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THE OPTIMAL POLARIZATIONS FOR ACHIEVING MAXIMUM CONTRAST IN RADAR IMAGES

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Abstract

There is considerable interest in determining the optimal polarizations that maximize contrast between two scattering classes in polarimetric radar images. In this paper a systematic approach is presented for obtaining the optimal polarimetric matched filter, i.e., that filter which produces maximum contrast between two scattering classes. The maximization procedure involves solving an eigenvalue problem where the eigenvector corresponding to the maximum contrast ratio is optimal polarimetric matched filter. To exhibit the physical significance of this filter, it is transformed into its associated transmitting and receiving polarization states, written in terms of horizontal and vertical vector components. For the special case where the transmitting polarization is fixed, the receiving polarization which maximizes the contrast ratio is also obtained. Polarimetric filtering is then applied to synthetic aperture radar images obtained from the Jet Propulsion Laboratory. It is shown, both numerically and through the use of radar imagery, that maximum image contrast can be realized when data is processed with the optimal polarimetric matched filter.

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I. Introduction

Contrast enhancement is a processing technique which modifies the input data structure so that either the human observer, computer, or other hardware devices can extract certain information from the processed data more readily after the change [*Huang*, 1969]. In this paper, the polarimetric properties of the radar return are utilized to enhance the contrast between two scattering classes. It is assumed that complete *a priori* statistical knowledge of the two scattering classes or types exists and the polarimetric signals backscattered from the two scattering classes are independent [*Chan*, 1981]. The processing requirement is then to determine the optimal transmitting and receiving polarization state which will maximize the separation of the average power returns between the two classes. Applying such a technique to radar imagery will allow for better discrimination of the two classes.

For two deterministic scatterers, completely characterized by 2×2 complex scattering matrices, *Kozlov* [1979] introduced a method for computing the optimal polarization state which involves transformation of the scattering matrix of each of the two objects into a preferred polarization basis. Solutions involving scattering matrix *co-pol* and *cross-pol* (polarization) *nulls* have also been presented by *Kennaugh* [1949], *Chan* [1981], *Boerner* [1982], *et al.*, [*Mieras*, 1983; *Nespor et al.*, 1984; *McCormick and Hendry*, 1985]. More recently, *Kostinski and Boerner* [1987] determined the transmitting and receiving polarization state which produced maximum contrast between two classes represented by their Graves power matrices. This technique involved maximizing the expected power return from one deterministic scattering class with respect to another.

Although these are viable procedures when dealing with deterministic scattering classes, they can not be utilized in the case of statistically distributed scattering objects, e.g., terrain clutter. A deterministic scatterer can be characterized by a scattering matrix, whereas nondeterministic scatterers must be represented either by an average Mueller matrix or equivalently, by a polarimetric covariance matrix, which are the second order statistics of the scattering matrix. To enhance the contrast between nondeterministic scatterers, the ratio of the average power returns from the two scattering classes must be maximized.

In the case of distributed scatterers in the clear, observability is limited only by background noise. Since the noise in the receiver channels is statistically independent and usually normalized at the same average power, the background noise is generally unpolarized. This implies that when the target-to-clutter ratio is maximized, under the assumption that the distributed scatterer represents the target, whereas background noise denotes clutter, it will be found that the background noise contribution has essentially a constant expected power level [*Giuli*, 1985]. Thus, for targets which are assumed to be in the clear, or independent of a clutter background, maximizing this ratio is equivalent to maximizing the target return power only. Target detection improvement which can be attained by such a procedure has been analyzed by resorting to a Gaussian target model [*Giuli*, 1982] derived from Huynen's target decomposition theorem [*Huynen*, 1970, 1978]. Moreover, *Van Zyl et al.*, [1987] have determined the optimal co-polarization state for maximum power return from an isolated, distributed scatterer, represented by its average Mueller matrix.

For the case of two scattering classes, both of which were either fully or partially polarized, *Ioannidis and Hammers* [1979] employed a Lagrange multiplier method to determine the transmitting and receiving polarization state that maximized a target's return in the presence of clutter. The target-to-clutter ratio, expressed in terms of average Mueller matrices, was maximized in order to determine the optimal transmitting and receiving antenna Stokes polarization vectors. *Van Zyl and Zebker et al.*, [1985; 1987] have introduced the polarization signature as a means of displaying polarimetric characteristics of various scatterers. They [*Zebker et al.*, 1987] numerically determined the polarization state which maximized contrast between two classes when the receiving polarization was fixed with respect to that of the transmitter, e.g., co- or cross-polarized returns.

It should be noted that all of the techniques previously discussed for polarimetric enhancement of a target's return in the presence of clutter have maximized the target-to-clutter ratio. However, maximization of the contrast between two classes is not necessarily the same as maximizing the target-to-clutter ratio. For example, the object scattering the most power generally is denoted to signify the target class, whereas the other scattering class is referred to as clutter, although this may not always be the case. If a target in severe clutter is considered, the clutter class may actually scatter more power than the target for some transmitting and receiving polarization states. Moreover, the classes can exhibit different polarimetric correlation coefficients between the receiver channels while having comparable radar cross sections. Thus, the notion of a target and clutter class, in some instances, is not well defined. Note also that, in general, maximization of the target-to-clutter ratio does not provide the same contrast between classes as maximization of the clutter-to-target ratio. Therefore, the problem addressed here is to select the larger of these two values and determine its corresponding polarization state. For this reason, the term contrast ratio will be adopted as opposed to using either the target-to-clutter or clutter-to-target ratio.

Consequently, the procedure implemented in this paper will determine the transmitting and receiving polarization state which produces maximum contrast, or separation in the average intensity, between the two scattering classes. To realize this objective, the contrast ratio will be maximized, i.e., the *maximum contrast ratio* is computed in order to obtain the optimal linear weighting vector or optimal polarimetric matched filter [*Cadzow*, 1980; *Novak et al.*, 1987; *Swartz* 1988]. Processing polarimetric synthetic aperture radar (SAR) images with this filter performs a polarization synthesis on the data which yields maximum contrast between classes.

In Section II, the polarimetric matched filter and the contrast ratio are defined. It is then shown how to realize the polarimetric matched filter in terms of an equivalent transmitting and receiving polarization pair. The contrast ratio is defined as a function of a

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linear weighting vector (polarization filter) and the polarimetric covariance matrices of the two classes. The method for achieving maximum contrast between classes, i.e., determining the maximum contrast ratio and is corresponding optimal polarimetric matched filter, as well as a closed form solution to this problem for the case in which the polarimetric covariance matrices contain four zero elements is considered in Section III. In Section IV, the case where the radar transmitting polarization state is fixed will be addressed. Here, the receiving polarization state is optimized so that the maximum contrast ratio is attained under this constraint. Discussion of the results obtained using optimal polarimetric matched filtering is the scope of Section V.

II. The Polarimetric Matched Filter and Contrast Ratio

Assume that two classes of statistically distributed scattering types exist. Each class is represented by a covariance matrix of the form $\overline{\overline{\Sigma}}_j = E\left[\overline{X}\ \overline{X}^{\dagger}\right]_j$ where $j = a, b$ represents class A and class B scatterers, respectively. Here $E[\cdot]$ denotes the expected value and superscript $\dagger$, the complex conjugate transpose operation. For the case of electromagnetic waves which are backscattered from a reciprocal media, $HV = VH$. Therefore, the polarimetric feature vector, $\overline{X}$, is expressed in a horizontal-vertical polarization basis as [*Kong et al., 1988*]

$$\overline{X} = \begin{bmatrix} HH \\ HV \\ VV \end{bmatrix} \tag{1}$$

The objective is to find the best linear weighting vector or polarimetric matched filter for processing an observed polarimetric feature vector; that is, the linear combination

$$Y = \overline{W}^{\dagger}\ \overline{X} \tag{2}$$

where

$$\overline{W} = \begin{bmatrix} W_{hh} \\ W_{hv} \\ W_{vv} \end{bmatrix} \tag{2a}$$

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which provides the maximum contrast ratio, r, between the two respective classes (class A and class B). The maximum contrast ratio is defined as

$$r = \text{MAX}\left\{\text{MAX}_{\overline{W}}(r_{ab}), \text{MAX}_{\overline{W}}(r_{ba})\right\} \tag{3a}$$

where

$$r_{ab} = \frac{\overline{W}^{\dagger}\,\overline{\overline{\Sigma}}_a\,\overline{W}}{\overline{W}^{\dagger}\,\overline{\overline{\Sigma}}_b\,\overline{W}} \tag{3b}$$

$$r_{ba} = \frac{\overline{W}^{\dagger}\,\overline{\overline{\Sigma}}_b\,\overline{W}}{\overline{W}^{\dagger}\,\overline{\overline{\Sigma}}_a\,\overline{W}} \tag{3c}$$

Note that from a physical point of view, the elements of the vector $\overline{W}$ in (2) are linear weighting coefficients which adjust the amplitude and phase of the polarimetric radar measurements. In equation (3b), the term r_{ab} denotes the contrast ratio of class A with respect to class B scatterers. The contrast ratio of class B with respect to class A scattering elements is expressed in (3c) as r_{ba}. The symbol MAX$\{\cdot\}$ signifies the maximum value of the argument, i.e., either r_{ab} or r_{ba} in this case, whereas $\text{MAX}_{\overline{W}}(r_{ab})$ indicates that a linear weighting vector W has been obtained which maximizes r_{ab} independently of r_{ba}, and $\text{MAX}_{\overline{W}}(r_{ba})$ implies that a different linear weighting vector has been found which maximizes r_{ba} independently of r_{ab}. Also, the numerator and denominator in (3b) and (3c) are obtained from (2) by taking the expected value of the return power from each class.

To demonstrate that the polarimetric matched filter, $\overline{W}$, directly corresponds to specific transmitting and receiving polarizations, we express equation (2) in terms of a monostatic reciprocal scattering matrix as

$$Y = \begin{bmatrix} H_r & V_r \end{bmatrix} \begin{bmatrix} HH & HV \\ HV & VV \end{bmatrix} \begin{bmatrix} H_t \\ V_t \end{bmatrix} \tag{4}$$

in which the values H_t and V_t, H_r and V_r represent the horizontal and vertical vector components of the transmitting and receiving polarization state, respectively. Also, without

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loss of generality, it is assumed that

$$|H_t|^2 + |V_t|^2 = 1 \qquad (5a)$$

$$|H_r|^2 + |V_r|^2 = 1 \qquad (5b)$$

Equating (2) and (4) yields

$$H_t H_r = W_{hh}^* \qquad (6a)$$

$$H_t V_r + V_t H_r = W_{hv}^* \qquad (6b)$$

$$V_t V_r = W_{vv}^* \qquad (6c)$$

Thus, given a general linear weighting vector, $\overline{W}$, its corresponding polarization state components, H_t, V_t, H_r, and V_r, can be completely specified through (6). This will be shown in Section III.

Note that equation (4) indicates the transmitting and receiving polarization vectors are reciprocal, i.e., the terms H_t and V_t can be interchanged with H_r and V_r without altering the measurement, Y. Therefore, in the case of reciprocal backscattering, the same contrast ratio will be obtained if the transmitting and receiving polarization vectors are exchanged.

III. The Optimal Polarimetric Matched Filter Required to Obtain Maximum Contrast Between Two Scattering Classes

In this section the optimal polarimetric matched filter which produces maximum contrast between two scattering classes is determined. It will then be transformed into the specific transmitting and receiving polarization state that a radar can utilize to realize the same maximum contrast ratio.

In order to compute the optimal polarimetric matched filter, (3b) and (3c) must be maximized. The linear weighting vector which corresponds to the maximum contrast ratio, shown in (3a), will be denoted as the optimal polarimetric matched filter. The

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maximization procedure makes use of the Lagrange multiplier technique. Details of this procedure were outlined by *Cadzow* [1980], although the steps will be repeated here for completeness. For example, in order to maximize r_{ab}, in (3b),

$$\mathrm{MAX}\left\{\overline{W}^{\dagger}\,\overline{\overline{\Sigma}}_a\,\overline{W}\right\} \tag{7a}$$

is determined under an arbitrary constraint

$$\overline{W}^{\dagger}\,\overline{\overline{\Sigma}}_b\,\overline{W} = 1 \tag{7b}$$

This reformulation is possible, without loss of generality, since the linear weighting vector can be multiplied by any arbitrary complex constant without affecting the contrast ratio. The solution to this constrained maximization problem is obtained by making use of the Lagrange multiplier concept, which reflects the constraint shown in (7b). Its solution will be a stationary point of the auxiliary functional

$$f(\overline{W}) = \overline{W}^{\dagger}\,\overline{\overline{\Sigma}}_a\,\overline{W} - \lambda\left[1 - \overline{W}^{\dagger}\,\overline{\overline{\Sigma}}_b\,\overline{W}\right] \tag{8}$$

in which λ is a scalar valued Lagrange multiplier. Specifically, the stationary points of this auxiliary functional are found first by representing the generally complex vector $\overline{W}$ in terms of its real and imaginary components, as $\overline{W}_R - i\,\overline{W}_I$. Then, taking the gradient of the auxiliary functional with respect to $\overline{W}_R$ and $\overline{W}_I$, setting the resulting equations equal to zero, i.e.,

$$\frac{\partial f(\overline{W})}{\partial \overline{W}_R} = 0 \tag{9a}$$

$$\frac{\partial f(\overline{W})}{\partial \overline{W}_I} = 0 \tag{9b}$$

yields the necessary condition for a maxima or minima to occur. Carrying out (9a) and (9b) leads to the eigenvalue equation

$$\overline{\overline{\Sigma}}_a\,\overline{W} = \lambda\,\overline{\overline{\Sigma}}_b\,\overline{W} \tag{10}$$

Note that the eigenvalue (Lagrange multiplier) λ, in (10), is the contrast ratio r_{ab} given by (3b), whereas $1/\lambda$ signifies the contrast ratio r_{ba} shown in (3c). Since the objective is to determine the maximum contrast ratio between classes, the values of the maximum and the reciprocal of the minimum eigenvalue must be compared and the larger of the two selected. The eigenvector which corresponds to this maxima is the optimal polarimetric matched filter that should be employed to process the radar polarimetry. Note that it is not required to maximize both (3b) and (3c). By extremizing (3b) then selecting the largest of either the maximum eigenvalue or the reciprocal of the minimum eigenvalue, both (3b) and (3c) have been simultaneously maximized.

In the event that the eigenvalues of (10) are degenerate, there will exist no preferred polarization basis for which the two objects expected power return can be separated. Assuming then, that the contrast ratios are not degenerate, the optimal polarimetric matched filter, $\overline{W}$, is interpreted to be the equivalent transmitting and receiving polarization state which a radar can utilize in order to detect the maximum contrast, or separation in average intensity, between classes.

The contrast optimization approach used for the case of a monostatic radar also can be applied to a polarimetric bistatic radar. Taking into account the fact that for bistatic scattering $HV \neq VH$ when defining $\overline{X}$ and $\overline{W}$ in (1) and (2), leads to 4×4 polarimetric covariance matrices which characterize the scattering classes. Then applying exactly the same method of solution yields the transmitting and receiving polarization state that maximizes contrast between scattering classes.

Once the optimal polarimetric matched filter is obtained, the corresponding transmitting and receiving polarization state can be calculated. Without loss of generality, the case when W_{hh} is not equal to zero will be shown. From (5) and (6), it is found that

$$H_t = \left[\left| \frac{W^*_{hv} \mp \sqrt{(W^*_{hv})^2 - 4(W_{hh}W_{vv})^*}}{2W^*_{hh}} \right|^2 - 1 \right]^{-1/2} \qquad (11a)$$

$$V_t = H_t \cdot \left[\frac{W^*_{hv} \mp \sqrt{(W^*_{hv})^2 - 4(W_{hh}W_{vv})^*}}{2W^*_{hh}} \right] \tag{11b}$$

$$H_r = \frac{\tau}{H_t} \cdot W^*_{hh} \tag{11c}$$

$$V_r = \frac{\tau}{H_t} \cdot \left[\frac{W^*_{hv} \mp \sqrt{(W^*_{hv})^2 - 4(W_{hh}W_{vv})^*}}{2} \right] \tag{11d}$$

where

$$\tau = \frac{1}{\sqrt{\tau_1 \cdot \tau_2 \cdot \tau_3}} \tag{12}$$

$$\tau_1 = \left| W^*_{hh} \right|^2 \tag{12a}$$

$$\tau_2 = 1 - \left| \frac{W^*_{hv} \mp \sqrt{(W^*_{hv})^2 - 4(W_{hh}W_{vv})^*}}{2W^*_{hh}} \right|^2 \tag{12b}$$

$$\tau_3 = 1 - \left| \frac{W^*_{hv} \mp \sqrt{(W^*_{hv})^2 - 4(W_{hh}W_{vv})^*}}{2W^*_{hh}} \right|^2 \tag{12c}$$

The absolute intensity value, I, is given by

$$I = \left| \tau \, \overline{W}^{\dagger} \, \overline{X} \right|^2 \tag{13}$$

where τ is the amplitude normalization constant given in (12).

The observed sign change in (11), i.e., $\pm$ or $\mp$, indicates the reciprocity of the transmitting and receiving polarization state, as previously mentioned. Also, the resulting transmitting and receiving polarization state is independent of any multiplicative constant effecting the matched filter. This is necessary since the general complex eigenvector solution to (10) can vary by a multiplicative complex constant; however, the resulting polarization state remains unaffected since this constant can be factored out.

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Finally, a comparison between the methods for contrast enhancement presented in this paper versus that originally proposed by *Ioannidis and Hammers* [1979] is in order. In their method, the target-to-clutter ratio was maximized to determine the optimal transmitting and receiving antenna Stokes polarization vectors. The *Ioannidis and Hammers'* method requires the use of three constraints in order to solve the maximization problem. One is similar to (7b) in that it constrains the denominator of target-to-clutter ratio to be equal to an arbitrary constant. The other two constrain the transmitting and receiving vectors to be antenna Stokes polarization vectors. This results in complex expressions which specify the optimal transmitting and receiving polarization state. In addition, they do not obtain the matched filter which corresponds to the optimal transmitting and receiving polarization state. Using *Cadzow's* method, only one constraint (7b) is needed to solve for the optimal polarimetric matched filter. In this case, the maximization procedure only requires solving the eigenvalue problem shown in (10). It should be pointed out that both methods yield identical results when polarimetric target and clutter classes are pre-specified.

The major difference between these two techniques is that *Ioannidis and Hammers'* method dealt with the specific problem of maximizing the target-to-clutter ratio by determining the optimal transmitting and receiving antenna Stokes polarization vectors, whereas the matched filtering approach used in this paper can be applied to a more general class of problems. That is, *Cadzow's* procedure extends to multi-channel, multi-frequency sensor data. Polarimetric contrast enhancement is considered here as a special case.

Closed Form Solution for the Case of a Covariance Matrix with Four Zero Elements

Thus far, the most general form of the polarimetric covariance matrix has been assumed, which is

$$\overline{\overline{\Sigma}}_j = \sigma_j \begin{bmatrix} 1 & \beta_j\sqrt{\epsilon_j} & \rho_j\sqrt{\gamma_j} \\ \beta_j^*\sqrt{\epsilon_j} & \epsilon_j & \xi_j\sqrt{\epsilon_j\gamma_j} \\ \rho_j^*\sqrt{\gamma_j} & \xi_j^*\sqrt{\epsilon_j\gamma_j} & \gamma_j \end{bmatrix} \tag{14a}$$

where $j = a, b$ represents the class A and class B parameters, respectively, and

$$\sigma = \sigma_{hh} \tag{14b}$$

$$\epsilon = \sigma_{hv} / \sigma \tag{14c}$$

$$\gamma = \sigma_{vv} / \sigma \tag{14d}$$

$$\rho = \frac{E\left[HH \cdot VV^*\right]}{\sigma\sqrt{\gamma}} = |\rho| \exp\left[i\phi_\rho\right] \tag{14e}$$

$$\beta = \frac{E\left[HH \cdot HV^*\right]}{\sigma\sqrt{\epsilon}} = |\beta| \exp\left[i\phi_\beta\right] \tag{14f}$$

$$\xi = \frac{E\left[HV \cdot VV^*\right]}{\sigma\sqrt{\epsilon\gamma}} = |\xi| \exp\left[i\phi_\xi\right] \tag{14g}$$

Here, the values σ_{hh}, σ_{hv}, and σ_{vv} denote the normalized backscatter cross section per unit area of the HH, HV, and VV returns [*Kong et al.*, 1988].

It has been rigorously shown using the random medium model [*Shin et al.*, 1986; *Borgeaud et al.*, 1987], that when each of the two scattering classes can be modeled as a uniform terrain cover, no average correlation exists between HH and HV returns, or between VV and HV returns. Therefore, the variables β and ξ, in (14), are both equal to zero and the polarimetric covariance matrices contain four zero elements. This implies that the terrain exhibits azimuthal symmetry from a statistical point of view. It should be pointed out that this effect has been experimentally observed at various sites by MIT Lincoln Laboratory in their polarimetric measurements at 35 GHz [*Borgeaud et al.*, 1987] (see Section V and Table 2). In this case the polarimetric covariance matrix can be expressed as

$$\overline{\overline{\Sigma}}_j = \sigma_j \begin{bmatrix} 1 & 0 & \rho_j\sqrt{\gamma_j} \\ 0 & \epsilon_j & 0 \\ \rho_j^*\sqrt{\gamma_j} & 0 & \gamma_j \end{bmatrix} \tag{15}$$

A closed form solution to the general eigenvalue problem in (10), based on the covariance matrix in (15), will be now presented. First, the eigenvalues for the matrix $\overline{\overline{\Sigma}}_b^{-1}\,\overline{\overline{\Sigma}}_a$ are determined. They can be expressed as

$$\lambda_1 = \frac{\epsilon_a\sigma_a}{\epsilon_b\sigma_b} \tag{16a}$$

$$\lambda_{2,3} = \frac{\sigma_a}{2\sigma_b\gamma_b\left(1-|\rho_b|^2\right)} \times \left[\gamma_a - \gamma_b - 2\sqrt{\gamma_b\gamma_a}\,|\rho_a|\,|\rho_b|\cos\left(\phi_{\rho_a}-\phi_{\rho_b}\right) \pm \sqrt{\left[\gamma_a+\gamma_b-2\sqrt{\gamma_b\gamma_a}\,|\rho_a||\rho_b|\cos\left(\phi_{\rho_a}-\phi_{\rho_b}\right)\right]^2 - 4\gamma_a\gamma_b\left(1-|\rho_b|^2\right)\left(1-|\rho_a|^2\right)}\right] \tag{16b}$$

Their corresponding eigenvectors are given as

$$\lambda_1 \longrightarrow \overline{W}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{17a}$$

$$\lambda_{2,3} \longrightarrow \overline{W}_{2,3} = \begin{bmatrix} 1 \\ 0 \\ \Upsilon_{2,3} \end{bmatrix} \tag{17b}$$

where

$$\Upsilon_{2,3} = \frac{\left[\gamma_a - \gamma_b - i\,2\sqrt{\gamma_b\gamma_a}\,|\rho_a|\,|\rho_b|\sin\left(\phi_{\rho_a}-\phi_{\rho_b}\right) \pm \sqrt{\left[\gamma_b-\gamma_a-2\sqrt{\gamma_b\gamma_a}\,|\rho_a||\rho_b|\cos\left(\phi_{\rho_a}-\phi_{\rho_b}\right)\right]^2 - 4\gamma_a\gamma_b\left(1-|\rho_b|^2\right)\left(1-|\rho_a|^2\right)}\right]}{2\left(\gamma_b\sqrt{\gamma_a}\rho_a - \gamma_a\sqrt{\gamma_b}\rho_b\right)} \tag{17c}$$

As in the case of the generalized solution, the eigenvector corresponding to the maximum of either the largest or the reciprocal of the smallest eigenvalue in (16) will produce maximum contrast between the two classes. Therefore, this eigenvector will be the optimal polarimetric matched filter which should be utilized to process the polarimetric feature vector.

IV. The Optimal Receiving Polarization State for a Fixed Transmitting Polarization

If presented with a situation where the radar transmitting polarization state is fixed, an optimal receiving polarization state can be determined which maximizes the contrast between the two classes. This problem arises in the case of radar systems which are not fully polarimetric, i.e., they transmit using only a single polarization, say horizontal, but receive the principle and cross-polarization components of the scattered response, say the horizontal and vertical returns. Applying the following technique will indicate how to coherently combine the horizontal and vertical returns such that the contrast between classes is maximized.

Assume that the values H_t and V_t are known and that the requirement shown in (5a) is satisfied. Thus, from (6), $\overline{W}$ may be written as

$$\overline{W} = \begin{bmatrix} (H_t H_r)^* \\ (H_t V_r)^* - (V_t H_r)^* \\ (V_t V_r)^* \end{bmatrix} = \begin{bmatrix} H_t^* & 0 \\ V_t^* & H_t^* \\ 0 & V_t^* \end{bmatrix} \begin{bmatrix} H_r^* \\ V_r^* \end{bmatrix} = \zeta \, \overline{R} \tag{18}$$

Substituting (18) for $\overline{W}$ in (3b) gives

$$r_{ab} = \frac{\overline{R}^\dagger \, \overline{\overline{Z}}_a \, \overline{R}}{\overline{R}^\dagger \, \overline{\overline{Z}}_b \, \overline{R}} \tag{19a}$$

where

$$\overline{\overline{Z}}_a = \zeta^\dagger \, \overline{\overline{\Sigma}}_a \, \zeta \tag{19b}$$

$$\overline{\overline{Z}}_b = \zeta^\dagger \, \overline{\overline{\Sigma}}_b \, \zeta \tag{19c}$$

Since $\overline{\overline{Z}}_a$ and $\overline{\overline{Z}}_b$ are hermitian symmetric, positive semidefinite matrices, (19a) can be extremized as in the previous section, to obtain the generalized eigenvalue problem

$$\overline{\overline{Z}}_a \, \overline{R} = \lambda \, \overline{\overline{Z}}_b \, \overline{R} \tag{20}$$

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Once $\bar{R}$ is obtained, it should be normalized so that (5b) holds. Application of the transform given in (18) will yield $\overline{W}$.

V. Results and Discussion

In order to present the optimal polarimetric matched filtering results in a compact format, orientation (ψ) and ellipticity (χ) angles (Fig. 1) are utilized to express the transmitting and receiving polarization states. Here, the definitions from *Kong* [1986] are adopted. Therefore, horizontal (H) and vertical (V) polarization states will have zero degree ellipticity angles, with orientation angles of 0° and 90°. Right (R) and left (L) polarization states are orientation independent with ellipticity angles of 45° and −45°, respectively. In addition, $0° \leq \psi \leq 180°$ and $-45° \leq \chi \leq 45°$.

A general polarization vector,

$$\bar{P} = \begin{bmatrix} P_h \\ P_v \end{bmatrix} = \begin{bmatrix} |P_h| \exp[i\phi_h] \\ |P_v| \exp[i\phi_v] \end{bmatrix} \quad (21)$$

written in terms of horizontal ($\hat{h}$) and vertical ($\hat{v}$) vector components, can be transformed into a normalized Stokes vector as follows

$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} |P_h|^2 + |P_v|^2 \\ |P_h|^2 - |P_v|^2 \\ 2|P_h||P_v| \cos\phi \\ 2|P_h||P_v| \sin\phi \end{bmatrix} = (|P_h|^2 + |P_v|^2) \begin{bmatrix} 1 \\ \cos 2\chi \cos 2\psi \\ \cos 2\chi \sin 2\psi \\ \sin 2\chi \end{bmatrix} \quad (22)$$

where $\phi = \phi_v - \phi_h$. Using this equation, the angles ψ and χ are obtained.

Two data bases were utilized to study the contrast problem, utilizing the techniques outlined in the previous sections. Table 1 gives polarimetric covariance statistics extracted from San Francisco Bay area, L-band (1.225 GHz) SAR data, collected by the Jet Propulsion Laboratory's airborne polarimeter [*Zebker* et al., 1987]. These covariance statistics were obtained from the training areas shown in Figure 2 and were utilized to generate the results shown in Tables 4, 5, and 7. Similarly, the experimental polarimetric covariance data, shown in Table 2, was supplied by the MIT Lincoln Laboratory [*Borgeaud et al.*, 1987; *Kong et al.*, 1988]. This data, collected at 35 GHz, was used to generate the results presented in Table 6. The MIT Lincoln Laboratory radar imaged a vegetation field consisting of grass or trees at a range of approximately 2 kilometers. Studies using this database indicate that essentially no correlation exists between the HH and HV, and between the HV and VV polarimetric returns, i.e., the terrain clutter exhibits azimuthal symmetry.

Next, a comparison is made which evaluates the performance obtained by processing polarimetric radar data using the optimal polarimetric matched filter versus other commonly used polarization states. Linear weighting vectors which correspond to commonly used transmitting and receiving polarization states are given in Table 3. The weighting vectors presented in this table were generated using equations (6a) through (6c), (21) and (22). These linear weighting vectors are expressed in a horizontal-vertical polarization basis.

Table 4 presents theoretical contrast ratios r_{ab} and r_{ba} obtained when utilizing the above mentioned transmitting and receiving polarization states (Table 3) as well as the optimal solution. Here, class A and B scatterers have been defined to denote the park and urban (city) regions, which were represented by their corresponding covariance matrices, $\bar{\bar{\Sigma}}_a$ and $\bar{\bar{\Sigma}}_b$, respectively. As previously discussed, for reciprocal backscattering the transmitting and receiving polarization state may be interchanged while maintaining the same contrast ratio. This is clearly indicated by the HV and VH results and the LR and RL results. The values r_{ab} and r_{ba}, which denote the maximum and reciprocal of the minimum eigenvalues found after solving (10), are expressed in terms of their corresponding orientation and ellipticity angles, ψ and χ. From Table 4 it is seen that the maximum contrast ratio between the two selected classes is 9.38 dB. Note that had only r_{ab} been maximized, a contrast ratio of 2.37 dB would have been realized. In some cases, though, this may be what is required. If the problem was only to make the park processed pixel intensity as large as possible with respect to that of the city, the transmitting and receiving polarization state corresponding to the average power ratio of 2.37 dB would be the appropriate matched filter which should be used to process data.

Table 5 shows the actual polarimetric contrast enhancement achieved when processing the radar measurements using the optimal polarimetric matched filter and various other polarization filters. Since the contrast ratios for r_{ba}, given in Table 4, are larger than those for r_{ab}, the linear weighting vectors which correspond to the contrast ratios for r_{ba} have been used to generate the results shown in Table 5. Thus, Table 5 contains the actual average processed pixel intensity realized for each of the two classes (urban and park areas) for both suboptimal polarization filters, i.e., transmitting and receiving polarization states which do not provide maximum contrast between classes, and the optimal polarimetric matched filter. In comparing the data in this table, it is seen that the quantitative measure of attainable contrast is the contrast ratio, which is the linear ratio of (or the logarithmic *distance* between) the average pixel intensity for the two respective classes. In the case of processing data with the optimal polarimetric matched filter, this distance is maximum. Thus after optimal processing, it is possible to more readily separate the two classes than prior to it, since the distance between the average value of pixel intensity has increased between the two classes. Also note that the contrast ratios, shown in Table 5, and those given in Table 4 for r_{ba}, are similar indicating a good match between the theoretical predictions and the processed results.

A further demonstration of contrast enhancement can be seen visually by comparing Figures 3A through 3D which show San Francisco Bay area images. This imagery has been synthesized utilizing some commonly employed linear polarization states, in addition to the transmitting and receiving polarization state required to produce maximum contrast between the park and urban regions. In all four of these images, the average processed pixel intensity of the park region was set to the baseline value, i.e, the minimum quantization level of the imaging system display, which was −20 dB. The maximum quantized intensity was −10 dB. By utilizing these quantization limits, some clipping of the higher and lower intensity levels has occurred. This procedure was implemented to compare more easily the contrast between images. Figure 3A illustrates the result of processing the San Francisco Bay polarimetry using the optimal polarimetric matched filter. The contrast ratio obtained between the city and park area, as previously indicated in Table 5, was 9.4 dB. Contrast ratios achieved using HH (Fig. 3B), VV (Fig. 3C), and HV (Fig. 3D) polarization filters were only 7.3, 5.4, and 2.6 dB, respectively. As indicated in Table 5, utilizing the optimal transmitting and receiving polarization state, i.e., the optimal polarimetric matched filter, to process data yields the maximum contrast ratio. Figure 3 also shows that the optimal and HH synthesized images appear somewhat similar; this is due to the fact that there is only a 2.1 dB difference between their contrast ratios. However, the optimal polarimetric matched filter always yields a larger contrast ratio between classes than when any other transmitting and receiving polarization states are utilized.

Contrast ratio results, obtained using the MIT Lincoln Laboratory data are presented in Table 6. As was the case in Table 4, theoretical contrast ratios are given for frequently employed polarization states as well as for the optimal solution. In this table, the tree and grass regions have been arbitrarily selected to denote class a and b scatterers, respectively. The optimal solution again is represented by its corresponding orientation and ellipticity angles ψ and χ, in which case, the values presented for r_{ab} and r_{ba}, shown in Table 6, signify the maximum and reciprocal of the minimum eigenvalues found when employing the equations shown in (16a) and (16b).

In Table 7, the optimal receiving polarization states required to produce the maximum contrast ratio between classes for various fixed transmitting polarization states are presented. These results show that by employing the optimal receiving polarization state, all contrast ratios have increased relative to those shown in Table 4 or 5. Thus, for a given transmitting polarization state, synthesizing imagery using the optimal receiving polarization state always yields a larger contrast ratio than when any other polarization state is used. However, the maximum contrast ratio is achieved only when both the optimal transmitting and receiving polarization state is employed.

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List of Figures

Figure 1 Generalized elliptic polarization state.

Figure 2 Training areas used to generate the covariance matrix elements for the park and urban (city) regions, shown in Table 1.

Figure 3 San Francisco Bay area images synthesized using the optimal polarimetric matched filter (A), *HH* (B), *VV* (C), and *HV* (D) polarization filters. The corresponding contrast ratios between the city and park region were 9.4, 7.3, 5.4, and 2.6 dB, respectively.

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	$\sigma(dB)$	ϵ	γ	ρ	ϕ_ρ	β	ϕ_β	ξ	ϕ_ξ
Park	-40.5	0.406	1.42	0.219	-0.463	0.168	0.590	0.090	-1.22
Urban	-33.2	0.137	0.907	0.404	2.98	0.792	0.001	0.414	-2.98

Table 1 Covariance matrix elements for park and urban (city) regions. Phase angles are given in radians.

	$\sigma(dB)$	ϵ	γ	ρ	ϕ_ρ
Trees	- 13.0	0.06	1.1	0.74	0.0
Grass	- 15.0	0.15	1.2	0.56	0.0

Table 2 Covariance matrix elements for a uniform terrain cover consisting of grass and tree regions.

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Transmitting Polarization	Receiving Polarization	Linear Weighting Vector : $\overline{W}$
H	H	$\begin{bmatrix} 1.0 \\ 0.0 \\ 0.0 \end{bmatrix}$
H	V	$\begin{bmatrix} 0.0 \\ 1.0 \\ 0.0 \end{bmatrix}$
V	H	$\begin{bmatrix} 0.0 \\ 1.0 \\ 0.0 \end{bmatrix}$
V	V	$\begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{bmatrix}$
L	L	$\begin{bmatrix} 0.5 \\ i \\ -0.5 \end{bmatrix}$
L	R	$\begin{bmatrix} 0.5 \\ 0.0 \\ 0.5 \end{bmatrix}$
R	L	$\begin{bmatrix} 0.5 \\ 0.0 \\ 0.5 \end{bmatrix}$
R	R	$\begin{bmatrix} 0.5 \\ -i \\ -0.5 \end{bmatrix}$

Table 3 Commonly utilized transmitting and receiving polarization states versus their corresponding linear weighting vectors (expressed in a horizontal-vertical polarization basis).

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Transmitting Polarization	Receiving Polarization	Contrast Ratio (dB) r_{ab}	Contrast Ratio (dB) r_{ba}
H	H	-7.30	7.30
H	V	-2.58	2.58
V	H	-2.58	2.58
V	V	-5.35	5.35
L	L	-6.94	6.94
L	R	-3.29	3.29
R	L	-3.29	3.29
R	R	-6.73	6.73
$\psi = 1.82$ $\chi = 3.72$	$\psi = 107.0$ $\chi = -1.64$	2.37	-2.37
$\psi = 48.7$ $\chi = -6.44$	$\psi = 150.3$ $\chi = 3.51$	-9.38	*9.38*

Table 4 Theoretical contrast ratios between classes when $\overline{\overline{\Sigma}}_a = Park$ and $\overline{\overline{\Sigma}}_b = Urban$ San Francisco Bay regions, respectively. Orientation and ellipticity angles are given in degrees. $\mathrm{MAX}_{\overline{W}}(r_{ab}) = 2.37$ dB. $\mathrm{MAX}_{\overline{W}}(r_{ba}) = 9.38$ dB, therefore $r = 9.38$ dB.

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Transmitting Polarization	Receiving Polarization	Urban Class (dB)	Park Class (dB)	Contrast Ratio (dB)
H	H	-33.2	-40.5	7.3
H	V	-41.8	-44.4	2.6
V	V	-33.6	-39.0	5.4
L	L	-34.2	-41.0	6.8
L	R	-38.6	-41.9	3.3
R	R	-34.0	-40.8	6.8
$\psi = 48.7$ $\chi = -6.44$	$\psi = 150.3$ $\chi = 3.51$	-34.3	-43.7	*9.4*

Table 5 Actual average pixel intensities and the contrast ratios between the park and urban San Francisco Bay regions when data was processed using commonly employed polarization filters and the optimal polarimetric matched filter.

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Transmitting Polarization	Receiving Polarization	Contrast Ratio r_{ab}	Contrast Ratio r_{ba}
H	H	2.00	-2.00
H	V	-1.98	1.98
V	H	-1.98	1.98
V	V	1.62	-1.62
L	L	-1.00	1.00
L	R	2.28	-2.28
R	L	2.28	-2.28
R	R	-1.00	1.00
$\psi = 0$, $\chi = 0$	$\psi = 90$, $\chi = 0$	-1.98	1.98
$\psi \simeq 0$, $\chi = -38.3$	$\psi \simeq 0$, $\chi = 38.3$	2.31	-2.31

Table 6 Theoretical contrast ratios between classes when trees = a and grass = b for class A and B scatterers, respectively. Orientation and ellipticity angles are given in degrees. $\mathrm{MAX}_{\overline{W}}(r_{ab}) = 2.30$ dB, $\mathrm{MAX}_{\overline{W}}(r_{ba}) = 1.99$ dB, therefore $r = 2.30$ dB.

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Transmitting Polarization	Receiving Polarization	Contrast Ratio (dB)
H	$\psi = 31.8$, $\chi = -8.64$	7.83
V	$\psi = 134.2$, $\chi = 4.34$	6.06
R	$\psi = 27.5$, $\chi = 26.1$	6.97
L	$\psi = 169.1$, $\chi = -21.4$	7.36
$\psi = 48.7$, $\chi = -6.44$	$\psi = 150.3$, $\chi = 3.51$	*9.38*

Table 7 Optimal receiving polarization state for a fixed transmitting polarization state when $\bar{\bar{\Sigma}}_a = Park$ and $\bar{\bar{\Sigma}}_b = Urban$ San Francisco Bay regions. Orientation and ellipticity angles are given in degrees.

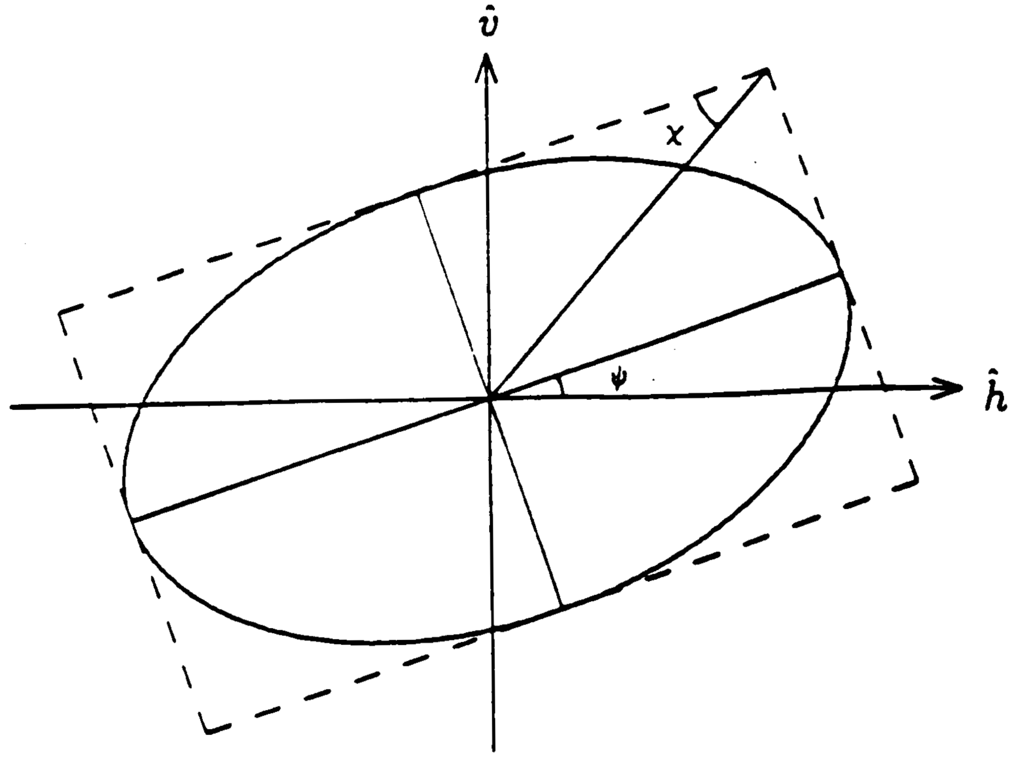

Figure 1 Generalized elliptic polarization state.

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Figure 2 Training areas used to generate the covariance matrix elements for the park and urban (city) regions, shown in table 1.

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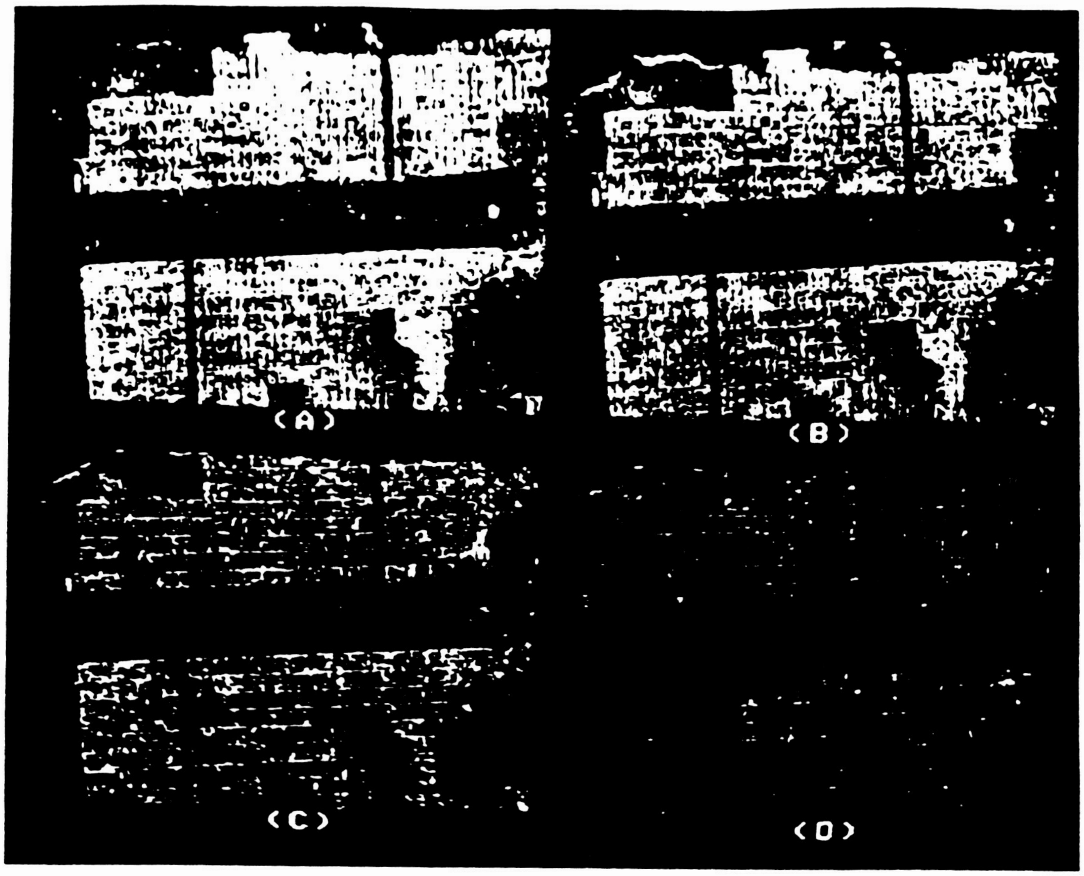

Figure 3 San Francisco Bay area images synthesized using the optimal polarimetric matched filter (A), HH (B), VV (C), and HV (D) polarization filters. The corresponding contrast ratios between the city and park region were 9.4, 7.3, 5.4, and 2.6 dB, respectively.