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EFFICIENT NUMERICAL TECHNIQUES FOR COMPLEX FLUID FLOWS*

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INTRODUCTION

The use of computational methods for the prediction of multi-dimensional recirculating flows has been continuously increasing over the recent years. As the numerical techniques become more powerful, they are being applied to even more challenging problems encountered in combustion chambers, gas turbines, rotating machinery, heat exchangers, and other devices. Although computation is far less expensive than full-scale testing, the cost of a computational run is still substantial. Therefore, attempts are continually being made for improving the accuracy and efficiency of numerical techniques so that the predictions of a given accuracy can be obtained at a modest cost.

The central feature in any flow prediction method is the treatment of the coupling between the momentum and continuity equations. In natural-convection flows, the energy equation also becomes strongly coupled with the momentum equations. Because of the nonlinear nature of the coupling, these equations are solved iteratively. Iterative methods are often prone to slow convergence, divergence, and extreme sensitivity to underrelaxation factors.

The aim of the present research is to develop more efficient and reliable solution schemes for the coupled flow equations. Such schemes will significantly reduce the expense of computing complex flows encountered in combustion chambers, gas turbines, heat exchangers, and other practical equipment.

In the work completed so far, a technique employing norm reduction in conjunctine with the successive-substitution and Newton-Raphson techniques has been developed. Also, a block-correction procedure for the flow equations is currently being formulated and tested.

NORM REDUCTION TECHNIQUES

The development of a number of methods for solving strongly coupled equations has been reported in reference 1. The recommended method there is a combination of the successive-substitution and Newton-Raphson methods coupled with a norm reduction technique.

The central idea of the method will now be described. The linearized discretization equations are solved by a direct method, such as the sparse-matrix LU decomposition. The linearization can be of two kinds. If the unknown coefficients are simply evaluated from the currently available values of the dependent variables, the linearization is called successive substitution. In the Newton-Raphson method, the anticipated change in the coefficients is taken into account via their first derivatives with respect to the dependent variables.

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The solution of the linearized equations predicts a change in the values of the variables. However, instead of accepting this change as is, it is multiplied by a constant. (This constant can be imagined to be a kind of under- or over-relaxation factor.) The value of this constant multiplier is found by requiring that the norm of the residual vector (i.e., the sum of the squares of the residuals of all the equations) be a minimum. This minimization search produces a kind of "optimum" underrelaxation of the dependent variables.

The norm minimization (or reduction) can be used with either successive-substitution or Newton-Raphson linearization. When the current estimates are close to the final solution, the Newton-Raphson technique is very efficient. But for initial guesses that are far from the solution, the technique often diverges. Therefore, the practice found satisfactory for all the flows tested was to employ the successive-substitution linearization until the norm became less than a small quantity and then to switch to the Newton-Raphson linearization until the final convergence. Because of the combination of the two linearization practices, the technique is called the hybrid method in reference 1.

The hybrid method and many other alternative schemes (some of which are based on the Broyden methods described in reference 2) were tested on two flow configurations: the flow in a driven cavity at different Reynolds numbers and the natural convection flow in an enclosure with hot and cold walls at different Rayleigh numbers.

For the driven cavity problem, solutions were obtained by the hybrid method in at least one-third the computer time required for the iterative method SIMPLER. For the natural convection problem, most methods either diverged or converged extremely slowly as the Rayleigh number was increased. The hybrid method, however, converged rapidly and required a very modest amount of computer time. By using the hybrid method, it was possible to obtain convergence in 25 iterations at a Rayleigh number of 10^7 when a zero initial guess was used for all variables. If the results for a lower Rayleigh number could be used as the initial guess, it was possible to obtain solutions at a Rayleigh number of 10^9. It is believed that, for the first time, solutions have been obtained for such a high Rayleigh number (with a Prandtl number of 0.71). Iterative methods such as SIMPLER failed to converge even after 1000 iterations.

BLOCK CORRECTION TECHNIQUE

Work is currently in progress on another approach for accelerating the convergence rate of an iterative procedure such as SIMPLER. In the proposed technique, the velocity and pressure values are adjusted through a block correction procedure. Here the calculation domain is considered to be composed of several large blocks. Each block contains a number of grid points (and hence control volumes). It is proposed that the values of a variable for grid points within a given block will receive a uniform correction. These corrections are calculated such that the integral conservation of momentum and continuity is satisfied for each block. The block-correction equations thus resemble the discretization equations for momentum and continuity but are formulated on much coarser grid. The solution of these equations by a direct or iterative method is rather straightforward.

Initial testing of this apprach shows that, for fine grids, there is a noticeable improvement in the computational effort required to obtain a converged solution.

REFERENCES

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