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UNMODELED DYNAMICS AND NONLINEAR CONTROL

Our research involves both theoretical and applicable results concerning systems of nonlinear ordinary differential equations and control of partial differential equations.

We present titles and abstracts of recent papers.

<u>Nonlinear Input–Output Systems</u>, L.R.Hunt, M.Luksic, R.Su; Nonlinear Analysis and Applications, Marcel Dekker, Inc., New York, V.Lakshmikantham, Ed., 1987, 261–266.

Recently, many researchers have considered the problem of classifying those nonlinear systems which are equivalent (in some proper sense) to linear systems. In addition to the theoretical results, applications have appeared in the areas of aircraft control, spacecraft maneuvering, robotics, and attitude control of a rotating satellite. One major theory involves the feedback equivalence of nonlinear control systems, without outputs, to a controllable linear system. Another deals with the state space and output space equivalence of nonlinear systems, without inputs, to an observable linear system with output injection (an algorithm that allows nonlinear systems with controls is also considered). The purpose of this paper is to present necessary and sufficient conditions that a nonlinear control system having outputs be feedback equivalent to a controllable linear system with linear outputs. A surprising discovery is that nonlinear feedback can be applied to overcome certain nonlinearities of the output equation, as well as those in the state equation in the following sense. After the dynamical state equation has been placed in linear form, particular nonlinearities in the output equation can be eliminated by state and input space coordinate changes followed by feedback.

Applications of Nonlinear Systems Theory to Control Design, L.R. Hunt and R. Villarreal; Dynamical Systems Approaches to Nonlinear Problems in Systems and Circuits, SIAM Proceedings, Fathi Salam, Ed., 1988,209–223.

For most applications in the control area, the standard practice is to approximate a nonlinear mathematical model by a linear system. Since the feedback linearizable systems contain linear systems as a subclass, we examine the procedure of approximating a nonlinear system by a feedback linearizable one. Because many physical plants (e.g. aircract at the NASA Ames Research Center) have mathematical models which are "close" to feedback linearizable systems, such approximations are certainly justified. We introduce results and techniques for measuring the "gap" between the model and its "truncated linearizable part." The topic of pure feedback systems is important in our study.

Nonlinear System Approximation, R.G. Goldthwait and L.R. Hunt; 26th Conference on Decision and Control, Los Angeles, 1987, 1752–1756.

We are interested in approximating a nonlinear system by a feedback linearizable system instead of a linear system. Two approaches presently exist in the literature. One involves the concept of involutivity to a certain order, and the other considers a canonical expansion and pure feedback approximation. We show the relationship between these two methods. This provides insight into lower order invariants in the equivalence problem for two nonlinear systems. Moreover, the output time responses for a nonlinear system and its feedback linearizable approximation are mentioned.

Nonlinear Filter Design, L.R. Hunt and P.W. Whitney; Mathematical Theory of Networks and Systems, C. Byrnes, C. Martin, and R. Saeks, Eds., North Holland, to appear.

Early work on the topic of nonlinear input–output system modeling is due to Volterra. Moreover, Wiener's research on nonlinear system identification is quite well known.

The purpose of this paper is to introduce a new technique for identifying nonlinear systems, and we begin with a single input -- single output system. Assuming the system is initially at rest, we calculate the first kernel (first convolution integral in the continuous case or first convolution sum in the discrete case). We then obtain a controllable and observable linear realization in a particular canonical form. We probe the actual nonlinear system with an appropriate input (or inputs) and determine the output (or outputs). For the linear system we compute the input that produces the same output. In the difference between the inputs to the nonlinear and linear systems, we find basic information about the nonlinear system. There is an interesting class of nonlinear systems for which this type of identification scheme should prove to be accurate.

Canonical Coordinates for Partial Differential Equations, L.R. Hunt and R. Villarreal, Systems & Control Letters 11, 1988, 159–165.

Suppose we consider the second order linear partial differential equation

$$-\frac{\partial u}{\partial t} + \sum_{j,k=1}^{n} A_{jk}(x)\frac{\partial^2 u}{\partial x_j \partial x_k} + \sum_{j=1}^{n} b_j(x)\frac{\partial u}{\partial x_j} = f$$

where A_{jk} and B_j are real C^∞ coefficients in $\mathbb{R}^n$. Assume that the matrix (A_{jk}) is symmetric, positive semidefinite, and of constant rank. When can we reduce an initial value problem for this equation to an initial value problem for an ordinary differential equation by using

i) coordinate changes on $\mathbb{R}^n$

ii) Fourier transforms?

We address this question by finding conditions under which coordinate changes exist that make the above equation either constant coefficient or of the Kolmogorov type. This latter equation can be attacked by Fourier transforms, resulting in a first order linear partial differential equation that can be reduced to ordinary differential equations.

<u>Parallels Between Control PDE's and Systems of ODE's,</u> L.R. Hunt and R. Villarreal , 27th Conference on Decision and Control, to appear.

System theorists understand that the same mathematical objects which determine controllability for nonlinear control systems of ordinary differential equations also determine hypoellipticity for linear partial differential equations. Moreover, almost any study of o.d.e. systems begins with linear systems. It is remarkable that Hörmander's paper on hypoellipticity of second order linear p.d.e.'s starts with equations due to Kolmogorov, which we show are analogous to the linear o.d.e.'s. Eigenvalue placement by state feedback for a controllable linear system can be paralleled for a Kolmogorov equation if an appropriate type of feedback is introduced. Results concerning transformations of nonlinear systems to linear systems are similar to results for transforming a linear p.d.e. to a Kolmogorov equation.

Simplifying Partial Differential Equations by Feedback, L.R. Hunt and R. Villarreal, submitted.

Given a partial differential equation, analysts often ask for conditions under which the equation takes a simplified form. For example, we consider coordinate changes that make the highest order part constant coefficient or Lie transformations that reduce the number of variables. However, if we view a partial differential equation as an input–output system, then the prospect of a simplifying feedback arises.

For systems of nonlinear control ordinary differential equations, much research and many applications have unfolded concerning the feedback equivalence with controllable linear systems. In a sense the well understood linear control system is an ideal model that we use for design. We consider second order linear partial differential equations with variable coefficients and propose for the parallel to the linear control system equations due to Kolmogorov. In these equations the second order spatial part is constant coefficient and the first order spatial part has linearly varying coefficients. We then examine the problem of feedback equivalence after introducing an interesting type of feedback.

Approximations of Nonlinear Control Systems, R.G. Goldthwait and L.R. Hunt, submitted.

There is a gap between mathematical models of physical systems and control design techniques. Many physical plants have nonlinear mathematical models, but most design schemes are based on linear systems (in either the time or frequency domain). In view of this a theory of exact "feedback linearization" for a class of nonlinear systems was developed and shown to be applicable in control system design. Since the feedback linearizable systems properly contain the linear systems and are accessible from the standpoint of linear design, we consider the problem of approximating a nonlinear system by a feedback linearizable one instead of the usual Taylor series linearization.

Nonlinear System Identification, L.R. Hunt, R.D. DeGroat, and D.A. Linebarger, submitted.

It is extremely difficult to identify general nonlinear systems because of the number of unknowns involved. Moreover, since most design techniques assume a linear model many of the nonlinearities that can be determined are essentially ignored. Why not develop nonlinear system identification techniques to reveal restricted classes of nonlinear systems that are mathematically tractable and design accessible? Some interesting classes are restricted by differential geometric conditions on a state space representation. The purpose of this paper is to develop methods to recognize related conditions from an input–output point of view.

In addition to the above papers, the following talks have been presented by the principal investigator.

"Nonlinear System Approximation", IEEE Conference on Decision and Control. Los Angeles, California, December, 1987.

"Canonical Coordinates and Feedback for Partial Differential Equations", International Conference on Theory and Applications of Differential Equations. Columbus, Ohio, March 1988.

"Approximations of Nonlinear Control Systems", Southern Methodist University, April 1988.

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