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STUDIES OF THE DYNAMICS OF THE TWIN-LIFT SYSTEM

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INTRODUCTION

There have been a number of studies over the past few years regarding the problems associated with coupling two helicopters together in such a way as to lift a single load, thus increasing the load that can be carried with a given helicopter (References 1-5). A specific configuration geometry shown in Figure 1 and referred to as the twin-lift system is of interest here. Preliminary flight demonstrations of this concept described in Reference 4 have indicated that while the configuration described here can be flown, the pilot workload is high. A simplified set of equations of motion were formulated for this dynamic system, describing what was assumed to represent the most critical aspect of the dynamics and control of such a system. An investigation of this reduced system is presented in Reference 1. Among other results, it was found that the stability of the system is critically dependent upon the location of the tether attachment point relative to the center of gravity of the helicopters and that a fairly rapidly divergent mode is associated with the attachment point located below the center of gravity of the helicopter. Further details of this study can be found in Reference 1.

This report extends these studies and presents the complete linearized equation of motion for the twin-lift system about a hovering trim condition and discusses the features of the additional modes of motion involved in the complete system that are not included in the formulation of Reference 1.

Two sets of equations of motion are presented in this report. Part I presents the equations of motion of the twin lift system assuming that the yaw angles of the helicopter are constant and that the helicopters have a specific orientation relative to the load system. Part II generalizes these results to permit each helicopter to have a yaw degree of freedom and more general geometric characteristics than in Part I. The derivation of the equations of motion is described in each section and the equations of motion are presented. It is assumed that both the helicopters, referred to as the master and slave, are identical and that the hovering trim condition consists of a rectangular frame orientation, with the tethers vertical as shown in Figure 1. Certain simplifications are made in the aerodynamic derivatives that are used to describe the helicopter aerodynamics. However it should be evident from the formulation how to add a more complete aerodynamic description of the helicopter to these equations if desired. One of the objectives of this study as well as the earlier one described in Reference 1 was to obtain physical insight into the important features of this dynamic system and in particular to note how the system differs from dynamics and control problems of a single helicopter carrying a single load, a problem that has been exhaustively studied in the past.

Using an aerodynamic description of the helicopters that neglects the aerodynamic derivatives that couple the longitudinal and lateral-directional dynamics it can be shown that in the case in which the tethers of both helicopters are equal in length, the symmetry of the system can be used to decouple the equations of motion of this complex system into simpler systems which can be used to gain physical insight and draw analogues with other systems. In particular it is shown that for the linearized system with the assumptions noted above and the helicopters oriented initially either with their longitudinal axes parallel or perpendicular to the spreader bar that the complete equations of motion presented in Part II reduced to the following sets:

The complete sixteen degree of freedom (DOF) system described in Part II reduces to four simpler systems:

1. Yaw motion of Slave Helicopter (1 DOF)
2. Yaw motion of Master Helicopter (1 DOF)
3. Planar motion equations (7 DOF). This corresponds to system motion in the plane of the paper in Figure 1. (Reference 1)
4. Non-Planar Motion equations (7 DOF). This is system motion out of the plane of the paper in Figure 1.

1. and 2. will not be discussed further as they are simple first order systems. The planar motion equations are the system examined at length in Reference 1. Features of the non-planar motion are considered in some detail later in the report. It also can be shown that symmetry can be used to divide the planar

and non-planar motion dynamics into simpler systems. Both the planar and non-planar systems can be divided into anti-symmetric and symmetric motions. The anti-symmetric motions involve "in-phase" motion of both helicopters and are essentially the modes involved in maneuvering the system from place-to-place. In the planar case this is a four-degree-of-freedom system which bears some similarities to a single helicopter with a sling load as can be seen from Reference 1. Two symmetric systems are present, one involving one degree of freedom vertical translation similar to the vertical translation of a single helicopter. The other symmetric motion involves two degrees of freedom and essentially is the same as the dynamics of a helicopter tethered to a fixed point on the ground (6). The stability characteristics of this motion are very sensitive to the location of the attachment point relative to the center of gravity of the helicopter.

The non-planar motion can also be divided into anti-symmetric and symmetric motions. The anti-symmetric motion (four degree of freedom) again involves "in-phase" motion of the helicopters and translation of the system as a whole, and is essentially the same as that of a single helicopter carrying a sling load (7). The sling load is a compound pendulum with the spreader bar mass and the load mass being the two loads. The symmetric motion (three degrees of freedom) involves rotation of the entire system about a vertical axis through the load. The non-planar motion is considered in more detail later in this report.

It is interesting also to note that this division into anti-symmetric and symmetric modes also indicates a separation of the control actions. If the pilots of the master and slave helicopter apply exactly equal inputs system motion occurs only in the anti-symmetric modes, the system motion is that required to move the system from place to place. If exactly equal and opposite inputs are applied to the master and slave helicopters then only symmetric modes are excited and the total system moves about a fixed point in space.

In general it may be desirable to consider a system in which the tether lengths are unequal in which case the decoupling into symmetric and anti-symmetric modes does not apply. However studies of the unequal tether case in Reference 1 do not indicate a strong influence of unequal tether length on the modes of motion.

NOMENCLATURE

Coordinates used in the development of the equations of motion are shown in Figures 1-4. Generally the following subscripts are used to distinguish slave characteristics from master characteristics.

$(\ )_S$, $(\ )_1$	slave associated quantities
$(\ )_M$, $(\ )_2$	master associated quantities
$(\ )_T$	associated with tether
$(\ )_B$	associated with spreader bar

Geometry of Load

$\Sigma(\ )$, $\Delta(\ )$	sum and difference coordinates
L	spreader bar length, ft
H_1	slave tether length, ft
H_2	master tether length, ft
Z	displacement of load below spreader bar, ft
h_1'	slave attachment point - center of gravity spacing, ft
h_2'	master attachment point - center of gravity space, ft
h', H	are used in the case of equal slave and master dimensions, ft

Masses

M_H	helicopter mass, slugs

M_L load mass, slugs

M_B spreader bar mass, slugs

I_{CMB} spreader bar moment of inertia, slug-ft^2

Dimensionless Parameters

$$\mu = \frac{M_B + M_L}{2M_H}$$

$$\beta = \frac{M_B}{M_L + M_B}$$

$$\delta = \frac{4I_{CMB}}{(M_B + M_L)L^2}$$

$$\epsilon_{y1} = \frac{h_1' M_H}{I_y}$$

Superscripts

( )* aerodynamic force derivatives in frame axes Moment derivatives with respect to attachment point. Velocities of helicopter center of gravity.

($\bar{}$) Stability derivative not divided by mass or inertia ($\bar{X}_u = \frac{\partial X}{\partial u}$)

The standard notation for stability derivatives is used, e.g.,

$$M = \frac{1}{I_y} \frac{\partial M}{\partial u}$$

$$X = \frac{1}{M_H} \frac{\partial X}{\partial u}$$

In this notation moment derivatives are relative to the helicopter center of gravity.

EQUATIONS OF MOTION: PART I

(Fourteen Degrees of Freedom)

Since the equations of motion for this dynamic system are quite lengthy, the derivation of a reduced set of equations of motion is described in this section and then extended to the general case in Part II. Here the tether attachment points are assumed to be located at the helicopter's center of gravity and the yaw angles of both the master and slave helicopters are assumed to be zero. That is, the yaw orientation of both helicopters remains constant at an initial orientation in space which is as shown in Figure 1. In Part II, both of these restrictions on the equations of motion are removed. The helicopters may be rotated 90 degrees from the orientation in Figure 1 by interchanging aerodynamic stability derivatives as noted later (1). Arbitrary initial yaw angles are included in the latter formulation. Previous studies of a seven degree of freedom model of this dynamic system are described in Reference 1. This set of degrees of freedom is referred to as the planar case. Motion is permitted only in the plane of the paper in Figure 1. This section therefore, describes the development of the equations of motion including motions out of the plane of Figure 1.

The coordinates used to develop the equations of motion are shown in Figure 2. The choice of a relative coordinate system is particularly convenient if a Lagrangian formulation is used. A space reference frame (X_G, Y_G, Z_G) is defined, with X_G parallel to the local gravity vector. The following coordinates are selected as degrees of freedom:

(x_s, y_s, z_s) translational displacements of the slave helicopter attachment point (center of mass) with respect to space, z_s is parallel to the local gravity vector and x_s is a displacement parallel to the longitudinal axis of the slave helicopter. The orientation of the longitudinal axis of the slave helicopter remains fixed in space.

Φ_{T1}, θ_{T1} Euler angles describing the orientation of the slave helicopter tether from the G axis system. Φ_{T1} is a rotation about the X_G axis and θ_{T1} is the second rotation about the deflected axis (Y_1')

Φ_{B1}, ψ_{B1} Euler angles describing the orientation of the spreader bar with respect to space. Φ_{B1} is a rotation about an axis parallel to X_G, and ψ_{B1} is a rotation about the deflected axis Z_B'.

θ_L Euler angle describing the orientation of the load triangle. θ_L is a rotation about the spreader bar axis Y_B where Y_B is parallel to the spreader bar axis.

Φ_{T2}, θ_{T2} Euler angles describing the orientation of the tether to the master helicopter. Φ_{T2} is a rotation about an axis parallel to X_G and θ_{T2} is a rotation about the deflected axis ψ_2'.

These ten degrees of freedom taken with the pitch and roll angles of each helicopter θ_M, ϕ_M, θ_s, ϕ_s comprise the fourteen degrees of freedom of the system. The subscripts M and s refer to the master and slave respectively.

The displacements of the four masses involved in the system, the mass of each helicopter M_H, the mass of the spreader bar M_B and the mass of the load M_L can be written in terms of the geometry of the sling,

H_1 the tether length of the slave helicopter

H_2 the tether length of the master helicopter

L the spreader bar length

Z the distance of the load mass below the spreader bar

and the rotation matrices

$$\Phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cs & s\phi \\ 0 & -s\phi & c\phi \end{bmatrix}$$

$$\theta = \begin{bmatrix} c\theta & 0 & -s\theta \\ 0 & 1 & 0 \\ s\theta & 0 & c\theta \end{bmatrix} \qquad \begin{matrix} c = \cos \\ s = \sin \end{matrix}$$

$$\psi = \begin{bmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The displacements of each mass are

a.) slave helicopter center of mass (attachmei.

$$\{P_s\} = \begin{Bmatrix} x_s \\ y_s \\ z_s \end{Bmatrix}$$

b.) bottom of slave tether

$$\{P_1\} = \{P_s\} + [\theta_{T1}\ \Phi_{T1}]^T \begin{Bmatrix} 0 \\ 0 \\ H_1 \end{Bmatrix}$$

c.) spreader bar center of mass

$$\{P_{CMB}\} = \{P_1\} + [\psi_{B1}\ \Phi_{B1}]^T \begin{Bmatrix} 0 \\ -L/2 \\ 0 \end{Bmatrix}$$

d.) bottom of master tether

$$\{P_2\} = \{P_2\} + [\Psi_{B1}\ \Phi_{B1}]^T \begin{Bmatrix} 0 \\ -L \\ 0 \end{Bmatrix}$$

e.) master helicopter center of mass (attachment point)

$$\{P_M\} = \{P_2\} + [\theta_{T2}\ \Phi_{T2}]^T \begin{Bmatrix} 0 \\ 0 \\ -H_2 \end{Bmatrix}$$

f.) load center of mass

$$\{P_{CML}\} = \{P_{CMB}\} + [\theta_L\ \psi_{B1}\ \Phi_{B1}]^T \begin{Bmatrix} 0 \\ 0 \\ Z \end{Bmatrix}$$

The kinetic energy of the system due to translation then can be written as

$$T = \frac{1}{2} M_H [\{\dot{P}_s\} \times \{\dot{P}_s\}^T] + \frac{1}{2} M_H [\{\dot{P}_M\} \times \{\dot{P}_M\}^T]$$

$$+ \frac{1}{2} M_B [\{\dot{P}_{CMB}\} \times \{\dot{P}_{CMB}\}^T] + \frac{1}{2} M_L [\{\dot{P}_{CML}\} \times \{\dot{P}_{CML}\}^T]$$

To this must be added the kinetic energy due to the moments of inertia of the helicopters and the moment of inertia of the spreader bar. The contributions of the helicopters assuming small angles are

$$T_{RH} = \frac{1}{2} I_y (\dot{\theta}_M^2 + \dot{\theta}_s^2) + \frac{1}{2} I_x (\dot{\phi}_M^2 + \dot{\phi}_s^2)$$

It is assumed that both the master and slave helicopter are identical. The contribution of the moment of inertia of the spreader bar assuming it to be a slender rod such that $I_{yy} = 0$ and $I_{xx} = I_{zz} = I_{CMB}$ is

$$T_{RB} = \frac{1}{2} I_{CMB} [(\dot{\Phi}_{B1} \cos\psi_{B1})^2 + \dot{\psi}_{B1}^2]$$

The potential energy of the system is

$$V = -M_H g\ z_s - M_B g\ (\text{z-component of } P_{CMB}) \\ - M_L g\ (\text{z-component of } P_{CML}) - M_H g\ (\text{z-component of } P_M)$$

Lagrange's equation may now be used to develop the equations of motion. Since a large amount of algebra is involved, MACYSMA was used to determine the equations of motion. The MACYSMA formulation treated only the translational kinetic energy terms. The equations of motion were linearized about an arbitrary initial geometric configuration of the system assumed that all initial velocities are zero, that is only the hovering trim state is considered. Certain simplifications in the kinetic energy expression arise due to the assumption that all initial velocities are zero. In this case, the derivative of the kinetic energy with respect to displacements involves only products of velocities and thus will be zero in the linearized hover case, i.e.,

$$\frac{\partial T}{\partial q_i} = 0$$

The following names were used for the variables in the MACYSMA program

tt	kinetic energy
faitl, ϕ_{T1},	initial value faitl0
fait2, ϕ_{T2}	
faibi, ϕ_{B1}	
saib2, ψ_{B1}	
satatl, θ_{T1}	
sitat2, θ_{T2}	
sitaℓ, θ_L	

Other symbols relate directly to the notation above. Thus MACYSMA is used to form the kinetic energy and then calculate

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right),$$

The potential energy derivatives are also calculated. Helicopter body rotation is uncoupled from this procedure and these degrees of freedom will give rise to four additional equations. The spreader bar moment of inertia gives contributions to the ψ_{B1} and Φ_{B1} equations

$$\Delta \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}_{B1}}\right) = I_{CMB}\,\ddot{\Phi}_{B1}$$

$$\Delta \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\psi}_{B1}}\right) = I_{CMB}\,\ddot{\psi}_{B1}$$

The effect of the external aerodynamic forces acting on the helicopter is obtained by calculating the virtual work,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_1} = Q_k$$

where $L = T - V$

and

$$\delta W = \Sigma\, Q_k\, \delta_{qk}$$

The external forces acting on the helicopter are expressed as

$$X_s,\ Y_s,\ Z_s,\ X_M,\ Y_M,\ Z_M$$

Note that these forces must be expressed in gravity axes and not body axes. The virtual work of the aerodynamic forces is:

$$\delta W = X_s\ \delta x_s + Y_s\ \delta y_s + Z_s\ \delta z_s + X_M\ \delta x_M + Y_M\ \delta y_M + Z_M\ \delta z_M$$

The displacements of the master helicopter must be expressed in terms of the coordinates. For small angular displacements:

$$\delta x_M = \delta x_s + H_1\ \delta\theta_{T1} + L\ \delta\psi_{B1} - H_2\ \delta\theta_{T2}$$

$$\delta y_M = \delta y_s - H_1\ \delta\phi_{T1} + H_2\ \delta\phi_{T2}$$

$$\delta z_M = \delta z_s - L\ \delta\phi_{B1} - H_1\ [\theta_{T1}\ \delta\theta_{T1} + \phi_{T1}\ \delta\phi_{T1}]$$

$$+ H_2\ [\theta_{T2}\ \delta\theta_{T2} + \phi_{T2}\ \delta\delta_{T2}]$$

Second order terms are retained in δz_M because the vertical aerodynamic forces Z_M and Z_s have non zero values in the equilibrium hovering state. The X and Y forces are zero in equilibrium. The equilibrium values of the Z forces are denoted Z_{Mo} and Z_{so}. Thus

$$\delta W = (\Delta X_s + \Delta X_M)\ \delta x_s + (\Delta Y_M + \Delta Y_s)\ \delta y_s + (Z_M + Z_s)\ \delta z_s$$
$$+ (-H_1 \Delta Y_M - H_1\ Z_{MO}\ \Phi_{T1})\ \delta\Phi_{T1} + (H_1 \Delta X_M - H_1 Z_{MO} \theta_{T1})\ \delta\theta_{T1}$$
$$+ (-L\ Z_M)\ \delta\Phi_{B1} + (L\Delta X_M)\ \delta\psi_{B1} + (0)\ \delta\theta_L$$
$$+ (H_2 \Delta Y_M + H_2\ Z_{MO}\ \Phi_{T2})\ \delta\Phi_{T2} + (-H_2 \Delta X_M + H_2\ Z_{MO}\ \theta_{T2})\ \delta\theta_{T2}$$

The equilibrium values of the vertical aerodynamic forces for the rectangular equilibrium state where all initial values of the angular coordinates are zero can be seen by inspection to be

$$Z_{MO} = -\ g\ [M_H + \frac{M_L + M_B}{2}]$$
$$Z_{sO} = -\ g\ [M_H + \frac{M_L + M_B}{2}]$$

Thus the virtual work contributions to the equations of motion are

x_s	$\Delta X_M + \Delta X_s$
y_s	$\Delta Y_M + \Delta Y_s$
z_s	$Z_{MO} + Z_{sO} + \Delta Z_M + \Delta Z_s = -\ g(2M_H + M_L + M_B) + \Delta Z_M + \Delta Z_s$
Φ_{T1}	$-H_1(\Delta Y_M + Z_{MO}\ \Phi_{T1}) = -\ H_1 \Delta Y_M + gH_1(M_H + \frac{M_L + M_B}{2})\ \Phi_{T1}$
θ_{T1}	$H_1(\Delta X_M - Z_{MO}\ \theta_{T1}) = H_1 \Delta X_M + gH_1(M_H + \frac{M_L + M_B}{2})\ \theta_{T1}$
Φ_{B1}	$-L(Z_{MO} + \Delta Z_M) = gL(M_H + \frac{M_L + M_B}{2}) - L\Delta Z_M$
ψ_{B1}	$L\Delta X_M$
θ_L	0

$$\Phi_{T2} \quad H_2(\Delta Y_M + Z_{MO}\ \Phi_{T2}) = -\ gH_2(M_H + \frac{M_L + M_B}{2})\ \Phi_{T2} + H_2\Delta Y_M$$

$$\theta_{T2} \quad -H_2(\Delta X_M - Z_{MO}\ \theta_{T2}) = -\ gH_2(M_H + \frac{M_L + M_B}{2})\ \theta_{T2} - H_2\Delta X_M$$

Note that some constant terms appear which will be balanced by terms on the right hand side and in addition there are contributions to the spring matrix due to the initial value of the Z-force. Only the dominant aerodynamic derivatives are included in this mode. The perturbations in the aerodynamic forces in a gravity fixed reference frame are:

$$\Delta X = X_u u + X_{B1}\ B_1\ - T_0\ \theta$$

$$\Delta Y = Y_v v + Y_{A1}\ A_1\ + T_0\ \Phi$$

$$\Delta Z = Z_w w + Z_{\theta c}\ \theta_c$$

For the slave helicopter

$$u_s = \dot{x}_s$$

$$v_s = \dot{y}_s$$

$$w_s = \dot{z}_s$$

For the master helicopter

$$u_M = \dot{x}_s - H_2\dot{\theta}_{T2} + H_1\ \dot{\theta}_{T1} + L\ \dot{\psi}_{B1}$$

$$v_M = \dot{y}_s + H_2\ \dot{\Phi}_{T2} - H_1\ \dot{\Phi}_{T2}$$

$$w_M = \dot{z}_s - L\ \dot{\Phi}_{B1}$$

The four equations describing the helicopter body motions are

$$I_y \ddot{\theta}_M = \Delta M_M$$

$$I_x \ddot{\phi}_M = \Delta L_M$$

$$I_y \ddot{\theta}_s = \Delta M_s$$

$$I_x \ddot{\phi}_M = \Delta L_s$$

where

$$\Delta M = \bar{M}_u u + \bar{M}_q q + \bar{M}_{B1} \quad B_1$$

$$\Delta L = \bar{L}_v v + \bar{L}_p p + \bar{L}_{A1} \quad A_1$$

where for simplicity aerodynamic coupling derivatives have been neglected.

Thus using these results with the output of the MACYSMA program the equations of motion can be written as in general as,

$$[M] \{\ddot{x}\} + [C] \{\dot{x}\} + K \{x\} = B \{u\}$$

where the fourteen motion variables are:

$$\{x\} = [x_s, y_s, z_s, \Phi_{T1}, \theta_{T1}, \phi_{B1}, \psi_{B1}, \theta_L, \phi_{T2}, \theta_{T2}, \theta_M, \theta_s, \phi_M, \phi_s]^T$$

and the controls are:

$$\{u\} = \{B_{1s}, B_{1M}, A_{1s}, A_{1M}, \theta_{Cs}, \theta_{CM}]^T$$

The [M], [C], [K] and [B] matrices can be obtained for this case from the more general matrices given in the next section. The equations have been normalized by mass and inertia. With the exception of the θ_L equation, the first ten equations have been divided by the helicopter mass (M_H). The load coordinate (θ_L)

equation is divided by the load mass (M_L). The pitch and roll moment equations are divided by the respective moments of inertia of the helicopter.

The following nondimensional quantities are introduced:

$$\mu = \frac{M_B + M_L}{2M_H},$$ ratio of spreader bar plus load mass to total helicopter mass

$$\beta = \frac{M_B}{M_L + M_B},$$ ratio of spreader bar mass to load mass plus spreader bar mass.

Note that

$$(1 - \beta) = \frac{M_L}{M_L + M_B}$$

$$(\mu)\ (1 - \beta) = \frac{M_L}{2M_H}$$

It can be noted from the M, C, and K matrices that the equations of motion for the complete fourteen degree-of-freedom system decouple into two seven degree-of-freedom systems, the planar dynamics studied in Reference 1 associated with the following degrees of freedom:

$$\{x_p\} = [y_s,\ z_s,\ \phi_{T1},\ \phi_{B1},\ \phi_{T2},\ \phi_M,\ \phi_s]^T$$

The remaining coordinates describe what will be referred to as the non-planar motion. It is associated with the following degrees of freedom

$$\{x_{NP}\} = [x_s,\ \theta_{T1},\ \psi_{B1},\ \theta_L,\ \theta_{T2},\ \theta_M,\ \theta_s]^T$$

Note that a slightly different notation is used in Reference 1 for the coordinates.

EQUATIONS OF MOTION: PART II

(Sixteen Degrees of Freedom)

The previous section described in detail the derivation of the equations of motion for the twin lift system. The formulation in Part I is restricted to the tether attachment points of both helicopters located at their respective centers of gravity. In addition the initial yaw orientation of each helicopter is assumed to be either perpendicular or parallel to the load suspension frame and the yaw angles are assumed to remain zero throughout the ensuing perturbed motion. The formulation is given for in Part I for the longitudinal axes of the helicopters perpendicular to the spreader bar in the initial state. It can be seen from Part I that the case in which the helicopters are rotated 90° is readily treated by interchanging the aerodynamic stability derivatives.

In this section, the formulation is extended to include vertical spacing between the tether attachment points and the centers of gravity of each helicopter. This dimension was shown to have an important effect on the stability of the twin lift system in the planar case. In addition, the initial orientation of the master and slave helicopters with respect to the load frame is arbitrary as shown in Figure 3. The following quantities are added in this section:

ψ_{So} trim or initial yaw angle of slave helicopter relative to frame (spreader bar)

ψ_{Mo} trim or initial yaw angle of master helicopter relative to frame (spreader bar)

$\Delta\psi_s$ change in yaw angle of slave helicopter measured from initial orientation of slave helicopter.

$\Delta\psi_M$ change in yaw angle of master helicopter relative to initial orientation of master helicopter.

The equations of motion are formulated such that it is assumed that the yaw motion of the master relative to the slave helicopter,

$$\Delta\psi_{MR} = \Delta\psi_M - \Delta\psi_s$$

is a small quantity. The change in yaw angle of the slave helicopter is not restricted to be small.

Two equations of motion are added to the dynamic system described in Part I.

$$\Delta\ddot{\psi}_s - N_r \, \Delta\dot{\psi}_s = N_{\delta_{TR}} \, \delta_{TR_s}$$

and

$$\Delta\ddot{\psi}_M - N_r \, \Delta\dot{\psi}_M = N_{\delta_{TR}} \, \delta_{TR_M}$$

These modifications to the equations of motion can be incorporated into the equations of motion in the following way. A frame axis system (X_F, Y_F, Z_F) is added which moves about Z_F with the change in yaw angle of the slave helicopter as shown Figure 4. Within the framework of the small angle assumption made in the linearization of the equations of motion, ψ_{B1} ca interpreted as the rotational displacement of the spreader relative to space. Note however, that the other angular coordinates describing the system motion rotate with $\Delta\psi_s$ helicopter body axes are now misaligned from the frame a

Figure 4, the relative spreader bar motion is also shown. Thus to summarize:

(X_F, Y_F, Z_F) Frame axes rotate with $\Delta\psi_S$. The frame motion variables defined in Part I are perturbations relative to this axis system. (Figure 4)

(X_{BS}, Y_{BS}, Z_{BS}) Slave helicopter body axes. Misaligned from frame axes by ψ_{SO} and rotate with $\Delta\psi_S$ and thus remain in a fixed orientation relative to the frame axes.

(X_{BM}, Y_{BM}, Z_{BM}) Master helicopter body axes. Misaligned from frame axes by ψ_{MO} and rotate with $\Delta\psi_M$. Thus it is assumed that in the disturbed motion that ($\Delta\psi_M$ - $\Delta\psi_S$) is a small quantity.

The equations of motion are now modified to account for the misalignment of the helicopter body axes and the frame axes. The stability derivatives of the helicopter are given in terms of velocities and forces oriented in the body axis directions. Note that these are not true body axes as they do not rotate with either the pitch or roll angles and thus the thrust vector must be accounted for in the expressions for the longitudinal and lateral aerodynamic forces. In addition, the equations must account for the fact that the tether attachment point is being used as the reference frame for the translational velocities.

First, the transformation of the aerodynamic derivatives is considered and then additional inertial and gravity terms due to the spacing between the attachment point and the center of gravity of each helicopter are considered. In this formulation only the dominant aerodynamic derivatives of the helicopter are considered. The formulation indicates how additional terms may be included. It is assumed that the following aerodynamic derivatives characterize the helicopter:

$$X_u,\ Y_v,\ M_u,\ M_q,\ L_v,\ L_p,\ N_r,\ Z_w$$

The following control derivatives are included

$$X_{B1}\ ,\ M_{B1}\ ,\ Y_{A1}\ ,\ L_{A1}\ ,\ N_{\delta_{TR}},\ Z_{\theta c}$$

The velocities of the helicopter center of gravity expressed in terms of attachment point velocities (u_T, v_T) and attachment point, center of gravity spacing (h') are:

$$u = u_T - h'\ \dot{\theta}$$

$$v = v_T + h'\ \dot{\phi}$$

The final formulation permits the spacing between the attachment point and the center of gravity to be different on each helicopter with the subscript 1 referring to the slave and the subscript 2 referring to the master.

The aerodynamic forces in the helicopter body/gravity axes are:

$$\Delta X_B = X_u u + X_{B1}\ \ B_1 - T_o\theta$$

$$\Delta Y_B = Y_v v + Y_{A1}\ \ A_1 + T_o\phi$$

The body motion equations are about the helicopter center of gravity are:

$$I_y\ddot{\theta} = \bar{M}_u u + \bar{M}_q\dot{\theta} + \bar{M}_{B1}\ \ B_1\ \ ,$$

$$I_x \ddot{\phi} = \bar{L}_v v + \bar{L}_p \dot{\phi} + \bar{L}_{A1} \quad A_1$$

The moment equations will be retained in body axes and the force equation will be transferred to frame axes.

$$\Delta X_F = \Delta X_B \cos\psi - \Delta Y_B \sin\psi$$

$$\Delta Y_F = \Delta Y_B \cos\psi + \Delta X_B \sin\psi$$

$$u_T = \dot{x}_s \cos\psi + \dot{y}_s \sin\psi$$

$$v_T = \dot{y}_s \cos\psi - \dot{x}_s \sin\psi$$

In the force equations, the effect of the spacing between the attachment point and the center of gravity can be neglected. This is consistant with neglecting the stability derivatives X_q and Y_p.

Thus using the above transformation

$$\Delta X_F = X_u^* \quad \dot{x}_s + X_v^* \dot{y}_s + (X_{B1} \cos\psi)\ B_1 - (Y_{A1} \sin\psi)\ A_1$$
$$+ T_o(-\theta \cos\psi - \phi \sin\psi)$$

$$\Delta Y_F = Y_u^* \quad \dot{x}_s + Y_v{}^* \dot{y}_s + (X_{B1} \sin\psi)\ B_1 + (Y_{A1} \cos\psi)\ A_1$$
$$+ T_o(-\theta \sin\psi + \phi \cos\psi)$$

where

$$X_u^* = X_u \cos^2\psi + Y_v \sin^2\psi$$

$$X_v^* = (X_u - Y_v) \sin\psi \cos\psi$$

$$Y_u^* = (X_u - Y_v) \sin\psi \cos\psi$$

$$Y_v^* = Y_v \cos^2\psi + X_u \sin^2\psi$$

If it is assumed that $X_u = Y_v$ then

$$X_u^* = Y_v^* = X_u$$

$$X_v^* = Y_u^* = 0$$

This is a reasonable simplification and is used in equations of motion presented later.

The linearized form of the aerodynamic forces is obtained by letting $\psi = \psi_{so}$ for the slave helicopter and $\psi = \psi_{Mo}$ for the master helicopter terms.

To account for the spacing between the attachment point and the helicopter center of gravity, the moment equations are written about the attachment point. Thus the stability derivatives about the attachment point are denoted

$$M^* = M - h' \Delta X_B'$$

$$L^* = L + h' \Delta Y_B'$$

Note that $\Delta X_B'$ and $\Delta Y_B'$ are body axis forces and do not contain the thrust vector.

The moment equations for the helicopter written about the attachment point are:

$$[I_y + h'^2 M_H] \ddot{\theta} - h' M_H \dot{u} - W h' \theta = M^*$$

$$[I_x + h'^2 M_H] \ddot{\phi} + h' M_H \dot{v} - W h' \phi = L^*$$

The following terms are added to the equations of motion to account for the displacement of the helicopter center of gravity from the attachment point. The inertial terms are:

$$\left(\frac{\Delta X_s}{M_H}\right) \quad -h_2' \, [\cos\psi_{Mo}\ddot{\theta}_M + \sin\psi_{Mo}\ddot{\Phi}_M] \; -h_1' \, [\cos\psi_{so} \, \ddot{\theta}_s + \sin\psi_{so}\ddot{\phi}_s]$$

$$\left(\frac{\Delta Y_s}{M_H}\right) \quad -h_2' \, [\cos\psi_{Mo}\ddot{\phi}_M - \sin\psi_{Mo}\ddot{\theta}_M] \; +h_1' \, [\cos\psi_{so}\ddot{\phi}_s - \sin\psi_{so}\ddot{\theta}_s]$$

$$\left(\frac{\phi_{T1}}{M_H}\right) \quad h_2' \, H_1(-\cos\psi_{Mo}\ddot{\phi}_M + \sin\psi_{Mo}\ddot{\theta}_M)$$

$$\left(\frac{\theta_{T1}}{M_H}\right) \quad h_2' \, H_1(-\cos\psi_{Mo}\ddot{\theta}_M - \sin\psi_{Mo}\ddot{\phi}_M)$$

$$\left(\frac{\psi_{B1}}{M_H}\right) \quad -h_2' \, L \, (\cos\psi_{Mo}\ddot{\theta}_M + \sin\psi_{Mo}\ddot{\phi}_M)$$

$$\left(\frac{\phi_{T2}}{M_H}\right) \quad h_2' \, H_2 \, (\cos\psi_{Mo}\ddot{\phi}_M - \sin\psi_{Mo}\ddot{\phi}_M)$$

$$\left(\frac{\theta_{T2}}{M_H}\right) \quad h_2' \, H_2 \, (\cos\psi_{Mo}\ddot{\theta}_M + \sin\psi_{Mo} \, \ddot{\phi}_M)$$

$$\left(\frac{\theta_M}{I_y}\right) \quad \left(h_2' \, \frac{M_H}{I_y}\right) \{-\ddot{x}_s \, \cos\psi_{Mo} - \ddot{y}_s \, \sin\psi_{Mo} + H_1(-\ddot{\theta}_{T1}\cos\psi_{Mo}$$
$$+ \ddot{\phi}_{T1} \, \sin\psi_{Mo}) + H_2(\ddot{\theta}_{T2} \, \cos\psi_{Mo} - \ddot{\phi}_{T2} \, \sin\psi_{Mo})$$
$$+ L(-\ddot{\psi}_{B1} \, \cos\psi_{Mo}) \quad + h_2'\ddot{\theta}_M\}$$

$$\left(\frac{\phi_M}{I_x}\right) \quad \left(h_2' \, \frac{M_H}{I_x}\right) \{-\ddot{x}_s\sin\psi_{Mo} + \ddot{y}_s \, \cos\psi_{Mo} + H_1(-\ddot{\theta}_{T1} \, \sin\psi_{Mo}$$
$$-\ddot{\phi}_{T1} \, \cos\psi_{Mo}) + H_2(\ddot{\theta}_{T2} \, \sin\psi_{Mo} + \ddot{\phi}_{T2} \, \cos\psi_{Mo})$$
$$+ L(-\ddot{\psi}_{B1} \, \sin\psi_{Mo}) \quad + h_2' \, \ddot{\phi}_M\}$$

$$\frac{\theta_s}{I_y} \quad h_1' \, \frac{M_H}{I_y} \{-\ddot{x}_s \, \cos\psi_{so} - \ddot{y}_s \, \sin\psi_{so} + h_1' \, \ddot{\theta}_s\}$$

$$\frac{\phi_s}{I_x} \qquad h_1' \frac{M_H}{I_x} \{-\ddot{x}_s \sin\psi_{so} + \ddot{y}_s \cos\psi_{so} + h_1' \ \ddot{\phi}_s\}$$

In addition the gravity terms must be resolved as shown in the equations of motion and the following terms added to the body moment equations:

$$\frac{\theta_M}{I_y} \qquad - g\ h_2' \frac{M_H}{I_y} \theta_M$$

$$\frac{\phi_M}{I_x} \qquad - g\ h_2' \frac{M_H}{I_x} \phi_M$$

$$\frac{\theta_s}{I_y} \qquad - g\ h_1' \frac{M_H}{I_y} \theta_s$$

$$\frac{\phi_s}{I_x} \qquad - g\ h_1' \frac{M_H}{I_x} \phi_s$$

Introducing these terms and the resolution effects for helicopter orientation as noted leads to equations of motion or the form

$$[M]\ \ddot{x} + [C]\ \dot{x} + [K]\ x = [B]\ u$$

where

$$x = \{x_s,\ y_s,\ z_s,\ \phi_{T1},\ \theta_{T1},\ \phi_{B1},\ \psi_{B1},\ \theta_L,\ \phi_{T2},\ \theta_{T2},\ \theta_M,\ \theta_s$$
$$\phi_M,\ \phi_s,\ \psi_M,\ \psi_s\}^T$$

$$u = \{B_{1s},\ B_{1M},\ A_{1s},\ A_{1M},\ \theta_{cs},\ \theta_{cM},\ \delta_{TRM},\ \delta_{TRs}\}^T$$

The system is described by sixteen degrees of freedom and there are eight controls. The matrix elements are given on the following pages. The yaw moment equations are gien in the text and not included in the matrices. The equations of motion for the simpler case described in Part I are obtained by choosing $\psi_{so} = \psi_{Mo} = 0$, or $\psi_{so} = \psi_{Mo} = \pi/2$, and eliminating the yaw moment equations.

$[M]$ MASS MATRIX

x_s	$2(1+\mu)$				$H_1(1+2\mu)$		$L(1+\mu)$	$Z(2\mu)(1-\beta)$		$-H_2$	$-h_2' \cos\psi_{mo}$	$-h_1' \cos\psi_{so}$	$-h_2' \sin\psi_{mo}$	$-h_1' \sin\psi_{so}$
y_s		$2(1+\mu)$		$-H_1(1+2\mu)$		$-Z(2\mu)(1-\beta)$			H_2		$-h_2' \sin\psi_{mo}$	$-h_1' \sin\psi_{so}$	$h_2' \cos\psi_{mo}$	$h_1' \cos\psi_{so}$
z_s			$2(1+\mu)$			$-L(1+\mu)$								
Φ_{T1}		$-H_1(1+2\mu)$		$H_1^2(1+2\mu)$		$ZH_1(2\mu)(1-\beta)$			$-H_1 H_2$		$h_2' H_1 \sin\psi_{mo}$		$-h_2' H_1 \cos\psi_{mo}$	
θ_{T1}	$H_1(1+2\mu)$				$H_1^2(1+2\mu)$		$H_1 L(1+\mu)$	$ZH_1(2\mu)(1-\beta)$		$-H_1 H_2$	$-h_2' H_1 \cos\psi_{mo}$		$-h_2' H_1 \sin\psi_{mo}$	
Φ_{B1}		$-Z(2\mu)(1-\beta)$	$-L(1+\mu)$	$ZH_1(2\mu)(1-\beta)$		$L^2+Z^2(2\mu)(1-\beta)+\frac{\mu}{2}L^2+\frac{I_{cmB}}{M_H}$								
ψ_{B1}	$L(1+\mu)$				$H_1 L(1+\mu)$		$L^2(1+\frac{\mu}{2})+\frac{I_{cmB}}{M_H}$	$ZL(\mu)(1-\beta)$		$-H_2 L$	$-h_2' L \cos\psi_{mo}$		$-h_2' L \sin\psi_{mo}$	
θ_L	Z				ZH_1		$\frac{1}{2} ZL$	Z^2						
Φ_{T2}		H_2		$-H_1 H_2$					H_2^2		$-h_2' H_2 \sin\psi_{mo}$		$h_2' H_2 \cos\psi_{mo}$	
θ_{T2}	$-H_2$				$-H_1 H_2$		$-H_2 L$			H_2^2	$h_2' H_2 \cos\psi_{mo}$		$h_2' H_2 \sin\psi_{mo}$	
θ_M	$-\epsilon_{y2} \cos\psi_{mo}$	$-\epsilon_{y2} \sin\psi_{mo}$		$H_1 \epsilon_{y2} \sin\psi_{mo}$	$-H_1 \epsilon_{y2} \cos\psi_{mo}$		$-L \epsilon_{y2} \cos\psi_{mo}$		$-H_2 \epsilon_{y2} \sin\psi_{mo}$	$H_2 \epsilon_{y2} \cos\psi_{mo}$	$1+h_2' \epsilon_{y2}$			
θ_S	$-\epsilon_{y1} \cos\psi_{so}$	$-\epsilon_{y1} \sin\psi_{so}$										$1+h_1' \epsilon_{y1}$		
Φ_M	$-\epsilon_{x2} \sin\psi_{mo}$	$\epsilon_{x2} \cos\psi_{mo}$		$-H_1 \epsilon_{x2} \cos\psi_{mo}$	$-H_1 \epsilon_{x2} \sin\psi_{mo}$		$-L \epsilon_{x2} \sin\psi_{mo}$		$H_2 \epsilon_{x2} \cos\psi_{mo}$	$H_2 \epsilon_{x2} \sin\psi_{mo}$			$1+h_2' \epsilon_{x2}$	
Φ_S	$-\epsilon_{x1} \sin\psi_{so}$	$\epsilon_{x1} \cos\psi_{so}$												$1+h_1' \epsilon_{x1}$

NOTES: ψ_M, ψ_S EQUATIONS NOT SHOWN (SEE TEXT)

$$\epsilon_{AB} = \frac{h_B' M_H}{I_A}$$

THIS IS A SYMMETRIC MATRIX PRIOR TO NORMALIZATION

NORMALIZATION: EQNS 1 - 7 DIVIDED BY M_H $(x_s \rightarrow \psi_{B1})$
EQN 8 DIVIDED BY M_L (θ_L)
EQNS 9, 10 DIVIDED BY M_H (Φ_{T2}, θ_{T2})
EQNS 11, 12 DIVIDED BY I_y (θ_M, θ_S)
EQNS 13, 14 DIVIDED BY I_x (Φ_M, Φ_S)

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[C] DAMPING MATRIX

x_s	$-2X_u^*$				$-H_1X_u^*$		$-LX_u^*$			$H_2X_u^*$				
y_s		$-2Y_v^*$		$H_1Y_v^*$					$-H_2Y_v^*$					
q_s			$-2Z_w$			LZ_w								
ϕ_{T_1}		$H_1Y_v^*$		$-H_1^2Y_v^*$					$H_1H_2Y_v^*$					
θ_{T_1}	$-H_1X_u^*$				$-H_1^2X_u^*$		$-H_1LX_u^*$			$H_1H_2X_u^*$				
ϕ_{R_1}			LZ_w			$-L^2Z_w$								
ψ_{R_1}	$-LX_u^*$				$-LH_1X_u^*$		$-L^2X_u^*$			$LH_2X_u^*$				
θ_L														
ϕ_{T_2}		$-H_2Y_v^*$		$H_1H_2Y_v^*$					$-H_2^2Y_v^*$					
θ_{T_2}	$H_2X_u^*$				$H_1H_2X_u^*$		$LH_2X_u^*$			$-H_2^2X_u^*$				
θ_M	$-M_{u_2}^*\cos\psi_{MO}$	$-M_{u_2}^*\sin\psi_{MO}$		$H_1M_{u_2}^*\sin\psi_{MO}$	$-H_1M_{u_2}^*\cos\psi_{MO}$		$-{}^{\prime}M_{u_2}^*\cos\psi_{MO}$		$-H_2M_{u_2}^*\sin\psi_{MO}$	$H_2M_{u_2}^*\cos\psi_{MO}$	$-M_{q_2}^*$			
θ_S	$-M_{u_1}^*\cos\psi_{SO}$	$-M_{u_1}^*\sin\psi_{SO}$										$-M_{q_1}^*$		
ϕ_M	$L_{v_2}^*\sin\psi_{MO}$	$-L_{v_2}^*\cos\psi_{MO}$		$H_1L_{v_2}^*\cos\psi_{MO}$	$H_1L_{v_2}^*\sin\psi_{MO}$		$LL_{v_2}^*\sin\psi_{MO}$		$-H_2L_{v_2}^*\cos\psi_{MO}$	$-H_2L_{v_2}^*\sin\psi_{MO}$			$-L_{p_2}^*$	
ϕ_S	$L_{v_1}^*\sin\psi_{SO}$	$-L_{v_1}^*\cos\psi_{SO}$												$-L_{p_1}^*$

NOTES:

FORCE DERIVATIVES IN FRAME AXES (SEE TEXT), $Y_u^* \approx X_v^* \approx 0$

MOMENT DERIVATIVES ABOUT ATTACHMENT POINT

$M_{u_1}^* = M_u - \epsilon_{y_1}X_u$ $\quad L_{v_1}^* = L_v + \epsilon_{x_1}Y_v$

$M_{u_2}^* = M_u - \epsilon_{y_2}X_u$ $\quad L_{v_2}^* = L_v + \epsilon_{x_2}Y_v$

$X_q = Y_p = 0$

$M_q^* = M_q\,,\quad L_p^* = L_p$ SEE TEXT FOR X_u^*, Y_v^*

[K] SPRING MATRIX

x_s											$g(1+\mu)\cos\Psi_{MO}$	$g(1+\mu)\cos\Psi_{SO}$	$g(1+\mu)\sin\Psi_{MO}$	$g(1+\mu)\sin\Psi_{SO}$
y_s											$g(1+\mu)\sin\Psi_{MO}$	$g(1+\mu)\sin\Psi_{SO}$	$-g(1+\mu)\cos\Psi_{MO}$	$-g(1+\mu)\cos\Psi_{SO}$
z_s														
Φ_{T1}				$g\mu H_1$							$-gH_1(1+\mu)\cdot\sin\Psi_{MO}$		$gH_1(1+\mu)\cdot\cos\Psi_{MO}$	
Θ_{T1}					$g\mu H_1$						$gH_1(1+\mu)\cdot\cos\Psi_{MO}$		$gH_1(1+\mu)\cdot\sin\Psi_{MO}$	
Φ_{B1}						$gZ(2\mu)(1-\beta)$								
Ψ_{B1}											$gL(1+\mu)\cdot\cos\Psi_{MO}$		$gL(1+\mu)\cdot\sin\Psi_{MO}$	
Θ_L								$g\ell$						
Φ_{T2}									$g\mu H_2$		$gH_2(1+\mu)\cdot\sin\Psi_{MO}$		$-gH_2(1+\mu)\cdot\cos\Psi_{MO}$	
Θ_{T2}										$g\mu H_2$	$-gH_2(1+\mu)\cdot\cos\Psi_{MO}$		$-gH_2(1+\mu)\cdot\sin\Psi_{MO}$	
Θ_M											$-g\varepsilon_{y2}$			
Θ_S												$-g\varepsilon_{y1}$		
Φ_M													$-g\varepsilon_{x2}$	
Φ_S														$-g\varepsilon_{x1}$

[B] CONTROL MATRIX

	B_{1S}	B_{1M}	A_{1S}	A_{1M}	θ_{CS}	θ_{CM}
x_S	$X_{B_1}\cos\psi_{SO}$	$X_{B_1}\cos\psi_{MO}$	$-Y_{A_1}\sin\psi_{SO}$	$-Y_{A_1}\sin\psi_{MO}$		
y_S	$X_{B_1}\sin\psi_{SO}$	$X_{B_1}\sin\psi_{MO}$	$Y_{A_1}\cos\psi_{SO}$	$Y_{A_1}\cos\psi_{MO}$		
z_S					Z_{θ_C}	Z_{θ_C}
ϕ_{T_1}		$-X_{B_1}H_1\cdot\sin\psi_{MO}$		$-Y_{A_1}H_1\cdot\cos\psi_{MO}$		
θ_{T_1}		$X_{B_1}H_1\cdot\cos\psi_{MO}$		$-Y_{A_1}H_1\cdot\sin\psi_{MO}$		
ϕ_{B_1}						$-LZ_{\theta_C}$
ψ_{B_1}		$X_{B_1}L\cos\psi_{MO}$		$-Y_{A_1}L\sin\psi_{MO}$		
θ_L						
ϕ_{T_2}		$X_{B_1}H_2\cdot\sin\psi_{MO}$		$Y_{A_1}H_2\cdot\cos\psi_{MO}$		
θ_{T_2}		$-X_{B_1}H_2\cdot\cos\psi_{MO}$		$Y_{A_1}H_2\cdot\sin\psi_{MO}$		
θ_M		$M^*_{B_{1,2}}$				
θ_S	$M^*_{B_{1,1}}$					
ϕ_M				$L^*_{A_{1,2}}$		
ϕ_S			$L^*_{A_{1,1}}$			

$$M^*_{B_{1,1}} = M_{B_1} - \epsilon_{y_1} X_{B_1}$$

$$M^*_{B_{1,2}} = M_{B_1} - \epsilon_{y_2} X_{B_1}$$

NON-PLANAR MOTION

It can be seen from the equations of motion developed in the previous sections that the complete equations of motion for the twin lift system decouple into independent sets of equations of motion if the lateral-longitudinal coupling terms due to aerodynamic stability derivatives are neglected in the helicopter models and the initial yaw angle of each helicopter is either 0 or 90° (longitudinal axis of the helicopters perpendicular or parallel to the spreader bar).

The uncoupled sets of equations of motion involve the following degrees-of-freedom if the initial orientation of the helicopters is such that their longitudinal axes are perpendicular to the frame ($\psi_{Mo} = \psi_{so} = 0$).

Planar Motion (seven degrees of freedom)

$$\{x_p\} = \{y_s,\ z_s,\ \phi_{T1},\ \phi_{B1},\ \phi_{T2},\ \phi_M,\ \phi_s\}^T$$

Non-planar Motion (seven degrees of freedom)

$$\{x_{Np}\} = \{x_s,\ \theta_{T1},\ \theta_{T2},\ \psi_{B1},\ \theta_L,\ \theta_M,\ \theta_s\}^T$$

The other two degrees of freedom, the yaw angles of each helicopter are uncoupled. If the initial yaw angles are 90°, then the pitch and roll angles are interchanged in the degrees freedom of the two cases.

The planar motion has been investigated extensively and results of this investigation are presented in Reference 1. this section the basic dynamics of the non-planar case are examined. The same physical parameters are used as in Re

1. These are listed in Table I. The helicopters used in the example correspond nominally to the UH-60, Blackhawk.

It was noted in the studies of the planar motion that if the tether lengths are equal ($H = H_1 = H_2$) and the attachment point center of gravity spacing is the same on both helicopters, then the dynamic system can be further subdivided into sets of symmetric and anti-symmetric motions and considerable insight gained regarding the dynamics of this complex system. The anti-symmetric modes involve the two helicopters moving "in-phase" and are essentially the modes involved in translating the entire system laterally, i.e., in the plane of the paper in Figure 1. The symmetric modes involve essentially "out-of-phase" motion of the two helicopters with the load stationary and are equivalent to those of a single helicopter tethered to a fixed point. See Reference 1 for additional discussion.

A similar division or decoupling of the modes can be made in the non-planar case. Again using the terminology anti-symmetric modes to refer to the case in which the two helicopters move "i phase", the following coordinate relationships are involved:

Anti-symmetric motion:

$$x_s,\ \theta_{T1} = \theta_{T2},\ \theta_L,\ \theta_M = \theta_s,\ (\psi_{B1} = 0)$$

This is a four degree of freedom system. The equations o are those of a helicopter with a sling load that acts li compound pendulum (7). Figure 5 shows the geometry of motion.

Symmetric motion:

$$\psi_{B1},\ \theta_{T1} = -\theta_{T2},\ \theta_M = -\theta_s,\ (\theta_L = 0)$$

This is a three degree of freedom motion that involves only rotation of the system about a vertical axis through the load, spreader bar center of mass as shown in Figure 6.

The next section shows how the equations are decoupled by the introduction of new coordinates.

Decoupling the Equations of Motion (Non-Planar case)

The equations of motion for the non-planar case can be written as

$$[M]\ \{\ddot{x}_{NP}\} + [C]\ \{\dot{x}_{NP}\} + [K]\ \{x_{NP}\} = [B]\ \{u\} \qquad (1)$$

where

$$\{x_{NP}\} = [x_s,\ \theta_{T1},\ \theta_{T2},\ \psi_{B1},\ \theta_L,\ \theta_M,\ \theta_s]^T \qquad (2)$$

The matrices [M], [C], [K] and [B] are given following this section for this reduced degree of freedom case.

The case in which the tethers are equal ($H = H_1 = H_2$) is considered and a new set of coordinates similar to those used in the planar case is introduced to decouple the equations. The decoupling procedure is as follows.

Noting that the translational displacement of the master helicopter can be written as

$$x_M = x_s + H(\theta_{T1} - \theta_{T2}) + L\psi_{B1} \qquad (3)$$

This coordinate is introduced and sum and difference coordinates are defined:

$$\Sigma x = \frac{x_s + x_M}{2} \qquad \Delta x = x_M - x_s$$

$$\Sigma\theta = \frac{\theta_s + \theta_M}{2} \qquad \Delta\theta = \theta_M - \theta_s \tag{4}$$

$$\Sigma\theta_T = \frac{\theta_{T1} + \theta_{T2}}{2} \qquad \Delta\theta_T = \theta_{T2} - \theta_{T1}$$

The load displacement θ_L is chosen as the seventh coordinate. The spreader bar yaw motion ψ_{B1} can be eliminated as a motion variable, using the equation for x_M given in equations (3),

$$\psi_{B1} = \frac{\Delta x}{L} + \frac{H}{L}\Delta\theta_T$$

Sum coordinates are associated with the anti-symmetric motion and difference coordinates are associated with the symmetric motion. The pitching moment equations for the master and slave helicopters can be written as

$$\ddot{\theta}_M - M_u^* \dot{x}_M^* - M_q^* \dot{\theta}_M - h' \frac{M_H}{I_y} \ddot{x}_M + h'^2 \frac{M_H}{I_y} \ddot{\theta}_M - gh' \frac{M_H}{I_y} \theta_M = M_{B1}^* B_{1M}$$

(5)

$$\ddot{\theta}_s - M_u^* \dot{x}_s^* - M_q^* \dot{\theta}_s - h' \frac{M_H}{I_y} \ddot{x}_s + h'^2 \frac{M_H}{I_y} \ddot{\theta}_s - gh' \frac{M_H}{I_y} \theta_s = M_{B1}^* B_{1s}$$

Sum and difference equations are obtained by adding and subtracting these equations.

$$\Sigma\ddot{\theta} - M_u^* \Sigma\dot{x}^* - M_q^* \Sigma\dot{\theta} - h' \frac{M_H}{I_y} \Sigma\ddot{x} + h'^2 \frac{M_H}{I_y} \Sigma\ddot{\theta} - gh' \frac{M_H}{I_y} \Sigma\theta = M_{B1}^* \Sigma B_1 \tag{6}$$

$$\Delta\ddot{\theta} - M_u^*\Delta\dot{x}^* - M_q^*\Delta\dot{\theta} - h'\frac{M_H}{I_y}\Delta\ddot{x} + h'^2\frac{M_H}{I_y}\Delta\ddot{\theta} - gh'\frac{M_H}{I_y}\Delta\theta = M_{B1}^*\Delta B_1 \tag{7}$$

Note that the pitching moment derivatives are expressed with respect to the tether point. In order to reform the other five equations it is convenient to introduce the following set of additional coordinates which describe the linearized displacements of the centers of mass of the spreader bar and load, and the master helicopter tether point.

$$\begin{aligned} x_M &= x_s - H\,\Delta\theta_T + L\psi_{B1}, \quad x_M^* = x_M - h'\,\theta_M \\ x_B &= x_s + H\,\theta_{T1} + \frac{L}{2}\psi_{B1}, \quad x_s^* = x_s - h'\,\theta_s \\ x_L &= x_s + H\,\theta_{T1} + \frac{L}{2}\psi_{B1} + Z\theta_L \end{aligned} \tag{8}$$

The equations of motion in terms of these coordinates are:

x_s equation,

$$M_H\ddot{x}_s + M_H\ddot{x}_M + M_B\ddot{x}_B + M_L\ddot{x}_L - \bar{X}_u(\dot{x}_s^* + \dot{x}_M^*) + T_o\,2\Sigma\theta - h'\,M_H\,2\Sigma\ddot{\theta} = \bar{X}_{B1}\,2\Sigma B_1 \tag{9}$$

θ_{T1} equation divided by H

$$M_H\ddot{x}_M + M_B\ddot{x}_B + M_L\ddot{x}_L - \bar{X}_u\dot{x}_M^* + g\left(\frac{M_L + M_B}{2}\right)\theta_{T1} + T_o\theta_M - h'M_H\,\ddot{\theta}_M = \bar{X}_{B1}B_{1M} \tag{10}$$

ψ_{B1} equation divided by L

$$M_H \ddot{x}_M + \frac{1}{2}(M_B \ddot{x}_B + M_L \ddot{x}_L) + \frac{I_{CMB}}{L} \ddot{\psi}_{B1} - \bar{X}_u x_M^* + T_o \theta_M - h' M_H \ddot{\theta}_M = \bar{X}_{B1} B_{1M} \tag{11}$$

θ_L equation divided by ZM_L

$$\ddot{x}_L + g\,\theta_L = 0 \tag{12}$$

θ_{T2} equation divided by H

$$-M_H \ddot{x}_M + X_u \dot{x}_M^* + g\left(\frac{M_L + M_B}{2}\right) \theta_{T2} - T_o\, \theta_M + h' M_H \ddot{\theta}_M = -\bar{X}_{B1} B_{1M} \tag{13}$$

Noting that

$$\psi_{B1} = \frac{1}{L}\Delta x + \frac{H}{L}\Delta\theta_T$$

$$x_B - x_L = -Z\theta_L \tag{14}$$

Adding the θ_{T1} and θ_{T2} equations (10 + 13)

$$M_B \ddot{x}_B + M_L \ddot{x}_L + g(M_L + M_B)\,\Sigma\theta_T = 0 \tag{15}$$

Adding the ψ_{B1} and θ_{T2} equations and multiplying by 2,

$$M_B \ddot{x}_B + M_L \ddot{x}_L + \frac{2I_{CMB}}{L} \ddot{\psi}_{B1} + g(M_L + M_B)\,\theta_{T2} = 0 \tag{16}$$

It may be noted upon comparing equations (15) amd (16) that if the moment of inertia of the spreader bar about its center of mass is neglected then

$$\theta_{T2} = (\Sigma\theta_T)$$

implying that $\theta_{T1} = \theta_{T2}$, that is, the two tether angles two are equal if the spreader bar inertia is neglected. In this case, one motion variable becomes redundant as does one equation of motion. x_B and x_L can be expressed as from (4), (8) and (14) as:-

$$
\begin{aligned}
x_B &= \Sigma x + H\,\Sigma\theta_T \\
x_L &= \Sigma x + H\,\Sigma\theta_T + Z\theta_L
\end{aligned}
\tag{17}
$$

The sum coordinate equations (anti-symmetric equations) are formed from the x_s equation (9), and the load equation (12), the sum pitching moment equations (6) and the $(\theta_{T1} + \theta_{T2})$ equation (15). Note that these substitutions transfer the pitching moment derivatives to the center of gravity. The anti-symmetric system equations of motion are:

$$
\begin{aligned}
\Sigma\ddot{x} - X_u\Sigma\dot{x}^* - h'\Sigma\ddot{\theta} + g(1+\mu)\,\Sigma\theta - g\mu\Sigma\theta_T &= X_{B1}\Sigma B_1 \\
- M_u\Sigma\dot{x}^* + \Sigma\ddot{\theta} - M_q\Sigma\dot{\theta} + \epsilon_y g\mu(\Sigma\theta - \Sigma\theta_T) &= M_{B1}\Sigma B_1 \\
\Sigma\ddot{x} + H\Sigma\ddot{\theta}_T + Z\ddot{\theta}_L + g\theta_L &= 0 \\
- g\Sigma\theta_T + \beta Z\ddot{\theta}_L + g\theta_L &= 0 \\
\Sigma x - h'\Sigma\theta - \Sigma x^* &= 0
\end{aligned}
\tag{18}
$$

The difference coordinate equations (symmetric mode equations) are obtained by subtracting two times the ψ_{B1} equation from the x_s equation, the difference moment equation, and a combination of the θ_{T1}, θ_{T2}, and ψ_{B1} equations. The symmetric system equations of motion are:

$$
\begin{aligned}
\Delta\ddot{x} - X_u\Delta\dot{x}^* + h'\Delta\ddot{\theta} + g(1+\mu)\,\Delta\theta - g\mu\,\Delta\theta_T &= X_{B1}\Delta B_1 \\
- M_u\Delta\dot{x}^* + \Delta\ddot{\theta} - M_q\Delta\dot{\theta} + \epsilon_y g\mu(\Delta\theta - \Delta\theta_T) &= M_{B1}\Delta B_1 \\
\delta\Delta\ddot{x} + \delta H\Delta\ddot{\theta}_T + g\Delta\theta_T &= 0 \\
\Delta x - h'\Delta\theta - \Delta x^* &= 0
\end{aligned}
\tag{19}
$$

Thus the new coordinates invoved in these uncoupled sets of equations of motion are:

$$X_{NP,AS} = \{\Sigma x,\ \Sigma\theta,\ \Sigma\theta_T,\ \theta_L\}^T$$

$$X_{NP,S} = \{\Delta x,\ \Delta\theta,\ \Delta\theta_T\}^T$$

The parameter δ is a non-dimensional expression of the spreader bar moment of inertia. If the spreader bar moment of inertia is equal to zero, $\delta = 0$, and the last equation in the symmetric case gives $\Delta\theta_T = 0$ and the symmetric set reduces to a two degree of freedom system.

Note that x refers to the displacement of the tether attachment point while x* refers to the helicopter center of mass displacement. The characteristics of these two systems are now considered.

Anti-symmetric Modes

It can be noted that the anti-symmetric equations (18) are the equations of motion of a helicopter coupled to a sling load which is a compound pendulum. The parameter β represents the ratio of the two supported masses, the spreader bar mass and the load mass. In the limit of no spreader bar mass ($\beta = 0$), $\Sigma\theta_T = \theta_L$, the load equations of motion reduce to those of a single pendulum of length (H + Z). Comparison with the planar anti-symmetric equations of motion shows that for h' = 0, Z = 0 (load on spreader bar) and $\beta = 0$, the equations of motion are identical if the following interpretation is made.

$$x_L' \rightarrow H(\Sigma\theta_T - \Sigma\theta) = H(\theta_L - \Sigma\theta)$$

The coupled frequencies of the double pendulum system can be determined from the characteristic equation of the two degree of freedom system for $\Sigma\theta_T$, θ_L. The characteristic equation of this system is,

$$\frac{HZ\beta}{g(H + Z)} S^4 + S^2 + \frac{g}{(H + Z)} = 0$$

When $\beta = 0$ (no spreader bar mass) there is a single frequency

$$\omega_{s0}^2 = \frac{g}{H + Z}$$

and when $\beta \to 1$ (no load mass) there are two frequencies

$$\omega_{s1}^2 = \frac{g}{H} \qquad \omega_{s2}^2 = \frac{g}{Z}$$

Figure 7 shows the variation of these uncoupled frequencies with β. At the nominal value of β, with load ($\beta = .054$) the two frequencies are:

$$\omega_{s1} = .826 \text{ rad/sec}$$

$$\omega_{s2} = 8.16 \text{ rad/sec}$$

There is a wide variation in one of the frequencies associated with the load as the load is reduced to zero ($\beta = 1.0$).

The anti-symmetric equations of motion are those primarily associated with maneuvering the system from place to place. The complete system involves these two sling modes coupled to the helicopter dynamics. For the nominal load ($\beta = .054$) the high frequency (ω_{s2}) associated with the sling is only weakly coupled.

<u>Symmetric Modes</u>

The symmetric motion as described by equations (19) show as noted above that if the spreader bar inertia distribution is neglected then $\delta \to 0$ and $\Delta\theta_T = 0$ indicating that the tethers remain vertical throughout the motion in this limit. The tether cable tension provides a restoring moment as indicated by the equations of motion. With the spreader bar inertia included there is an angular oscillation associated with the spreader bar. The frequency of this motion uncoupled from the helicopters is obtained from the last of equations (19):

$$\omega_\psi^2 = \frac{g}{H\delta} = \frac{3g}{H\beta}$$

where

$$\delta = \frac{4I_{CMB}}{(M_B + M_L)L^2} \cong \frac{\beta}{3}$$

The nominal value of β with load is .054 and therefore with H = 13.25 ft, this frequency is:

$$\omega_\psi = 12.07 \text{ rad/sec}$$

The spring in this motion is tension in each tether. Since this frequency is high compared to the helicopter dynamics it is probably quite a good approximation to determine the influence of the spreader bar inertia on the modes associated with the helicopter dynamics by using a quasistatic approximation.

$$\Delta\ddot{x} \cong -\frac{g}{\delta}\,\Delta\theta_T = -H\omega_\psi{}^2\Delta\theta_T \tag{20}$$

If the entire system is moving at a frequency ω, then the differential tether angle is

$$\Delta\theta_T \cong \left(\frac{\omega}{\omega_\psi}\right)^2 \frac{\Delta x}{H}$$

Thus the tethers tend to remain relatively near vertical as indicated by the limit $\delta \to 0$, and the spreader bar follows the helicopter motion, i.e.,

$$\psi_{B1} = \frac{\Delta x}{L}\left(1 + \left(\frac{\omega}{\omega_\psi}\right)^2\right) \approx \frac{\Delta x}{L}$$

The equations of motion assuming quasi-static bar motion are ($\beta \ll 1$) obtained from equations (19) and (20),

$$(1 + \delta\mu)\,\Delta\ddot{x} - X_u \Delta\dot{x}^* - h'\Delta\ddot{\theta} + g(1 + \mu)\,\Delta\theta \equiv X_{B1}\,\Delta B_1 \qquad (21)$$

$$\epsilon_y \delta\mu\Delta\ddot{x} - M_u \Delta\dot{x}^* + \Delta\ddot{\theta} - M_q \Delta\dot{\theta} + \epsilon_y g\mu\,\Delta\theta \equiv M_{B1}\,\Delta B_1$$

Thus fully loaded with $\delta = .018$, $\mu = .43$ the effect quasi-static tether rotation, proportional to $\delta\mu$, on the helicopter modes is quite small. With no load, μ is very small and it would also be expected that the coupling between the angular motion of the spreader bar and the helicopter motion is weak. Note that the uncoupled frequency of the spreader bar motion varies significantly with load due to the tension variation as shown in Figure 8.

The limiting mode shapes in the symmetric motion with variation in spreader bar inertia are shown in Figure 9. It also can be seen that in the limit of very large spreader bar inertia, $\Delta x = H\ \Delta\theta_T$ and the anti-symmetric equations become identical to the symmetric planar case. That is, the spreader bar tends to remain fixed in space and a divergent mode will occur for the attachment point located below the center of gravity of the helicopter. This limit can be readily seen by changing coordinates in the difference equations replacing $\Delta\theta_T$ by ψ_{B1}. However the actual system will tend to be far from this limit for the physical parameters typical of this system.

In the practical case where the non-dimensional parameter measuring the spreader bar inertia (δ) is small the characteristic equation for the anti-symmetric coupled motion, neglecting $\delta\mu$ terms in equation (21), which is equivalent to assuming that the tethers remain vertical is,

$$(s)\ (s - X_u)\ (s - M_q) + g(1 + \mu)\ M_u + \epsilon_y g\mu(s - X_u) \cong 0$$

For ϵ_y positive, there is a stabilizing tendency, i.e., since in this limit the tethers are vertical an effective attitude stability is provided by tether tension acting at the attachment point below the center of gravity of the helicopter. Placing the attachment point above the center of gravity acts like an unstable attitude feedback. This stabilizing trend is opposite to that shown for the planar symmetric case in Reference 1.

Figure 10 shows numerical values for the modes of motion for the non-planar system with load (the symmetric and anti-symmetric modes) as a function of tether attachment point location relative to the helicopter center of mass. The uncoupled helicopter modes for this sample calculation based on the parameters in Table I are -.82 and +.172 ± .572i. The destabilizing effect of locating the attachment point above the center of gravity can be seen from the figure. The movement of the complex pair is very similar to that obtained with attitude feedback.

$[M]$ MASS MATRIX NON-PLANAR

X_s	$2(1+\mu)$	$H_1(1+2\mu)$	$L(1+\mu)$	$Z(2\mu)(1-\beta)$	$-H_2$	$-h_2'$	$-h_1'$
θ_{T1}	$H_1(1+2\mu)$	$H_1^2(1+2\mu)$	$H_1L(1+\mu)$	$ZH_1(2\mu)(1-\beta)$	$-H_1H_2$	$-h_2'H_1$	
Ψ_{B1}	$L(1+\mu)$	$H_1L(1+\mu)$	$L^2(1+\frac{\mu}{2}) + \frac{I_{CMB}}{M_H}$	$ZL(\mu)(1-\beta)$	$-H_2L$	$-h_2'L$	
θ_L	Z	ZH_1	$\frac{1}{2}ZL$	Z^2			
θ_{T2}	$-H_2$	$-H_1H_2$	$-H_2L$		H_2^2	$h_2'H_2$	
θ_M	$-\epsilon_{y2}$	$-H_1\epsilon_{y2}$	$-L\epsilon_{y2}$		$H_2\epsilon_{y2}$	$1+h_2'\epsilon_{y2}$	
θ_S	$-\epsilon_{y1}$						$1+h_1'\epsilon_{y1}$

$\{ \Psi_{M0} = \Psi_{S0} = 0 \}$

$[C]$ DAMPING MATRIX NON-PLANAR

$$
\begin{array}{c}
X_S \\ \Theta_{T1} \\ \Psi_{B1} \\ \Theta_L \\ \Theta_{T2} \\ \Theta_M \\ \Theta_S
\end{array}
\left[
\begin{array}{ccccccc}
-2X_u & -H_1X_u & -LX_u & & H_2X_u & & \\
-H_1X_u & -H_1^2X_u & -H_1LX_u & & H_1H_2X_u & & \\
-LX_u & -LH_1X_u & -L^2X_u & & LH_2X_u & & \\
 & & & & & & \\
H_2X_u & H_1H_2X_u & LH_2X_u & & -H_2^2X_u & & \\
-M_u^* & -H_1M_u^* & -LM_u^* & & H_2M_u^* & -M_q^* & \\
-M_u^* & & & & & & -M_q^*
\end{array}
\right]
$$

$\{\Psi_{M0} = \Psi_{S0} = 0\}$

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$[K]$ SPRING MATRIX NON-PLANAR

X_S						$g(1+\mu)$	$g(1+\mu)$
Θ_{T1}		$g\mu H_1$				$gH_1(1+\mu)$	
Ψ_{B1}						$gL(1+\mu)$	
Θ_L				gZ			
Θ_{T2}					$g\mu H_2$	$-gH_2(1+\mu)$	
Θ_M						$-g\epsilon_{Y2}$	
Θ_S							$-g\epsilon_{Y1}$

$\{\Psi_{MO} - \Psi_{SO} = 0\}$

[B] CONTROL MATRIX NON-PLANAR

	B_{1S}	B_{1M}					
X_S	X_{B1}	X_{B1}					
θ_{T1}		$X_{B1} H_1$					
ψ_{B1}		$X_{B1} L$					
θ_L							
θ_{T2}		$-X_{B1} H_2$					
θ_M		M^*_{B1}					
θ_S	M^*_{B1}						

TABLE I

NUMERICAL VALUES OF SYSTEM PARAMETERS

Helicopter Mass $M_H = 435$ slugs

Load Mass $M_H = 353$ slugs

Spreader Bar Mass $M_B = 20$ slugs

Slave Tether $H_1 = 13.25$ ft

Master Tether $H_2 - 13.25$ ft

Spreader Bar Length $L = 68.9$ ft

Load distance below spreader bar $Z = 34.5$ ft

Tether Attachment point/CG spacing $h' = 3.5$ ft (nominal)

Helicopter moment of inertia in pitch $I_y = 43{,}000$ slug ft^2

Helicopter Stability Derivatives (referenced to CG)

$$x_u = -.0602 \text{ sec}^{-1}$$

$$M_q = -.415 \text{ sec}^{-1}$$

$$M_u = .00538 \text{ (ft-sec)}^{-1}$$

Non-dimensional parameters:

$$\mu = \frac{M_B + M_L}{2M_H} = 0.43$$

$$\beta = \frac{M_B}{M_L + M_B} = .054$$

$$\delta = \frac{4I_{CMB}}{(M_B + M_L)\ L^2} \cong \frac{M_B}{3(M_B + M_L)} = .018$$

(uniform mass distribution)

$$\epsilon_y = \frac{h'M_H}{I_y} = .030$$

SUMMARY

A full set of equations of motion for the twin lift system, linearized about a hover trim condition have been derived and presented. It is shown that this full set of equations of motion decouples into simpler sets of equations of motion if the aerodynamic coupling derivatives of the helicopters are neglected. One of these decoupled sets of equations of motion (referred to as the planar set) was studied at length in Reference 1. The other decoupled set (referred to as the non-planar set) is examined here. It is shown that when the geometric configuration of the twin-lift is symmetric that a further decoupling is possible into anti-symmetric and symmetric sets of equations. One set of these reduced equations of motion referred to as the anti-symmetric set is directly equivalent to the longitudinal motion of a single helicopter with a sling load. The second set, referred to as the symmetric set corresponds to rotation of the entire system without load motion. As in the case of the planar symmetric motion, the location of the tether attachment point influences the stability of the non-planar symmetric mode. The trend is opposite however in the non-planar case in that an attachment point below the helicopter center of gravity gives a favorable effect on the stability. The effect is not as strong however as the unfavorable effect on the planar symmetric mode shown in Reference 1.

REFERENCES

1. Curtiss, H. C., Jr. and Warburton, F. W.: Stability and Control of the Twin Lift Helicopter System. Journal of the American Helicopter Society, Vol. 30, No. 2, April 1985.

2. Carter, E. S.: Implications of Heavy Lift Helicopter Size Effect Trends and Multilift Options for Filling the Need. Proceedings of the 8th European Rotorcraft Forum, Aix-en-Provence, France, September 1982.

3. Sheridan, P. F., et al.: Feasibility Study for Multiple Helicopter Heavy Lift Systems, Report R-136, VERTOL Aircraft Corp. Philadelphia, PA, October 1957.

4. Meek, T., Chesley, G. B.: Twin Helicopter Lift System Study and Feasibility Demonstration. Report SER 64323, Sikorsky Aircraft Co., Stratford, CT, December 1970 for U. S. Army Aviation Materiel Directorate, Fort Eustis, VA.

5. Cicolani, L. S., and Kanning G.: General Equilibrium Characteristics of a Dual-Lift Helicopter System, NASA TP 2615, July 1986.

6. Kaufman, L., and Schultz, E. R.: The Stability and Control of Tethered Helicopters, Journal of the American Helicopter Society, Vol. 7, No. 4, October 1962.

7. Wolkovitch, J. and Johnston, D. E.: Automatic Control Considerations for Helicopters and VTOL Aircraft With and Without Sling Loads, STI Technical Report 138-1, November 1965.

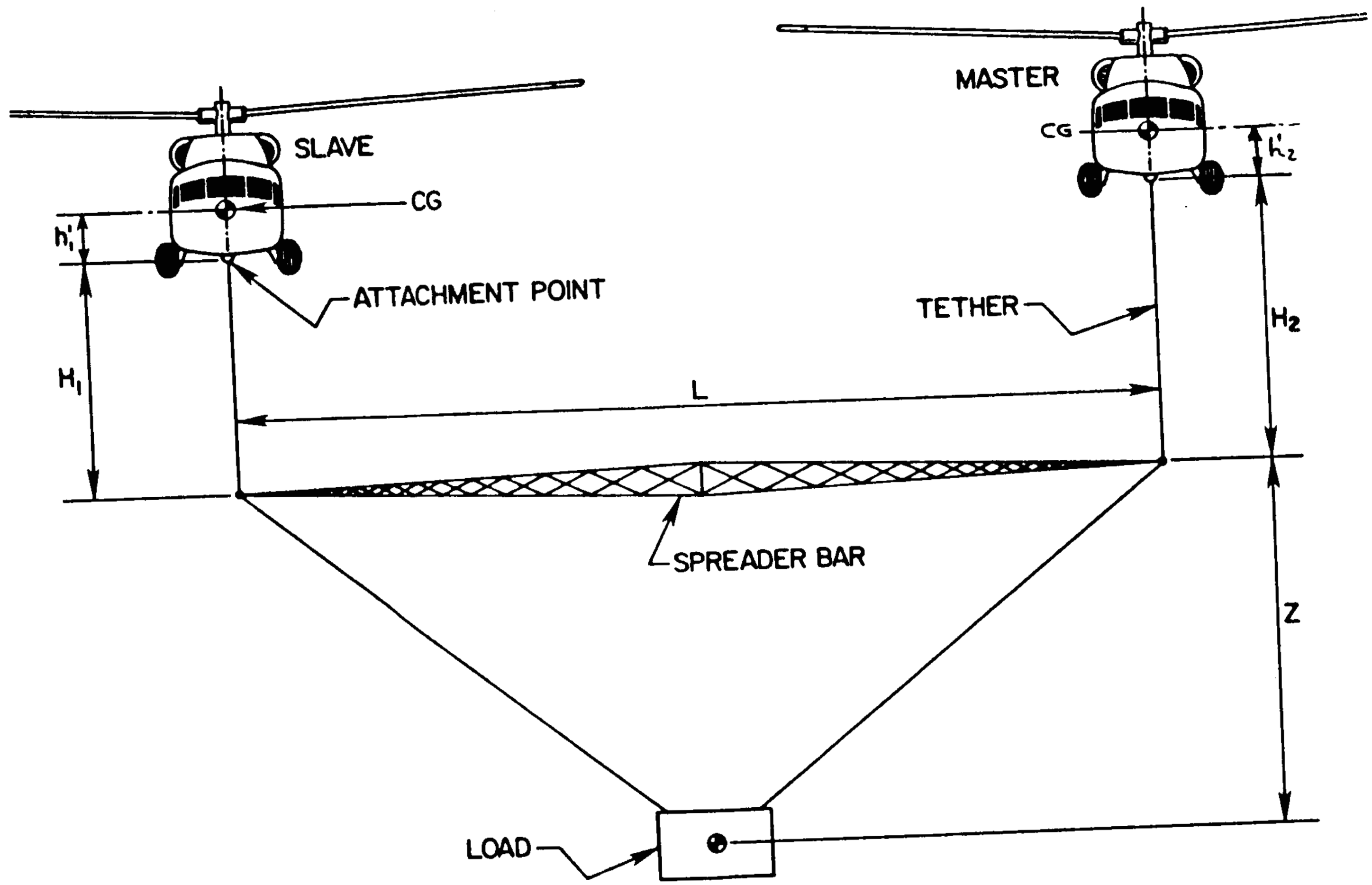

Figure 1: Twin Lift Configuration,

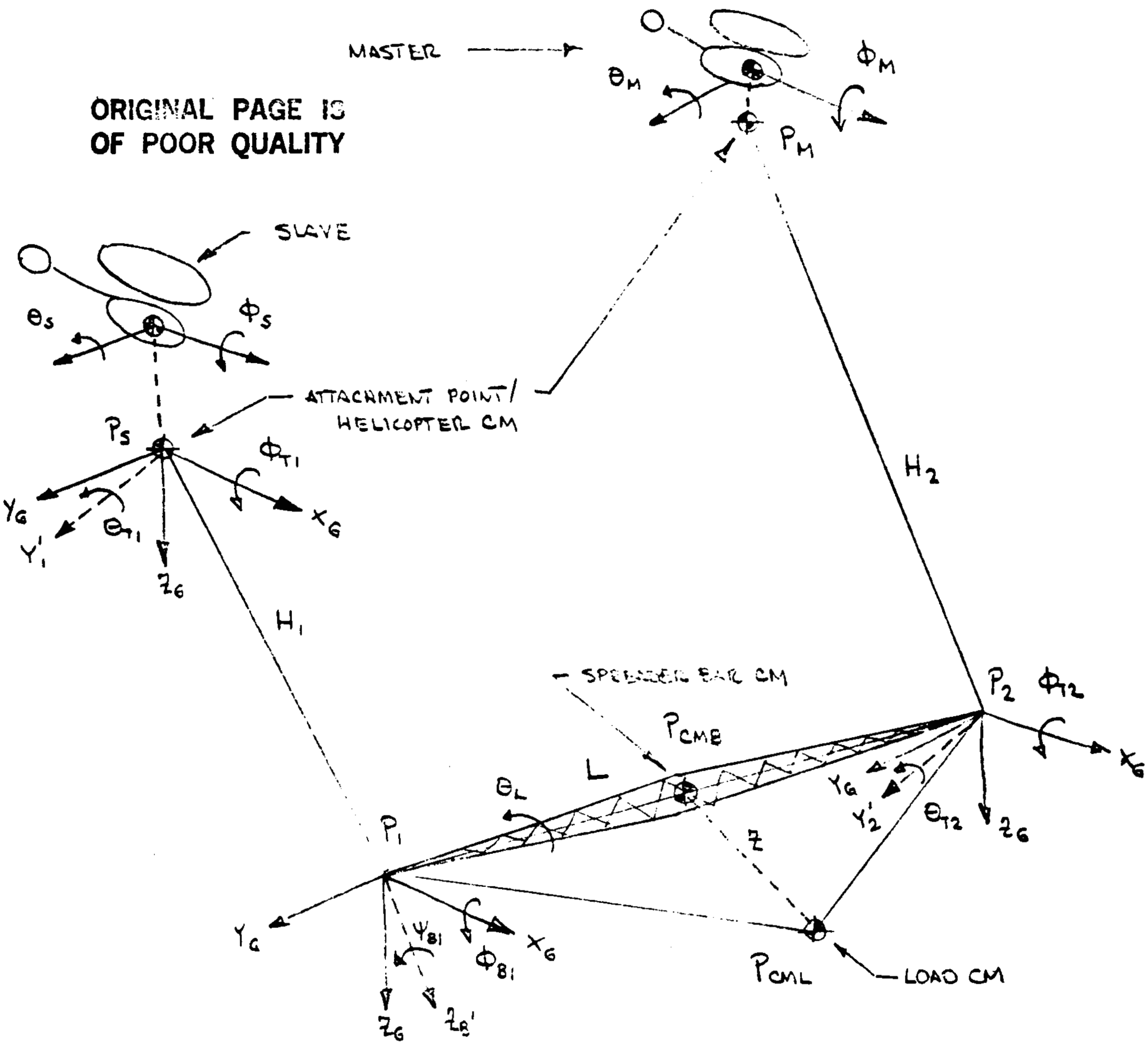

FIGURE 2: COORDINATE DEFINITIONS PART I

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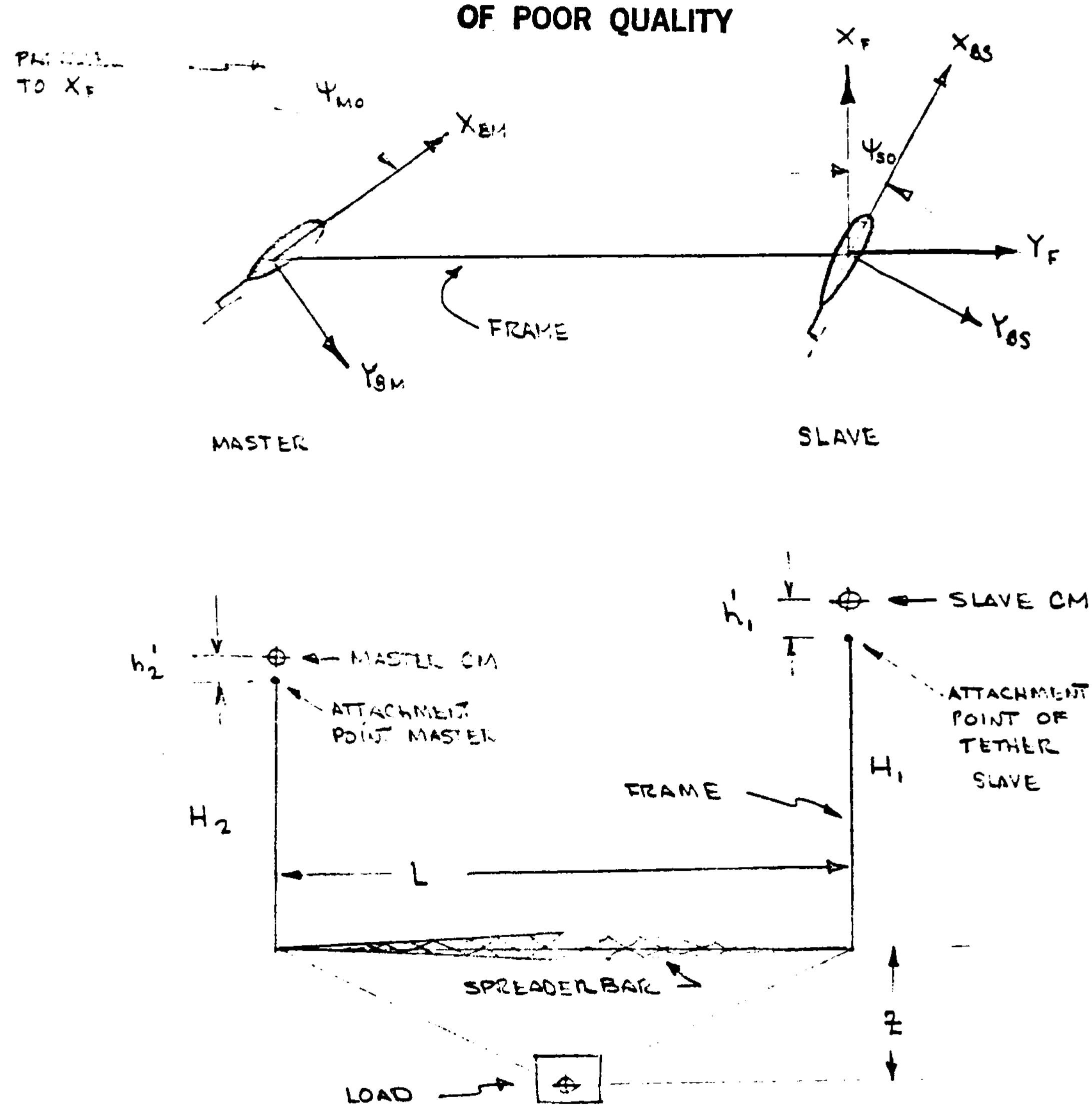


Ψ_{SO} INITIAL YAW ANGLE OF SLAVE HELICOPTER RELATIVE TO FRAME

Ψ_{MO} INITIAL YAW ANGLE OF MASTER HELICOPTER RELATIVE TO FRAME

$$\Theta_{T1} = \phi_{T1} = \phi_{B1} = \Psi_{B1} = \Theta_L = \Theta_{T2} = \phi_{T2} = 0$$

(FRAME ORIENTATION COORDINATES)

FIGURE 3: INITIAL OR TRIM CONFIGURATION OF TWIN LIFT SYSTEM (HOVER TRIM)

(X_F, Y_F, Z_F) FEATHER AXES, ROTATE WITH $(\Delta\Psi_S)$

(X_{BS}, Y_{BS}, Z_{BS}) SLAVE BODY AXES, ROTATE WITH $(\Delta\Psi_S)$

(X_{BM}, Y_{BM}, Z_{BM}) MASTER-BODY AXES, ROTATE WITH $(\Delta\Psi_M)$

NOTE:-

THE HELICOPTER BODY AXES DO NOT ROTATE WITH ϕ OR θ.

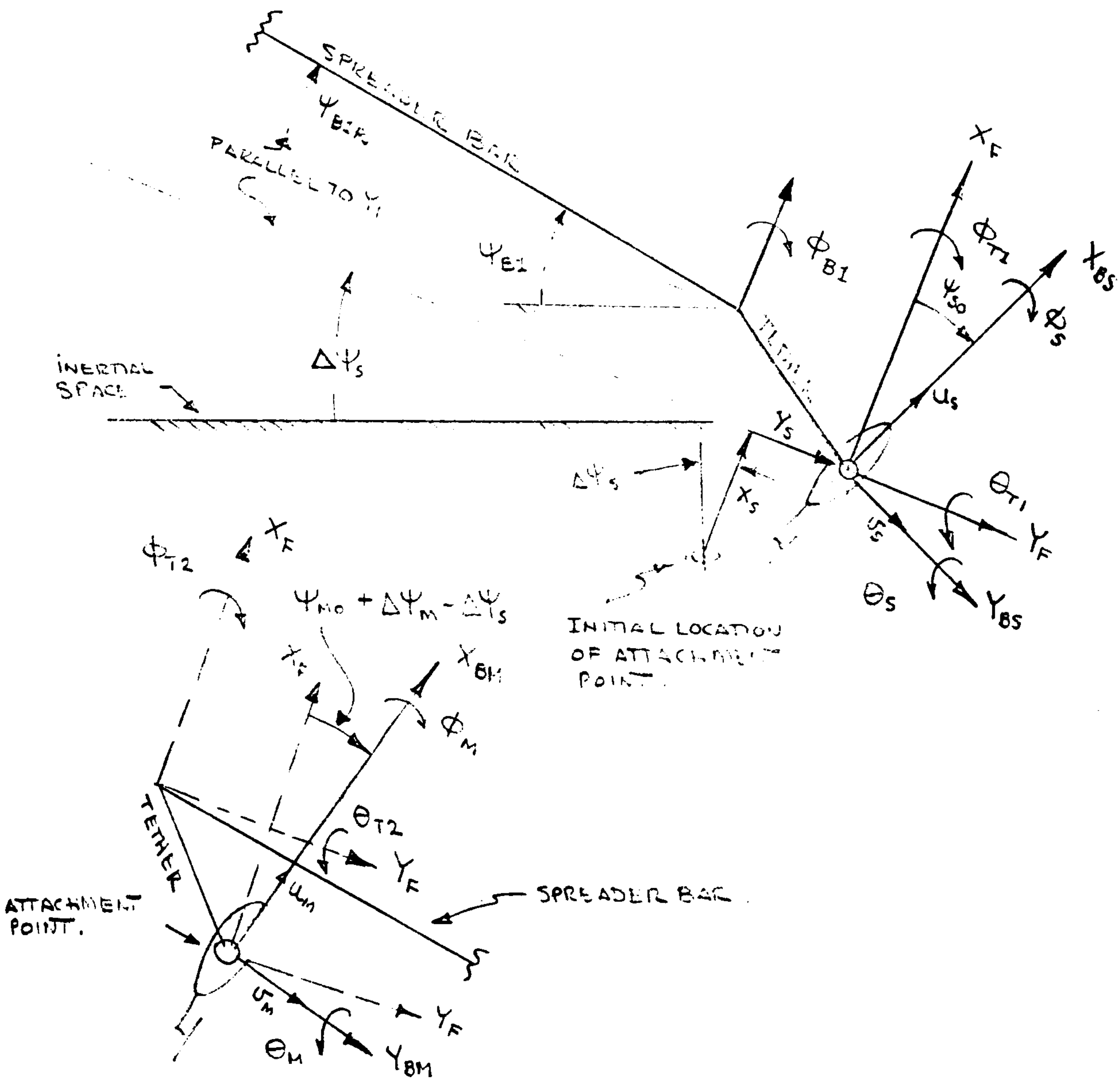
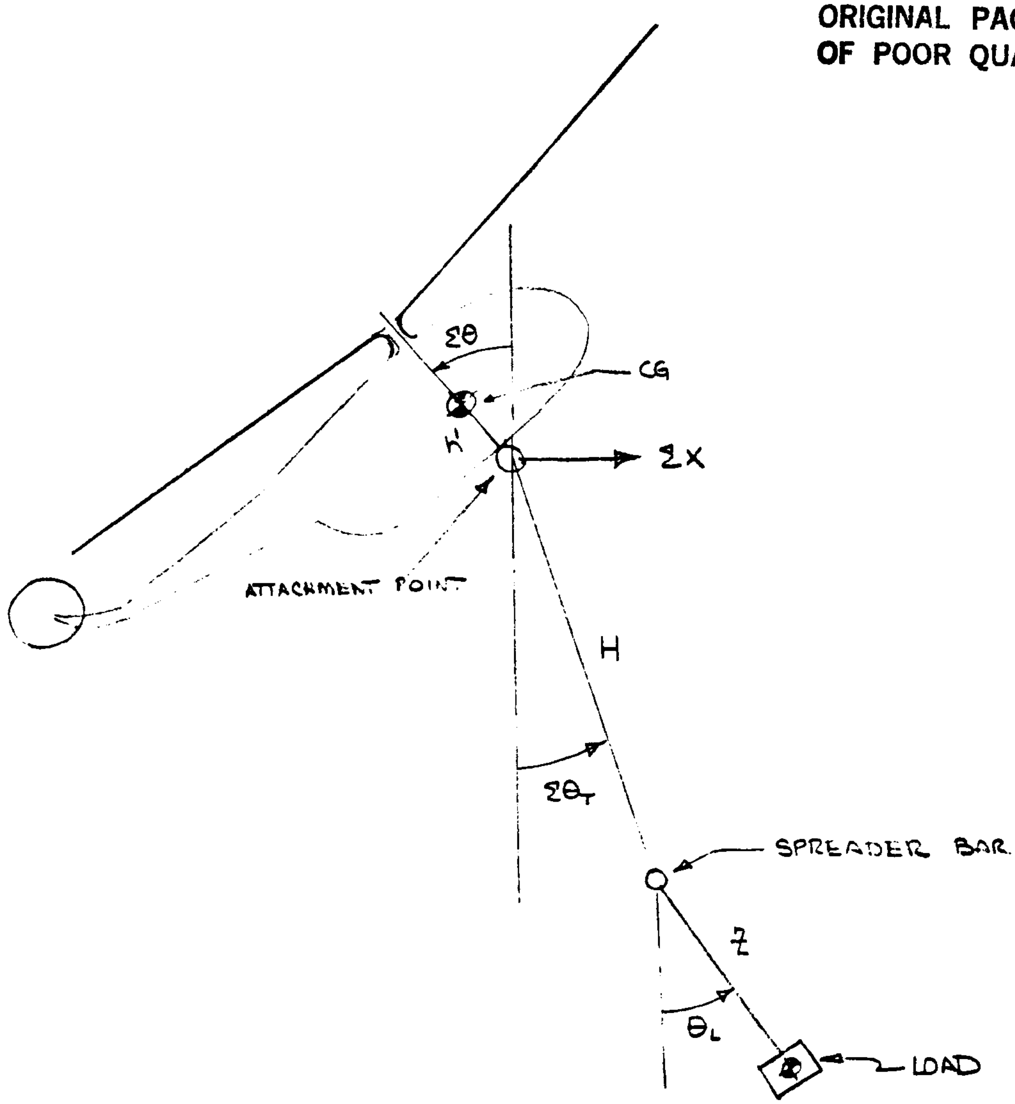


FIGURE 4: PETURBED AXIS SYSTEM GEOMETRY. PART II



DEGREES OF FREEDOM:

ΣX , $\Sigma \Theta$, $\Sigma \Theta_T$, Θ_L

FIGURE 5 : ANTI-SYMMETRIC MOTION

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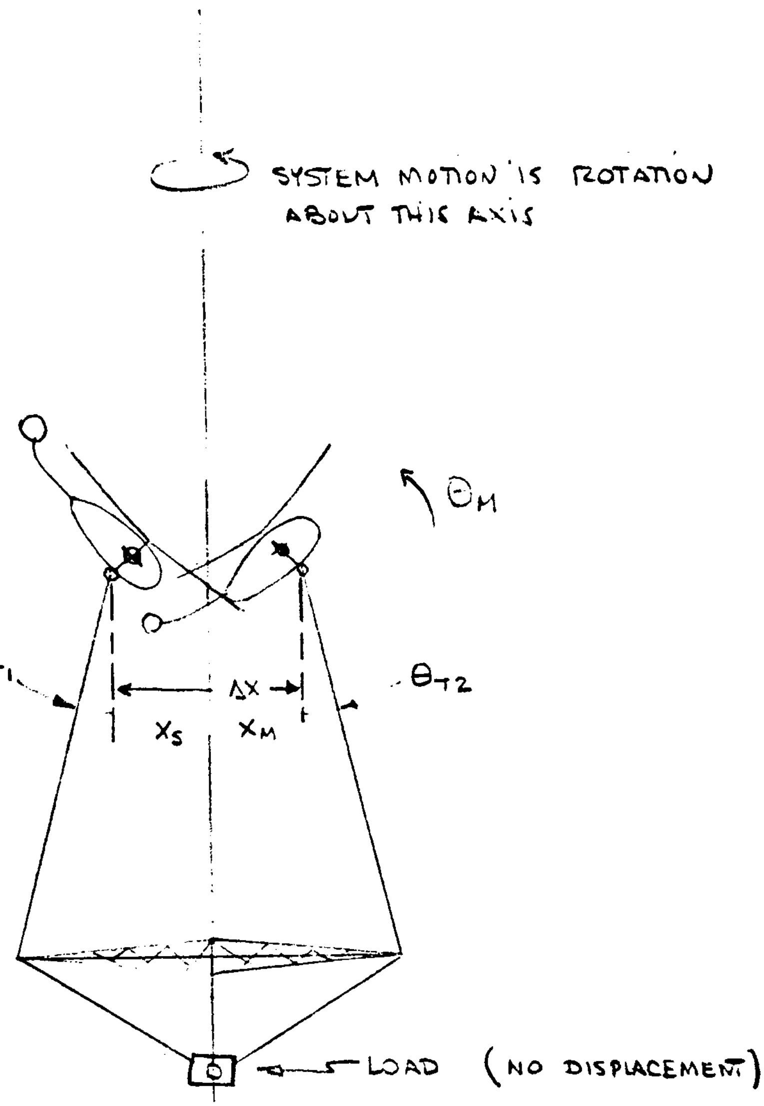

DEGREES OF FREEDOM :-

ΔX , $\Delta\theta$, $\Delta\theta_T$

FIGURE 6: SYMMETRIC MOTION

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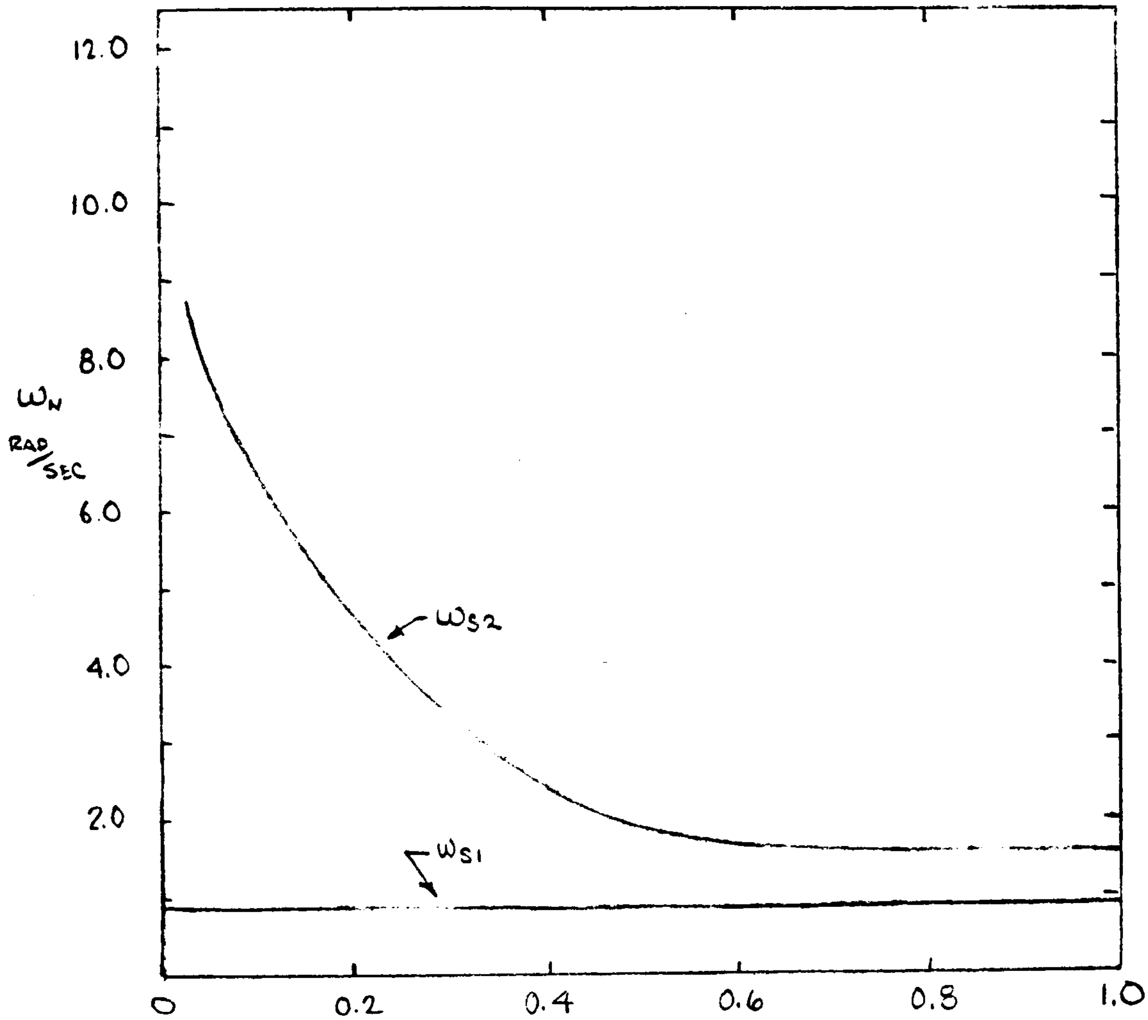


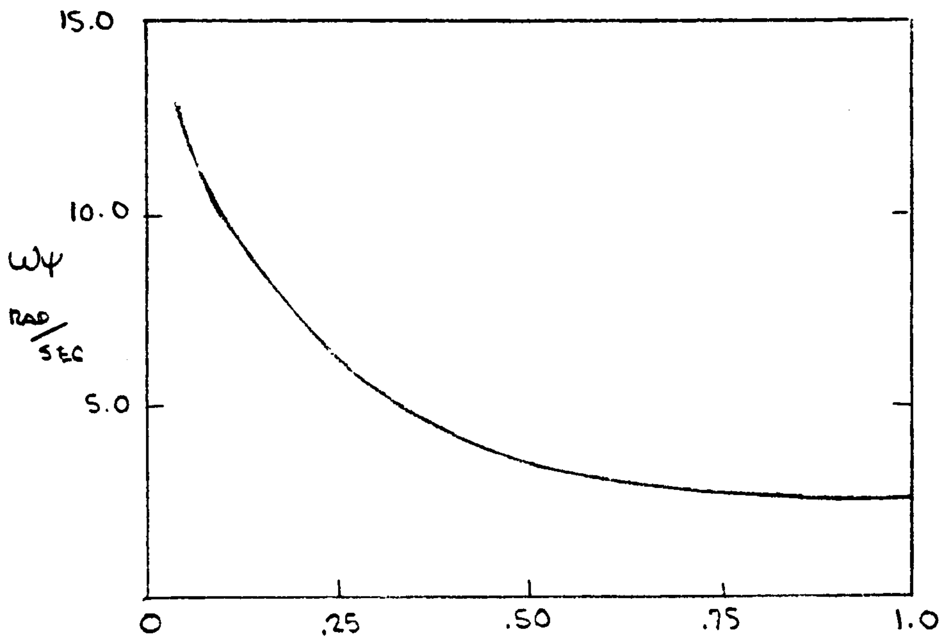
$$\beta = \frac{\text{SPREADER BAR MASS}}{\text{SPREADER BAR MASS + LOAD MASS}}$$

$$\omega_{S1} \rightarrow \sqrt{\frac{g}{Z+H}} \quad \text{AS } \beta \rightarrow 0 \qquad \omega_{S2} \rightarrow \sqrt{\frac{g(H+Z)}{\beta H Z}} \quad \beta \rightarrow 0$$

$$\sqrt{\frac{g}{Z}} \quad \text{AS } \beta \rightarrow 1 \qquad \rightarrow \sqrt{\frac{g}{H}} \quad \beta \rightarrow 1.0$$

NOMINAL LOADED CASE $\beta = .051$

FIGURE 7: UNCOUPLED SLING LOAD FREQUENCIES AS A FUNCTION OF MASS RATIO β.



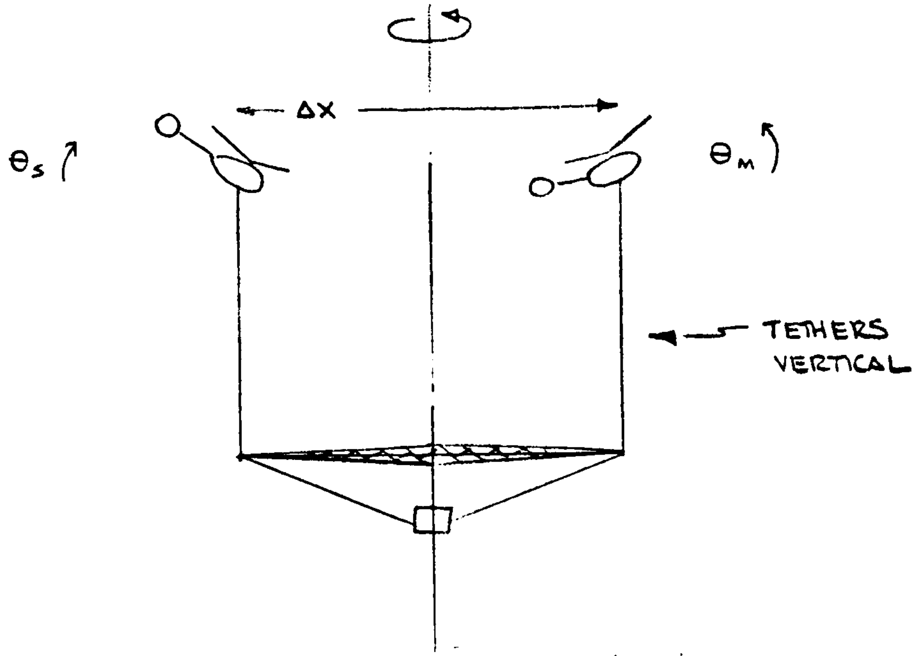
$$\beta = \frac{\text{SPREADER BAR MASS}}{\text{SPREADER BAR MASS + LOAD MASS}}$$

SPREADER BAR MASS DISTRIBUTION UNIFORM

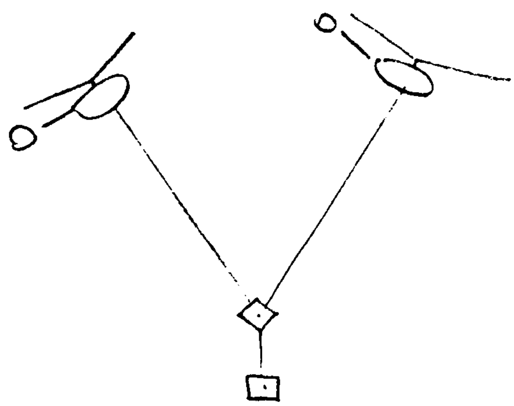
$$\delta = \frac{\beta}{3}$$

$$\omega_\psi = \sqrt{\frac{g}{\delta H}}$$

FIGURE 8: SPREADER BAR TORSION MODE (UNCOUPLED) AS A FUNCTION OF MASS RATIO β.



SYMMETRICAL MODE SHAPE IN LIMIT $I_{CMB} = 0 \ (\delta \to 0)$



SYMMETRICAL MODE SHAPE IN LIMIT $I_{CMB} \to \infty \ (\delta \to \infty)$

FIGURE 9: LIMITING CASES OF SYMMETRICAL MODE SHAPES

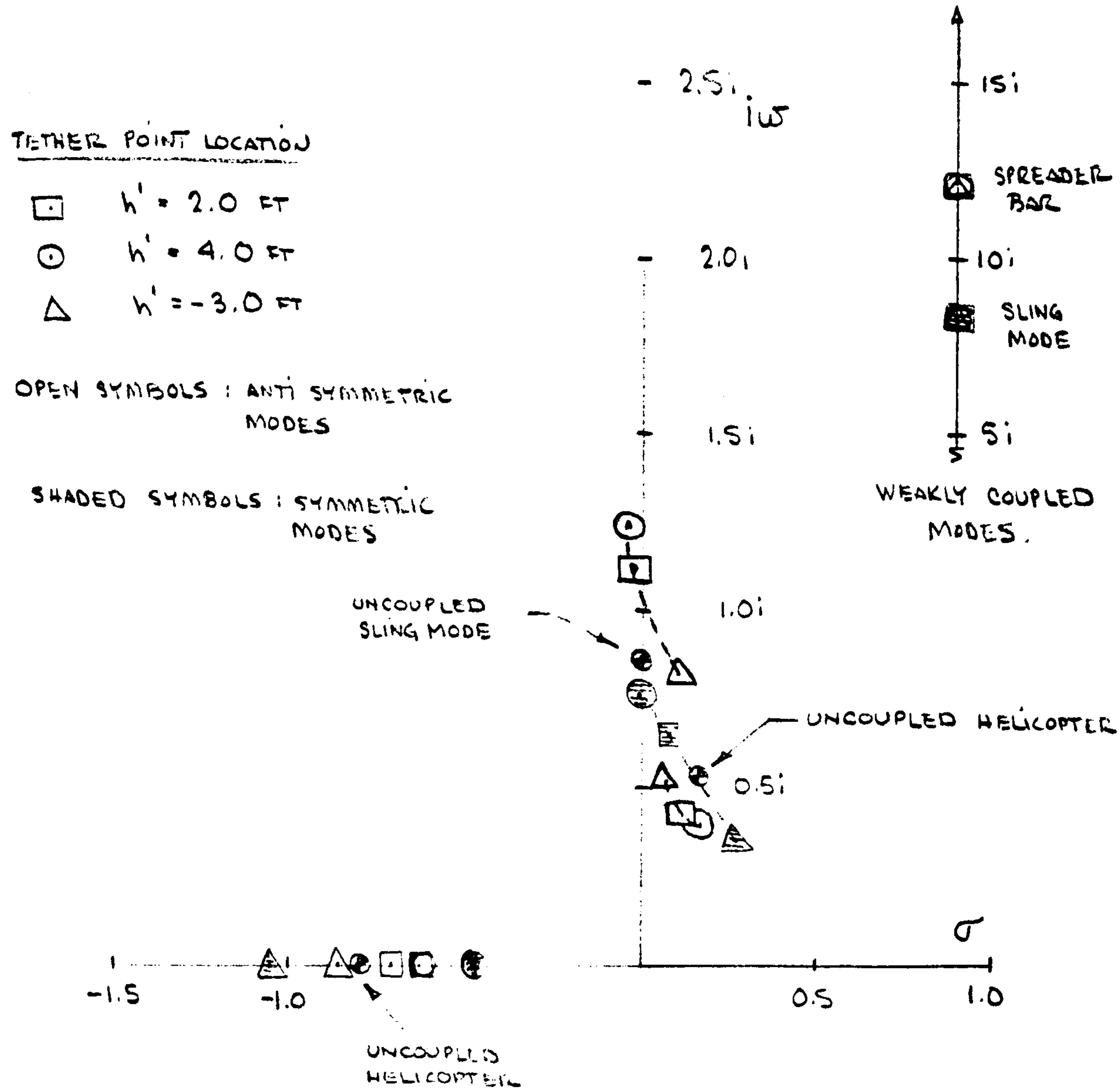


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FIGURE 10: TYPICAL MODES OF MOTION NON-PLANAR SYSTEM WITH LOAD.