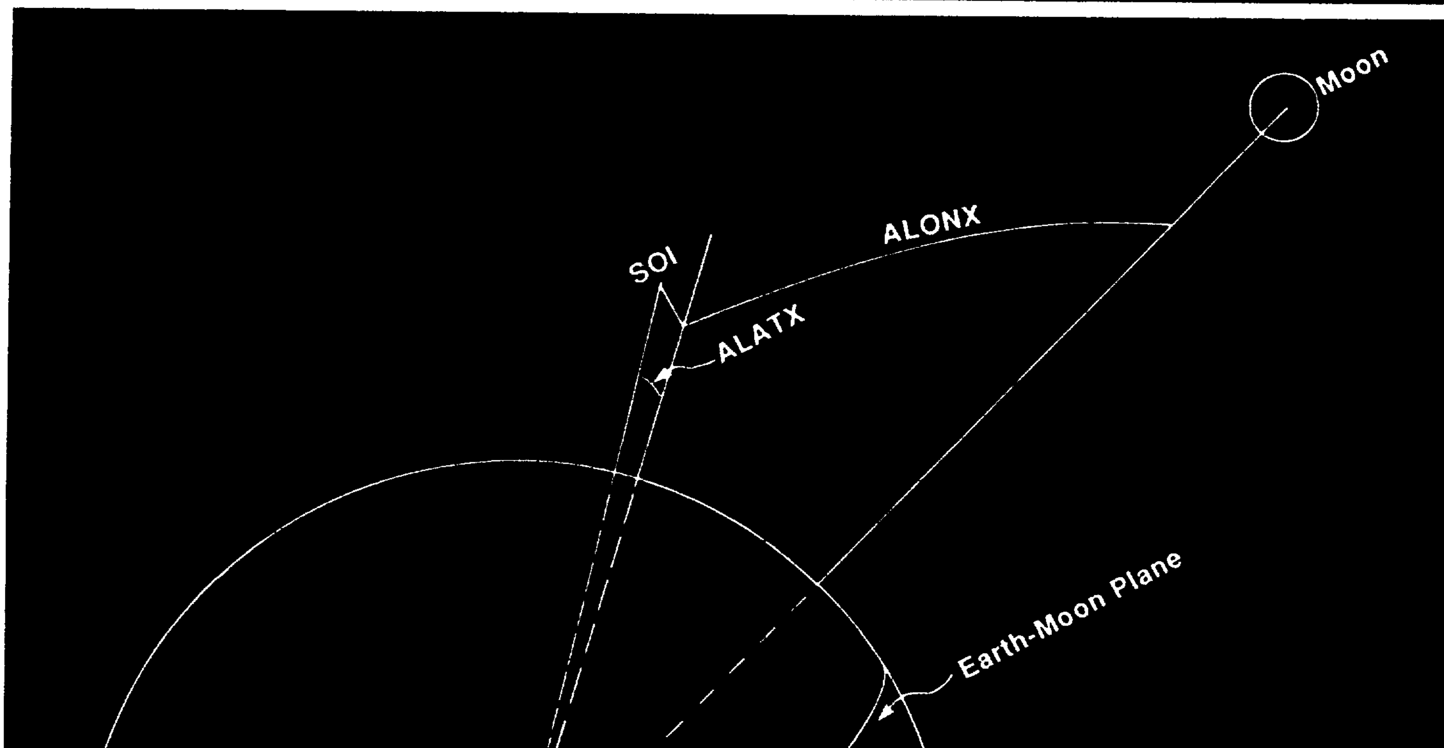
CR-172085

PLANECHG
Earth Orbit to Lunar Orbit Delta V Estimation Program User and Technical Documentation

LBSS



(NASA-CR-172085) PLANECHG: EARTH ORBIT TO LUNAR ORBIT DELTA V ESTIMATION PROGRAM. USER AND TECHNICAL DOCUMENTATION (Eagle Engineering) 103 p CSCL 22C

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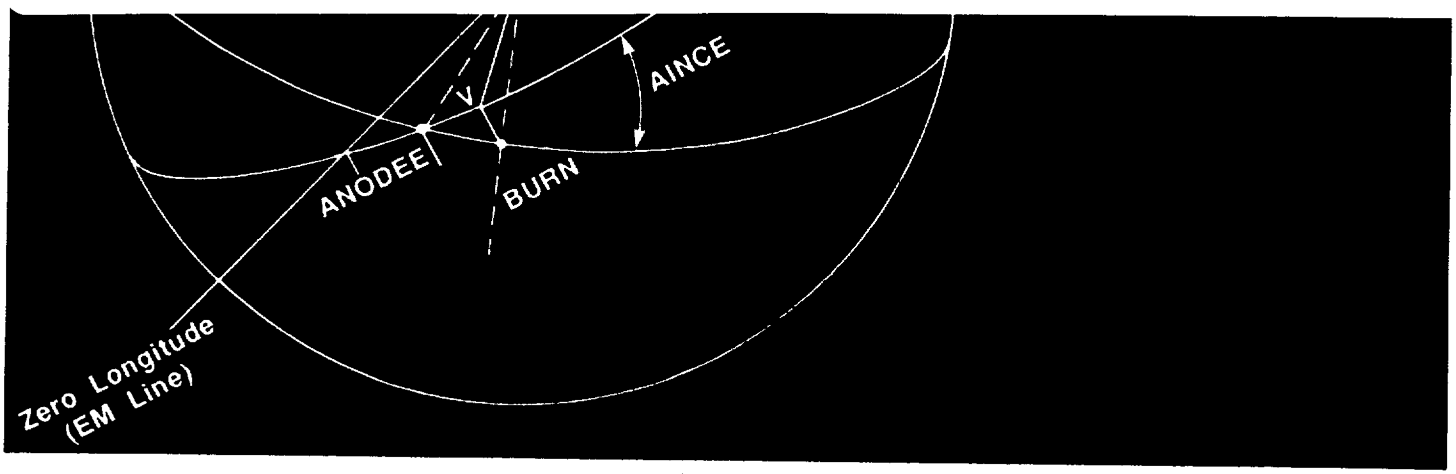

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EAGLE

PLANECHG

User and Technical Documentation

National Aeronautics and Space Administration
Lyndon B. Johnson Space Center

Advanced Projects Office

Eagle Engineering, Inc.
Houston, Texas
September, 1988

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Eagle Engineering Report No. 88-214

Forward

This program is a tool that engineers and planners will need to plan a future return to the Moon.

Dr. John Alred was the NASA Technical Monitor for the contract under which this program was produced. Mr. Andy Petro was the NASA Task Manager for this particular task.

Mr. Bill Stump was the Eagle Project Manager for the contract under which this program was produced. The program was written by Mr. Jack Funk, originally in Quick Basic, and translated into FORTRAN by Mr. Mike D'Onofrio and Mr. Bill Engblom. Mr. Steve Erickson prepared the documentation.

Table of Contents

1.0 Introduction

The program PLANECNG calculates velocities for Earth-to-Moon or Moon-to-Earth trajectories. The flight to be analyzed originates in a circular orbit of any inclination and altitude about one of the bodies, and culminates in a circular orbit of any inclination and altitude about the other body. An intermediate ΔV and plane change occurs at the Lunar sphere of influence (SOI), the region where the vehicle is near its lowest velocity in the trajectory, and therefore where it is able to make the plane change with the lowest ΔV.

A given flight may penetrate the SOI at a number of points. Each point has associated with it a unique set of ΔV's and total velocity. This program displays the velocities, in matrix form, for a representative set of SOI penetration points. An SOI point is identified by projecting Lunar latitude and longitude onto the SOI. The points reported for a given flight are defined by the user, who provides a starting longitude and latitude, and an increment for each. A matrix is built with ten longitudes forming the columns and 19 latitudes forming the rows. This matrix is presented in six different reports, each report containing different velocity or node information in the body of the matrix.

Section 2.0 of this document describes the inputs provided by the user to define the flight profile. Section 3.0 describes the contents of the six reports that are produced as outputs. Section 4.0 contains the instructions to execute this program. The appendices provide a look at the structure and details of the program code.

2.0 Program Inputs

The following paragraphs discuss the inputs provided by the user. The prompt is the message displayed by the program onto the screen. The input variable is the variable assigned to the user's response. The description provides information about how to respond to the prompt.

1. Prompt: INPUT OUTBOUND OR RETURN
   Input variable: MD
   Description: Enter "OUTBOUND" for Earth-to-Moon trajectory calculations. Enter "RETURN" for Moon-to-Earth trajectory calculations. Be sure to type the double quotation marks.

2. Prompt: INPUT NODE OPTION 1 OR 2
   Input variable: NP
   Description: Given that the Earth-Moon trajectory will intercept the SOI at a particular latitude and longitude, and given a desired destination circular orbit inclination (greater than or equal to the SOI latitude), there exist two destination orbits that provide a solution (see figure 2-1). Node option 1 directs the program to concern itself with the orbit in which the SOI longitude is less than 90° away from the destination orbit's ascending node. Node option 2 directs the program to concern itself with the orbit in which the SOI longitude is greater than 90° away from the destination orbit's ascending node.

3. Prompt: INPUT PERIGEE ALTITUDE OF EARTH ORBIT (NMI)
   Input variable: HPE
   Description: Enter the height above the Earth's surface of the Earth circular orbit, in nautical miles.

4. Prompt: INPUT PERIGEE ALTITUDE OF LUNAR ORBIT (NMI)
   Input variable: HPM
   Description: Enter the height above the Lunar surface of the Lunar circular orbit, in nautical miles.

5. Prompt: INPUT EARTH DEPARTURE JULIAN DATE
   Input variable: TIMJ
   Description: Enter the origin trans-SOI injection date (Earth departure date for outbound trajectories, and Lunar departure date for return trajectories) in Julian day format, where January 1, 2000 is day 2,451,545. Refer to Section C of "The Astronomical Almanac of the Year 1988". Day 2451545 is the default value if zero is entered in this field.

6. Prompt: INITIAL LONGITUDE
   Input variable: ALONI
   Description: Enter initial sphere of influence longitude for the output matrices. This value will become the heading for column 1 of the matrices.

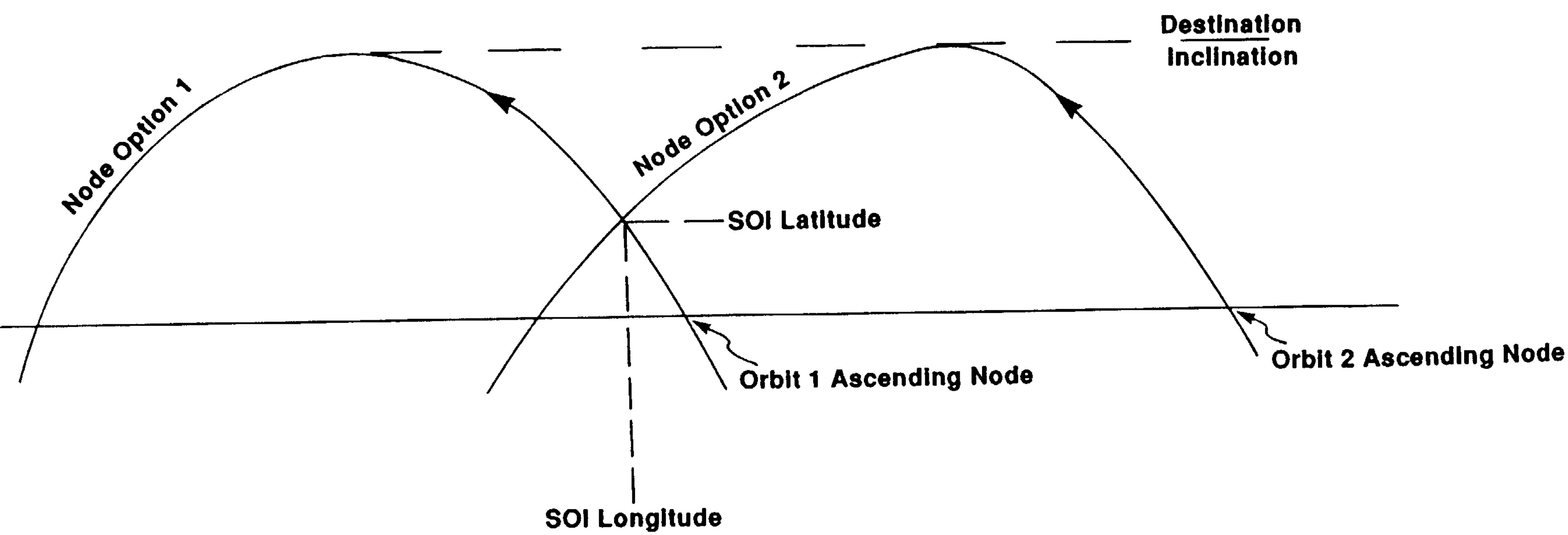

Node Option 1
Node Option 2
Destination Inclination
SOI Latitude
Orbit 1 Ascending Node
Orbit 2 Ascending Node
SOI Longitude

Figure 2-1

7. Prompt: INPUT INCREMENT FOR THE MAP
Input variable: DELLON
Description: Enter longitude increments for the output matrices. Applied to the initial longitude, this value defines the subsequent column headings of the matrices. Longitudes for outbound trajectories should be between zero and -90 degrees. Longitudes for return trajectories should be between zero and +90 degrees.

8. Prompt: INPUT INITIAL LATITUDE
Input variable: ALATI
Description: Enter initial sphere of influence latitude for the output matrices. This value will become the heading for row 1 of the matrices.

9. Prompt: INPUT INCREMENT FOR THE MAP
Input variable: DELLAT
Description: Enter latitude increments for the output matrices. Applied to the initial latitude, this value defines the subsequent row headings of the matrices.

10. Prompt: INPUT EARTH ORBIT TO LUNAR ORBIT INCLINATION
Input variable: AINCEO
Description: Enter Earth circular orbit inclination, in degrees. This is not what is typically considered inclination (i.e., a measurement taken from the Earth's equatorial plane), but rather the angle between the plane of the low Earth orbit and the plane of the Moon's orbit about the Earth.

11. Prompt: INPUT LUNAR ORBIT TO LUNAR ORBIT INCLINATION
Input variable: AINCM
Description: Enter Lunar circular orbit inclination, in degrees. This is the angle between the plane of the low Lunar orbit and the plane of the Moon's orbit about the Earth.

12. Prompt: INPUT FLIGHT TIME
Input variable: FTIM
Description: Enter the desired total flight time from origin trans-SOI injection to destination circular orbit injection, in hours.

3.0 Program Outputs

This section describes the contents and shows a sample of the six reports generated by the program.

Report #1

Velocity Map For Inbound/Outbound Trajectories

The top section of this report repeats the input values entered by the user. The second section is a 10 X 19 matrix of total velocities required to fly the profile described by the inputs. Each cell corresponds to a particular latitude and longitude on the Lunar sphere of influence. A plane change burn may occur at any one of these coordinates, and the value of the corresponding cell is the total velocity required for the flight if the burn occurs at that location. It is the sum of the transfer orbit injection ΔV, sphere of influence plane change ΔV, and destination circular orbit injection ΔV. The axes of the matrix are determined by the initial longitude, latitude, and increments specified by the user.

The third section of the report is a summary of key data corresponding to the matrix cell containing the lowest total velocity. This data includes:

- X-, Y-, and Z-components of velocity at the sphere of influence just before and just after the plane change burn (VX, VY, VZ).
- Total magnitude of the velocity at the sphere of influence just before and just after the plane change burn (VEL).
- Flight path angle at the sphere of influence just before and just after the plane change burn (GAMA).
- Azimuth of the sphere of influence point from the Earth and from the Moon (AZM).
- Earth and Lunar orbit inclinations (AINC).
- Transfer trajectory and circular orbit insertion ascending/descending nodes (ANODE).
- Earth-to-SOI and SOI-to-Moon times of flight (TIME).
- Transfer trajectory and circular orbit insertion ΔV's (DVCIR).
- Sphere of influence ΔV (DVPHER).
- Total ΔV (DVTOTAL).

VELOCITY MAP FOR OUTBOUND TRAJECTORIES NODE OPTION, 2
DATE 7-SEP-88 TIME 08:35:41

JUILIAN DAY 2451545. PERIGEE ALT (NMI) EARTH = 250. MOON = 60.
TRANSLUNAR FLIGHT TIME (HR) = 60.0 INCL EARTH 30.0 INCL MOON = 30.0

ALON> / ALAT	0.0	-10.0	-20.0	-30.0	-40.0	-50.0	-60.0	-70.0	-80.0	-90.0
90.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
80.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
70.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
60.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
50.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
40.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
30.0	16011.	15685.	15512.	15563.	15857.	16333.	16926.	17556.	18216.	18876.
20.0	15698.	15254.	14950.	14946.	15305.	15894.	16583.	17309.	18041.	18764.
10.0	15444.	14868.	14351.	14213.	14757.	15518.	16320.	17125.	17917.	18689.
0.0	15369.	14745.	14100.	13727.	14544.	15394.	16238.	17070.	17883.	18672.
-10.0	15506.	14964.	14518.	14428.	14883.	15591.	16367.	17160.	17948.	18719.
-20.0	15829.	15427.	15173.	15180.	15496.	16033.	16687.	17391.	18113.	18831.
-30.0	16357.	16099.	15976.	16032.	16277.	16682.	17206.	17781.	18398.	19026.
-40.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-50.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-60.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-70.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-80.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-90.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

MINIMUM VELOCITY PRINT NODE OPTION 2
DATE 7-SEP-88 TIME 08:35:41

```
FLIGHT TIME =  60. LAT =    0.0 LON =  -30.0 LUNAR AINC =  30.0 EARTH AINC =  30.0
BODY    VX      VY      VZ      VEL    GAMA    AZM     AINC    ANODE   TIME   DVCIR
MOON   3641.   700.   -199.   4400.  -84.8   120.0    30.0    150.0    7.5    3248. DVPHER = 229.
EARTH  3799.    793.   -336.   3895.   80.1   120.0    30.0           3.2   52.5   10250. DVTOTAL =
13727.
```

Report #2

Delta Velocity At Sphere Of Influence Map For Translunar/Transearth Trajectories

This report is a matrix of delta velocities that occur at the sphere of influence. Each cell corresponds to a particular latitude and longitude on the sphere of influence at which a plane change burn may occur.

DELTA VELOCITY AT SPHERE OF INFLUENCE MAP FOR TRANSLUNAR TRAJECTORIES NODE OPTION 2

DATE 7-SEP-88 TIME 08:35:41

TRANSLUNAR FLIGHT TIME (HR) = 60.0 INCL EARTH 30.0 INCL MOON = 30.0

ALON>	0.0	-10.0	-20.0	-30.0	-40.0	-50.0	-60.0	-70.0	-80.0	-90.0
ALAT										
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
30.0	3024.4	2622.4	2363.5	2362.3	2639.5	3135.6	3703.5	4417.9	5112.7	5767.7
20.0	2689.2	2126.3	1731.4	1671.2	2012.9	2623.8	3368.3	4139.9	4908.1	5655.9
10.0	2443.6	1707.3	1051.7	808.4	1381.4	2210.4	3069.9	3959.5	4785.8	5581.0
0.0	2371.9	1586.6	757.5	228.9	1124.6	2088.3	2989.1	3906.0	4752.5	5564.4
-10.0	2505.6	1844.2	1263.3	1070.5	1553.2	2325.6	3157.4	3995.0	4816.9	5610.8
-20.0	2891.8	2378.3	2037.5	1991.3	2289.3	2845.2	3511.6	4258.7	5012.5	5722.8
-30.0	3439.0	3109.9	2943.6	2949.8	3178.2	3562.3	4166.0	4711.5	5326.3	5947.1
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Report #3

Ascending Node Of Lunar Orbit Map For Translunar/Transearth Trajectories

This report is a matrix of ascending nodes at the Moon. Each matrix cell displays an ascending node corresponding to a trajectory that penetrates the SOI at the stated SOI latitude and longitude. The node describes the Lunar longitude at which the SOI-Moon trajectory passes through the Earth-Moon plane during Lunar circular orbit insertion for outbound flights, or during trans-SOI injection for return flights.

ASCENDING NODE OF LUNAR ORBIT MAP FOR TRANSLUNAR TRAJECTORIES NODE OPTION 2
DATE 7-SEP-88 TIME 08:35:41
TRANSLUNAR FLIGHT TIME (HR) = 60.0 INCL EARTH 30.0 INCL MOON = 30.0

ALON> ALAT	0.0	-10.0	-20.0	-30.0	-40.0	-50.0	-60.0	-70.0	-80.0	-90.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
30.0	270.0	260.0	250.0	240.0	230.0	220.0	210.0	200.0	190.0	180.0
20.0	219.1	209.1	199.1	189.1	179.1	169.1	159.1	149.1	139.1	129.1
10.0	197.8	187.8	177.8	167.8	157.8	147.8	137.8	127.8	117.8	107.8
0.0	180.0	170.0	160.0	150.0	140.0	130.0	120.0	110.0	100.0	90.0
-10.0	162.2	152.2	142.2	132.2	122.2	112.2	102.2	92.2	82.2	72.2
-20.0	140.9	130.9	120.9	110.9	100.9	90.9	80.9	70.9	60.9	50.9
-30.0	90.0	80.0	70.0	60.0	50.0	40.0	30.0	20.0	10.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Report #4

Ascending Node Of Earth-Moon Transfer Trajectory Map For Translunar/Transearth Trajectories

This report is a matrix of ascending nodes at the Earth. Each matrix cell displays an ascending node corresponding to a trajectory that penetrates the SOI at the stated SOI latitude and longitude. The node describes the Earth longitude at which the Earth-SOI trajectory passes through the Earth-Moon plane during Earth circular orbit insertion for return flights, or during trans-SOI injection for outbound flights.

ASCENDING NODE OF EARTH-MOON TRANSFER TRAJECTORY MAP FOR TRANSLUNAR TRAJECTORIES

NODE OPTION 2

DATE 7-SEP-88 TIME 08:35:41

TRANSLUNAR FLIGHT TIME (HR) = 60.0 INCL EARTH 30.0 INCL MOON = 30.0

ALON>	0.0	-10.0	-20.0	-30.0	-40.0	-50.0	-60.0	-70.0	-80.0	-90.0
ALAT										
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30.0	5.52	6.48	7.41	8.28	9.07	9.75	10.30	10.71	10.96	11.04
20.0	3.77	4.81	5.81	6.76	7.61	8.35	8.95	9.39	9.66	9.76
10.0	1.91	3.00	4.05	5.04	5.94	6.71	7.34	7.80	8.09	8.18
0.0	0.00	1.10	2.17	3.18	4.09	4.87	5.51	5.98	6.27	6.37
-10.0	-1.91	-0.82	0.23	1.22	2.11	2.89	3.52	3.98	4.26	4.36
-20.0	-3.77	-2.73	-1.73	-0.78	0.07	0.81	1.41	1.85	2.13	2.22
-30.0	-5.52	-4.56	-3.63	-2.76	-1.98	-1.29	-0.74	-0.33	-0.08	0.00
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Report #5

Delta Velocity At Earth For Earth-Moon Transfer Trajectory Map For Translunar/Transearth Trajectories

This report is a matrix of trajectory velocities at the Earth. Each cell corresponds to a particular targeted latitude and longitude on the SOI at which a plane-change burn may occur. For Earth-to-Moon trajectories, the values in these cells are the trans-SOI insertion trajectory ΔV's. For Moon-to-Earth trajectories, the values in these cells are the Earth circular orbit insertion ΔV's. This program assumes a departure from and arrival to circular orbit.

DELTA VELOCITY AT EARTH FOR EARTH-MOON TRANSFER TRAJECTORY MAP FOR TRANSLUNAR TRAJECTORIES

NODE OPTION 2

DATE 7-SEP-88 TIME 08:35:41

TRANSLUNAR FLIGHT TIME (HR) = 60.0 INCL EARTH 30.0 INCL MOON = 30.0

ALON>	0.0	-10.0	-20.0	-30.0	-40.0	-50.0	-60.0	-70.0	-80.0	-90.0
ALAT										
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
30.0	10287.	10281.	10280.	10288.	10303.	10325.	10348.	10388.	10430.	10471.
20.0	10270.	10262.	10263.	10271.	10286.	10309.	10341.	10378.	10421.	10471.
10.0	10263.	10252.	10251.	10257.	10275.	10301.	10333.	10375.	10420.	10471.
0.0	10260.	10249.	10246.	10250.	10271.	10299.	10331.	10374.	10419.	10471.
-10.0	10263.	10255.	10253.	10259.	10278.	10304.	10337.	10375.	10420.	10471.
-20.0	10278.	10269.	10269.	10277.	10293.	10317.	10345.	10383.	10428.	10471.
-30.0	10296.	10289.	10292.	10299.	10314.	10333.	10372.	10400.	10437.	10479.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

Report #6

Delta Velocity At Moon For Earth-Moon Transfer Trajectory Map For Translunar/Transearth Trajectories

This report is a matrix of trajectory velocities at the Moon. Each cell corresponds to a particular targeted latitude and longitude on the SOI at which a plane-change burn may occur. For Earth-to-Moon trajectories, the values in these cells are Lunar circular orbit insertion ΔV's. For Moon-to-Earth trajectories, the values in these cells are the trans-SOI insertion trajectory ΔV's.
Lunar circular orbits at the specified altitudes are assumed.

DELTA VELOCITY AT MOON FOR EARTH-MOON TRANSFER TRAJECTORY MAP FOR TRANSLUNAR TRAJECTORIES

NODE OPTION 2

DATE 7-SEP-88 TIME 08:35:41

TRANSLUNAR FLIGHT TIME (HR) = 60.0 INCL EARTH 30.0 INCL MOON = 30.0

ALON>	0.0	-10.0	-20.0	-30.0	-40.0	-50.0	-60.0	-70.0	-80.0	-90.0
ALAT										
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
30.0	2699.	2781.	2868.	2913.	2915.	2872.	2874.	2750.	2673.	2637.
20.0	2738.	2866.	2956.	3004.	3005.	2961.	2874.	2791.	2712.	2637.
10.0	2738.	2909.	3049.	3147.	3100.	3007.	2918.	2790.	2712.	2637.
0.0	2737.	2909.	3097.	3248.	3149.	3007.	2918.	2790.	2712.	2637.
-10.0	2738.	2865.	3002.	3098.	3052.	2961.	2873.	2790.	2712.	2637.
-20.0	2659.	2780.	2867.	2912.	2914.	2871.	2830.	2749.	2673.	2637.
-30.0	2622.	2700.	2741.	2783.	2785.	2787.	2668.	2670.	2635.	2600.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

4.0 Program Execution Instructions

The following instructions describe the steps to be taken by the user to execute this program.

A. Obtain access to the DEC VAX minicomputer and sign on with user identification.

B. At the $ prompt, RUN PLANECNG.

C. When prompted by the program, enter the program inputs. See section 2.0 for a discussion of the inputs.

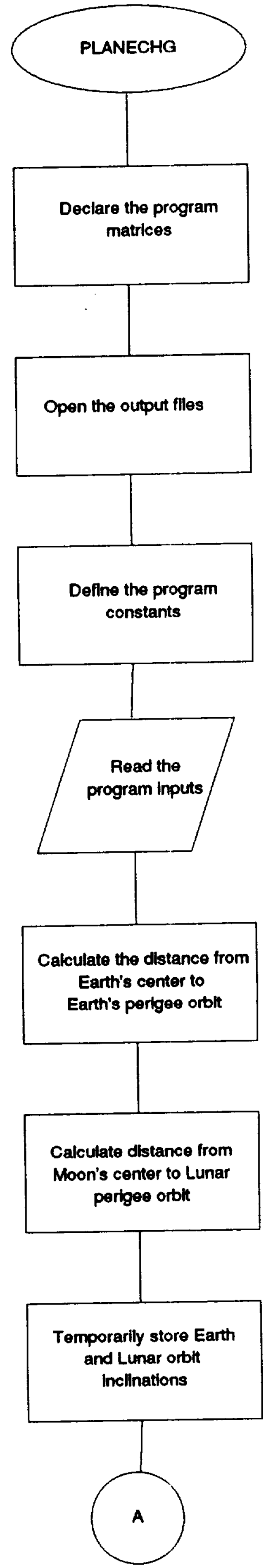
D. After the last input has been entered, the program will execute for approximately 10 to 30 minutes. When it is completed, the message "FORTRAN STOP" will appear.

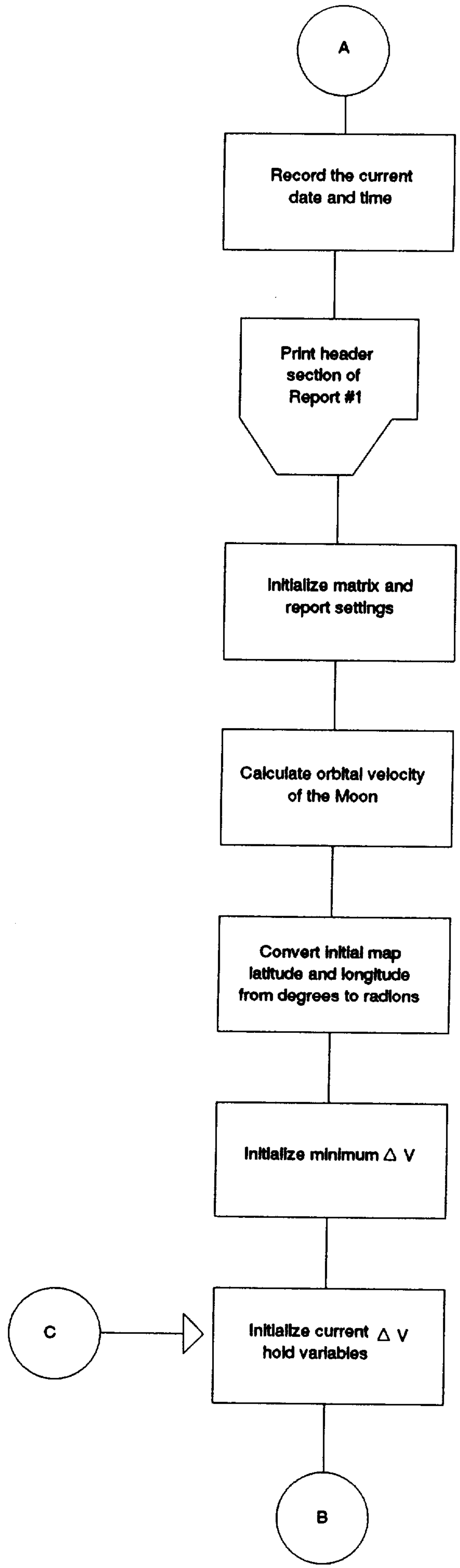
E. The program outputs will be placed in a file named PLANECNG.OUT;### where ### is a system generated version number of the report. To print the reports, type the following at the $ prompt:

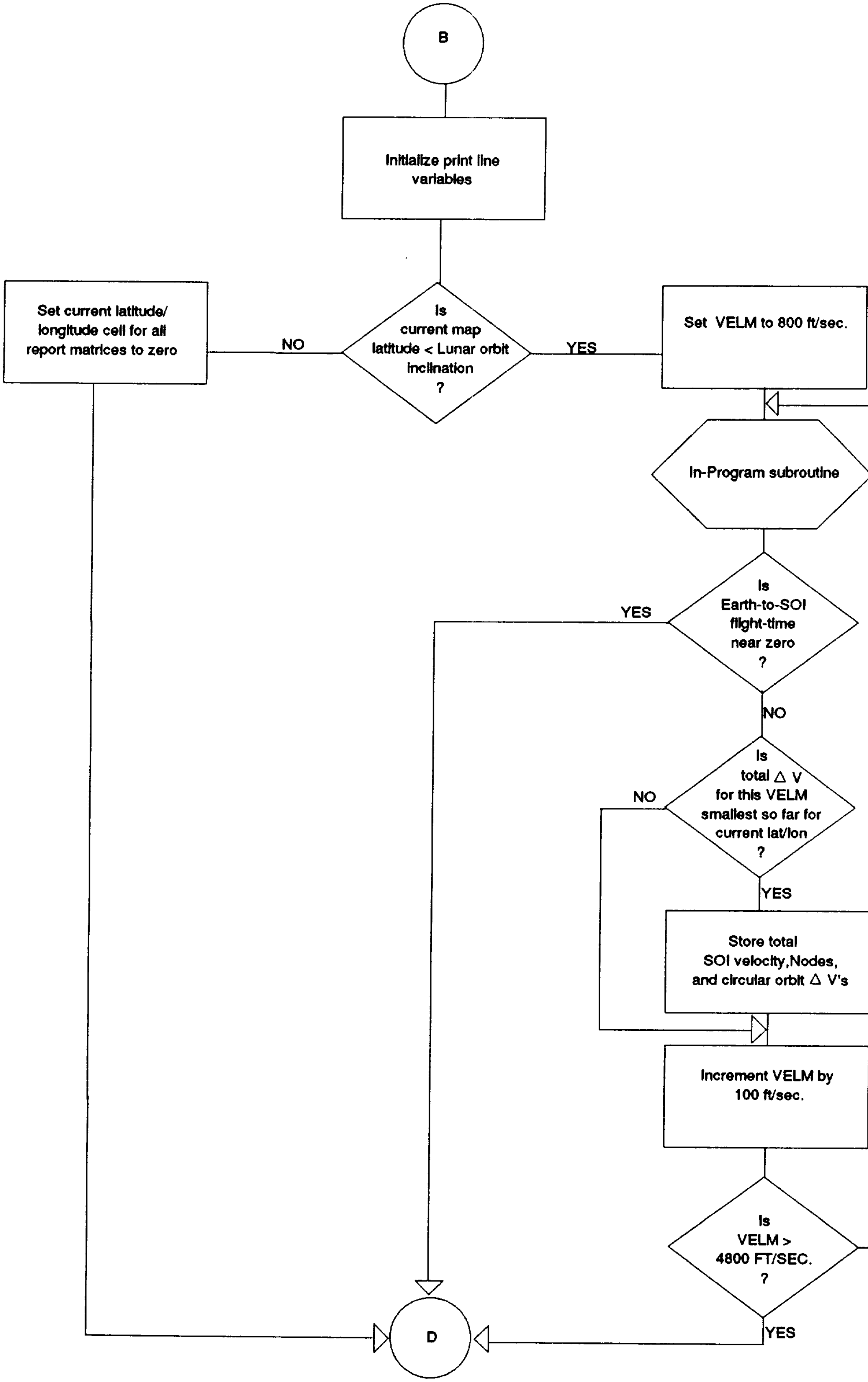
    TYPE PLANECNG.OUT;###

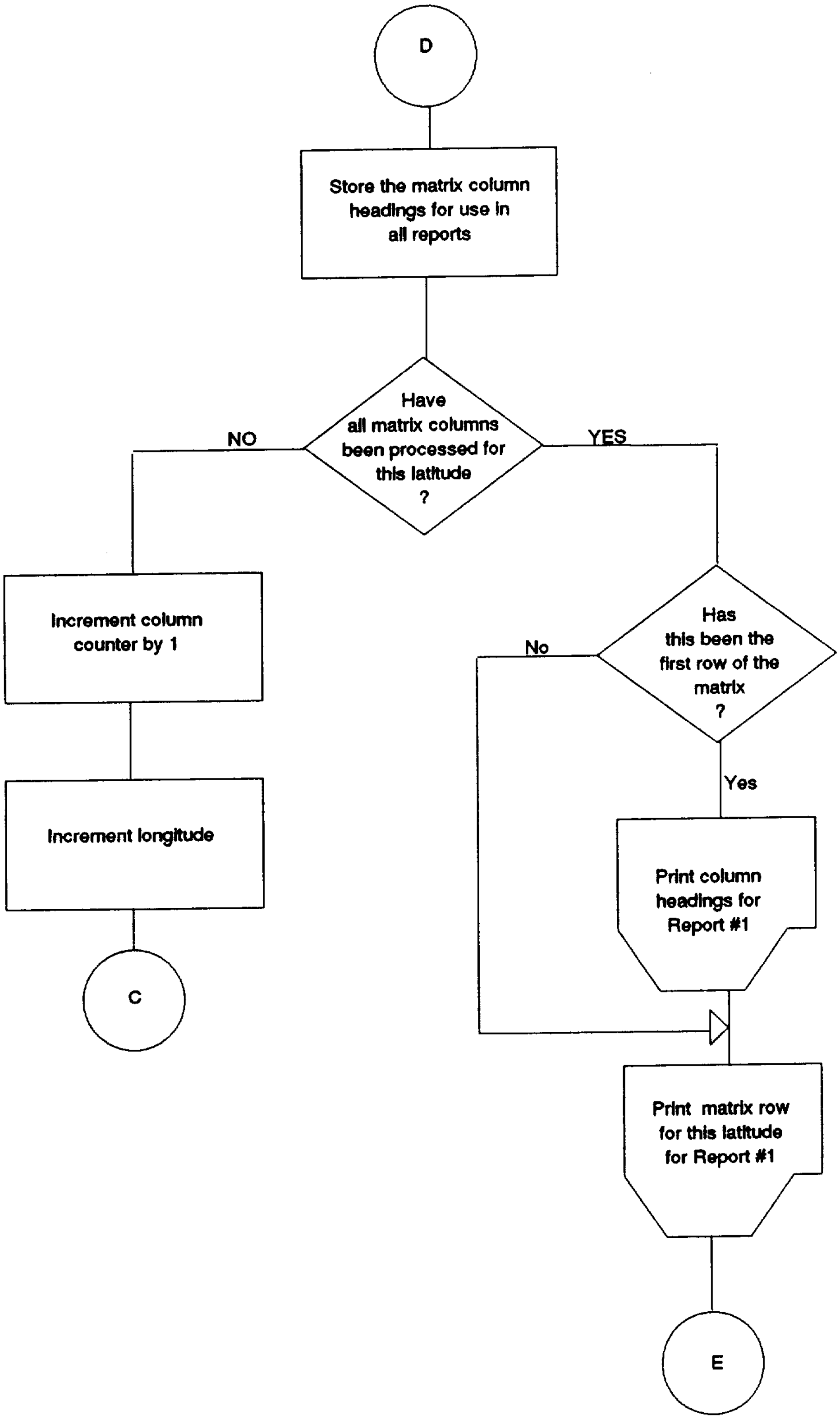
F. To re-execute the program with new parameters, begin again at step (B) above.

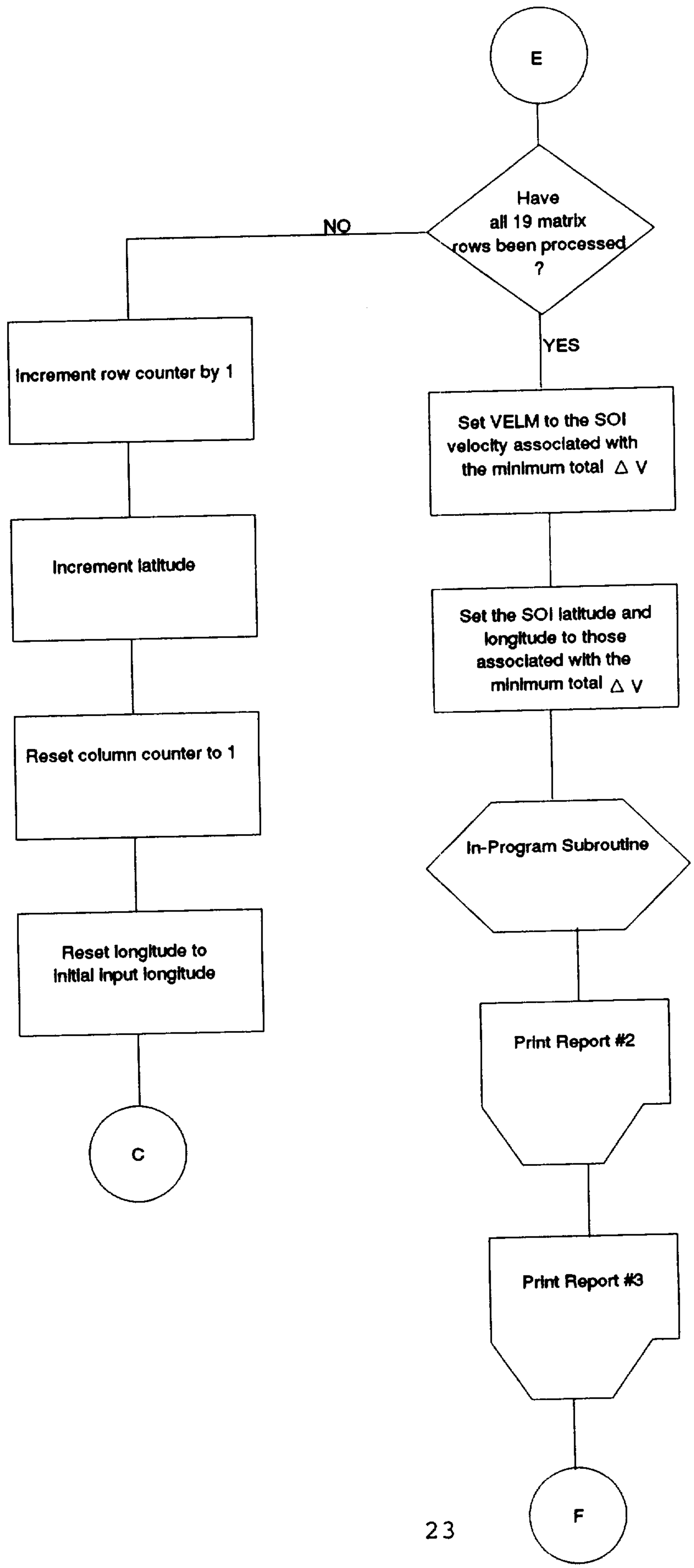
Appendix A - Program Flow Chart

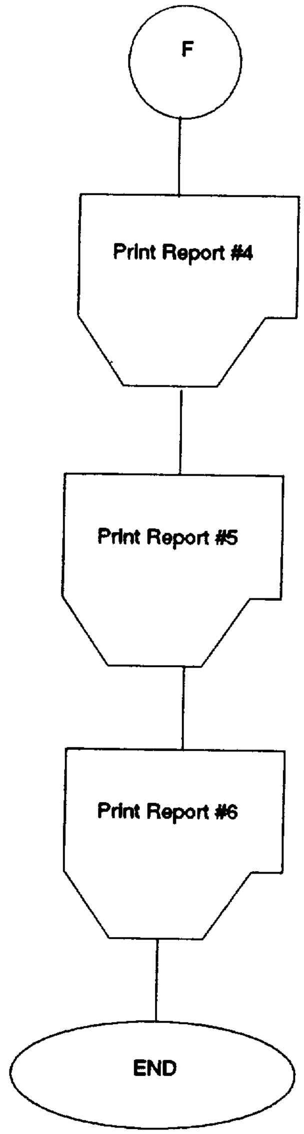

PLANECHG
Declare the program matrices
Open the output files
Define the program constants
Read the program inputs
Calculate the distance from Earth's center to Earth's perigee orbit
Calculate distance from Moon's center to Lunar perigee orbit
Temporarily store Earth and Lunar orbit inclinations
A

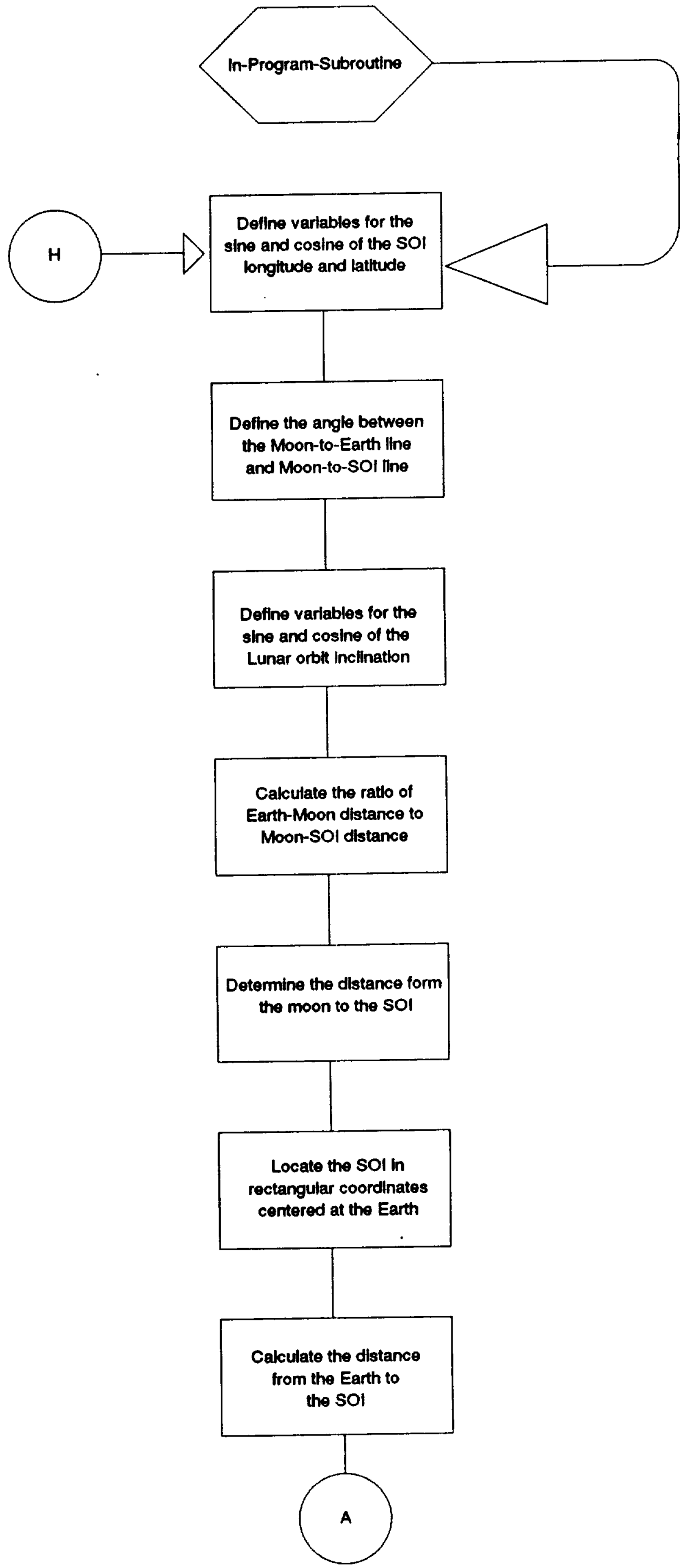

A
Record the current date and time
Print header section of Report #1
Initialize matrix and report settings
Calculate orbital velocity of the Moon
Convert initial map latitude and longitude from degrees to radions
Initialize minimum Δ V
C
Initialize current Δ V hold variables
B

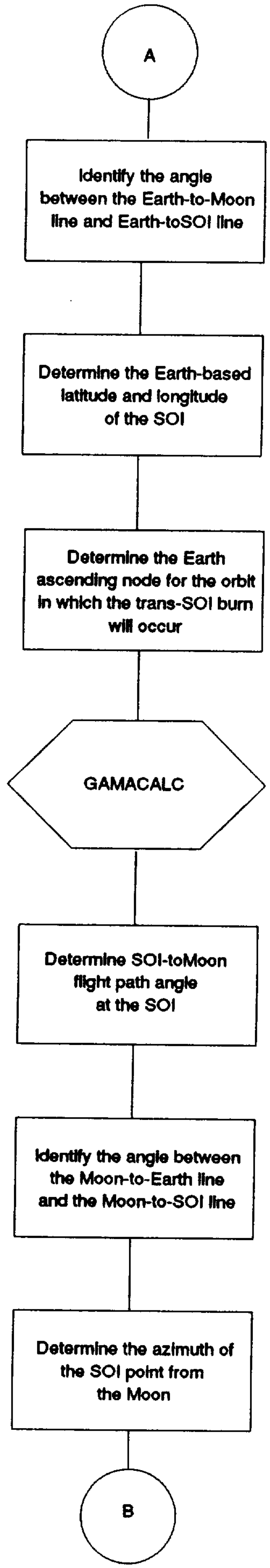

B
Initialize print line variables
Is current map latitude < Lunar orbit inclination ?
NO
Set current latitude/ longitude cell for all report matrices to zero
YES
Set VELM to 800 ft/sec.
In-Program subroutine
Is Earth-to-SOI flight-time near zero ?
YES
NO
Is total Δ V for this VELM smallest so far for current lat/lon ?
NO
YES
Store total SOI velocity,Nodes, and circular orbit Δ V's
Increment VELM by 100 ft/sec.
Is VELM > 4800 FT/SEC. ?
NO
YES
D


D
Store the matrix column headings for use in all reports
Have all matrix columns been processed for this latitude ?
NO
YES
Increment column counter by 1
Increment longitude
C
Has this been the first row of the matrix ?
No
Yes
Print column headings for Report #1
Print matrix row for this latitude for Report #1
E

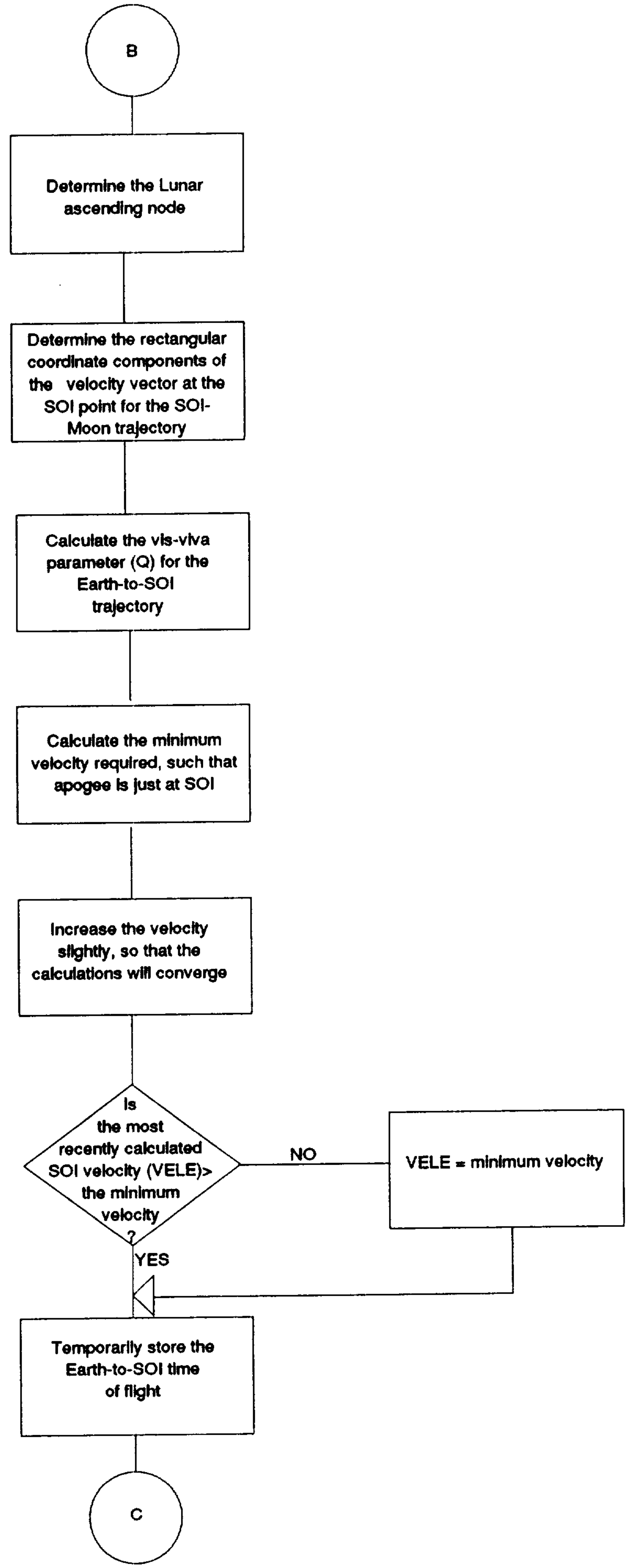

E
Have all 19 matrix rows been processed ?
NO
YES
Increment row counter by 1
Increment latitude
Reset column counter to 1
Reset longitude to initial input longitude
C
Set VELM to the SOI velocity associated with the minimum total Δ V
Set the SOI latitude and longitude to those associated with the minimum total Δ V
In-Program Subroutine
Print Report #2
Print Report #3
F

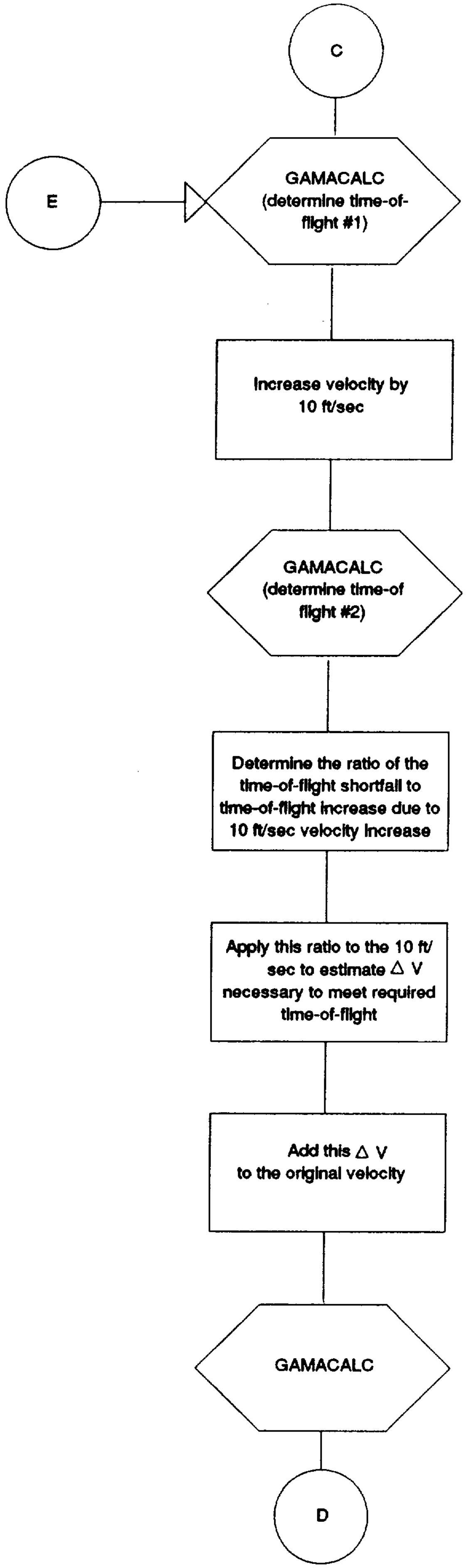

F
Print Report #4
Print Report #5
Print Report #6
END

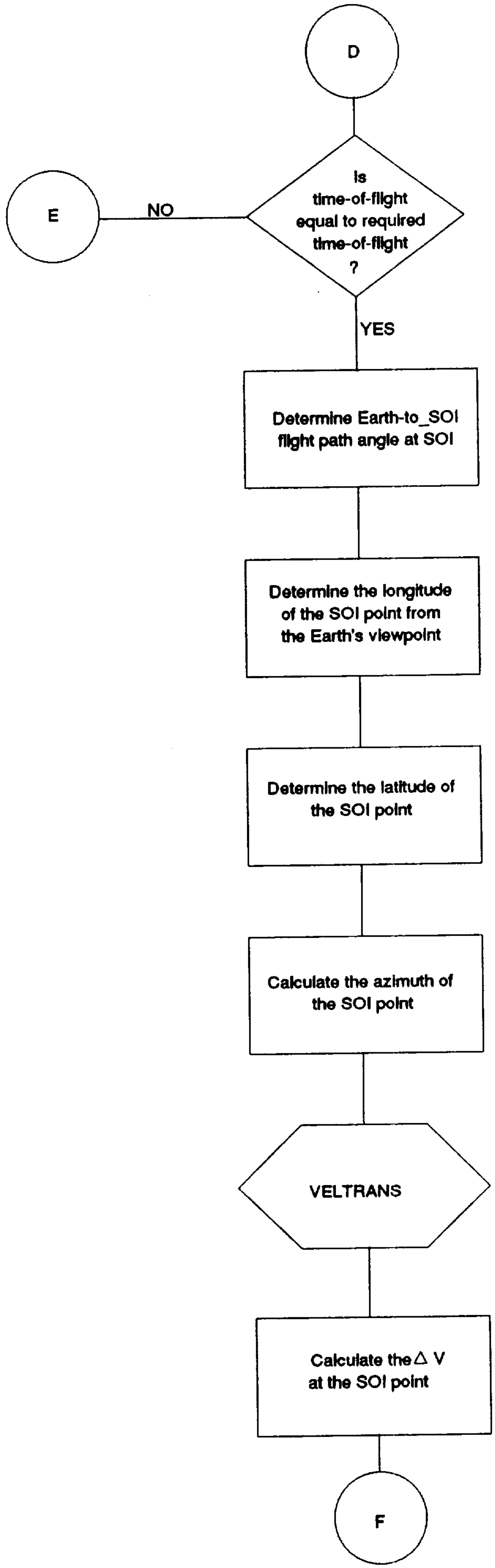

In-Program-Subroutine
H
Define variables for the sine and cosine of the SOI longitude and latitude
Define the angle between the Moon-to-Earth line and Moon-to-SOI line
Define variables for the sine and cosine of the Lunar orbit inclination
Calculate the ratio of Earth-Moon distance to Moon-SOI distance
Determine the distance form the moon to the SOI
Locate the SOI in rectangular coordinates centered at the Earth
Calculate the distance from the Earth to the SOI
A

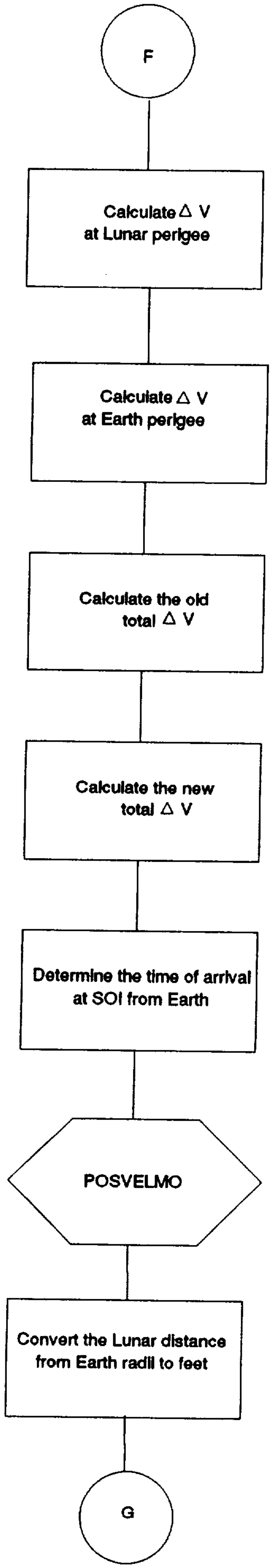


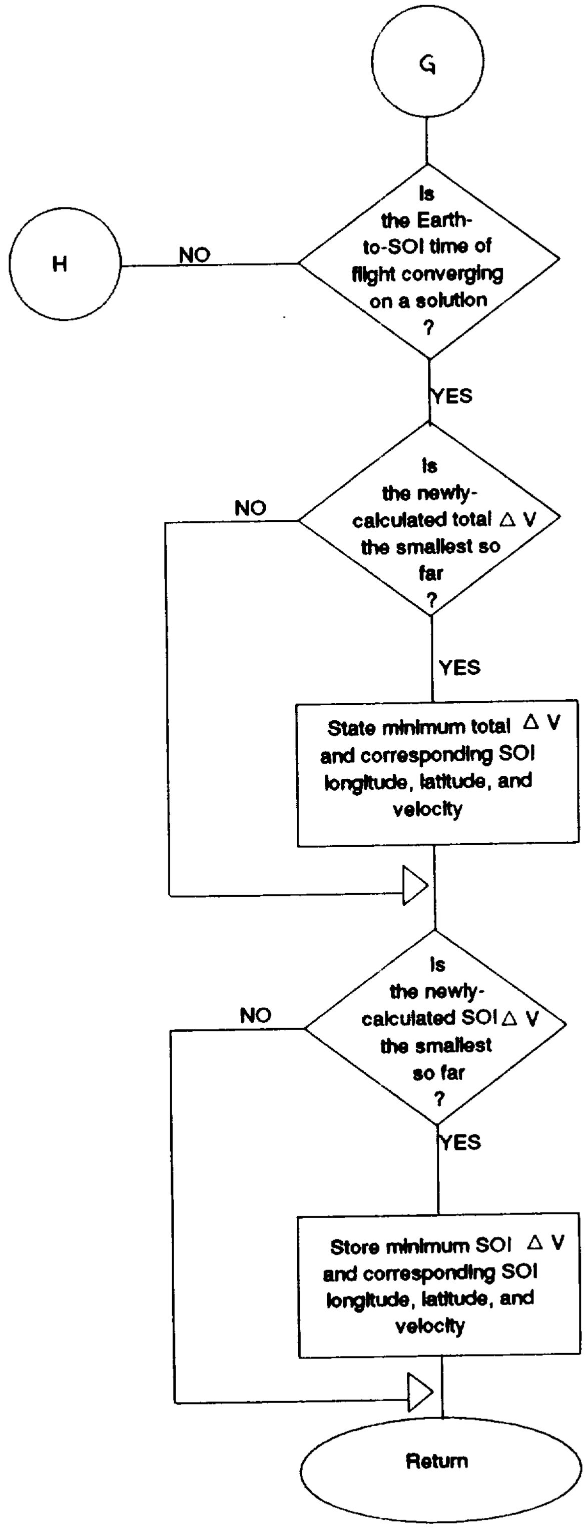
A
Identify the angle between the Earth-to-Moon line and Earth-toSOI line
Determine the Earth-based latitude and longitude of the SOI
Determine the Earth ascending node for the orbit in which the trans-SOI burn will occur
GAMACALC
Determine SOI-toMoon flight path angle at the SOI
Identify the angle between the Moon-to-Earth line and the Moon-to-SOI line
Determine the azimuth of the SOI point from the Moon
B

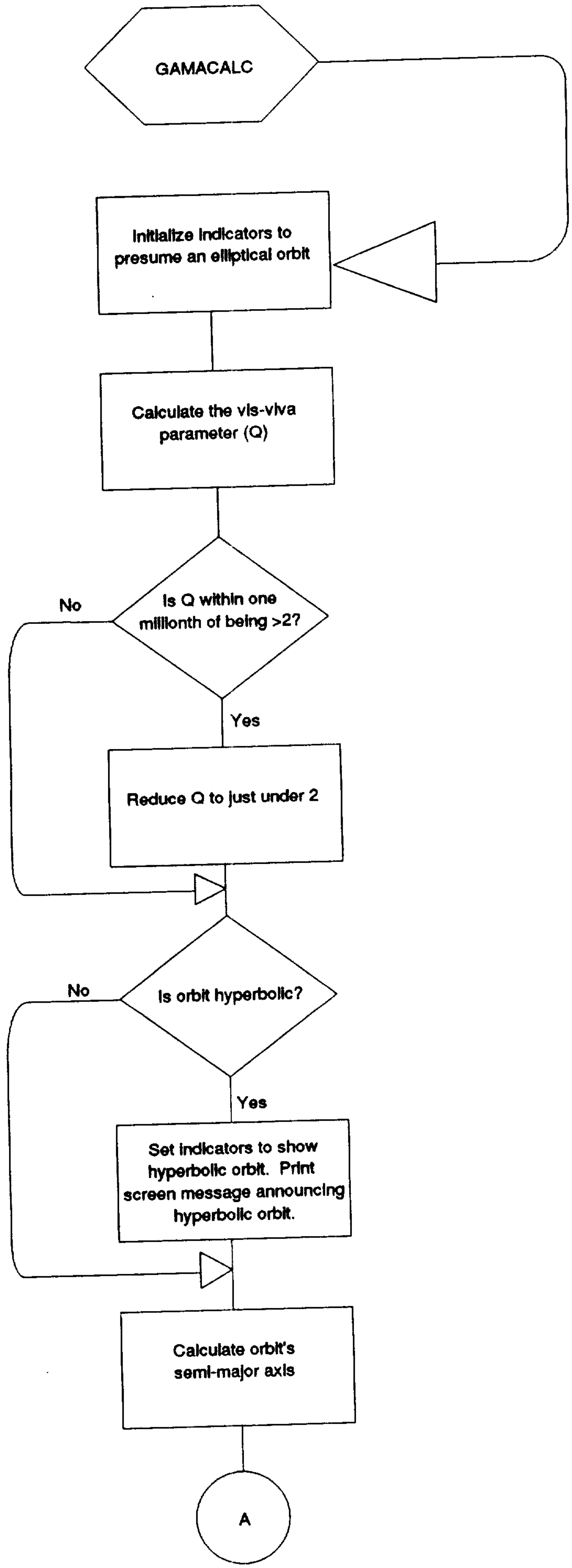

B
Determine the Lunar ascending node
Determine the rectangular coordinate components of the velocity vector at the SOI point for the SOI-Moon trajectory
Calculate the vis-viva parameter (Q) for the Earth-to-SOI trajectory
Calculate the minimum velocity required, such that apogee is just at SOI
Increase the velocity slightly, so that the calculations will converge
Is the most recently calculated SOI velocity (VELE)> the minimum velocity ?
NO
VELE = minimum velocity
YES
Temporarily store the Earth-to-SOI time of flight
C

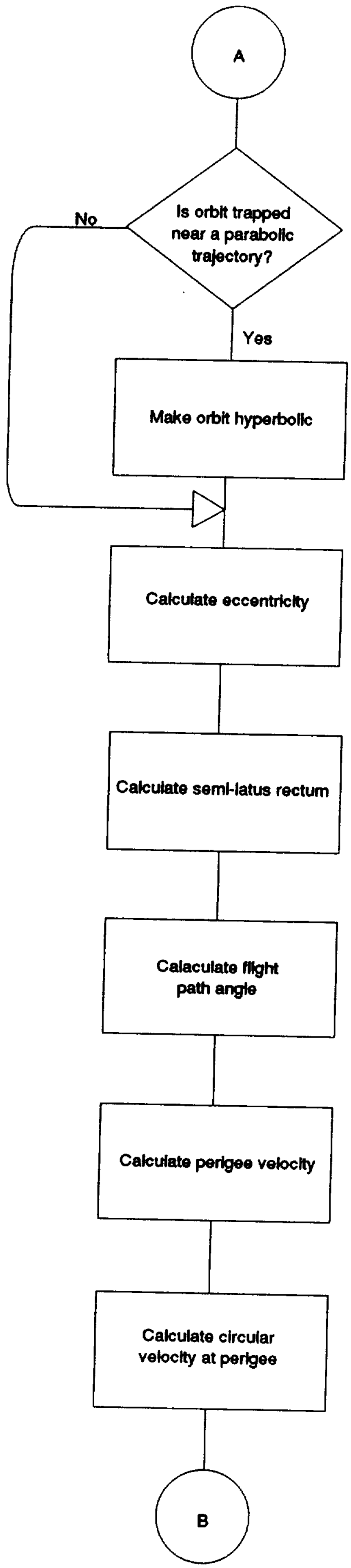

C
E
GAMACALC (determine time-of-flight #1)
Increase velocity by 10 ft/sec
GAMACALC (determine time-of flight #2)
Determine the ratio of the time-of-flight shortfall to time-of-flight increase due to 10 ft/sec velocity increase
Apply this ratio to the 10 ft/sec to estimate Δ V necessary to meet required time-of-flight
Add this Δ V to the original velocity
GAMACALC
D

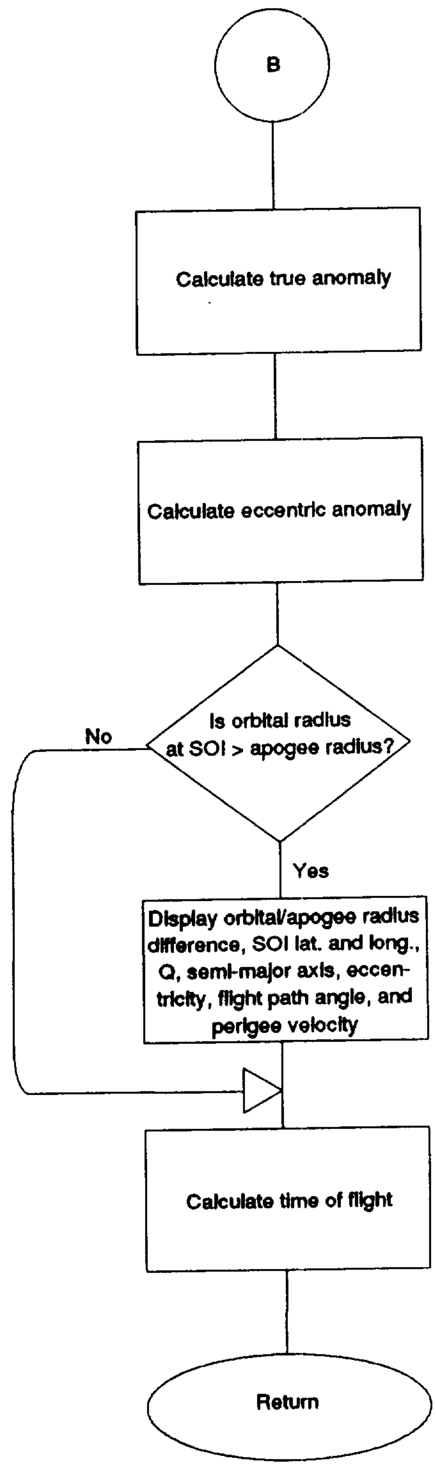

D
E
NO
Is time-of-flight equal to required time-of-flight ?
YES
Determine Earth-to_SOI flight path angle at SOI
Determine the longitude of the SOI point from the Earth's viewpoint
Determine the latitude of the SOI point
Calculate the azimuth of the SOI point
VELTRANS
Calculate the Δ V at the SOI point
F

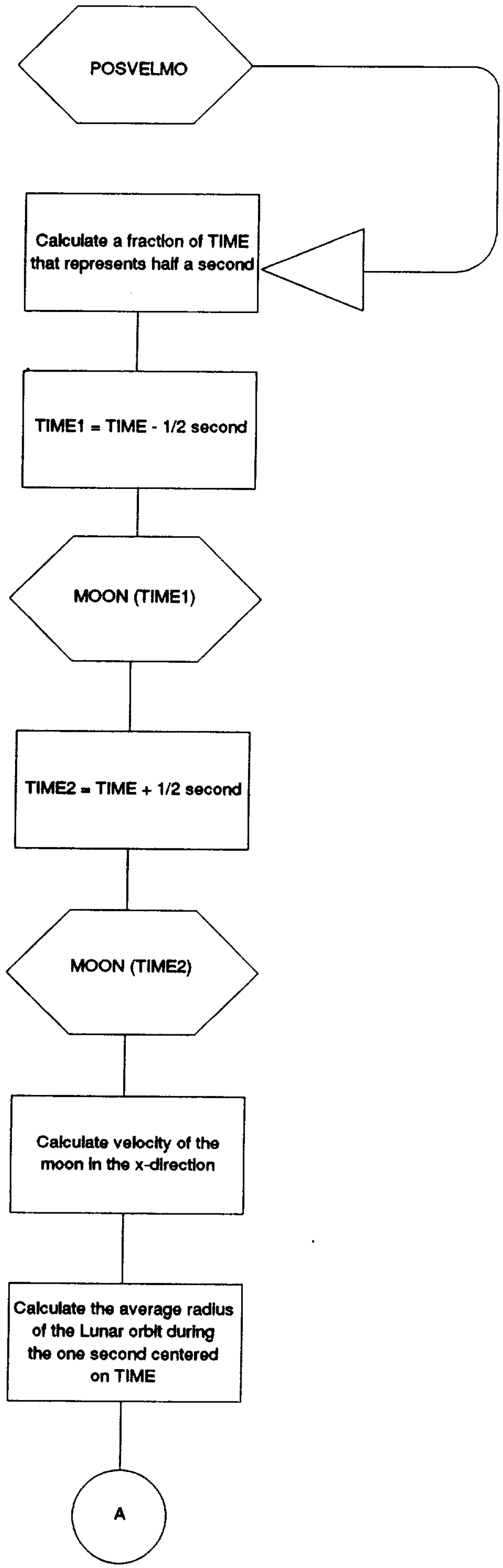

F
Calculate Δ V
at Lunar perigee
Calculate Δ V
at Earth perigee
Calculate the old
total Δ V
Calculate the new
total Δ V
Determine the time of arrival
at SOI from Earth
POSVELMO
Convert the Lunar distance
from Earth radii to feet
G

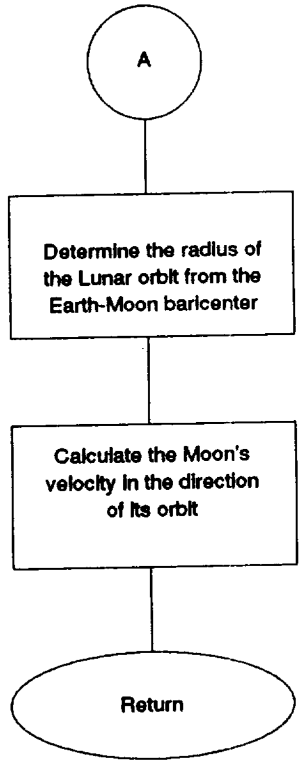

G
H
NO
Is the Earth-to-SOI time of flight converging on a solution ?
YES
NO
Is the newly-calculated total Δ V the smallest so far ?
YES
State minimum total Δ V and corresponding SOI longitude, latitude, and velocity
NO
Is the newly-calculated SOI Δ V the smallest so far ?
YES
Store minimum SOI Δ V and corresponding SOI longitude, latitude, and velocity
Return

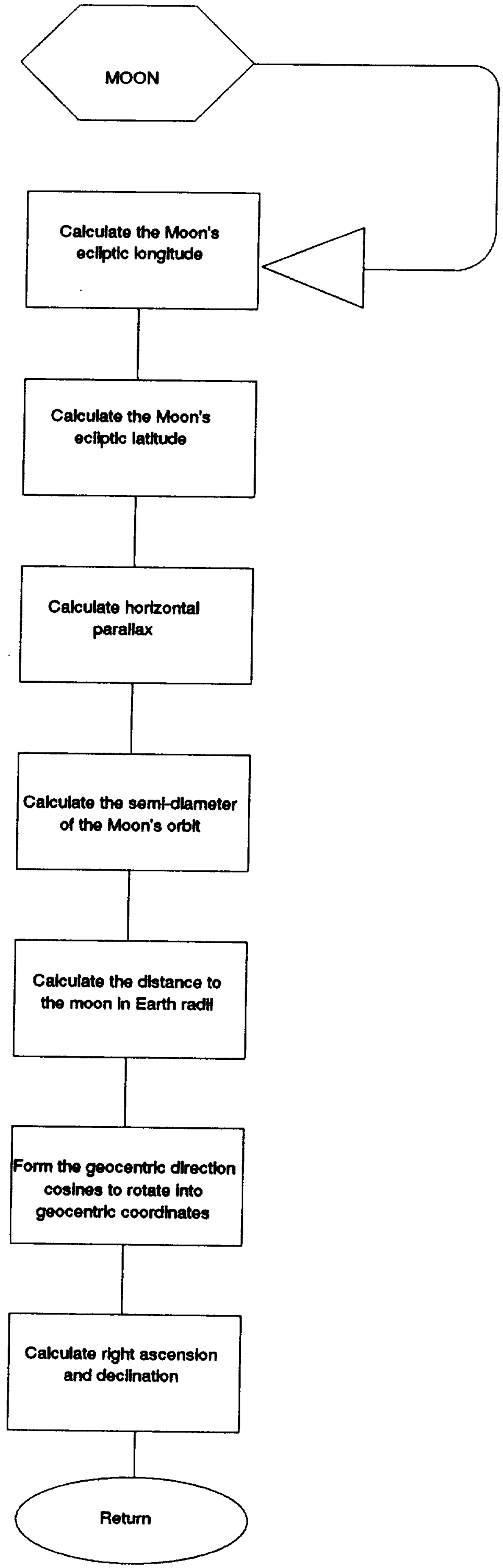

GAMACALC
Initialize indicators to presume an elliptical orbit
Calculate the vis-viva parameter (Q)
Is Q within one millionth of being >2?
No
Yes
Reduce Q to just under 2
Is orbit hyperbolic?
No
Yes
Set indicators to show hyperbolic orbit. Print screen message announcing hyperbolic orbit.
Calculate orbit's semi-major axis
A

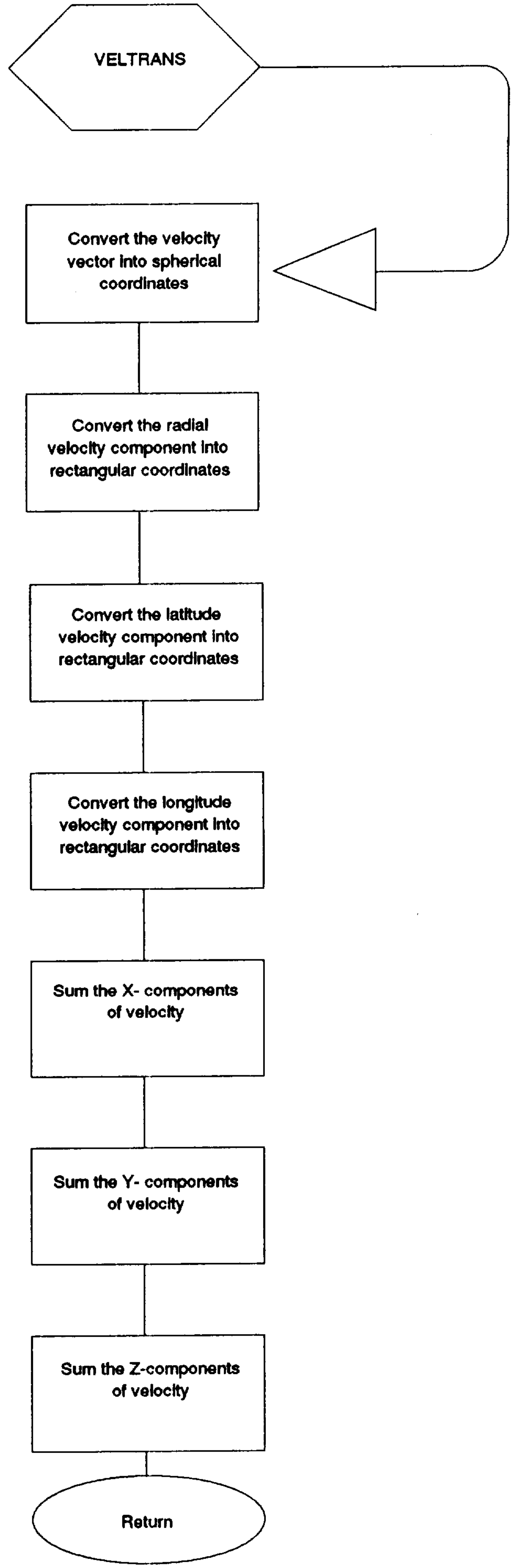

A
Is orbit trapped near a parabolic trajectory?
No
Yes
Make orbit hyperbolic
Calculate eccentricity
Calculate semi-latus rectum
Calaculate flight path angle
Calculate perigee velocity
Calculate circular velocity at perigee
B


B
Calculate true anomaly
Calculate eccentric anomaly
Is orbital radius at SOI > apogee radius?
No
Yes
Display orbital/apogee radius difference, SOI lat. and long., Q, semi-major axis, eccentricity, flight path angle, and perigee velocity
Calculate time of flight
Return


POSVELMO
Calculate a fraction of TIME that represents half a second
TIME1 = TIME - 1/2 second
MOON (TIME1)
TIME2 = TIME + 1/2 second
MOON (TIME2)
Calculate velocity of the moon in the x-direction
Calculate the average radius of the Lunar orbit during the one second centered on TIME
A


A
Determine the radius of the Lunar orbit from the Earth-Moon baricenter
Calculate the Moon's velocity in the direction of its orbit
Return


MOON
Calculate the Moon's ecliptic longitude
Calculate the Moon's ecliptic latitude
Calculate horizontal parallax
Calculate the semi-diameter of the Moon's orbit
Calculate the distance to the moon in Earth radii
Form the geocentric direction cosines to rotate into geocentric coordinates
Calculate right ascension and declination
Return


VELTRANS
Convert the velocity vector into spherical coordinates
Convert the radial velocity component into rectangular coordinates
Convert the latitude velocity component into rectangular coordinates
Convert the longitude velocity component into rectangular coordinates
Sum the X- components of velocity
Sum the Y- components of velocity
Sum the Z-components of velocity
Return

Appendix B - Program Code Listing

```
C *** JACK FUNK'S PLANE CHANGE PROGRAM *** TO 87-57  TASK 1.4
C    MAIN
C
     IMPLICIT REAL*16 (A-H,O-Z)
     CHARACTER*10 MD, HEAD, TRAJ, TRAJM, TRAJE
     CHARACTER*8 TIMP
     CHARACTER*9 DATP
     DIMENSION DELV(10,19), VELMOUT(10,19),ALONO(10),ALATP(19),
    * PAGE1(10), PAGE2(10,19), PAGE3(10,19),PAGE4(10,19),
    * PHIM(2), PAGE5(10,19), PAGE6(10,19)
C
C    OPEN OUTPUT FILE
C
     OPEN (UNIT = 1,FILE = 'PLANECNG.OUT' ,STATUS = 'NEW')
C    OUTPUT TO FILE; IP:  OUTPUT TO SCREEN; IS: OUTPUT TO DEBUG
     IP   = 1
     IS   = 5
     DPR  = 57.29578
     PI   = 3.1415926535
     CMUE  = 1.407647E+16
     CMUM  = 1.731432E+14
     FTNM  = 6076.115
     REE   = 20925741.
     REMO  = 5.7039E+6
     RREM  = 207559 * FTNM
     FTM   = .3048
     PRINT *, 'INPUT OUTBOUND OR RETURN '
     READ  *, MD
     IF   (MD .EQ. 'RETURN' )THEN
      MOOD = -1
      HEAD = 'TRANSEARTH'
     ELSE
      MOOD = 1
      HEAD = 'TRANSLUNAR'
     ENDIF
     PRINT *,'INPUT NODE OPTION 1 OR 2 '
     READ  *, NP
  10 CONTINUE
     PRINT *, 'INPUT PERIGEE ALTITUDE OF EARTH ORBIT (NMI) '
     READ  *, HPE
     PRINT *, 'INPUT PERIGEE ALTITUDE OF LUNAR ORBIT (NMI) '
     READ  *, HPM
     RPE   =  HPE * FTNM + REE
     RPM   =  HPM * FTNM + REMO
     PRINT *, 'INPUT EARTH DEPARTURE JULIAN DATE '
     READ  *, TIMJ
     IF (QABS (TIMJ ) .LE. 0.001 ) TIMJ = 2451545.
     PRINT *, 'LONGITUDE FOR OUTBOUND TRAJECTORIES SHOULD BE
    * BETWEEN 0 AND -90 DEG'
```

```
      PRINT *, 'AND RETURN TRAJECTORIES BETWEEN 0 AND +90 DEG'
      PRINT *, 'INITIAL LONGITUDE'
      READ *, ALONI
      PRINT *, 'INPUT INCREMENT FOR MAP '
      READ *, DELLON
      PRINT *, 'INPUT INITIAL LATITUDE'
      READ *, ALATI
      PRINT *, 'INPUT INCREMENT FOR MAP '
      READ *, DELLAT
      PRINT *, 'INPUT EARTH ORBIT TO LUNAR ORBIT INCLINATION '
      READ *, AINCEO
      AINCEO = AINCEO / DPR
      PRINT *, 'INPUT LUNAR ORBIT TO LUNAR ORBIT INCLINATION '
      READ *, AINCM
      AINCM =  AINCM / DPR
C   PAGE-1
      AINC  = AINCM
      AINCE = AINCEO
      PRINT *, 'INPUT FLIGHT TIME  '
      READ *, FTIM
      CALL DATE(DATP)
      CALL TIME(TIMP)
 15   CONTINUE
      WRITE (IP,7) MD, NP
 7    FORMAT (T12,' VELOCITY MAP FOR ',A8,' TRAJECTORIES
     * NODE OPTION  ',I1)
      WRITE (IP,17) DATP, TIMP
 17   FORMAT (T27,'DATE',A10,'   TIME',A10)
      WRITE (IP,27) TIMJ, HPE, HPM
 27   FORMAT (/,' JUILIAN DAY ', F8.0,'   PERIGEE ALT (NMI) EARTH = ',
     *F4.0, '  MOON = ', F4.0)
      WRITE (IP,37) FTIM, AINCEO * DPR, AINCM * DPR
 37   FORMAT (' TRANSLUNAR FLIGHT TIME (HR) = ',F5.1,'  INCL
     *EARTH ' F5.1, '  INCL MOON = ', F5.1)
      IPRINT = 0
      II    = 1
      NN    = 1
      DVMIN  = 99999.
      YDLO   = QSQRT (CMUE / RREM)
      ALAT   = ALATI / DPR
      ALON   = ALONI / DPR
      VELM   = VELMI
      DVSIM  = 99999.
 21   CONTINUE
      VEL        = VELM
      DELV(NN,II)  = 99999.
      PAGE1(NN)   = 0.0
C   PAGE-2
      PAGE2(NN,II) = 0.0
```

```
      PAGE3(NN,II) = 0.0
      PAGE4(NN,II) = 0.0
      PAGE5(NN,II) = 0.0
      PAGE6(NN,II) = 0.0
C     PRINT *,' '
C     PRINT *,' ALAT  ALON   VEL   DVT    DVSI  MOON  EARTH
C    *TIMEE  TIMEM  RREM'
 22  CONTINUE
C
C    CALCULATE DATA FOR POINT IF LATITUDE IS LESS THAN INCLINATION
C    LOAD ZEROS IN DELTA VEL
     IF(QABS(QSIN(ALAT )) .LE. QABS (QSIN(AINCM))+ .00001) THEN
      DO 8 NV = 0, 40
       VELM  = 800. + 100. * QFLOAT(NV )
       VEL   = VELM
       ICALL = 1
       GOTO 25
 1000   CONTINUE
       IF (QABS(TIEM) .LE. .0000000000000001) GOTO 23
C       STORE VALUES FOR OUTPUT
       IF(DVTOTAL .LT. DELV(NN,II) ) THEN
        DELV (NN,II)   = DVTOTAL
        PAGE1 (NN)     = DVTOTAL
        PAGE2 (NN,II)  = DELVEL
        VELMOUT (NN,II) = VELM
C        PAGE-3
        ALATP (II)     = ALAT  * DPR
        PAGE3 (NN,II)  = ANODEM * DPR
        PAGE4 (NN,II)  = ANODEE * DPR
        PAGE5 (NN,II)  = DVCIRE
        PAGE6 (NN,II)  = DVCIRM
C        WRITE (IS,57) ALAT * DPR, ALON * DPR, VELMOUT (NN,II)
C   *    ,DELV(NN,II), PAGE2(NN,II), TRAJM, TRAJE, TIEM,
C   *    TIMM, RREMER
C 57      FORMAT (1X,F6.1,1X,F6.1,2X,F7.0,1X,F8.1,1X,F7.1,1X,
C   *          A5,1X,A5,1X,F6.1,1X,F6.1,1X,F8.3)
C        WRITE (IS,67) VXE, VXM, VYE, VYM, VZE, VZM
C 67      FORMAT (' VXE ',F7.1,' VXM ',F7.1,' VYE ',F7.1,' VYM ',
C   *           F7.1, ' VZE ', F7.1, 'VZM ',F7.1)
      ENDIF
 8   CONTINUE
    ELSE
     PAGE1 (NN)    = 0.0
     DELV  (NN,II) = 0.0
     PAGE2 (NN,II) = 0.0
     PAGE3 (NN,II) = 0.0
     PAGE4 (NN,II) = 0.0
     PAGE5 (NN,II) = 0.0
     PAGE6 (NN,II) = 0.0
```

```
   ENDIF
23   CONTINUE
   ALONO (NN)    = ALON * DPR
   IF (NN .EQ. 10 .AND. IPRINT .EQ. 0 ) THEN
      WRITE  (IP,77)  ALONO(1), ALONO(2), ALONO(3), ALONO(4),
   *   ALONO(5), ALONO(6), ALONO(7),ALONO(8),ALONO(9),ALONO(10)
77      FORMAT (/,' ALON> ', 10(2X,F5.1 ))
      WRITE (IP,87)
87      FORMAT (' ALAT')
   ENDIF
   IF (NN .LT. 10) THEN
    ALON = (ALONI + DELLON * QFLOAT (NN ) ) / DPR
    NN   = NN + 1
    GOTO 21
   ENDIF
   IPRINT = 1
   WRITE (IP,97) ALAT * DPR, PAGE1(1),PAGE1(2),
  *PAGE1(3),PAGE1(4),PAGE1(5),PAGE1(6),
  *PAGE1(7),PAGE1(8),PAGE1(9),PAGE1(10)
 97  FORMAT (1X, F5.1, 2X, 10(1X,F6.0))
   IF (II .LT. 19) THEN
    ALAT = (ALATI + DELLAT * QFLOAT (II ) ) / DPR
    II   = II+1
    NN   = 1
    ALON = ALONI / DPR
    GOTO 21
   ENDIF
   VELM  = VELMMIN
   ALAT  = ALATMIN
   ALON  = ALONMIN
   ICALL = 2
   GOTO 25
 2000 CONTINUE
   WRITE  (IP,107) NP
 107  FORMAT (///,T23,'MINIMUM VELOCITY PRINT NODE OPTION ',I1)
   WRITE  (IP,117) DATP, TIMP
 117  FORMAT (T27,'DATE',A10,' TIME',A10)
   WRITE  (IP,127) FTIM, ALAT * DPR, ALON * DPR, AINCEO * DPR,
  * AINCM * DPR
 127  FORMAT (/,' FLIGHT TIME = ',F4.0,' LAT = ',F6.1,' LON = ',F6.1,
  *       ' LUNAR AINC = ', F5.1,' EARTH AINC = ',F5.1)
   WRITE  (IP,137)
 137  FORMAT (' BODY   VX   VY   VZ   VEL  GAMA  AZM
  *AINC  ANODE  TIME  DVCIR')
   WRITE  (IP,147) VXM,VYM,VZM,VMAGM,GAMAM * DPR,
  *AZMM * DPR, AINCM * DPR, ANODEM * DPR, TIMM, DVCIRM, DELVEL
 147  FORMAT (' MOON ',4(1X,F6.0),1X,F5.1,1X,F6.1,2X,F5.1,2X,F6.1,
  * 1X,F5.1,1X,F7.0,' DVPHER = ',F7.0)
   WRITE  (IP,1475) VXE, VYE, VZE, VELE, GAMAE * DPR, AZME * DPR,
```

```
*          AINCE * DPR, ANODEE * DPR, TIEM, DVCIRE, DVTOTAL
1475 FORMAT (' EARTH',4(1X,F6.0),1X,F5.1,1X,F6.1,2X,F5.1,2X,F6.1,
  * 1X,F5.1,1X,F7.0,' DVTOTAL = ', F7.0)
   WRITE (IP,157) CHAR(12)
157  FORMAT (' ',A1)
   WRITE  (IP,167) HEAD,NP
167  FORMAT (T10,'DELTA VELOCITY AT SPHERE OF INFLUENCE MAP
  *FOR ',A10,' TRAJECTORIES NODE OPTION ',I1)
   WRITE  (IP,177) DATP,TIMP
177  FORMAT (T27,'DATE',A10,'  TIME',A10)
   WRITE  (IP,187)  HEAD,FTIM, AINCEO * DPR, AINCM * DPR
187  FORMAT ('  ',A10,' FLIGHT TIME (HR) = ',F6.1,' INCL EARTH ',
  *F6.1,'  INCL MOON = ',F5.1)
   WRITE  (IP,197) ALONO(1), ALONO(2), ALONO(3), ALONO(4),
  *ALONO(5),ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
197  FORMAT (/,' ALON> ',10(2X, F5.1))
18  CONTINUE
   WRITE  (IP,207)
207  FORMAT (' ALAT')
   DO 28 NPI = 1 , 19
    WRITE (IP,217) ALATP(NPI), PAGE2(1,NPI),PAGE2(2,NPI),
  *  PAGE2(3,NPI),PAGE2(4,NPI),PAGE2(5,NPI),PAGE2(6,NPI),
  *  PAGE2(7,NPI),PAGE2(8,NPI),PAGE2(9,NPI),PAGE2(10,NPI)
217    FORMAT(1X,F5.1,1X,10(1X,F6.1))
28  CONTINUE
   WRITE  (IP, 227 ) CHAR(12)
227  FORMAT (' ', A1)
   WRITE  (IP, 237 ) HEAD , NP
237  FORMAT (T6,'ASCENDING NODE OF LUNAR ORBIT MAP FOR ',A10,
  *  ' TRAJECTORIES NODE OPTION ',I1)
   WRITE  (IP, 247 ) DATP, TIMP
247  FORMAT (T27,'DATE',A10,'   TIME',A10)
   WRITE  (IP, 257 ) HEAD, FTIM, AINCEO * DPR, AINCM * DPR
257  FORMAT ('  ',A10,'  FLIGHT TIME (HR) = ',F6.1,'  INCL EARTH ',
  *F6.1,'  INCL MOON = ',F6.1)
   WRITE  (IP,267) ALONO(1), ALONO(2), ALONO(3), ALONO(4),
  *ALONO(5),ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
267  FORMAT (/,' ALON> ', 10(2X, F5.1))
   WRITE (IP,277)
277  FORMAT (' ALAT')
   DO 48 NPI = 1 , 19
    WRITE (IP,287) ALATP(NPI),PAGE3(1,NPI),PAGE3(2,NPI),
  *  PAGE3(3,NPI),PAGE3(4,NPI),PAGE3(5,NPI),PAGE3(6,NPI),
  *  PAGE3(7,NPI), PAGE3(8,NPI),PAGE3(9,NPI),PAGE3(10,NPI)
287    FORMAT (' ',F5.1,1X,10(2X,F5.1))
48  CONTINUE
   WRITE  (IP, 297) CHAR(12)
297  FORMAT (' ',A1)
   WRITE  (IP, 307 ) HEAD,NP
```

```
307 FORMAT (T6,'ASCENDING NODE OF EARTH-MOON TRANSFER TRAJECTORY
   *MAP FOR ',A10,' TRAJECTORIES NODE OPTION ',I1)
    WRITE (IP, 317 ) DATP,TIMP
317 FORMAT (T27,'DATE',A10,'  TIME',A10)
    WRITE (IP, 327 ) HEAD, FTIM, AINCEO * DPR, AINCM * DPR
327 FORMAT (1X, A10,' FLIGHT TIME (HR) = ',F5.1,' INCL EARTH ',
   *          F5.1,' INCL MOON = ',F5.1)
    WRITE (IP, 337) ALONO(1),ALONO(2),ALONO(3),ALONO(4),ALONO(5),
   *ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
337 FORMAT (/,' ALON> ', 10(2X, F5.1))
    WRITE (IP,347)
347 FORMAT (' ALAT')
    DO 58 NPI = 1 , 19
     WRITE (IP,357) ALATP(NPI),PAGE4(1,NPI),PAGE4(2,NPI),
   * PAGE4(3,NPI),PAGE4(4,NPI),PAGE4(5,NPI),PAGE4(6,NPI),
   * PAGE4(7,NPI),PAGE4(8,NPI),PAGE4(9,NPI),PAGE4(10,NPI)
357  FORMAT(1X,F5.1,2X,10(1X,F6.2))
58  CONTINUE
    WRITE(IP,367) CHAR(12)
367 FORMAT(' ',A1)
    WRITE(IP,377) HEAD, NP
377 FORMAT(T2,'DELTA VELOCITY AT EARTH FOR EARTH-MOON TRANSFER
   * TRAJECTORY MAP FOR ',A10,' TRAJECTORIES NODE OPTION ',I1)
    WRITE(IP,387) DATP, TIMP
387 FORMAT(T27,'DATE ', A10,'  TIME ', A10)
    WRITE(IP,397) HEAD, FTIM, AINCEO * DPR, AINCM * DPR
397 FORMAT('  ',A10,' FLIGHT TIME (HR) = ',F6.1,' INCL EARTH ',
   *F6.1,' INCL MOON = ',F6.1)
    WRITE(IP,407) ALONO(1), ALONO(2), ALONO(3), ALONO(4),
   *ALONO(5), ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
407 FORMAT(/,' ALON> ', 10(2X, F5.1))
    WRITE(IP,417)
417 FORMAT(' ALAT')
    DO 68 NPI = 1, 19
     WRITE(IP,427) ALATP(NPI), PAGE5(1,NPI), PAGE5(2,NPI),
   * PAGE5(3,NPI), PAGE5(4,NPI), PAGE5(5,NPI), PAGE5(6,NPI),
   * PAGE5(7,NPI), PAGE5(8,NPI), PAGE5(9,NPI), PAGE5(10,NPI)
427 FORMAT (1X,F5.1,2X,10(1X,F6.0))
68  CONTINUE
    WRITE(IP,437) CHAR(12)
437 FORMAT(' ',A1)
    WRITE(IP,447) HEAD, NP
447 FORMAT(T2,'DELTA VELOCITY AT MOON FOR EARTH-MOON TRANSFER
   * TRAJECTORY MAP FOR ',A10,' TRAJECTORIES NODE OPTION ',I1)
    WRITE(IP,457) DATP, TIMP
457 FORMAT(T27,'DATE',A10,'  TIME',A10)
    WRITE(IP,467) HEAD, FTIM, AINCEO * DPR, AINCM * DPR
467 FORMAT(1X,A10,' FLIGHT TIME (HR) = ',F5.1,' INCL EARTH ',
   *F5.1,' INCL MOON = ',F5.1)
```

```
      WRITE(IP,477) ALONO(1), ALONO(2), ALONO(3), ALONO(4), ALONO(5),
     * ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
 477  FORMAT(/,' ALON> ', 10(2X, F5.1))
      WRITE(IP,487)
 487  FORMAT(' ALAT')
      DO 78 NPI = 1, 19
       WRITE(IP,497) ALATP(NPI), PAGE6(1,NPI), PAGE6(2,NPI),
     * PAGE6(3,NPI), PAGE6(4,NPI), PAGE6(5,NPI), PAGE6(6,NPI),
     * PAGE6(7,NPI), PAGE6(8,NPI), PAGE6(9,NPI), PAGE6(10,NPI)
 497   FORMAT(1X,F5.1,2X,10(1X,F6.0))
 78   CONTINUE
      GOTO 3000
 25   CONTINUE
      COSALAT = QCOS (ALAT )
      SINALAT = QSIN (ALAT )
      COSALON = QCOS (ALON )
      SINALON = QSIN (ALON )
      COSPHI  = COSALAT * COSALON
      COSAINC = QCOS (AINCM / DPR )
      SINAINC = QSIN (AINCM / DPR )
      RMR     = COSPHI +QSQRT (COSPHI ** 2. - (1. - CMUE / CMUM ))
      RRM     = RREM / RMR
      XXM     = RRM  * COSALAT * COSALON
      XX      = -XXM + RREM
      YY      = -RRM * COSALAT * SINALON
      ZZ      =  RRM * SINALAT
      YYM     = -YY
C     WRITE (IS,507) XX,YY,ZZ
C 507 FORMAT(' XYZ POSITION AT SPHERE ',F11.0,1X,F11.0,1X,F11.0)
      RRE     = QSQRT(XX**2.+YY**2.+ZZ**2.)
      COSANGA = XX / RRE
      SINANGA = QSQRT (YY ** 2. + ZZ ** 2. ) / RRE
      ANGA    = QATAN (SINANGA / COSANGA )
      ALONX   = QATAN (YY / XX )
      ALATX   = QATAN (ZZ / QSQRT (XX ** 2. + YY **2. ) )
      ANODEE  = ALONX - QASIN (QTAN (-ALATX ) / QTAN (AINCE ) )
C     END SPHERE OF INFLUENCE CALC
      AAMIN  = (RPM + RRM ) / 2.
      EEMIN  = (RRM - RPM ) / (RPM + RRM )
      VVMIN  = QSQRT (CMUM *(1. -EEMIN ) / RRM )
C     WRITE(IS,517) VVMIN
C 517 FORMAT (' MINIMUM VEL AT SI = ',F8.4)
      AAMINE = (RPE + RRE ) / 2.
      EEMINE = (RRE - RPE ) / (RPE +RRE )
      VVMINE = QSQRT (CMUE * (1. - EEMINE ) / RRE )
 30   CONTINUE
      CALL GAMACALC(RPM,VELM,RRM,CMUM,COSGAM,VPM,VCIRMP,TIMM,
     *TRAJM,DPR,ALAT,ALON,FTNM)
      GAMAM  = -QFLOAT (MOOD ) * QATAN (QSQRT (1.-COSGAM**2.)/ COSGAM)
```

```
   TEMP = ZZ/RRM/QSIN(AINCM)
   IF (ABS(TEMP) .GT. 1.) THEN
    IF (TEMP .GT. 1.) THEN
     PHIM(1) = PI/2
    ELSE
     PHIM(1) = -PI/2
    ENDIF
   ELSE
    PHIM(1) = QASIN (ZZ/RRM/QSIN(AINCM))
   ENDIF
   IF (ZZ .EQ. 0.) THEN
    PHIM(2) = PI
   ELSE
    PHIM(2) = (PI - PHIM (1 ) * ZZ/QABS(ZZ)) * ZZ/QABS(ZZ)
   ENDIF
   AZMM   = QATAN2(QCOS(AINCM),QSIN(AINCM)*QCOS(PHIM(NP)))
   AVM    = QATAN2(QTAN(ALAT)/QTAN(AINCM),QCOS(PHIM(NP))/QCOS(ALAT))
   ANODEM  = ALON-AVM
   IF(ANODEM .LT.0.)  ANODEM = ANODEM + 2.*PI
   CALL   VELTRANS(VELM,GAMAM,AZMM,ALAT,ALON,VXM,VYM,VZM,VMAGM,-
DPR)
   VXM = VXM+XDLO
   VYM = VYM+YDLO
   QQEMIN = 2.1*RPE/(RRE+RPE)
   VELEMIN = QSQRT(QQEMIN*CMUE/RRE)
   IF(VELE .LT. VELEMIN) VELE = VELEMIN
   TIEMS = TIEM
C   TIMEES CHANGED TO TIEMS, TIMEE CHANGED TO TIEM
 35  CONTINUE
   CALL GAMACALC(RPE,VELE,RRE,CMUE,COSGAME,VPE,VCIRE,
  *TIME1,TRAJE,DPR, ALAT,ALON,FTNM)
   VELE2 = VELE + 10.
   CALL GAMACALC(RPE,VELE2,RRE,CMUE,COSGAME,VPE,VCIRE,
  *TIME2,TRAJE,DPR,ALAT,ALON,FTNM)
   DELVELE = 10./(TIME2-TIME1)*(FTIM-TIME1-TIMM)
   IF(DELVELE .LT. 500.) THEN
    VELE =VELE + DELVELE
   ELSE
    VELE = VELE + DELVELE/QABS(DELVELE)*500.
   ENDIF
   CALL GAMACALC(RPE,VELE,RRE,CMUE,COSGAME,VPE,VCIRE,TIEM,
  *TRAJE,DPR,ALAT,ALON,FTNM)
   IF(TIEM .EQ. 0.) GOTO 60
   TIMT = TIEM+TIMM
   IF(ABS(FTIM-TIMT) .GT. 1.) GOTO 35
   GAMAE = QFLOAT(MOOD)*QATAN(QSQRT(1.-COSGAME**2.)/COSGAME)
   ALONE = 180. / DPR + ALONX
   ALATE = QATAN(ZZ/QSQRT(XX**2.+YY**2.))
 50  CONTINUE
```

```
      AINCE = AINCEO
C     'CALCULATION OF EARTH ORBIT AZIMUTH AT SPHERE OF INFLUENCE
      IF(AINCE .NE. 0.0) THEN
       PHIE = PI-QASIN(ZZ/(RRE*SIN(AINCE)))
       AZME = QATAN2(1./QTAN(AINCE),QCOS(PHIE))
      ELSE
       AZME = PI/2
      END IF
C
      CALL VELTRANS(VELE,GAMAE,AZME,ALATE,ALONE,VXE,VYE,VZE,VELE,
     * DPR)
      DELVEL = QSQRT((VXE-VXM)**2.+(VYE-VYM)**2.+(VZE-VZM)**2.)
      DVCIRM = VPM - VCIRMP
      DVCIRE = VPE - VCIRE
      DVE = QSQRT (VCIRE ** 2. + VPE**2.-2. * VCIRE*VPE*
     *COS (AINCEO - AINCE ) )
      DVTSAV  = DVTOTAL
      DVTOTAL = DELVEL + DVCIRM + DVCIRE
C       CALCULATE POSITION AND VELOCITY OF MOON
C       ITTERATE FOR FLIGHT TIM TO SPHERE OF INFLUENCE
      TIM = (TIMJ-2451545.)/36525.+TIEM/876600.
      CALL POSVELMO(TIM,RREMER,XDLO,YDLO)
      RREM = RREMER*20295741.
      IF(ABS(TIEM-TIEMS) .GT. .1) GOTO 25
      IF(DVTOTAL.LT. DVMIN)THEN
       DVMIN = DVTOTAL
       ALONMIN = ALON
       ALATMIN = ALAT
       VELMMIN=VELM
      ENDIF
      IF(DELVEL .LT. DVSIM)THEN
       DVSIM = DELVEL
       ALONSIM = ALON
       ALATSIM = ALAT
       VELMSIM = VELM
      ENDIF
 60   CONTINUE
      IF (ICALL.EQ.1) GOTO 1000
      IF (ICALL.EQ.2) GOTO 2000
3000  STOP
      END


       SUBROUTINE XYZPOS(RRX,ALAT,ALON,XX,YX,ZX)
       IMPLICIT REAL * 16 (A-Z)
       XX = -RRX*COS(ALAT)*COS(ALON)+RREM
       YX = -RRX*COS(ALAT)*SIN(ALON)
       ZX =  RRX*SIN(ALAT)
       RETURN
```

```
      END


      SUBROUTINE POSVELMO(TIM,RRM,XDLO,YDLO)
      IMPLICIT REAL * 16 (A-H,O-Z)
      DELT = 0.5/36525./24./3600.
      T1 = TIM-DELT
      CALL MOON(T1,RAM,DECM,RM1)
      T2 = TIM+DELT
      CALL MOON(T2,RAM,DECM,RM2)
      XDLO = (RM2-RM1)*20925741.
      RRM = (RM2+RM1)/2.
      RRM = RRM
      RRMB = RRM -7.412789E-01
      YDLO = 200570.2/RRMB
      RETURN
      END



      SUBROUTINE MOON (T, RAM, DECM, RM)
C     FINDS LOCATION OF MOON IN EQUATORIAL COORDS. AT ANY TIM
C     REF:   '87 ASTRONOMICAL ALMANAC
C           T IS JULIAN CENTURIES SINCE YEAR 2000
C           LAM IS MOON'S ECLIPTIC LONGITUDE
C           BETA IS MOON'S ECLIPTIC LATITUDE
C           PIE IS HORIZONTAL PARALLAX
C           RM IS DIST. TO MOON IN EARTH RADII
C           RAM IS RT. ASCENSION OF MOON
C           DECM IS MOON'S DECLINATION
C           SD IS SEMIDIAMETER OF MOON'S ORBIT

      IMPLICIT REAL * 16 (A-Z)
C     PRINT *, ' MOON'

      P = 3.1415926535
      C = P / 180

      LAM = C*218.32+C*481267.883*T+C* 6.29 * QSIN(C * 134.9 + C *
     *477198.85 * T) - C * 1.27 * QSIN(C * 259.2 - C * 413335.38 *
     *T) + c * .66 * QSIN(C * 235.7 + c * 890534.23*T)

      LAM = LAM + c * .21 * QSIN(c * 269.9 + c * 954397.7*T) - c *
     *.19 *  QSIN(c * 357.5 + c * 35999.05 * T) - c * .11 *
     *SIN(c * 186.6 + c * 966404.05*T)

      beta = c*5.13*QSIN(c*93.3 + c * 483202.03 * T) + c *
     * .28 * QSIN(C * 228.2 +  C * 960400.87 * T) -c*.28*QSIN
```

```
  *(c*318.3+c* 60003.18*T)-c*.17*QSIN(c*217.6-c*407332.2 * T)

   pie = c*.9508+c*.0518*COS(c*134.9 + c * 477198.85 * T) +
  *c*.0095*QCOS(c*259.2-c*413335.38*T)+c* .0078*COS(c * 23
  *5.7+c*890534.23*T)+c*.0028*QCOS(c*269.9+c*954397.7* T)

   SD = .2725 * pie
   RM = 1. / QSIN(pie)

   l    = QCOS(beta) * QCOS(LAM)
   M    = .9175 * QCOS(beta) * QSIN(LAM) - .3978 * QSIN(beta)
   n    = .3978 * QCOS(beta) * QSIN(LAM) + .9175 * QSIN(beta)
   RAM  = QATAN2(M, l)
   DECM = QASIN(n)
   RETURN
   END

   SUBROUTINE GAMACALC(RPX,VV,RRX,CMUX,COSGAMX,VPX,VCIRX,
  *TIMX,TRAJ,DPR,ALAT,ALON,FTNM)
   IMPLICIT REAL * 16 (A-H, O-Z)
   CHARACTER*10 TRAJ
   IHYPER = 1
   TRAJ = 'ELIPT'
   QQX = RRX*VV**2/CMUX
   IF(QQX-2 .LT. 1.0E-06) QQX = QQX - 1.0E-06
   IF (QQX .GT. 2.) THEN
    IHYPER = -1
    TRAJ = 'HYPER'
C     PRINT *,' ***** TRAJECTORY IS HYPERBOLIC '
   ENDIF
   AAX = RRX/(2.-QQX)
   IF (AAX. GT . 1.0E12 .OR. AAX. LT. -1.0E12) AAX = -1.0E12
   EEX      = 1.-RPX/AAX
   PPX      = AAX*(1. -EEX ** 2.)
   COSGAMX  = QSQRT(RPX/RRX*(1+EEX)/QQX)
   GAMAX    = QACOS(COSGAMX)
   VPX      = QSQRT(CMUX*(1+EEX)/RPX)
   VCIRX    = QSQRT(CMUX/RPX)
   COSTHETAX = (PPX/RRX-1)/EEX
   THETAX   = QACOS(COSTHETAX)
   COSAEX   = (EEX+COSTHETAX)/(1+EEX*COSTHETAX)
   ERRRP    = RRX-AAX*(1+EEX)
   IF(ERRRP .GT. 0. .AND. AAX .GT. 0.) THEN
    WRITE (IS,537) ERRRP/6076.1155, ALAT*DPR, ALON*DPR
 537  FORMAT (' RADIUS > APOGEE BY NMI ',F7.5, F8.0, F7.5)
    WRITE (IS,547) QQX, AAX/FNTM, EEX, GAMAX*DPR, VPX
 547  FORMAT (' QQX = ',F7.5,' AAX = ',F8.0,' EEX = ',F7.5,
  *  ' GAMAX = ', F5.1,' VPX = ',F7.1)
   ENDIF
```

```
      IF(QQX .GT. 2.) GOTO 101
C     CALC FLIGHT TIM FOR ELIPTICAL ORBITS
       IF (ERRRP .GT. 0.) THEN
        TIMX = 0.
        GOTO 199
       ENDIF
       AEX    = QACOS(COSAEX)
       SINAEX = QSQRT(1-COSAEX**2.)
       TIMX  = QSQRT(AAX ** 3. / CMUX)*(AEX-EEX*SINAEX)/3600.
       GOTO 199
 101  CONTINUE
C   CALC FLIGHT TIM FOR HYPERBOLIC ORBITS
      COSHF = COSAEX
      SINHF = QSQRT(COSHF**2.-1.)
      FFX   = QLOG(COSHF+QSQRT(COSHF**2.-1.))
      TIMX = QSQRT(-1. *AAX*AAX*AAX/CMUX)*(EEX*SINHF-FFX)/3600.
 199  CONTINUE
      RETURN
      END


      SUBROUTINE VELTRANS(VEL,GAMA,AZM,ALAT,ALON,VXX,VYX,VZX,VMAG,
     *             DPR)
      IMPLICIT REAL * 16 (A-Z)
      RRD   = VEL * QSIN (GAMA )
      RPHID = VEL * QCOS (GAMA )
      VLON  = RPHID * QSIN (AZM )
      VLAT  = RPHID * QCOS (AZM )
      VXR   = -RRD * QCOS (ALAT ) * QCOS (ALON )
      VYR   = -RRD * QCOS (ALAT ) * QSIN (ALON )
      VZR   =  RRD * QSIN (ALAT )
      VXLA  = VLAT * QSIN (ALAT ) * QCOS (ALON )
      VYLA  = VLAT * QSIN (ALAT ) * QSIN (ALON )
      VZLA  = VLAT * QCOS (ALAT )
      VXLO  = VLON * QSIN (ALON )
      VYLO = -VLON * QCOS (ALON )
      VZLO = 0.0
      VXX  = VXR + VXLA + VXLO
      VYX  = VYR + VYLA + VYLO
      VZX  = VZR + VZLA + VZLO
      VMAG = QSQRT (VXX **2. + VYX ** 2.+ VZX ** 2. )
      RETURN
      END
```

Appendix C - Program Variables

INPUT VARIABLE	DESCRIPTION
AINCEO	Earth orbit inclination (degrees). This is the angle between the plane of the low Earth orbit and the plane of the Moon's orbit about the Earth.
AINCM	Lunar orbit inclination (degrees). This is the angle between the plane of the low Lunar orbit and the plane of the Moon's orbit about the Earth.
ALATI	Initial Sphere of Influence longitude for map (degrees)
ALONI	Initial Sphere of Influence latitude for map (degrees)
DELLAT	Incremental latitude for map (degrees)
DELLON	Incremental longitude for map (degrees)
FTIM	Flight time for trajectory (hours)
HPE	Holding perigee altitude of Earth orbit (nautical miles)
HPM	Holding perigee altitude of Lunar orbit (nautical miles)
MD	Leg of trip for which to perform calculations (outbound or return)
NP	Node option. Selection of "1" directs the program to concern itself with destination circular orbits in which the SOI longitude is less than 90° away from the destination orbit's ascending node. Selection of "2" directs the program to concern itself with destination circular orbits in which the SOI longitude is greater than 90° away from the the destination orbit's ascending node.
TIMJ	Earth departure Julian date (where January 1, 2000 is day 2,451,545. Refer to Section C of "The Astronomical Almanac of the Year 1988").

CONSTANT	VALUE	DESCRIPTION
C	$\Pi/180$	Degrees per radian (deg./rad.)
CMUE	1.407647E+16	Gravitational parameter of the Earth (ft^3/sec^2)
CMUM	1.731432E+14	Gravitational parameter of the Moon (ft^3/sec^2)
DPR	57.29578	Degrees per radian (deg./rad.)
FTM	0.3048	Meters per foot (m/ft)

FTNM	6,076.115	Feet per nautical mile (ft/nmi)
P	3.1415927	Π (dimensionless)
PI	3.141593	Π (dimensionless)
REE	20,925,741	Radius of the Earth (ft)
REMO	5,703,900	Radius of the Moon (ft)
RREM	1,261,152,353	Earth-Moon distance of centers (ft)

VARIABLE	DESCRIPTION
AAMIN	Semi-major axis of the SOI-to-Moon trajectory
AAMINE	Semi-major axis of the Earth-to-SOI trajectory
AAX	Semi-major axis of one of the transfer orbits
AEX	Eccentric anomaly of one of the transfer orbits
AINC	Lunar orbit inclination
AINCE	Earth orbit inclination
ALAT	Initial latitude for map (radians)
ALATE	Latitude of SOI point, measured from the Earth
ALATMIN	Latitude of SOI point associated with the minimum total ΔV
ALATP(•)	ALAT in degrees
ALATSIM	Latitude of SOI point associated with the minimum SOI ΔV
ALATX	Latitude of the SOI point, measured from the Earth
ALON	Initial longitude for map (radians)
ALONE	Longitude of SOI point, measured from zero longitude at Earth (-X direction)
ALONMIN	Longitude of SOI point associated with the minimum total ΔV
ALONO(•)	Variable that contains the print matrix column headings (longitudes)
ALONSIM	Longitude of SOI point associated with the minimum SOI ΔV
ALONX	Longitude of the SOI point, measured from the Earth-Moon line at the Earth
ANGA	The angle between the Earth-to-Moon line and the Earth-to-SOI-point line.
ANODEE	Longitude of the Earth ascending or descending node
ANODEM	Longitude of the Lunar ascending or descending node
AVM	Angle between the Lunar node and the projection of the SOI point onto the Earth-Moon plane (υ)

AZM	Azimuth of the SOI point
AZME	Azimuth (from the Earth) of the SOI point
AZMM	Azimuth (from the Moon) of the SOI point
BETA	Moon's ecliptic latitude
CMUX	Earth or Moon gravitational parameter
COSAEX	Cosine of the eccentric anomaly of one of the transfer orbits
COSAINC	Cosine of the Lunar orbit inclination
COSALAT	Cosine of the SOI point latitude
COSALON	Cosine of the SOI point longitude
COSANGA	Cosine of the angle between the Earth-to-Moon line and the Earth-to-SOI-point line
COSGAM	Cosine of the flight path angle at SOI of the SOI-to-Moon trajectory
COSGAME	Cosine of the flight path angle at SOI of the Earth-to-SOI trajectory
COSGAMX	Cosine of the flight path angle at SOI of one of the transfer orbits
COSHF	Hyperbolic cosine of the eccentric anomaly of one of the transfer orbits
COSPHI	Cosine of the angle between the Moon-to-Earth line and the Moon-to-SOI-point line
COSTHETAX	Cosine of the true anomaly of one of the transfer orbits
DATP	Today's date
DECM	Declination of the Moon
DELT	A fraction of TIM that represents a half-second
DELV(•,•)	Total ΔV for pairs of SOI longitudes and latitudes
DELVEL	ΔV at the SOI point
DELVELE	Estimated ΔV required for the Earth-to-SOI trajectory in order to satisfy the time-of-flight requirement

DVCIRE	ΔV between Earth circular orbit and Earth-to-SOI trajectory
DVCIRM	ΔV between Lunar circular orbit and Moon-to-SOI trajectory
DVE	Unused plane change variable
DVMIN	Hold variable for minimum total ΔV
DVSIM	Hold variable for minimum SOI ΔV
DVTOTAL	Total ΔV for flight
DVTSAV	Temporary storage for total ΔV
EEMIN	Eccentricity of SOI-to-Moon trajectory
EEMINE	Eccentricity of Earth-to-SOI trajectory
EEX	Eccentricity of one of the transfer orbits
ERRRP	Difference between orbital range at SOI and apogee range
FFX	Hyperbolic eccentric anomaly
GAMA	Flight path at SOI of one of the transfer orbits
GAMAE	Flight path at SOI of Earth-to SOI trajectory
GAMAM	Flight path at SOI of SOI-to-Moon trajectory
GAMAX	Flight path angle at SOI of one of the transfer orbits
HEAD	Heading that indicates leg of journey ("TRANSEARTH" or "TRANSLUNAR")
ICALL	Flag to track occurance of call to in-program subroutine.
II	Counter (1-19), representing increments in latitude for the output matrices
IHYPER	Indicator that describes whether an orbit is hyperbolic
IP	Output number indicating output to file
IPRINT	Flag for print block that contains matrix column headers
IS	Output number indicating output to screen
L	A geocentric direction cosine

LAM	Moon's ecliptic longitude
M	A geocentric direction cosine
MOOD	Leg of journey (-1: return. +1: outbound)
N	A geocentric direction cosine
NN	Counter (1-10) representing increments in longitude for the output matrices
NPI	Iteration counter. Used to iterate through 19 matrix rows during report printing
NV	Iteration counter. Used to iterate through 41 values of VELM for each latitude/longitude combination, in search of the minimum ΔV.
PAGE1(•)	Total ΔV for a given SOI longitude and latitude. Cell values for Report #1 matrix.
PAGE2(•,•)	SOI ΔV for a given SOI longitude and latitude. Cell values for Report #2 matrix.
PAGE3(•,•)	Longitude for the Lunar ascending or descending node for a given SOI longitude and latitude. Cell values for Report #3 matrix.
PAGE4(•,•)	Longitude for the Earth ascending or descending node for a given SOI longitude and latitude. Cell values for Report #4 matrix.
PAGE5(•,•)	ΔV between Earth circular orbit velocity and trans-Lunar trajectory velocity for a given SOI longitude and latitude. Cell values for Report #5 matrix.
PAGE6(•,•)	ΔV between Lunar circular orbit velocity and trans-Earth trajectory velocity for a given SOI longitude and latitude. Cell values for Report #6 matrix.
PHIE	ANGA + 180°. Used to determine the azimuth of the SOI point where 0° Earth longitude is in the -X direction.
PHIM(•)	Angle between the Moon-to-Earth line and the Moon-to-SOI-point line
PIE	Horizontal parallax
PPX	Semi-latus rectum of one of the transfer orbits
QQEMIN	Vis-viva parameter for the Earth-to-SOI-point trajectory
QQX	Vis-viva parameter for an orbit
RAM	Right ascension of the Moon

RM	Distance from the Earth to the Moon in Earth radii
RMR	Ratio of Earth-Moon distance to Moon-SOI distance
RM1	Distance from the Earth to the Moon in Earth radii
RM2	Distance from the Earth to the Moon in Earth radii
RPE	Distance from Earth's center to Earth perigee orbit
RPHID	Orbital path component of the velocity vector
RPM	Distance form Moon's center to Lunar perigee orbit
RPX	Distance from Earth or Moon center to perigee
RRD	Radial component of the velocity vector
RRE	Distance from the Earth to the SOI point
RREMER	Moon's distance from the Earth (Earth radii)
RRM	Distance from the Moon to the SOI point, when the Moon is at its mean distance from the Earth
RRMB	Distance of the Moon from the Earth-Moon baricenter
RRX	Distance from Earth or Moon to SOI
SD	Semi-diameter of the Moon's orbit
SINAEX	Sine of the eccentric anomaly of one of the transfer orbits
SINAINC	Sine of the Lunar orbit inclination
SINALAT	Sine of the latitude of the SOI point
SINALON	Sine of the longitude of the SOI point
SINANGA	Sine of the angle between the Earth-to-Moon line and the Earth-to-SOI point line
SINHF	Hyperbolic sine of the eccentric anomaly of one of the transfer orbits
T	Number of Julian centuries since the year 2000 AD
TEMP	Temporary hold variable

THETAX	True anomaly of one of the transfer orbits
TIEM	Earth-to-SOI time of flight (seconds)
TIEMS	Temporary storage for Earth-to-SOI time of flight
TIM	Time of arrival at SOI point from Earth, in centuries since the year 2000 AD
TIME1	Earth-to-SOI time of flight (seconds)
TIME2	Earth-to-SOI time of flight (seconds)
TIMM	Time of flight from SOI to Lunar perigee
TIMP	Time Now
TIMT	Total time of flight
TIMX	Time of flight from Earth or Moon perigee to SOI
TRAJ	Text that describes whether an orbit is hyperbolic or elliptical
TRAJE	Text that describes whether the Earth-to-SOI trajectory is hyperbolic or elliptical
TRAJM	Text that describes whether the SOI-to-Moon trajectory is hyperbolic or elliptical
T1	One-half second before TIM
T2	One-half second after TIM
VCIRE	Velocity of Earth circular orbit
VCIRMP	Velocity of Lunar circular orbit
VCIRX	Velocity of the Earth or Lunar circular orbit
VEL	Velocity at SOI of one of the transfer orbits
VELE	Velocity at SOI of the Earth-to-SOI trajectory
VELE2	Ten feet per second more than VELE
VELEMIN	Minimum velocity required such that apogee of the trajectory is just at SOI
VELM	Velocity at SOI of the SOI-to-Moon trajectory

VELMMIN	Velocity at SOI of the SOI-to-Moon trajectory associated with the minimum total ΔV
VELMOUT(•,•)	Velocity at SOI of the SOI-to-Moon trajectory, for a given SOI longitude and latitude
VELMSIM	Velocity at SOI of the SOI-to-Moon trajectory associated with the minimum SOI ΔV
VLAT	Latitude component of the velocity vector
VLON	Longitude component of the velocity vector
VMAG	Velocity vector magnitude
VMAGM	Velocity at SOI of the SOI-to-Moon trajectory
VPE	Perigee velocity of the Earth-to-SOI trajectory
VPM	Perigee velocity of the SOI-to-Moon trajectory
VPX	Perigee velocity for one of the transfer orbits
VV	Velocity at SOI of one of the transfer orbits
VVMIN	Velocity at SOI point (apogee) of SOI-to-Moon trajectory
VVMINE	Velocity at SOI point (apogee) of Earth-to-SOI trajectory
VXE	X-coordinate of velocity vector at SOI for Earth-to-SOI trajectory
VXLA	X-component of the latitude component of the velocity vector
VXLO	X-component of the longitude component of the velocity vector
VXM	X-coordinate of velocity vector at SOI for SOI-to-Moon trajectory
VXR	X-component of the radial component of the velocity vector
VXX	Total X-component of the velocity vector
VYE	Y-coordinate of velocity vector at SOI for Earth-to-SOI trajectory
VYLA	Y-component of the latitude component of the velocity vector
VYLO	Y-component of the longitude component of the velocity vector

VYM	Y-coordinate of velocity vector at SOI for SOI-to-Moon trajectory
VYR	Y-component of the radial component of the velocity vector
VYX	Total Y-component of the velocity vector
VZE	Z-coordinate of velocity vector at SOI for Earth-to-SOI trajectory
VZLA	Z-component of the latitude component of the velocity vector
VZLO	Z-component of the longitude component of the velocity vector
VZM	Z-coordinate of velocity vector at SOI for SOI-to-Moon trajectory
VZR	Z-component of the radial component of the velocity vector
VZX	Total Z-component of the velocity vector
XDLO	X-coordinate of the velocity of the Moon
XX	Distance in the X-direction from the Earth to the SOI point's X-coordinate
XXM	Distance in the X-direction from the Moon to the SOI point's X-coordinate
YDLO	Y-coordinate of the velocity of the Moon
YY	Distance in the Y-direction from the Earth-Moon line to the SOI point's Y-coordinate
ZZ	Distance in the Z-direction from the Earth-Moon plane to the SOI point

Appendix D - Detailed Program Description

This section is a step-by-step description of the lines of code in program PLANECNG. Refer to Appendix B for the actual program listing, and figures J-1 through J-3 for supporting illustrations.

1. Declare the matrices DELV, VELMOUT, PAGE1, PAGE2, PAGE3, PAGE4, PAGE5, PAGE6, PHIM, ALONO, and ALATP.

2. Open the output files.

3. Define the program constants.

4. Read the program inputs.

    a. MD (leg of trip for which to perform calculations; "outbound" or "return")

    b. NP (destination node option)

    c. HPE (perigee altitude of Earth orbit)

    d. HPM (perigee altitude of Lunar orbit)

    e. TIMJ (Earth departure Julian date. Default is 2451545, representing Jan. 1, 2000 AD)

    f. ALONI, DELLON (Initial longitude and increment for output map)

    g. ALATI, DELLAT (Initial latitude and increment for output map)

    h. AINCEO (Angle between the plane of the low Earth orbit and the Earth-Moon plane)

    i. AINCM (Angle between the plane of the low Lunar orbit and the Earth-Moon plane)

    j. FTIM (flight time)

5. Calculate the distance from the Earth's center to Earth perigee orbit.
    RPE = HPE * FTNM + REE

6. Calculate the distance from the Moon's center to Lunar perigee orbit.
    RPM = HPM * FTNM + REMO

7. Store the inclinations in temporary variables (AINCE and AINC).

8. Record the current date and time (DATP and TIMP).

9. Print the headings for Report #1, "Velocity Map for Inbound/Outbound Trajectories".

10. Echo back the input values onto Report #1.

11. Initialize the velocity map matrix coordinates to 1,1 (II = rows, or latitude increments; NN = columns, or longitude increments). Set the flag IPRINT to zero, which has the effect of causing the longitude increments to be printed as a subheading. Initialize the minimum ΔV hold variable (DVMIN) to 99999.

12. Calculate the orbital velocity of the Moon.

$$YDLO = \sqrt{\mu E / RREM}$$

13. Convert the initial map latitude and longitude from degrees to radians (ALAT and ALON).

14. Initialize DVSIM at 99999. This variable will be used to hold the lowest ΔV for the trajectory, selected from all combinations of latitude and longitude.

15. Initialize the current Total ΔV matrix cell (DELV) to 99999.

16. Initialize with zeros the print line variables for the current cell on each report (PAGE1, PAGE2, PAGE3, PAGE4, PAGE5, PAGE6).

17. Determine whether the current latitude for the map is less than the Lunar orbit inclination. If the absolute value of the map latitude is greater than the inclination, then set the current latitude/longitude cell for all the report matrices to zero. Otherwise, perform the following iteration 41 times, incrementing the value of VELM by 100 feet per second each iteration, beginning at 800 ft/sec and ending at 4800 ft/sec.

    a. Call the in-program subroutine given the current latitude, longitude, and VELM to determine ΔV, flight times, and nodes.

    b. Leave the iteration if the Earth-to-SOI flight time is near zero.

    c. If the total ΔV for this iteration of VELM is the smallest so far for the current latitude/longitude, then store the following information in the current longitude/latitude cell of the appropriate report matrix:

    - Total ΔV (DVTOTAL)
    - SOI ΔV (DELVEL)
    - Velocity at SOI for the SOI-to-Moon trajectory (VELMOUT)
    - Earth and Lunar ascending/descending nodes (ANODEE, ANODEM)
    - Earth and Lunar circular orbit ΔV's (DVCIRE, DVCIRM)

18. Store the matrix column heading (longitude increments) for use in all the output reports.

19. If all the matrix columns have been processed for this latitude, and if this has been the first row of the matrix, print the matrix column headings for Report #1.

20. If all ten matrix columns have not been processed for this latitude, increment the column counter by one and increment the longitude by the amount specified by the user in the inputs. Return to step 15 to process the next longitude/latitude combination.

21. If all ten matrix columns have been processed for this latitude, print the matrix row for this latitude for Report #1.

22. If all 19 matrix rows have not been processed, increment the row counter by one, increment the latitude by the amount specified by the user in the inputs, reset the column counter to one, reset the longitude to the initial input longitude, and return to step 15 to process the next longitude/latitude combination.

23. If all 19 matrix rows have been processed, continue with the following steps.

24. Set VELM to the SOI velocity associated with the minimum total ΔV.

25. Set the SOI latitude (ALAT) and longitude (ALON) to those associated with the minimum total ΔV (ALATMIN, ALONMIN).

26. Call the in-program subroutine to determine the orbital characteristics associated with the minimum total ΔV.

27. Print the remaining five reports.

Appendix E - In-Program Subroutine Description

This section is a line-by-line description of the lines of code in the subroutine that is imbedded in program PLANECHG, beginning at line #25. Refer to Appendix B for the actual code listing, and figures J-2 through J-4 for supporting illustrations.

I. Establish the key Earth, Moon, and Sphere of Influence distances and angles.

A. Calculate the distance from the Moon to the SOI point.

1. Define variables for the sine and cosine of the SOI latitude and longitude (COSALAT, COSALON, SINALAT, SINALON).

2. Given a point on the SOI defined by the latitude and longitude, define the angle Φ between the Moon-to-Earth line and the Moon-to-SOI line.

$$\cos(\Phi) = \cos(\text{lat}) * \cos(\text{lon})$$

3. Define variables for the sine and cosine of the Lunar orbit inclination (COSAINC and SINAINC).

4. Calculate the ratio of Earth-Moon distance to Moon-SOI distance (RMR = REM / RM).

a. $RRE^2 = RRM^2 + RREM^2 - 2 * RRM * RREM * \cos(\Phi)$

b. $\mu E/RRE^2 = \mu M/RRM^2 \quad \therefore \quad RRE^2/RRM^2 = \mu E/\mu M$

(because μ/R^2 = gravitational acceleration, and by definition, at the sphere of influence, acceleration towards the Earth equals acceleration towards the Moon).

c. $RRE^2/RRM^2 = \mu E/\mu M = 1 + RREM^2/RRM^2 - 2*(RREM/RRM)*\cos(\Phi)$
$= 1 + RMR^2 - 2*RMR*\cos(\Phi)$

d. $0 = -(\mu E/\mu M) + 1 + RMR^2 - 2*RMR*\cos(\Phi)$
$= RMR^2 - 2*RMR*\cos(\Phi) + 1 - (\mu E/\mu M)$

e. For the quadratic formula:

$$a = 1, \quad b = -2*\cos(\Phi), \quad c = 1 - (\mu E/\mu M)$$

$$RMR = (2*\cos(\Phi) \pm \sqrt{4\cos^2 - 4(1 - \mu E/\mu M)}\,)/2$$

$$RMR = \cos(\Phi) + \sqrt{\cos^2 - 1 + \mu E/\mu M}$$

5. Determine the distance from the Moon to the SOI.

$$RRM = RREM / RMR$$

B. Determine an Earth-based rectangular coordinate system such that the X-direction is on the line from the Earth to the Moon; the Y-direction is in the direction of the Moon's orbit; and the Z-direction is perpendicular to X and Y in right-hand coordinates. Project the SOI point onto the X-Y plane (Earth-Moon plane) in order to calculate the X- and Y-coordinates.

a. Determine the distance from the Moon to the X-intercept.

$$XXM = RRM * \cos(\Phi) = RRM * COSALAT * COSALON$$

b. Determine the distance from the Earth to the X-intercept.

$$XX = RREM - XXM$$

c. Determine the Y-coordinate. Note that longitude increases in the negative Y direction.

$$YY = -RRM * COSALAT * SINALON$$

d. Determine the Z-coordinate.

$$ZZ = RRM * SINALAT$$

C. Calculate the distance from the Earth to the SOI point.

$$RRE = \sqrt{XX^2 + YY^2 + ZZ^2}$$

D. Identify the angle between the Earth-to-Moon line and the Earth-to-SOI line.

a. $\cos(ANGA) = XX / RRE$

b. $\sin(ANGA) = \sqrt{YY^2 + ZZ^2} / RRE$

c. $ANGA = \tan^{-1}(\sin(ANGA) / \cos(ANGA))$

E. Determine the Earth-based latitude and longitude of the SOI point.

a. $ALONX = \tan^{-1}(YY / XX)$

b. $ALATX = \tan^{-1} (ZZ / \sqrt{XX^2 + YY^2})$

F. Determine the Earth ascending node for the orbit in which the trans-SOI burn will occur.

a. Given the latitude of the burn at Earth (negative SOI latitude) and the inclination of the Earth orbit, spherical coordinate trigonometry states that the longitude from the node to the burn point is

$$\upsilon = \sin^{-1} (\tan(-ALATX) / \tan(AINCE))$$

b. Longitude of the burn point (measured from the Earth-Moon line) is ALONX.

c. The node is calculated by subtracting the longitude in (a) above from the longitude in (b).

$$ANODEE = ALONX - \sin^{-1}(\tan(-ALATX) / \tan(AINCE))$$

II. For the SOI-to-Moon trajectory, calculate velocity at the SOI point, ΔV at Lunar circular orbit, Lunar ascending/descending node, and time of flight.

A. Calculate flight path angle, ΔV at Lunar circular orbit, and time of flight.

a. Call the subroutine GAMACALC to determine the cosine of the flight path angle, Lunar perigee velocity, Lunar circular orbit velocity, and time of flight.

b. The cosine of the flight path angle will be between zero and one, corresponding to angles between zero and 90. In reality, Moon-to-SOI flight path angles will range from zero to 90, but SOI-to-Moon angles will range from zero to -90. Gamma must be derived from cos(GAM) and multiplied by -MOOD (from the input: +1 for outbound and -1 for return) to get the correct flight path angle. The arctan function is used instead of arccos to evaluate gamma because of that function's better debugging diagnostics capabilities.

$$GAMAM = -MOOD * \tan^{-1}(\sqrt{1 - COSGAM^2} / COSGAM)$$

B. Identify the angle between the Moon-to-Earth line and the Moon-to-SOI line.

a. Calculate $\Phi(1)$ for node option 1.

$$\Phi(1) = \sin^{-1}(\sin(latitude) / \sin(inclination)$$
$$\Phi(1) = \sin^{-1}((ZZ / RRM) / \sin(AINCM))$$

b. Calculate $\Phi(2)$ for node option 2.

$$\Phi(2) = \Pi - \Phi(1) \text{ if ZZ is positive, or}$$
$$\Phi(2) = -\Phi(1) - \Pi \text{ if ZZ is negative, or}$$
$$\Phi(2) = \Pi \text{ if ZZ is zero.}$$

C. Determine the azimuth of the SOI point.

$$AZMM = \tan^{-1}(\cot(AINCM) / \cos(\Phi(NP)))$$
$$= \tan^{-1}(\cos(AINCM) / (\sin(AINCM) * \cos(\Phi(NP))))$$

where NP is the node option from the input.

D. Determine the Lunar ascending node.

a. Calculate ν (upsilon) for a Lunar orbit that has inclination AINCM and that passes through the SOI point.

$$AVM = \tan^{-1}(\sin(ALAT) / (\cos(\Phi) * \tan(AINCM)))$$
$$= \tan^{-1}((\tan(ALAT) * \cos(ALAT)) / (\cos(\Phi) * \tan(AINCM)))$$
$$= \tan^{-1}((\tan(ALAT) / \tan(AINCM)) / (\cos(\Phi) / \cos(ALAT)))$$

b. Determine the node.

$$ANODEM = ALON - AVM$$

E. Determine the components of the velocity vector at the SOI point.

a. Call the subroutine VELTRANS to determine the X-, Y-, and Z-components of the velocity vector, from the viewpoint of the moon.

b. Add the X- and Y-components of the Moon's velocity to determine the SOI velocity from the viewpoint of the Earth.

$$VXM = VXM + XDLO \qquad VYM = VYM + YDLO$$

III. For the Earth-to-SOI trajectory, calculate velocity at the SOI point, ΔV at Earth circular orbit, and time of flight.

A. Modify the velocity at SOI, if necessary, to bring the trajectory up to at least the minimum velocity required to intercept the SOI.

a. Calculate the vis-viva parameter QQEMIN for the Earth-to-SOI trajectory.

$$QQEMIN = 2 - (RRE / a) \quad \text{where } a = (RPE + RRE) / 2$$
$$QQEMIN = 2 - ((2 * RRE) / (RPE + RRE))$$
$$QQEMIN = 2 * (1 - (RRE / (RPE + RRE)))$$
$$QQEMIN = 2 * (RPE / (RPE + RRE))$$

b. Calculate the minimum velocity required, such that apogee is just at SOI.

$$QQEMIN \equiv (V^2 * RRE) / CMUE$$
$$\therefore\ VELEMIN = \sqrt{(QQEMIN * CMUE) / RRE}$$

c. Increase the velocity slightly so that the calculations converge.

d. Compare the most recently calculated SOI velocity (VELE) with the minimum velocity. If it is too low, bring it up to the minimum.

B. Temporarily store (until step IV-D) the calculated Earth-to-SOI time of flight.

C. Further increase the velocity at SOI, if necessary, to meet the Earth-to-SOI time of flight requirement (but capping the ΔV at 500 ft/sec). Calculate the new SOI flight path angle, ΔV at Earth circular orbit, and time of flight.

a. Call the subroutine GAMACALC to determine the time of flight from Earth perigee to SOI (TIMEE1), given the velocity VELE.

b. Increase the velocity by 10 ft/sec, and call GAMACALC again to determine the new time of flight (TIMEE2).

c. For the Earth-to-SOI leg, determine the ratio of time-of-flight shortfall to the time-of-flight increase that was due to the 10 ft/sec increase in velocity.

Shortfall/Increase = (FTIME-TIMEE1-TIMEM) / (TIMEE2 - TIMEE1)

Apply this ratio to the increased velocity of 10 ft/sec to estimate the ΔV necessary to meet the required time of flight.

DELVELE = 10 * (FTIME - TIMEE1 - TIMEM) / (TIMEE2 - TIMEE1)

d. Cap this ΔV at 500 ft/sec, and add it to the original velocity.

e. Call the subroutine GAMACALC to determine the new Earth-to-SOI time of flight for the revised velocity.

D. Compare the total Earth-to-Moon time of flight to the required time of flight. If the total is short, return to step III-C.

E. Determine the components of the velocity vector at the SOI point.

a. Calculate the flight path angle at SOI (GAMAE) for the Earth-to-SOI trajectory (see step II-A-b for a discussion of this).

b. Determine the longitude of the SOI from the Earth's viewpoint.

ALONE = ALONX + 180 (since zero longitude is on the Earth-Moon line on the side of the Earth away from the Moon).

c. Determine the latitude of the SOI point.

$\text{ALATE} = \tan^{-1} (\text{ZZ} / \sqrt{\text{XX}^2 + \text{YY}^2})$

d. Calculate the azimuth of the SOI point

$\text{AZME} = \tan^{-1} (\cot(\text{AINCE}) / \cos(\text{PHIE}))$

$\text{AZME} = \tan^{-1} (1 / (\tan(\text{AINCE}) * \cos(\text{PHIE})))$

where $\text{PHIE} = \Pi - \sin^{-1}((\text{ZZ}/\text{RRE})/\sin(\text{AINCE}))$ -- see step II-B-b for a discussion of this.

e. Call the subroutine VELTRANS to determine the X-, Y-, and Z-components of velocity at SOI for the Earth-to-SOI trajectory.

IV. Combine the SOI-to-Moon calculations with the Earth-to-SOI calculations to identify total flight characteristics.

A. Calculate total ΔV

a. Calculate ΔV at the SOI point required to patch the Earth-to-SOI trajectory into the SOI-to-Moon trajectory. (This value is calculated by determining the difference between each of the components of the two trajectories).

$$DELVEL = \sqrt{(VXE - VXM)^2 + (VYE - VYM)^2 + (VZE - VZM)^2}$$

b. Calculate ΔV at Lunar perigee.

DVCIRM = VPM - VCIRMP

c. Calculate ΔV at Earth perigee.

DVCIRE = VPE - VCIRE

d. Save the old total ΔV.

DVTSAV = DVTOTAL

e. Calculate the new total ΔV.

DVTOTAL = DELVEL + DVCIRM + DVCIRE

B. Determine the distance from the Moon to the Earth for this trajectory just calculated. Use this value in subsequent iterations of the in-program subroutine, if necessary.

a. Determine the time of arrival at SOI from Earth, in centuries since the year 2000 AD.

TIM = Departure date + Time of flight - 2000

where Departure date = TIMJ / 36525 days per century
Time of flight = TIEM hrs / 876600 hrs per century
The year 2000 = Day 2451545 / 36525 days per century.

b. Call the subroutine POSVELMO to determine the Moon's distance from the Earth and velocity at the time of arrival at SOI from the Earth.

c. Convert Lunar distance from Earth radii to feet.

RREM = RREMER * 20295741 ft/radii.

C. Compare the value of Earth-to-SOI time of flight saved in step III-B above with the new value of time of flight calculated in step III-C above. If the difference has not yet converged (to less than 0.1 hour), reiterate this in-program subroutine beginning at step I-A. Otherwise, continue.

D. If the newly calculated total ΔV is the smallest ΔV calculated so far, then store it in the variable DVMIN. Also store its corresponding SOI longitude (ALONMIN), SOI latitude (ALATMIN), and SOI velocity for the SOI-to-Moon trajectory (VELMMIN).

E. If the newly calculated SOI ΔV is the smallest calculated so far, then store it in the variable DVSIM. Also store its corresponding SOI longitude (ALONSIM), SOI latitude (ALATSIM), and SOI velocity for the SOI-to-Moon trajectory (VELMSIM).

Appendix F - Description of Subroutine GAMACALC

The subroutine GAMACALC receives the parameters orbital perigee radius (RPX), orbital velocity at SOI (VV), orbital radius at SOI (RRX), and the gravitational parameter of the body being orbited (CMUX). It calculates and returns time of flight (TIMEX), the cosine of the flight path angle (COSGAMX), velocity at periapses (VPX), circular velocity at periapses (VCIRX), and an indicator describing whether the orbit is elliptical or hyperbolic (TRAJ).

1. Initialize indicators to presume an elliptical orbit. (IHYPER, TRAJ).

2. Calculate the vis-viva parameter
   $QQX = (RRX * VV^2) / CMUX.$

3. If the orbit is just barely hyperbolic (QQX is within one-millionth of 2), force the calculation to consider it elliptical (reduce QQX to just under 2).

4. If the orbit is still hyperbolic, reset the indicators to show this. Print a message on the screen announcing a hyperbolic orbit.

5. Calculate the semi-major axis of the orbit
   $AAX = RRX / (2-QQX).$

6. If the semi-major axis is very large (greater than 10^{12}) or very small (less than -10^{12}), the orbit is trapped near a parabolic trajectory. Make it hyperbolic:
   $AAX = -10^{12}.$

7. Calculate the orbit eccentricity
   $EEX = 1 - (RPX/AAX).$

8. Calculate the semi-latus rectum
   $PPX = AAX * (1 - EEX^2).$

9. Calculate the flight path angle

   a. The angular momentum of the orbit at perigee is
      HP = RPX * VELPERIGEE * cos(GAMAX).
      But at perigee, GAMAX is zero, so
      HP = RPX * VELPERIGEE.

   b. At SOI, angular momentum is
      HX = RRX * VELX * cos(GAMAX).

   c. Angular momentum is constant along a given orbit, so
      HP = HX
      RPX * VELPERIGEE = RRX * VELX * cos(GAMAX)
      $GAMAX = \cos^{-1}((RPX * VELPERIGEE) / (RRX * VELX))$

d. The velocity at perigee is

$$VELPERIGEE = \sqrt{(CMUX * RAPOGEE) / (AAX * RPX)}.$$

Since RAPOGEE / AAX = (1 + EEX), then

$$VELPERIGEE = \sqrt{CMUX * (1 + EEX) / RPX} \text{ or}$$

$$RPX * VELPERIGEE = \sqrt{RPX * (1 + EEX) * CMUX}.$$

e. Substituting (d) into (c) above yields

$$GAMAX = \cos^{-1}(\sqrt{RPX * (1 + EEX) * CMUX} / (RRX * VELX))$$

f. Substituting from (2) above yields

$$GAMAX = \cos^{-1}(\sqrt{RPX * (1 + EEX)) / (RRX * QQX)}).$$

10. Calculate the perigee velocity

$$VPX = \sqrt{CMUX * (1 - EEX) / RPX}.$$

11. Calculate the circular velocity at perigee

$$VCIRX = \sqrt{CMUX / RPX}.$$

12. Calculate the true anomaly

$$THETAX = \cos^{-1}(((PPX / RRX) - 1) / EEX)$$

13. Calculate the eccentric anomaly

$$AEX = \cos^{-1}((EEX + \cos(THETAX)) / (1 + EEX * \cos(THETAX))).$$

14. Compare the orbital radius at SOI (RRX) to the apogee radius (AAX * (1 + EEX)). If the orbital radius at SOI is greater than the apogee radius, print a message on the screen indicating the difference in nautical miles. Also display the following:

a. SOI latitude (ALAT)
b. SOI longitude (ALON)
c. vis-viva parameter (QQX)
d. semi-major axis (AAX)
e. eccentricity (EEX)
f. flight path angle (GAMAX)
g. perigee velocity (VPX).

15. Calculate the time of flight.

a. If the orbit is elliptical:

$$TIMX = \sqrt{AAX^3/CMUX} * (AEX - EEX * \sin(AEX)).$$

b. If the orbit is hyperbolic:

$$TIMX = \sqrt{-AAX^3/CMUX}*(EEX*\sinh(EEX) - \log(\cosh(EEX) + \sinh(EEX))).$$

Appendix G - Description of Subroutine POSVELMO

The subroutine POSVELMO receives the parameter TIME (number of Julian centuries from the year 2000) and returns the moon's position and velocity at that time. Specifically, it returns the moon's distance from the Earth's center, in Earth radii (RRM); velocity in the x-direction (along the Earth-Moon line), in feet per second (XDLO); and velocity in the y-direction (direction of the moon's orbit), in feet per second (YDLO).

1. Calculate a fraction of TIME that represents half a second.
   DELT = 0.5 seconds / (36525 days/century * 24 hrs/day * 3600 seconds/hr)
   = 1.58440E-10 centuries.

2. Call the subroutine MOON with the parameter (TIME minus DELTA) to determine the moon's distance in Earth radii at half a second before TIME. This distance is RM1.

3. Call the subroutine MOON with the parameter (TIME plus DELTA) to determine the moon's distance in Earth radii at half a second after TIME. This distance is RM2.

4. Calculate the velocity of the moon in the x- (radial) direction.
   XDLO = [(RM2 Earth radii - RM1 Earth radii) / (1 sec)] * 20,925,741 ft/radii.

5. Calculate the average radius of the Lunar orbit during the one second centered on TIME.
   RRM = (RM1 + RM2) / 2.

6. Determine the radius of the Lunar orbit from the Earth-Moon baricenter.
   RRMB = RRM - 0.7412789 Earth radii.

7. Calculate the moon's velocity in the direction of its orbit.

   a. Moon's apogee (Apo) = 62.83308 Earth radii.

   b. Moon's perigee (Per) = 55.68264 Earth radii.

   c. Eccentricity (e) = (Apo - Per) / (Apo + Per) = 0.06033.

   d. Semi-latus rectum (p) = Apo($1 - e^2$) = 62.60439 Earth radii
      = 1,310,038,967 feet.

   e. Earth's gravitational parameter (mu) = 1.407646822E+16 ft^3/sec^2.

   f. Angular momentum (h) = $\sqrt{mu * p}$
      = 4.29427E+12 ft^2/sec * (1 earth radii / 20,925,672.57 ft)
      = 205,215.4 ft*Earth radii/second.

   g. Y-velocity (YDLO) = h / RRMB = 205,215.4/RRMB.

Appendix H - Description of Subroutine MOON

The subroutine MOON receives the parameter T (number of Julian centuries from the year 2000) and returns the approximate location of the moon in geocentric coordinates at that time. Specifically, it returns the right ascension of the moon (RAM), declination of the moon (DECM), and distance to the moon in Earth radii (RM). The formulae are from The Astronomical Almanac For The Year 1984, page D46. All degrees are converted to radians with the conversion factor $C = \pi/180$.

1. Calculate the ecliptic coordinates of the moon.

   a. Moon's ecliptic longitude

$$
\begin{aligned}
LAM = \; & 218^\circ.32 + 481267^\circ.833T \\
& + 6^\circ.29 * \sin(134^\circ.9 + 477198^\circ.85T) \\
& - 1^\circ.27 * \sin(259^\circ.2 - 413335^\circ.38T) \\
& + 0^\circ.66 * \sin(235^\circ.7 + 890534^\circ.23T) \\
& + 0^\circ.21 * \sin(269^\circ.9 + 954397^\circ.70T) \\
& - 0^\circ.19 * \sin(357^\circ.5 + 35999^\circ.05T) \\
& - 0^\circ.11 * \sin(186^\circ.6 + 966404^\circ.05T)
\end{aligned}
$$

   b. Moon's ecliptic latitude

$$
\begin{aligned}
BETA = \; & 5^\circ.13 * \sin(93^\circ.3 + 483202^\circ.03T) \\
& + 0^\circ.28 * \sin(228^\circ.2 + 960400^\circ.87T) \\
& - 0^\circ.28 * \sin(318^\circ.3 + 6003^\circ.18T) \\
& - 0^\circ.17 * \sin(217^\circ.6 - 407332^\circ.20T)
\end{aligned}
$$

   c. Horizontal parallax

$$
\begin{aligned}
PIE = \; & 0^\circ.9508 \\
& + 0^\circ.0518 * \cos(134^\circ.9 + 477198^\circ.85T) \\
& + 0^\circ.0095 * \cos(259^\circ.2 - 413335^\circ.38T) \\
& + 0^\circ.0078 * \cos(235^\circ.7 + 890534^\circ.23T) \\
& + 0^\circ.0028 * \cos(269^\circ.9 + 954397^\circ.70T)
\end{aligned}
$$

   d. Semi-diameter of moon's orbit

$$SD = 0.2725 * PIE$$

   e. Distance to the moon in Earth radii

$$RM = 1 / \sin(PIE)$$

2. Form the geocentric direction cosines to rotate into geocentric coordinates.

   a. $l = \cos(BETA)\cos(LAM)$
   b. $m = 0.9175*\cos(BETA)\sin(LAM) - 0.3978*\sin(BETA)$
   c. $n = 0.3978*\cos(BETA)\sin(LAM) + 0.9175*\sin(BETA)$

   where $l = \cos(DECM)\cos(RAM)$, $m = \cos(DECM)\sin(RAM)$, $n = SIN(DECM)$.

3. Then:

   a. $RAM = \arctan(m/l)$ [right ascension]
   b. $DECM = \arcsin(n)$ [declination]

The errors will rarely exceed 0.2 Earth radii in distance (RM), $0^{\circ}.3$ in right ascension (RAM), and $0^{\circ}.2$ in declination (DECM).

Appendix I - Description of Subroutine VELTRANS

The subroutine VELTRANS converts an orbital velocity vector into rectangular coordinates (see figure J-5). Parameters received by the subroutine are velocity (VEL), flight path angle (GAMA), azimuth (AZM), latitude above the Earth-Moon plane (ALAT), and longitude from the Earth-Moon line (ALON). A set of intermediate calculations is performed to express the velocity vector in terms of a radial component, a latitudinal component, and a longitudinal component. Each of these three components is further resolved into x-, y-, and z-components. Finally, all three x-components, all three y-components, and all three z-components are summed to provide the total x-, y-, and z-components of velocity.

1. Conversion of velocity vector into spherical coordinates.

From the geometry, the radial component of velocity, $\dot{R}$, is calculated to be
VEL * sin(GAMA). (See figure J-6).
The component along the orbital path, $R\dot{\Phi}$, is
VEL * cos(GAMA).
This orbital path component of velocity is further resolved into a latitude component, $\dot{\mathrm{LAT}}$, and a longitude component, $\dot{\mathrm{LON}}$ (see figure J-7). Again, from the geometry,
$\dot{\mathrm{LON}} = R\dot{\Phi}$ * sin(AZM) and
$\dot{\mathrm{LAT}} = R\dot{\Phi}$ * cos(AZM).

2. Conversion of spherical coordinates into rectangular coordinates.

a. Conversion of radial component into rectangular coordinates.

Refer to figure J-8. The projection of R onto the x-y plane is
$\dot{R}$ * cos(LAT).
The negative x-component of this is
$\dot{R}$ * cos(LAT) * cos(LON)
so the x-component, $\dot{X}$, is
-$\dot{R}$ * cos(LAT) * cos(LON).
The negative y-component of $\dot{R}$ is
$\dot{R}$ * cos(LAT) * sin(LON)
so the y-component, $\dot{Y}$, is
-$\dot{R}$ * cos(LAT) * sin(LON).
The z-component, $\dot{Z}$, is
$\dot{R}$ * sin(LAT).

b. Conversion of latitude component into rectangular coordinates.

Refer to figures J-9 and J-10. $\dot{\mathrm{LAT}}$ is perpendicular to the radial vector, R. A line in the z-direction that meets the tip of $\dot{\mathrm{LAT}}$ and intersects the radius vector produces the angles a and b, where
b = 90 - LAT and
a + b + 90 = 180. Therefore,
a = LAT.
From the geometry, the z-component of $\dot{\mathrm{LAT}}$, $\mathrm{Z}\dot{\mathrm{LAT}}$, is
$\dot{\mathrm{LAT}}$ * cos(LAT).

The projection of $\dot{LAT}$ onto the x-y plane is

$\dot{LAT} * \sin(LAT)$. (see figure J-11).

The x-component of this, $XL\dot{A}T$, is

$\dot{LAT} * \sin(LAT) * \cos(LON)$.

The y-component of this, $YL\dot{A}T$, is

$\dot{LAT} * \sin(LAT) * \sin(LON)$.

c. Conversion of longitude component into rectangular coordinates.

Refer to figures J-12 and J-13. $\dot{LON}$ is always parallel to the x-y plane, so the z-component of $\dot{LON}$, $ZL\dot{O}N$, is always zero. Using the same proof as in (b) above, it can be seen that the angle between $\dot{LON}$ and the y-component of $\dot{LON}$ is equal to LON. From the geometry, the x-component of $\dot{LON}$, $XL\dot{O}N$, is

$\dot{LON} * \sin(LON)$.

The negative y-component of $\dot{LON}$ is

$\dot{LON} * \cos(LON)$,

so the y-component, $YL\dot{O}N$, is

$-\dot{LON} * \cos(LON)$.

3. Sum of the rectangular coordinates.

All of the x-, y-, and z-components are summed to provide the complete rectangular coordinates of the velocity vector.

$VXX = \dot{X} + XL\dot{A}T + XL\dot{O}N$

$VYX = \dot{Y} + YL\dot{A}T + YL\dot{O}N$

$VZX = \dot{Z} + ZL\dot{A}T + ZL\dot{O}N$.

Appendix J - Illustrations

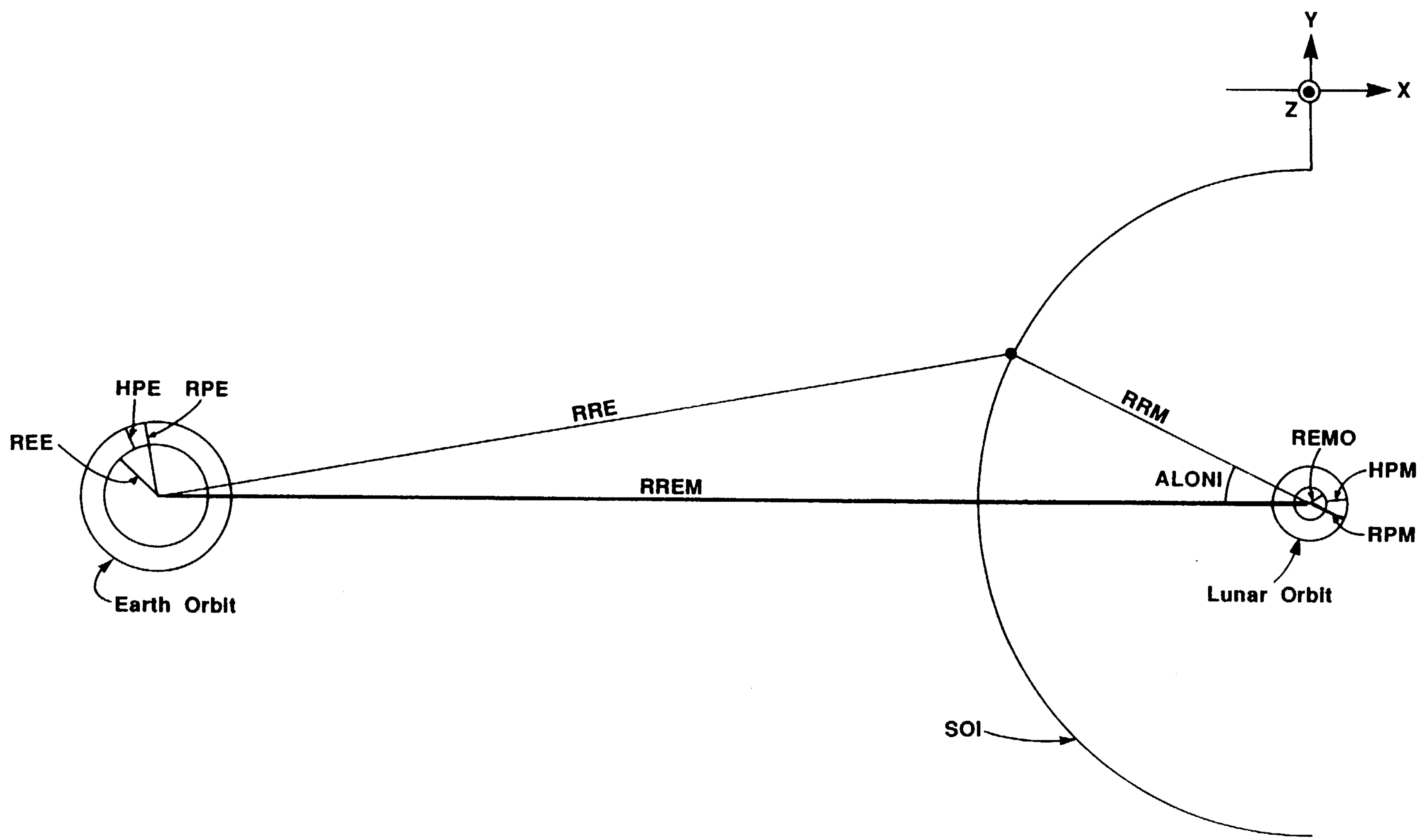

Y
X
Z
HPE
RPE
REE
Earth Orbit
RRE
RREM
RRM
ALONI
REMO
HPM
RPM
Lunar Orbit
SOI

Figure J-1

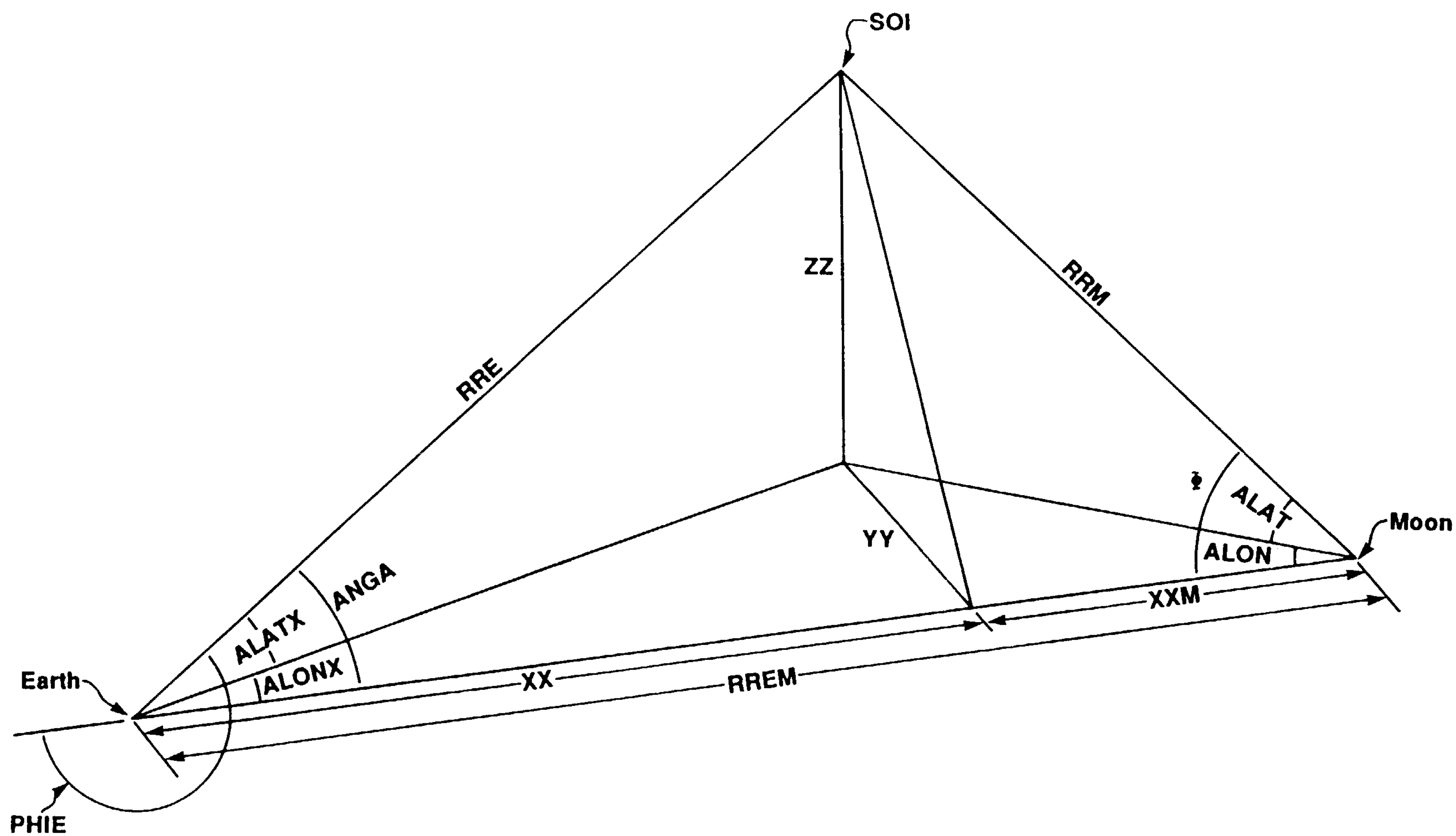

SOI
ZZ
RRE
RRM
YY
ALAT
ALON
Moon
ANGA
ALATX
ALONX
Earth
XX
RREM
XXM
PHIE

Figure J-2

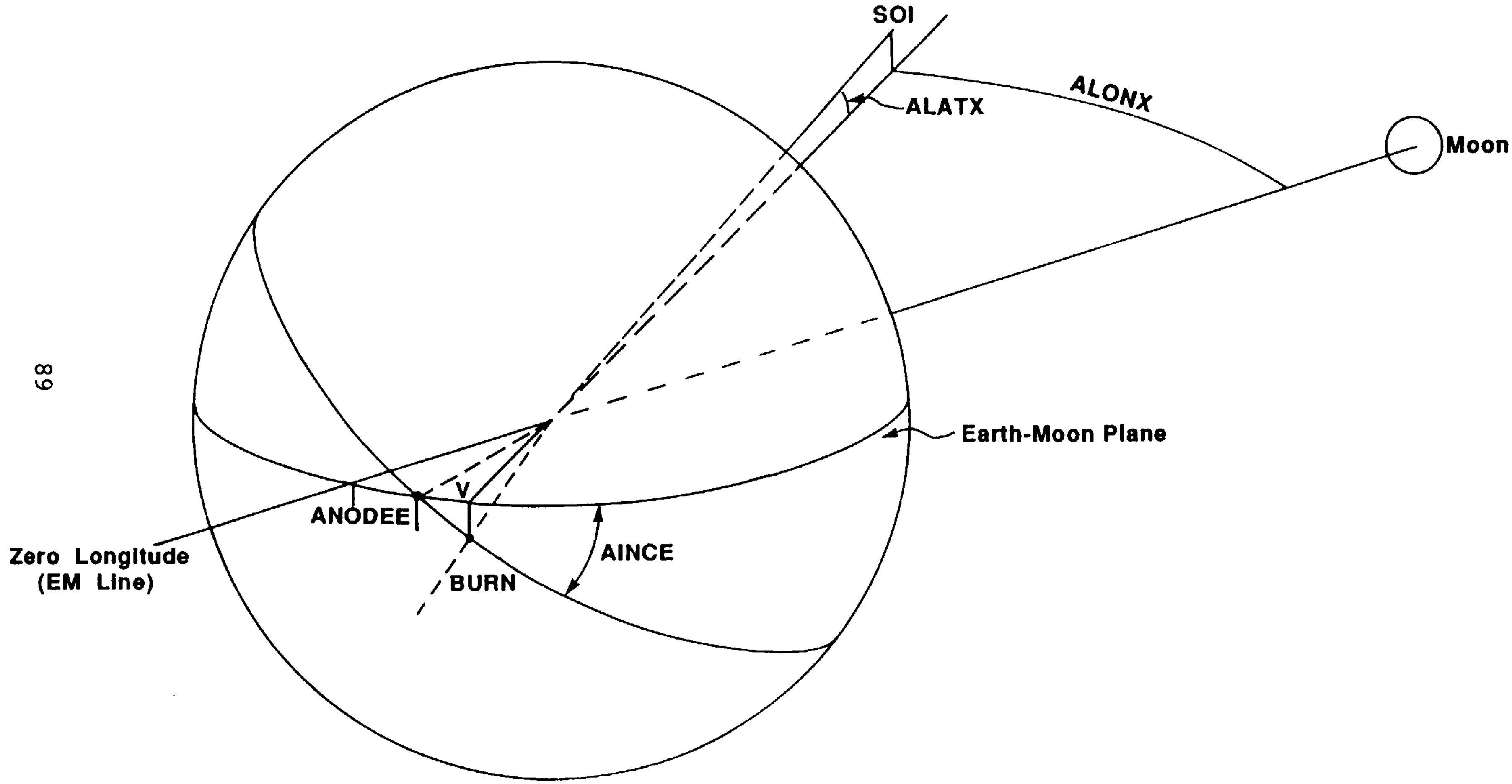

SOI
ALONX
ALATX
Moon
Earth-Moon Plane
V
ANODEE
Zero Longitude
(EM Line)
AINCE
BURN

Figure J-3

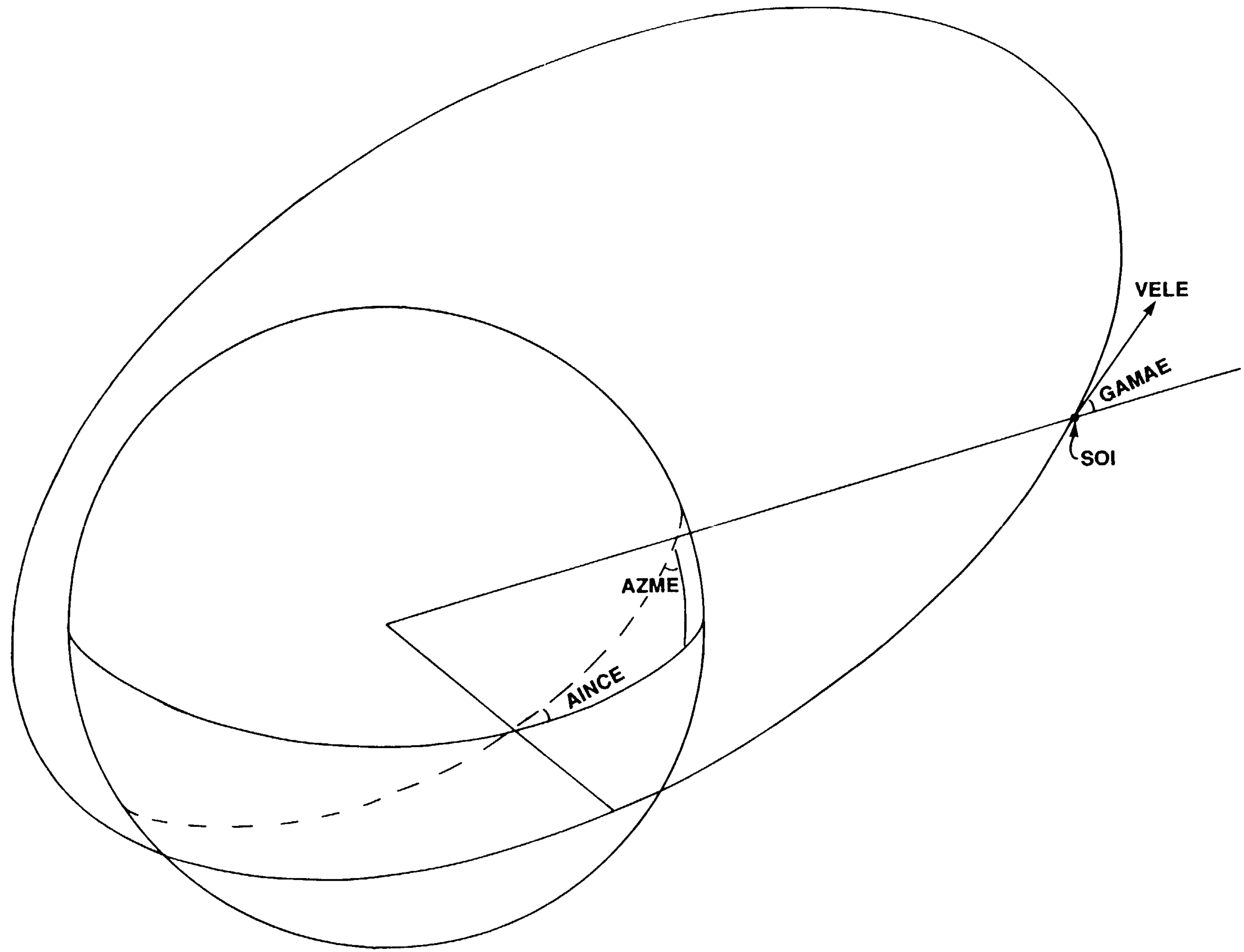


Figure J-4

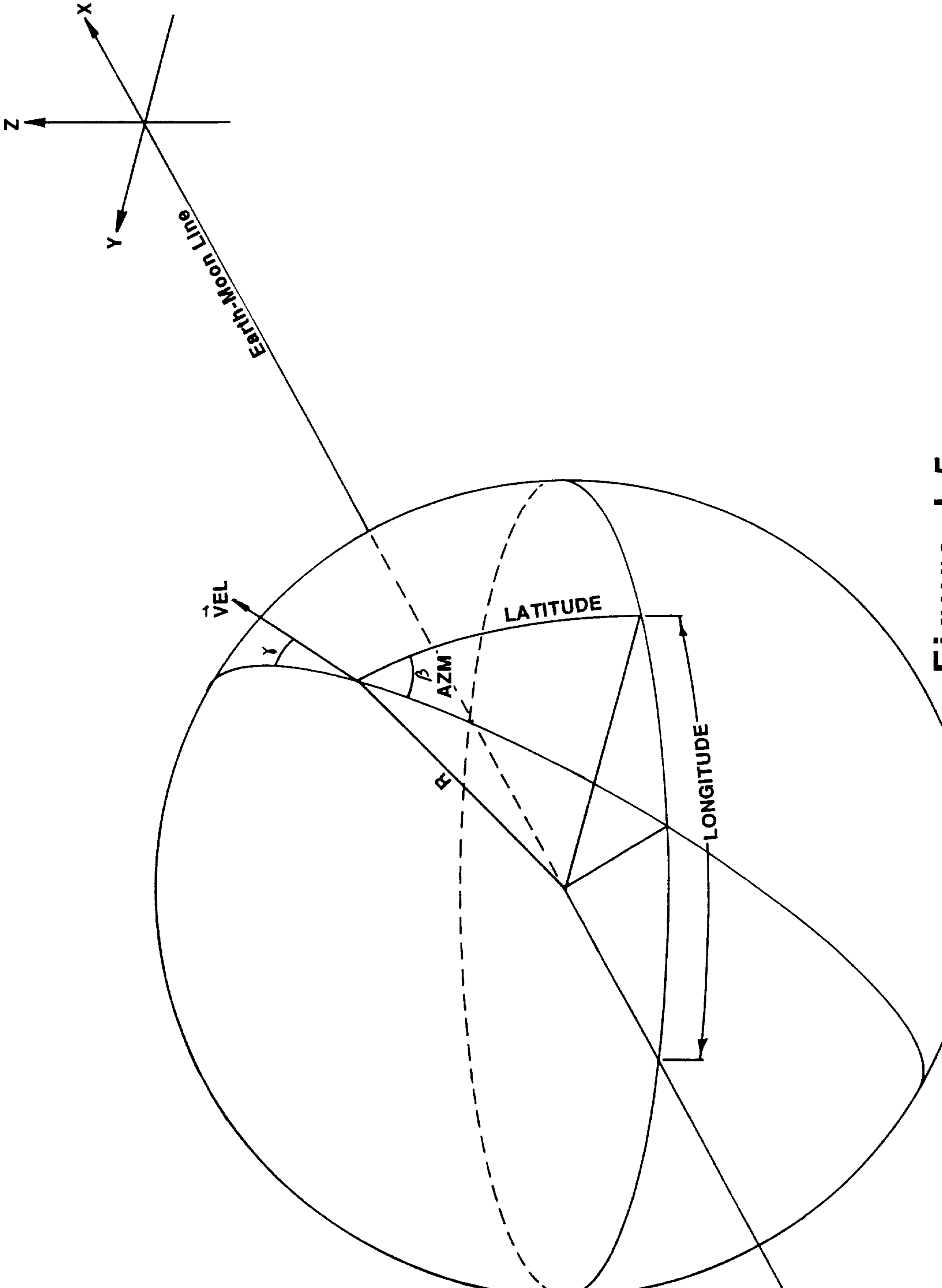


Figure J-5

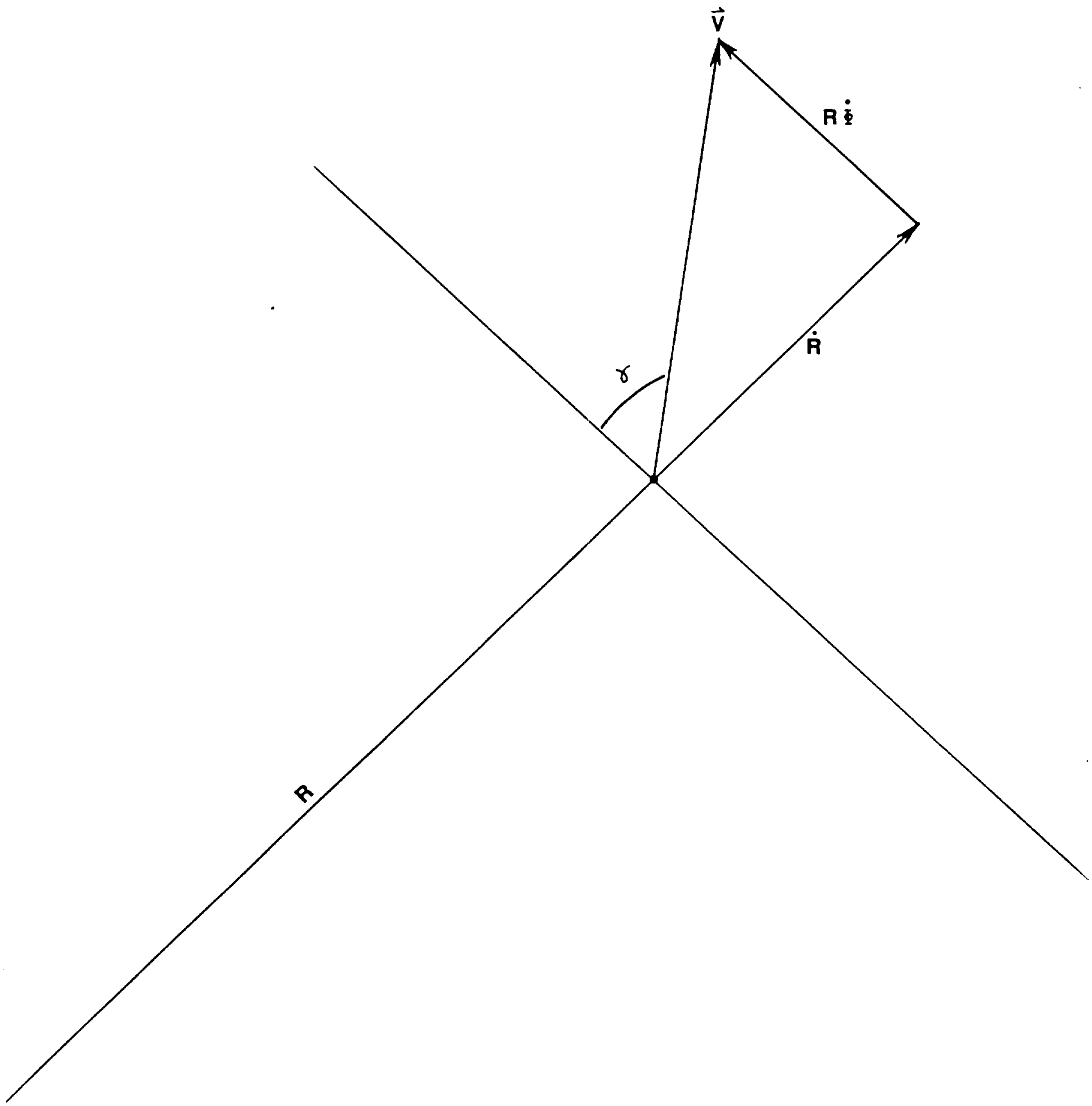


Figure J-6

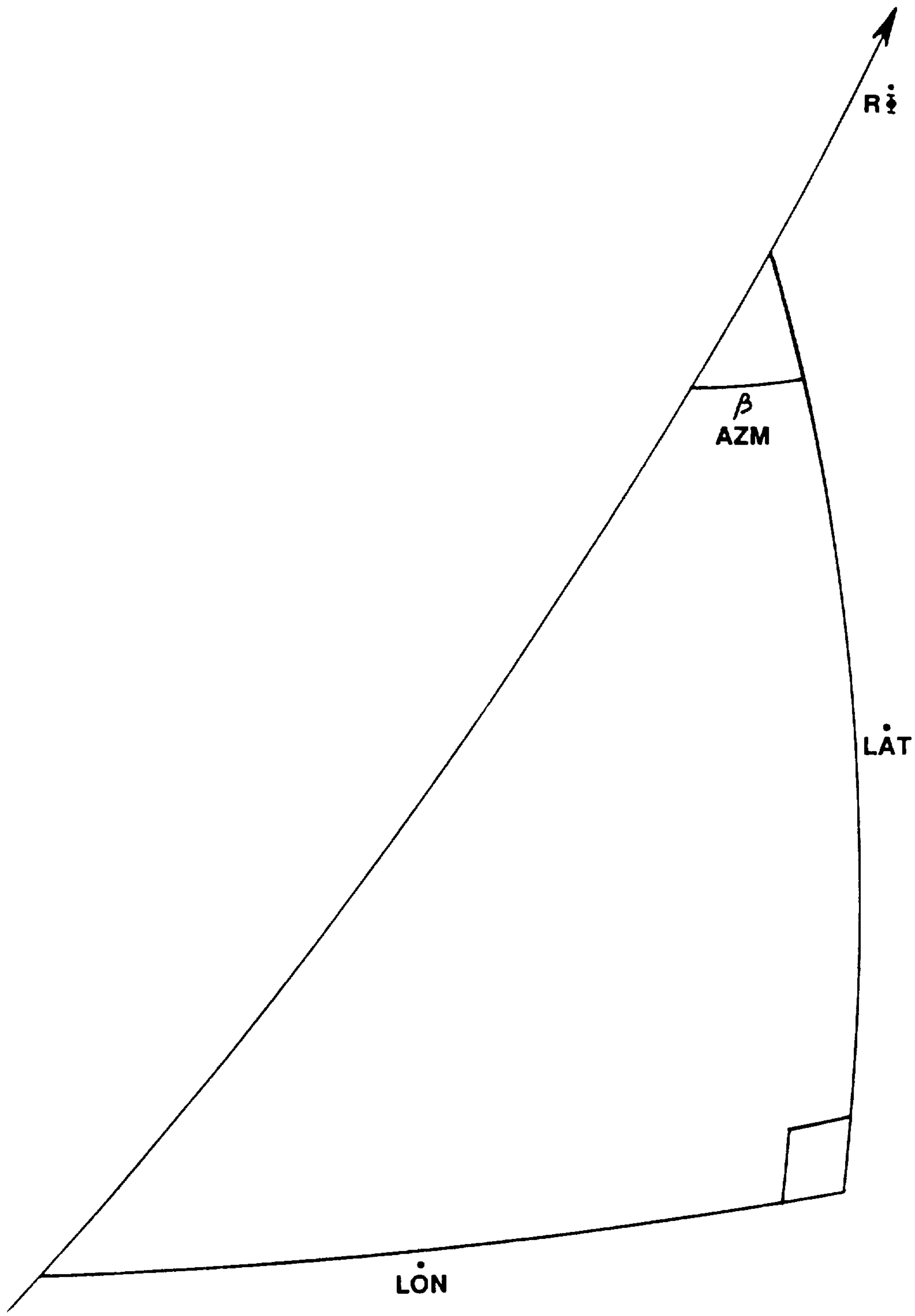


Figure J-7

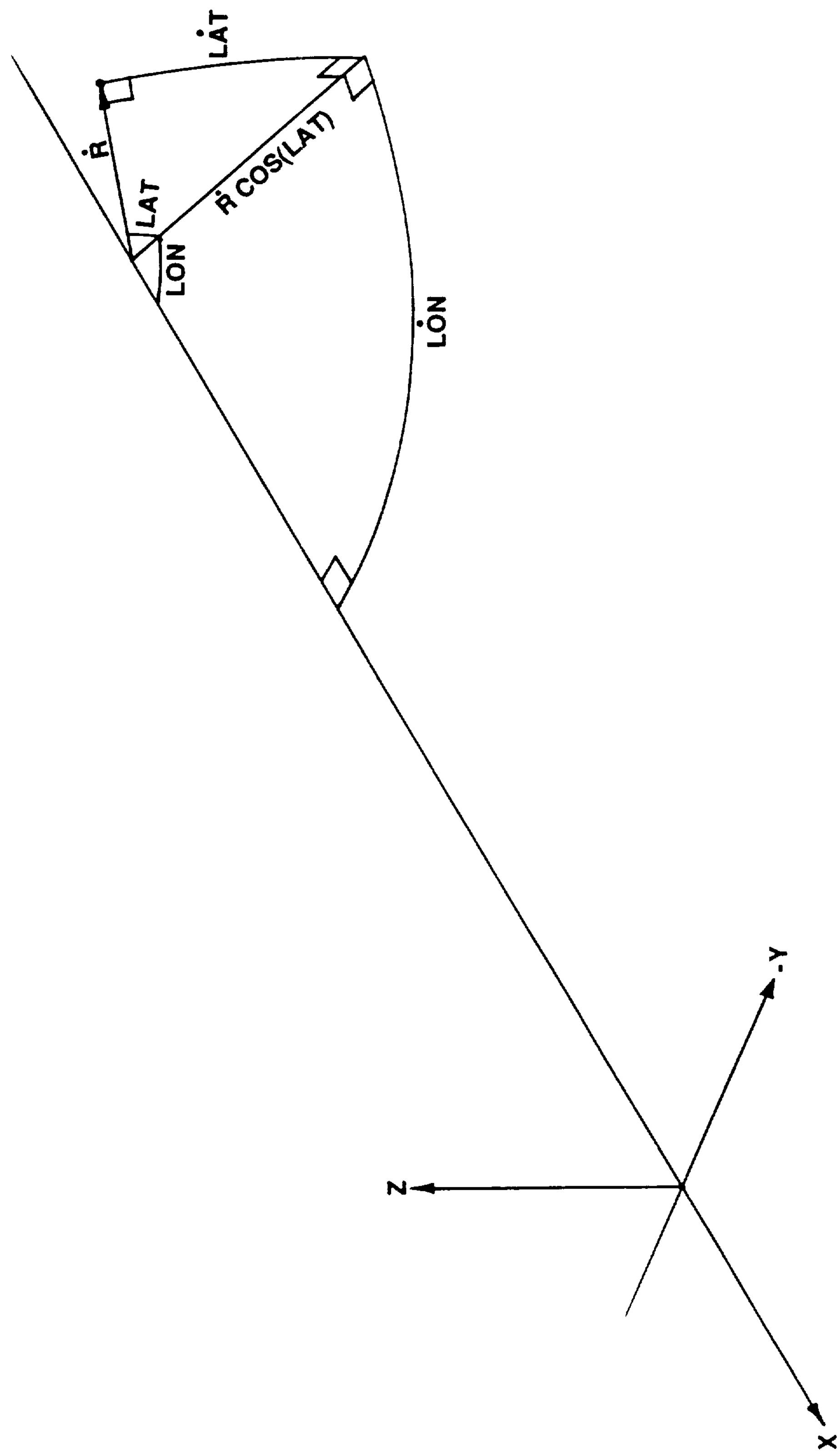


Figure J-8

C-2

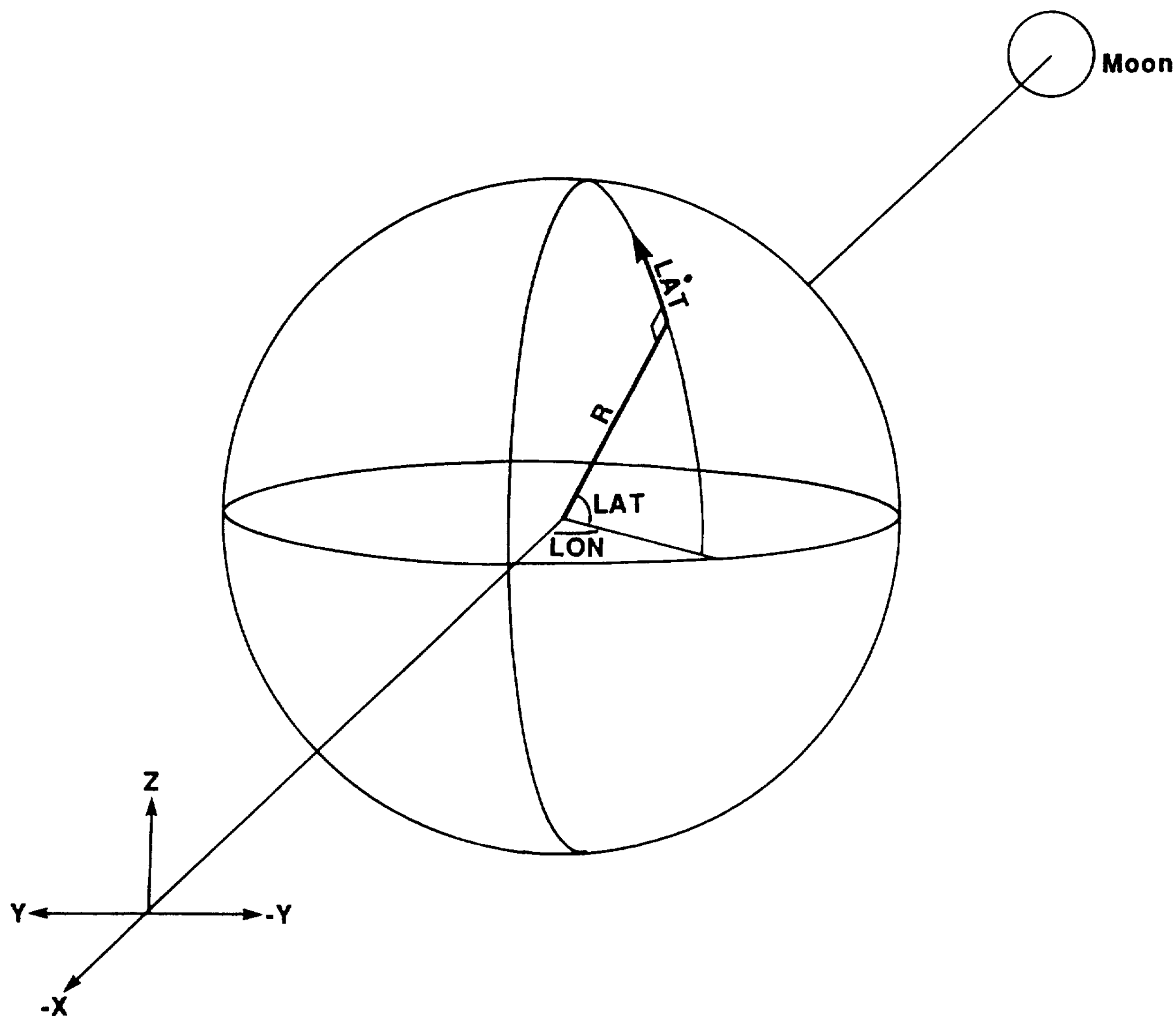


Figure J-9

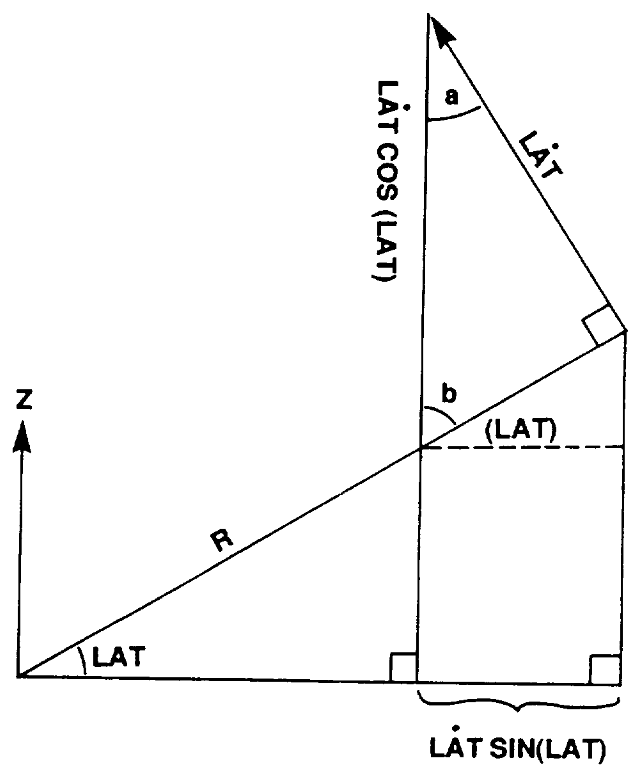


Figure J-10

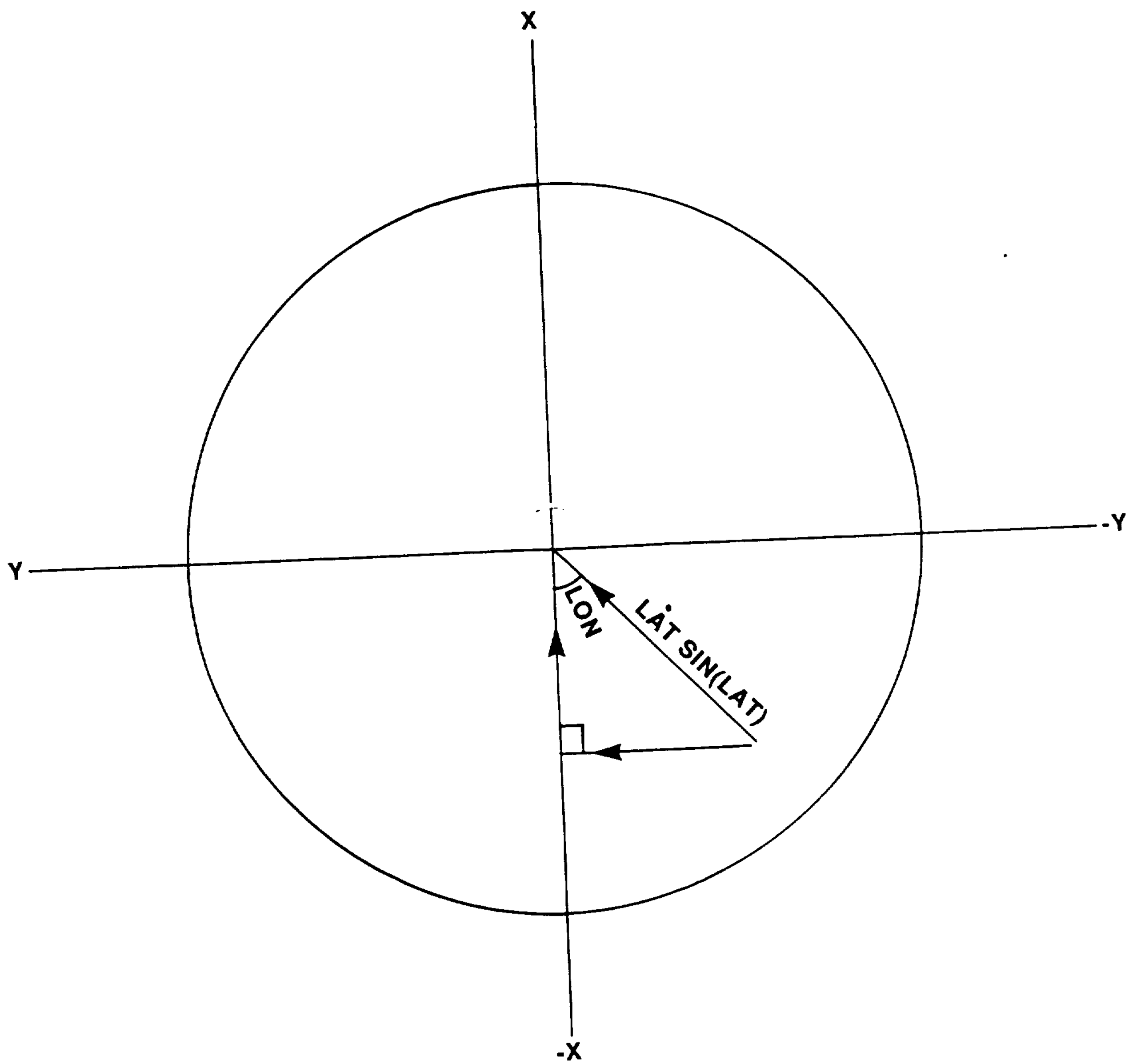


Figure J-11

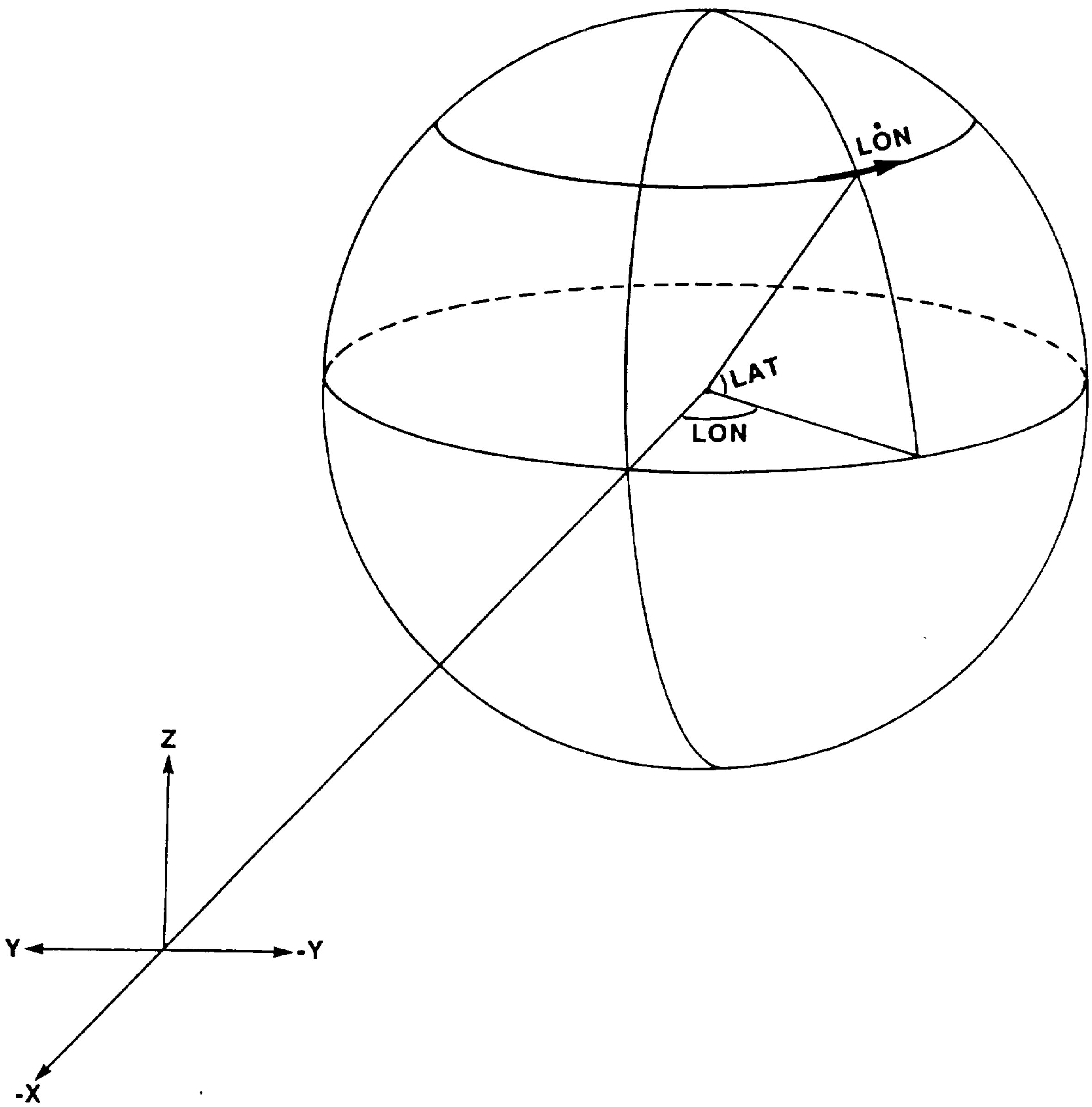


Figure J-12

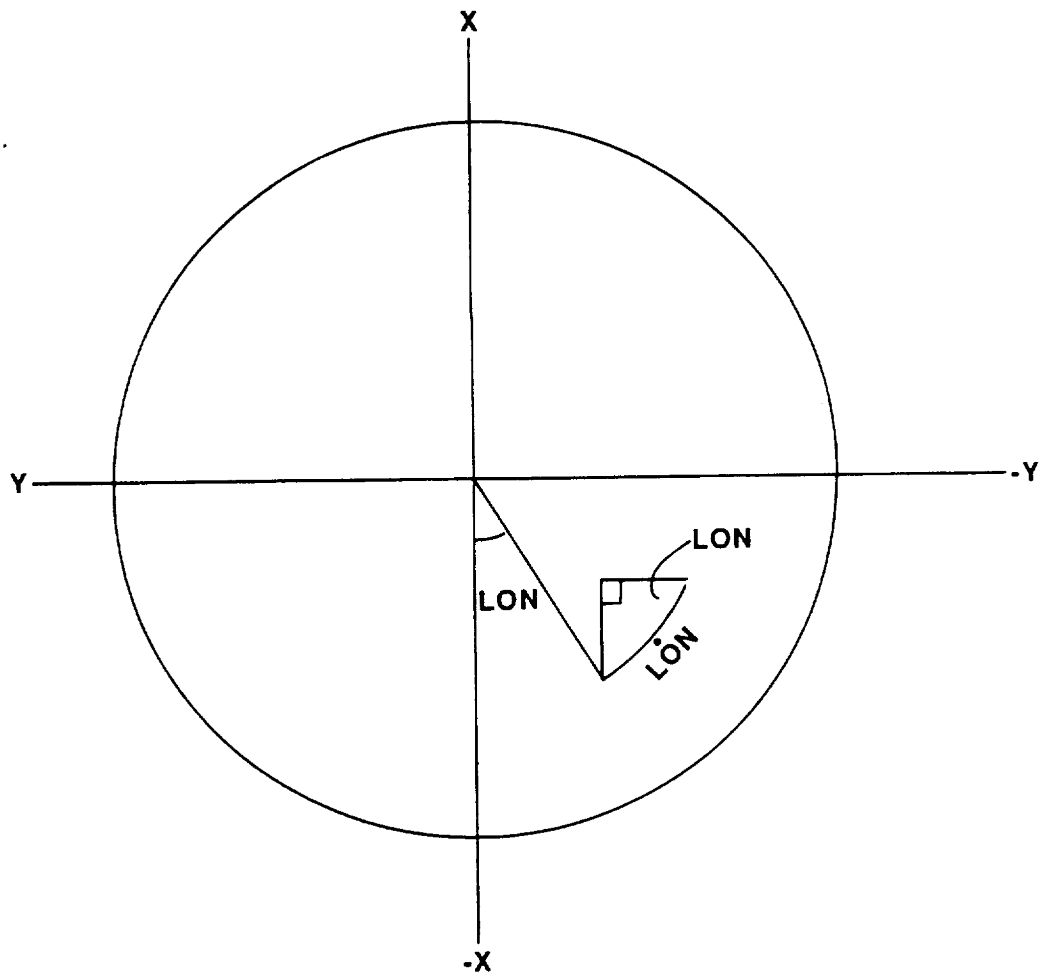


Figure J-13