due to drag are equal and opposite, i.e. momentum exchange is conserved. The slip velocity of two phase flow is then equated with the modified mean sedimentation velocity due to gravity. Resulting in, for a single spherical particle:

$$\frac{9\mu}{2a^2} (1 - 6.55 \phi^D) (\underline{V}^D - \underline{V}^C) = (\rho^D - \rho^C) \underline{g} \tag{8}$$

This result is then further generalized with the assumption that if this is the potential for interphase momentum exchange due to drag on a single sphere, then the drag force per unit volume of mixed flow should be simply this potential normalized by the local volume fraction or concentration of the particulate phase:

$$\hat{M}^D_{drg} = -\hat{M}^C_{drg} = \frac{9\mu}{2a^2} \phi^D (1 + 6.55 \phi^D) (\underline{V}^D - \underline{V}^C) \tag{9}$$

The caveats are obvious: The single particle analysis is grounded in the assumption of Stokes flow for the motion or slip of the dispersed phase relative to the continuous phase. Unlike the slip velocity of two phase flow, the sedimentation velocity at a given concentration is a constant, being the result of a constant potential; gravity. The modification to the mean sedimentation velocity by the hydrodynamic interactions with the other particles of the dispersion has presupposed that the dispersion is uniform, i.e. gradients of dispersed phase concentration are not present.

Lift Forces

It is observed that certain classes of laminar shear flows will result in demixing of the dispersed phase [30],[31], i.e. the dispersed phase will become distributed in a non-uniform manner, despite the fact that initially the flow may have been homogeneous with respect to dispersed phase concentration. Analysis of these phenomena have been made using single particle hydrodynamics [32] and using continuum two phase flow theory with the inclusion of lift forces in the description of the interphase momentum exchange [11],[12],[15],[16],[33]. It is generally accepted that the lift forces are those potentials, accounted for within the interphase momentum exchange which produce dispersed phase motions or migrations transverse to the slip or velocity difference between the phases. These lift forces arise through the interaction of the slip velocity, the rotation or spin of the particles of the dispersed phase and the shearing or gradients of the continuous phase velocity. If the exchange of momentum between the dispersed and continuous phase is conserved, then the lift force potentials act equally and in opposite sense on both phases.

A variety of single particle analysis, with the resulting identification of certain lift forces have been performed [23],[34],[35],[36],[37],[38]. The lift force on a single particle, as outlined by Saffman, [35] is the most commonly used in models of momentum exchange due to lift forces in continuum theories of demixing two phase flow. Saffman's analysis identifies the lift force:

$$L = \frac{K\mu a^2}{\nu^{1/2}} u(y) \left(\frac{\partial u(y)}{\partial y}\right)^{1/2} \tag{10}$$

WHERE ν :   FLUID KINEMATIC VISCOSITY

$u(y)$:   FLUID VELOCITY (IN PARALLEL FLOW)

Unlike the assumption of Stokes flow used in the analysis of and subsequent application to momentum exchange due to drag, Saffman's lift analysis begins with the Navier-Stokes equations. However, the Reynolds numbers for the slip velocity, the particle spin and the fluid shear are all constrained to order unity or smaller. Hence, even though the Saffman's analysis retains the inertial terms of the Navier-Stokes equations, the flow is not inertially dominated. The solution requires the matching of the "inner" and "outer" asymptotic expansions of the flow equations in an analysis technique pioneered before numerical approximation coupled with computational methods became available [39]. In Saffman's analysis the field variables for the flow about a single sphere are expanded about the radial position from the sphere center. The inner expansion has as a boundary condition the no slip requirement at the sphere surface. Since the sphere is spinning, the no slip boundary condition constrains the fluid to have the angular velocity of the sphere surface. Hence, the particle spin enters the calculation implicitly, despite the fact that due to the level of truncation, it does not appear explicitly in the expression for lift. The necessity for an outer expansion results from the non-convergent nature of the solutions to the inner expansion as the distance from the sphere center approaches the infinite.

The outer expansion embodies a second boundary condition, at a some large distance from the sphere center; namely, the undisturbed (by particles) velocity field as radial position approaches the infinite. In fact the primary difference in most single particle analysis of lift is whether the boundary condition for the fluid velocity field in the outer expansion is constrained by a wall condition [32], a quiescent fluid [34] or by the rate of fluid strain [35]. The assumptions implicit in Saffman's analysis of the lift force on a single sphere include: the flow is uniform and parallel, the slip velocity is parallel to the plane of the fluid shear, the shear or velocity gradients of the fluid are linear and the particle spin vector lies in the plane of the fluid shear, but is normal to the slip vector. The resulting force is normal to the plane of the fluid shear (and slip vector) as well as being normal to the spin vector of the particle. If, in terms of the slip velocity, the particle lags behind the fluid the lift will produce a migration of the particle into the faster, adjacent fluid and vice versa if the particle leads the fluid. In other words, the sense of the lift force depends on the sense of both the gradients of the fluid velocity and the slip velocity.

Saffman's result for the lift on a single spherical particle has been generalized for the analysis of the momentum exchange due to lift forces in continuum theories of de-mixing two phase flow [11],[12],[15]. The lift force on the dispersed phase is normalized by the number of particles in a unit volume of two phase flow. The resulting generalized form of the momentum exchange due to lift being written:

$$\underline{M}_m^D = -\underline{M}_m^C = \frac{3K(\rho^C \mu)^{1/2}}{4\pi a} \phi^D \underline{D}^C |\underline{D}^C|^{-1/2} \cdot (\underline{V}^D - \underline{V}^C) \tag{11}$$

This form of the momentum exchange due to lift has been used in calculations of parallel, de-mixing two phase flows using continuum theories. However, it is not clear that this general form will reduce directly to Saffman's result for a one dimensional, parallel flow, both in terms of magnitude and the directional nature of the lift force. In addition, despite the fact that the intentions are well motivated, the meaning of operations such as absolute value, square root and division by a second order tensor valued variable is not clear.

IV. Summary

In conclusion, it has been shown that single particle hydrodynamics is the only source presently used to derive and justify forms for the interphase momentum exchange models within continuum theories of laminar multiphase.

Four generic classes of momentum exchange models can be identified: Drag, Lift, Inertial or Virtual Mass effects and Inertial History or Basset forces. The latter two categories are still advent in nature and have not yet assumed a role in the models of interphase momentum exchange in applications of continuum theories of two phase flow. On the other hand, examples of Drag and Lift forces in applied momentum exchange models are numerous, though not without obvious caveats and inconsistencies.

It is encouraging that the requirements of emerging technologies based on an understanding of multiphase flow processes has motivated such a great deal of work on generalized models for interphase momentum exchange.

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