Nonlinearities in Spacecraft Structural Dynamics

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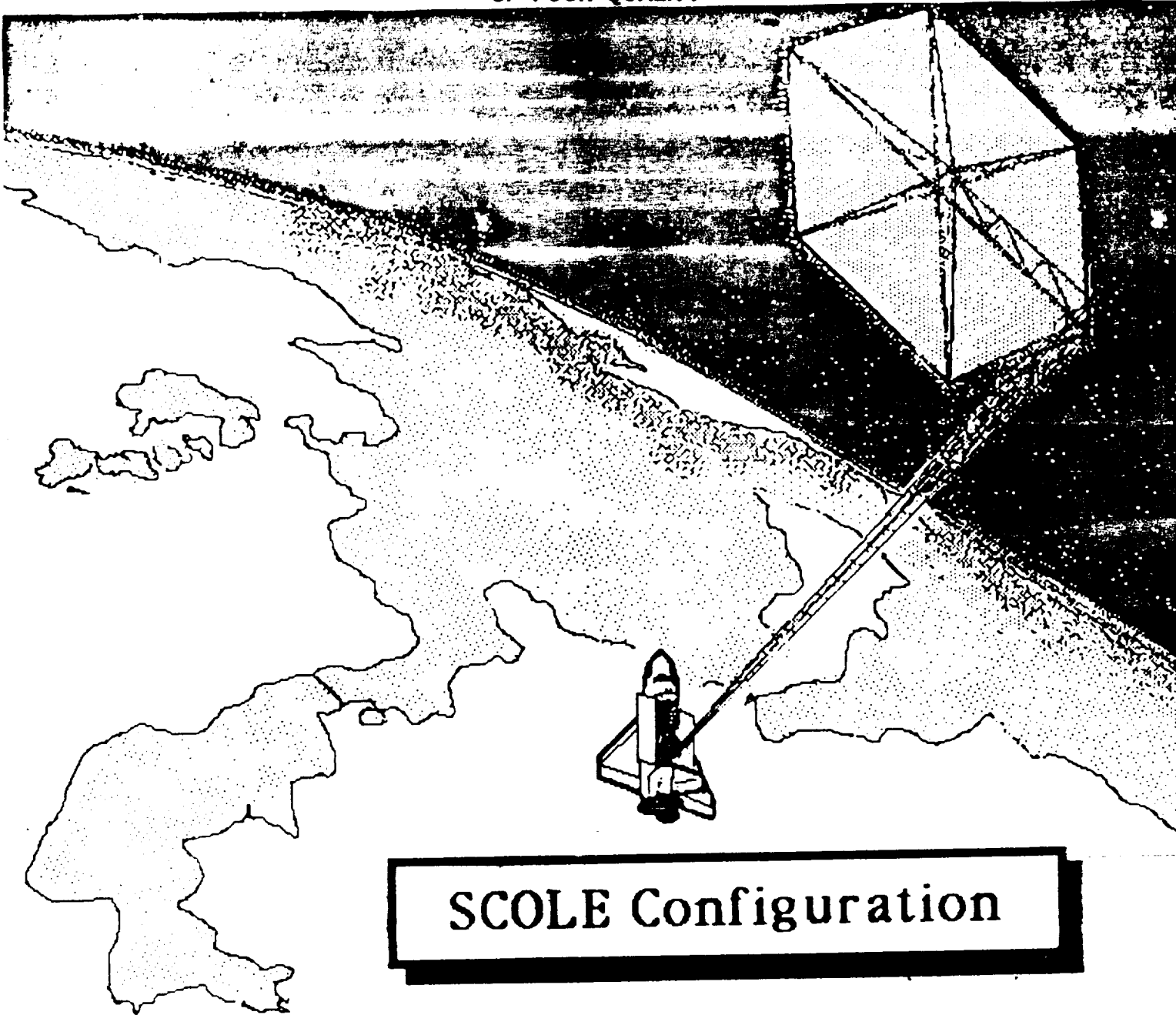and

Kelly Latimer

U.S. Air Force and G.W.U.
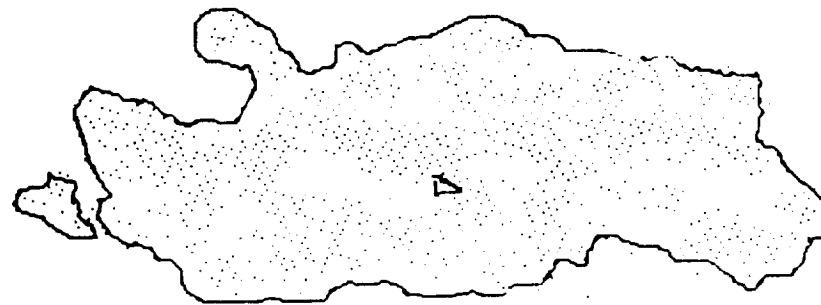
4th Annual SCOLE Workshop
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OUTLINE

- SCOLE Configuration - Equations of Motion

- Modeling Error Sources

- Approximate Solutions

- Comparison of Model Accuracy

- Linear & Nonlinear Damping

- Experimental Results

- Future Work

SCOLE Configuration

Equations of Motion

Shuttle (and Reflector) Body

$$\dot{w}_1 = -I_1^{-1}\left(\tilde{w}_1 I_1 w_1 - M_1 - M_{1,Beam}\right)$$

$$\dot{v}_1 = \left(F_1 + F_{1,Beam}\right)/m_1$$

$$\dot{T}_1^T = -\tilde{w}_1 T_1^T$$

Roll (and Pitch) Beam Bending

$$pA_\phi \frac{d^2 u_\phi}{dt^2} - CI_\phi \frac{d^3 u_\phi}{ds^2 dt} + EI_\phi \frac{d^4 u_\phi}{ds^4} = \sum_{n=1}^{4} \left[f_{\phi,n} \delta(s-s_n) \right.$$
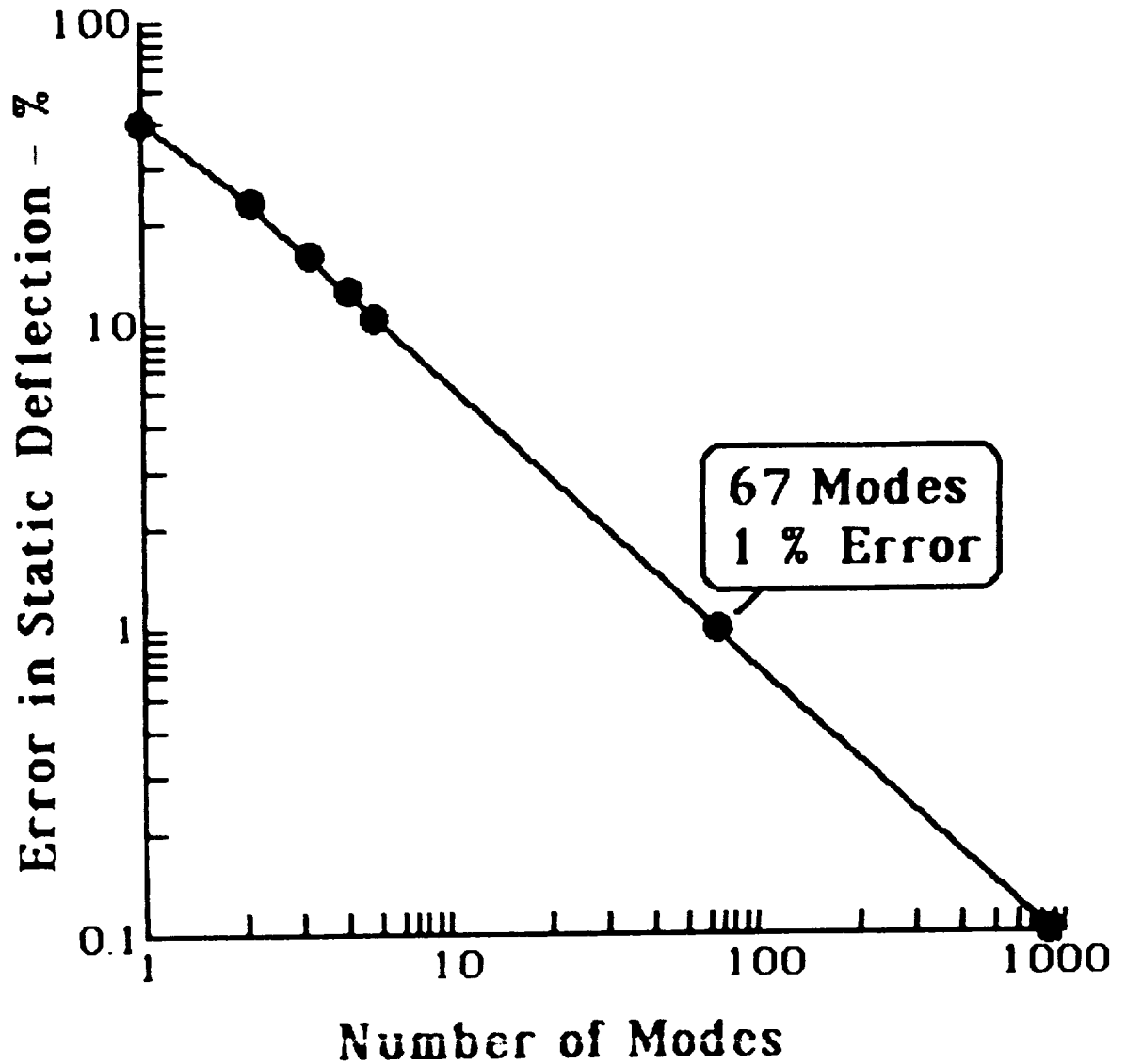$$\left. + g_{\phi,n} \frac{d\delta}{ds}(s-s_n) \right]$$

Yaw Beam Torsion

$$pI_\psi \frac{d^2 u_\psi}{dt^2} + CI_\psi \frac{d^3 u_\psi}{ds^2 dt} - GI_\psi \frac{d^2 u_\psi}{ds^2} = \sum_{n=1}^{4} g_{\psi,n} \delta(s-s_n)$$
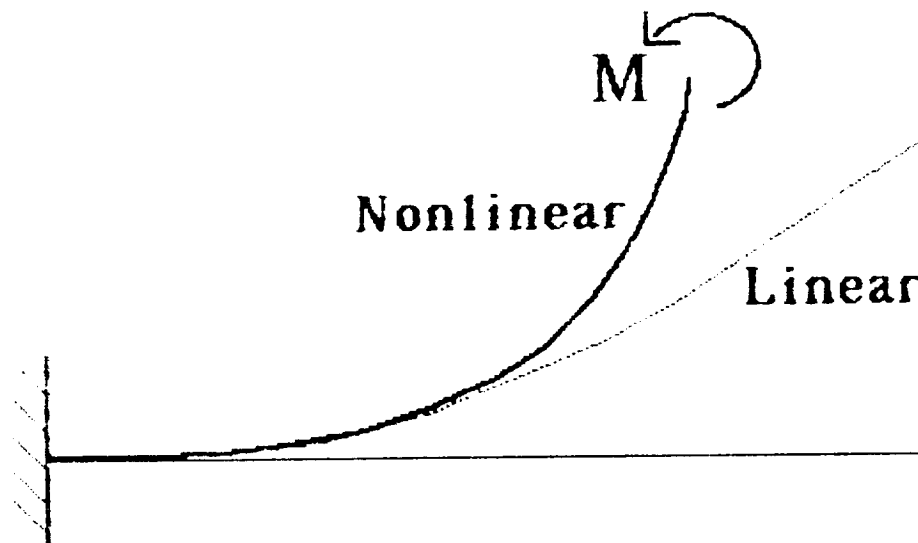
Beam Elongation

$$pA \frac{d^2 u_z}{dt^2} + C_z A \frac{d^2 u_z}{ds\,dt} - EA \frac{d^2 u_z}{ds^2} = \sum_{n=1}^{4} f_{z,n} \delta(s-s_n)$$

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Static Deflection Error

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Deflection, y/L	Error, e/y
.05	.17 %
.10	.67 %
.20	2.7 %
.30	6.0 %
.40	10.6 %
.50	16.4 %

Lumped-Mass Model

- Exact Static Deflection

- Approximates Low-Frequency Modes

- Nonlinear Kinematics

- Linearized State Space, Modal Model

- Classical Damping(Working Proportional)

- Extended to n-Body Network

Body j

y

x

$r_{j,i}$

y

z

z

x

R_j

$u_{j,i}$

$Beam_{j,i}$

$L_{j,i}$

z

x

Body i

y

y

x

R_i

y

z

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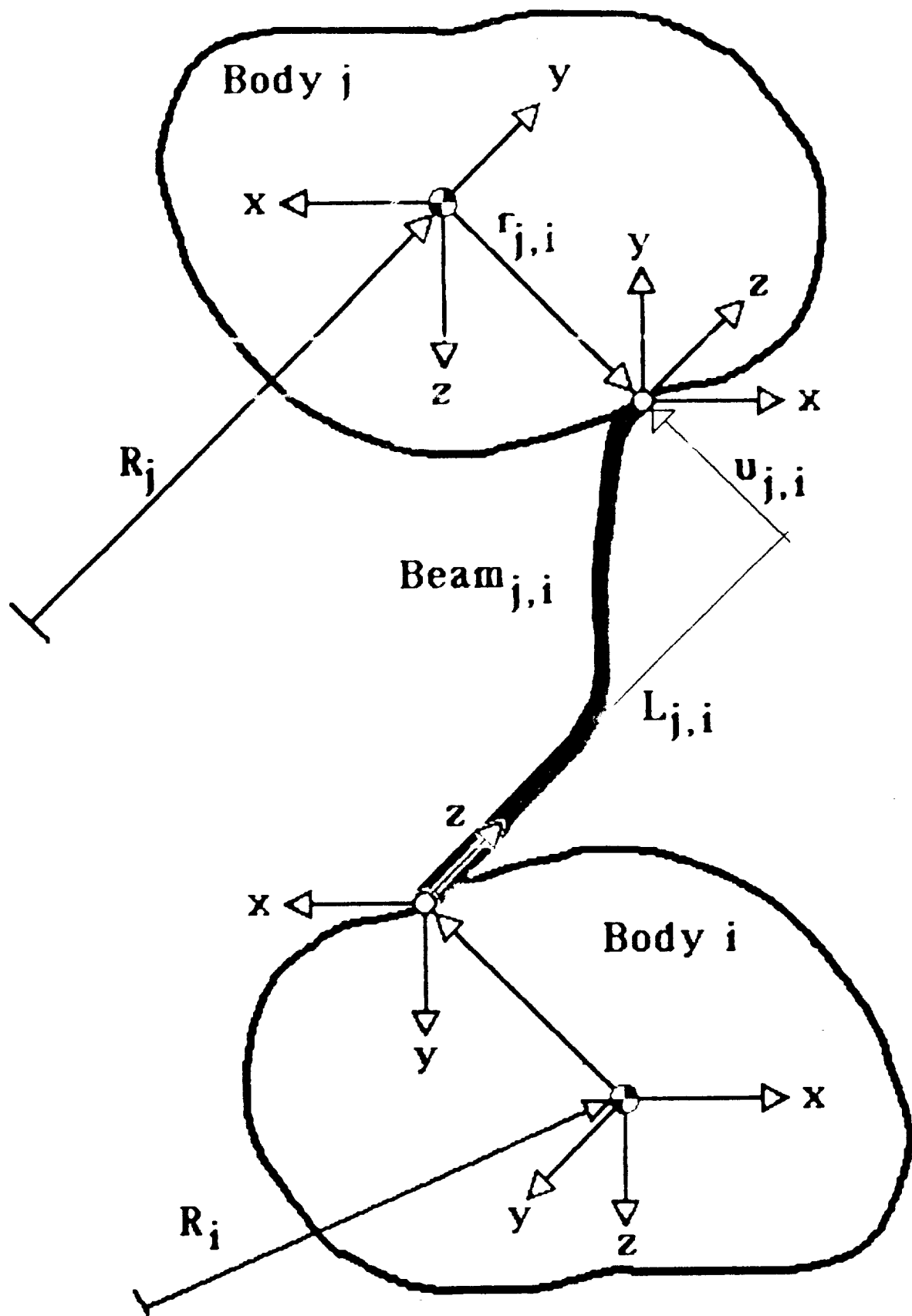
Stiffness Matrices

$$M_{\acute{u}} = \begin{bmatrix} -\dfrac{4EI}{L} - \dfrac{2WL}{15}^{*} & 0 & 0 \\[2em] 0 & -\dfrac{4EI}{L} - \dfrac{2WL}{15}^{*} & 0 \\[2em] 0 & 0 & -\dfrac{GJ}{L} \end{bmatrix}$$

$$M_{u} = \begin{bmatrix} 0 & \dfrac{6EI}{L^2} + \dfrac{W}{10}^{*} & 0 \\[2em] \dfrac{6EI}{L^2} + \dfrac{W}{10}^{*} & 0 & 0 \\[2em] 0 & 0 & 0 \end{bmatrix}$$

* Gravity Effect

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Stiffness Matrices

$$F_u = \begin{bmatrix} -\dfrac{12EI}{L^3} - \dfrac{6W^*}{5L} & 0 & 0 \\[2em] 0 & -\dfrac{12EI}{L^3} - \dfrac{6W^*}{5L} & 0 \\[2em] 0 & 0 & -\dfrac{EA}{L} \end{bmatrix}$$

$$F_{\acute{u}} = \begin{bmatrix} 0 & \dfrac{6EI}{L^2} & 0 \\[2em] \dfrac{6EI}{L^2} & 0 & 0 \\[2em] 0 & 0 & 0 \end{bmatrix}$$

* Gravity Effect

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Asymptotic Approximation

- Motion Approaches Clamp-Clamped System as Mode Number Increases

- Accuracy Increases with Increasing Mode Number

- Explicit Expressions for Modal Frequencies and Mode Shapes

- First Variation Approximation for Motion of End Bodies

- Singular Perturbation Technique can be used to Improve Approximate Solutions

Comparison of Modal Frequencies

	EXACT	FINITE ELEMENT		
MODE NO.	REF.5&6	REF. 9	REF. 10	REF. 11
1	.278025624	.278	.277	.2740
2	.313776751	.317	.314	3229
3	.812326353	.726	.805	.7494
4	1.18366347	1.226	1.175	1.244
5	2.05047101	2.069	2.028	2.052
6	4.75561758	4.77	4.617	
7	5.51248431	5.52	5.388	
8	12.2598619	12.4	11.782	
9	12.8877037	13.0	12.513	
10	23.5359367	24.2	14.670	
11	24.2568205	24.7	22.968	
12	26.4794890	26.2	23.490	
13	38.9199260	45.4	37.568	
14	39.4643489	45.9	38.146	
15	45.1313668	56.3	44.653	
16	57.90		45.161	
17	57.92			
18	80.72			
19	80.72			

16% ERROR

Comparison of Modal Frequencies

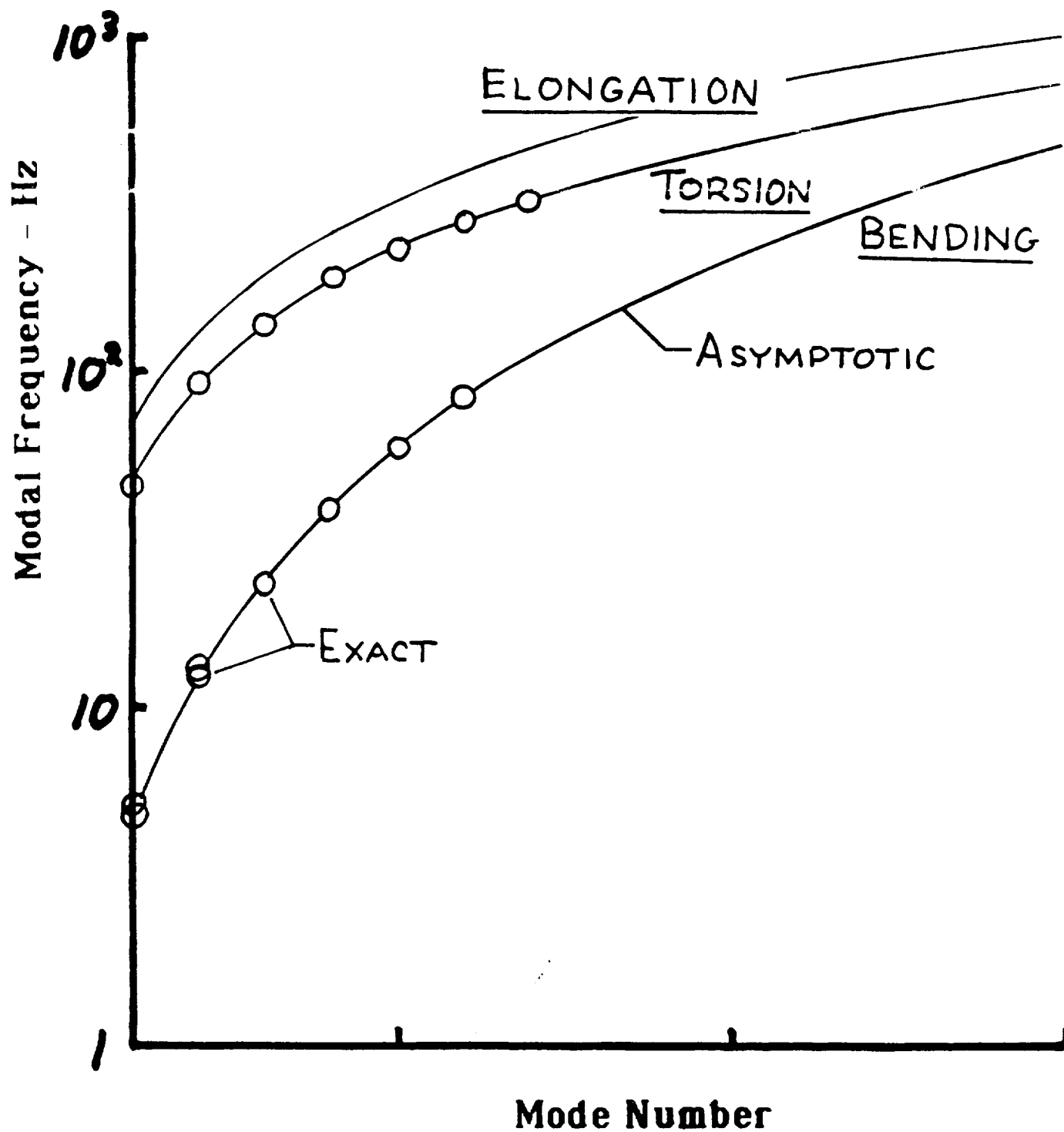| MODE NO.	EXACT REF.5&6	Lumped Mass	Asymptotic
1	.278025624	.258	
2	.313776751	.370	
3	.812326353	.926	
4	1.18366347	1.79	
5	2.05047101	2.57	
6	4.75561758		4.23885
7	5.51248431		4.23885
8	12.2598619		11.88805
9	12.8877037		11.88805
10	23.5359367		23.313674
11	24.2568205		23.313674
12	26.4794890		
13	38.9199260		38.534998
14	39.4643489		38.534998
15	45.1313668		
16	57.90[*]		57.455629
17	57.92[*]		57.455629
18	80.72[*]		80.24802
19	80.72[*]		80.24802

51 % Error

30 % Error

.0 % Error

[*] - Uncoupled (Reference 3).

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Comparison of Modal Frequencies

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Comparison of Deflections

End Deflection (vertical axis, from 10^{-5} to 10^{-1})

REFLECTOR

ASYMPTOTIC

EXACT

SHUTTLE

EXACT

ASYMPTOTIC

Mode Number

81
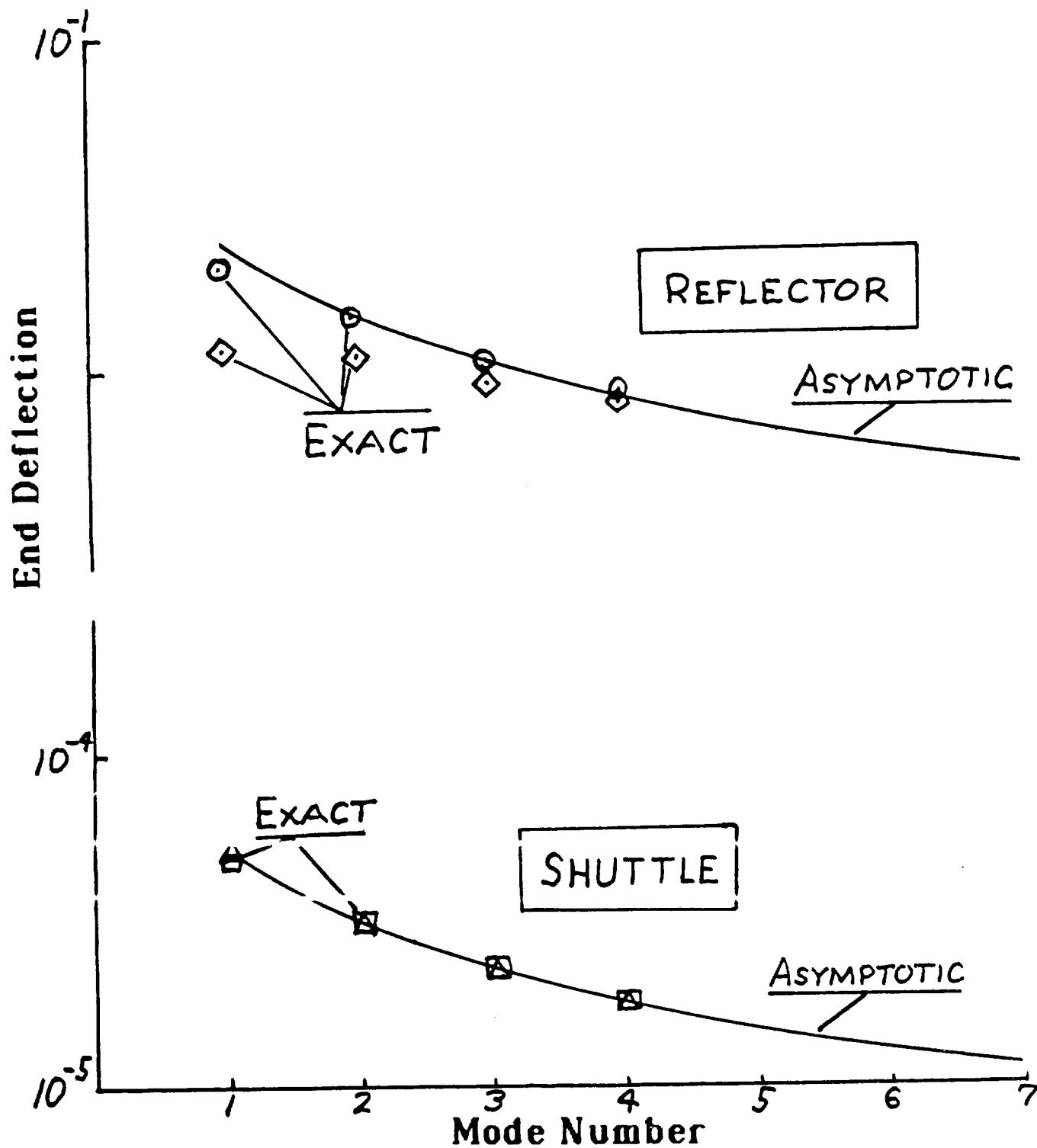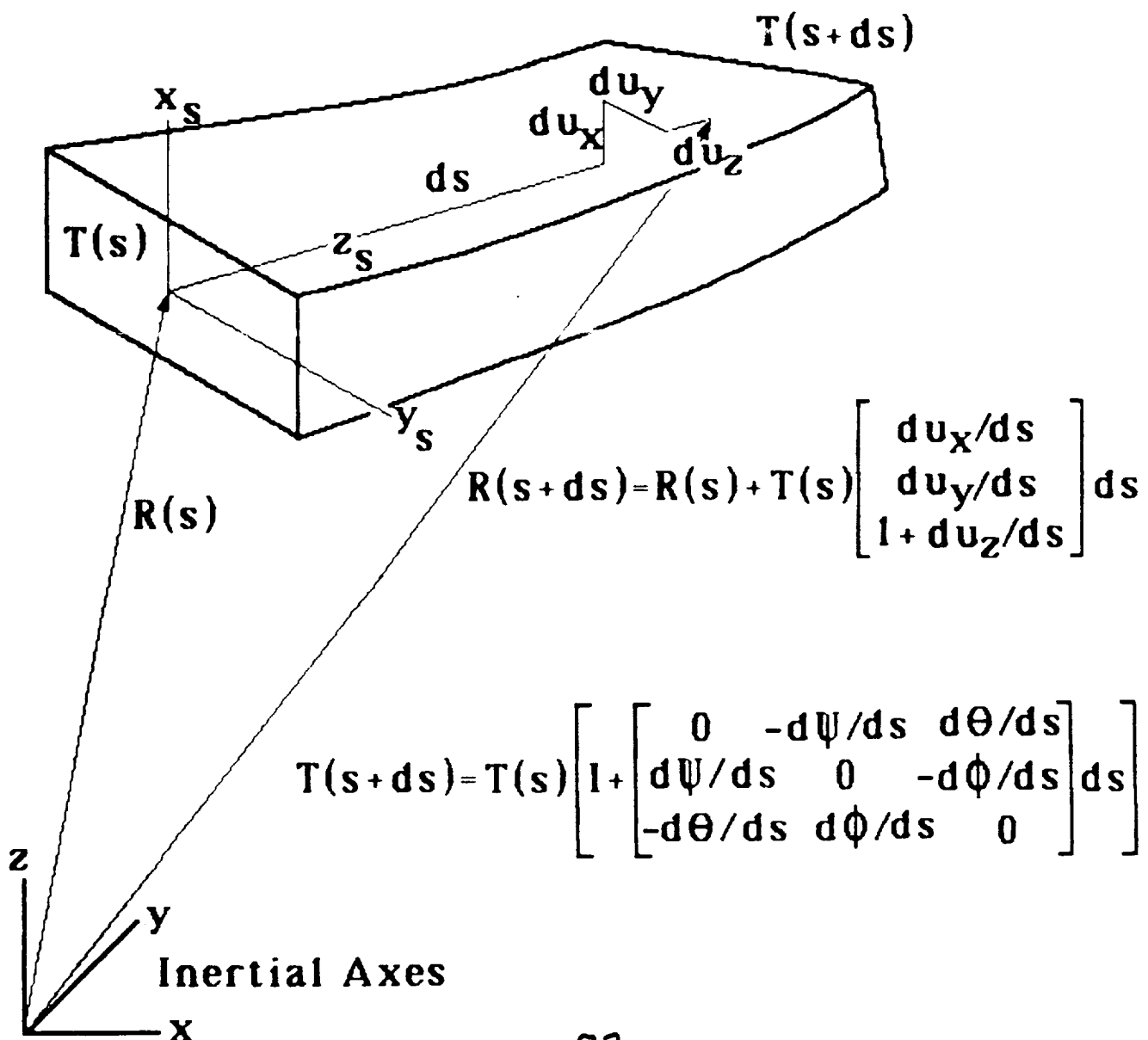
A 3-Dimensional Beam Equation

- Includes Nonlinear Kinematics

- Makes No Small Angle Approximation

- Bending(2 Directions), Torsion & Elongation

$$R(s+ds) = R(s) + T(s) \begin{bmatrix} du_x/ds \\ du_y/ds \\ 1 + du_z/ds \end{bmatrix} ds$$

$$T(s+ds) = T(s) \left[I + \begin{bmatrix} 0 & -d\psi/ds & d\theta/ds \\ d\psi/ds & 0 & -d\phi/ds \\ -d\theta/ds & d\phi/ds & 0 \end{bmatrix} ds \right]$$

Inertial Axes

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It follows that the deflection vector of the beam is:

$$R(s) = \int_0^s T(s') \begin{bmatrix} du_x/ds' \\ du_y/ds' \\ 1+du_z/ds' \end{bmatrix} ds'$$

and the direction cosine of the cross section axes is given by:

$$\frac{dT(s)}{ds} = T(s) \begin{bmatrix} 0 & -d\psi/ds & d\theta/ds \\ d\psi/ds & 0 & -d\phi/ds \\ -d\theta/ds & d\phi/ds & 0 \end{bmatrix}$$

The forces and moments applied to the beam are related to the beam deformations by:

$$\begin{bmatrix} dFx \\ dFy \\ dFz \\ dMx \\ dMy \\ dMz \end{bmatrix} = \begin{bmatrix} -k\,GA & & & & & \\ & -k\,GA & & & & \\ & & -EA & & & \\ \hline & & & -EI_{xx} & & \\ & & & & -EI_{yy} & \\ & & & & & -EI_{zz} \end{bmatrix} \begin{bmatrix} du_x/ds \\ du_y/ds \\ du_z/ds \\ d\phi/ds \\ d\theta/ds \\ d\psi/ds \end{bmatrix} ds$$

$$dF = F_u du + F_e de$$
$$dM = M_u du + M_e de$$

Where

$$du = \begin{bmatrix} du_x/ds \\ du_y/ds \\ du_z/ds \end{bmatrix} ds \qquad de = \begin{bmatrix} d\phi/ds \\ d\theta/ds \\ d\psi/ds \end{bmatrix} ds$$

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The incremental force can be related to deformation of the beam.

$$\frac{dR}{ds} = \frac{R(s+ds)-R(s)}{ds} = T\begin{bmatrix}0\\0\\1\end{bmatrix} + T\begin{bmatrix}du_x/ds\\du_y/ds\\du_z/ds\end{bmatrix}$$

$$\begin{bmatrix}du_x/ds\\du_y/ds\\du_z/ds\end{bmatrix} = T^T\frac{dR}{ds} - \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\frac{dF}{ds} = \frac{F(s+ds)-F(s)}{ds} = F_uT^T\frac{dR}{ds} - F_u\begin{bmatrix}0\\0\\1\end{bmatrix} + F_e\frac{de}{ds}$$

Similarly, for the incremental moment..

$$\frac{dM}{ds} = M_uT^T\frac{dR}{ds} - M_u\begin{bmatrix}0\\0\\1\end{bmatrix} + M_e\frac{de}{ds} + \frac{dF}{ds}$$

The equations of motion for the beam element are:

$$m\frac{d^2R}{dt^2} = T\frac{d}{ds}\left[\frac{dF}{ds}\right] + TF$$

and

$$I_0\frac{d^2e}{dt^2} = -\frac{\widetilde{de}}{dt}I_0\frac{de}{dt} + \frac{d}{ds}\left[\frac{dM}{ds}\right] + M$$

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The equations of motion for the beam element become:

$$m\frac{d^2R}{dt^2} = T\frac{d}{ds}\left[F_u\left(T^T\frac{dR}{ds} - \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix}\right) + F_e\frac{de}{ds}\right] + T\,F$$

$$I_0\frac{d^2e}{dt^2} = -\frac{\widetilde{de}}{dt}I_0\frac{de}{dt} + \frac{d}{ds}\left[M_u\left(T^T\frac{dR}{ds} - \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix}\right) + M_e\frac{de}{ds}\right]$$

$$+ F_u\left(T^T\frac{dR}{ds} - \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix}\right) + F_e\frac{de}{ds} + M$$

Where

$$e = \text{the arc direction cosine}[T] = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$$

$$\frac{de}{dt} = \begin{bmatrix} d\phi/dt \\ d\theta/dt \\ d\psi/dt \end{bmatrix}$$

$$\frac{dT}{dt} = T\frac{\widetilde{de}}{dt}$$

$$\frac{dT}{ds} = T\frac{\widetilde{de}}{ds}$$

m = the mass per unit length

I_0 = the moment of inertia per unit length

$$= \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$

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For the case in which deflections are small and the
end of the beam is aligned with the inertial axes:

$$T \approx 1 + \tilde{e} = \begin{bmatrix} 1 & -dU & d\theta \\ d\psi & 1 & -d\phi \\ -d\theta & d\phi & 1 \end{bmatrix}$$

$$\frac{dR}{ds} = T \begin{bmatrix} du_x/ds & 0 \\ du_y/ds & + & 0 \\ du_z/ds & 1 \end{bmatrix} \approx \begin{matrix} du_x/ds \\ du_y/ds \\ du_z/ds \end{matrix} + \begin{bmatrix} \theta \\ -\phi \\ 1 \end{bmatrix}$$

$$R \approx R_0 + u + \int_0^s \begin{bmatrix} \theta \\ -\phi \\ 1 \end{bmatrix} ds$$

The linearized equations become:

$$m\frac{d^2R}{dt^2} = \frac{d}{ds}\left[F_u\left(\frac{dR}{ds} - \begin{vmatrix} \theta \\ -\phi \\ 1 \end{vmatrix}\right) + F_e\frac{de}{ds} \right] + F + \tilde{e}F$$

$$I_0\frac{d^2e}{dt^2} = \frac{d}{ds}\left[M_u\left(\frac{dR}{ds} - \begin{vmatrix} \theta \\ -\phi \\ 1 \end{vmatrix}\right) + M_e\frac{de}{ds} \right] + F_u\left(\frac{dR}{ds} - \begin{vmatrix} \theta \\ -\phi \\ 1 \end{vmatrix}\right) + F_e\frac{de}{ds} + M$$

For bending only, in a single plane the equations of motion become those for the Timoshenko beam.

$$m\frac{d^2R_x}{dt^2} = \frac{d}{ds}\left[kGA\left(\frac{dR_x}{ds} - \theta\right)\right] + F_x$$

$$I_{xx}\frac{d^2\theta}{dt^2} = \frac{d}{ds}\left[EI_{xx}\frac{dR_x}{ds}\right] - kGA\left(\frac{dR_x}{ds} - \theta\right) + M_x$$

If rotary inertia effects are neglected the result is the Bernoulli-Euler beam.

$$m\frac{d^2R_x}{dt^2} = -\frac{d^2}{ds^2}\left[EI_{xx}\frac{d^2R_x}{ds^2}\right] + F_x$$

Kelvin-Voight Damping
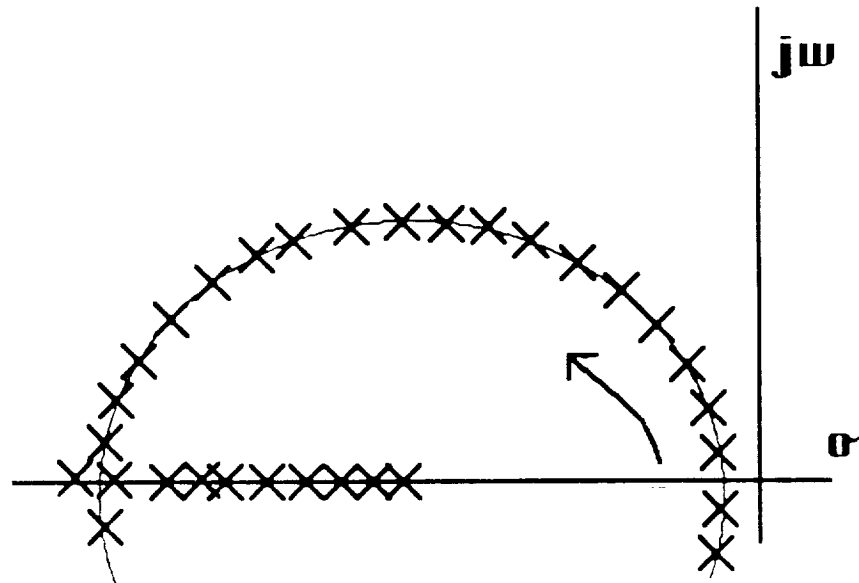
Bernoulli-Euler Beam Equation with Kelvin-Voight
Damping

$$EIu'''' + C\dot{u}'''' + m\ddot{u} = 0$$

<u>Allows Separation of Variables</u>

<u>Theoretical Basis for Damping</u>

<u>Locus of Modal Characteristics</u>

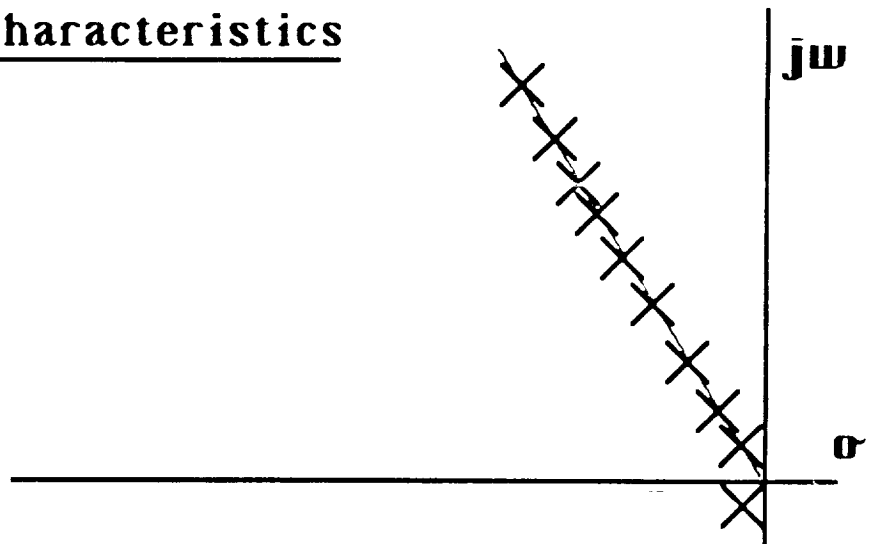## EXCESSIVE DAMPING AT HIGH MODE NUMBERS !!!!

Proportional Damping

Bernoulli-Euler Beam Equation with Proportional Damping

$$EIu'''' + C\dot{u}'' + m\ddot{u} = 0$$

Allows Separation of Variables for Pinned and Infinite End Conditions

Lacks Theoretical Basis for Damping

Locus of Modal Characteristics

REASONABLE DAMPING AT HIGH MODE NUMBERS

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Piano-Wire Damping

Viscous Damping Ratio

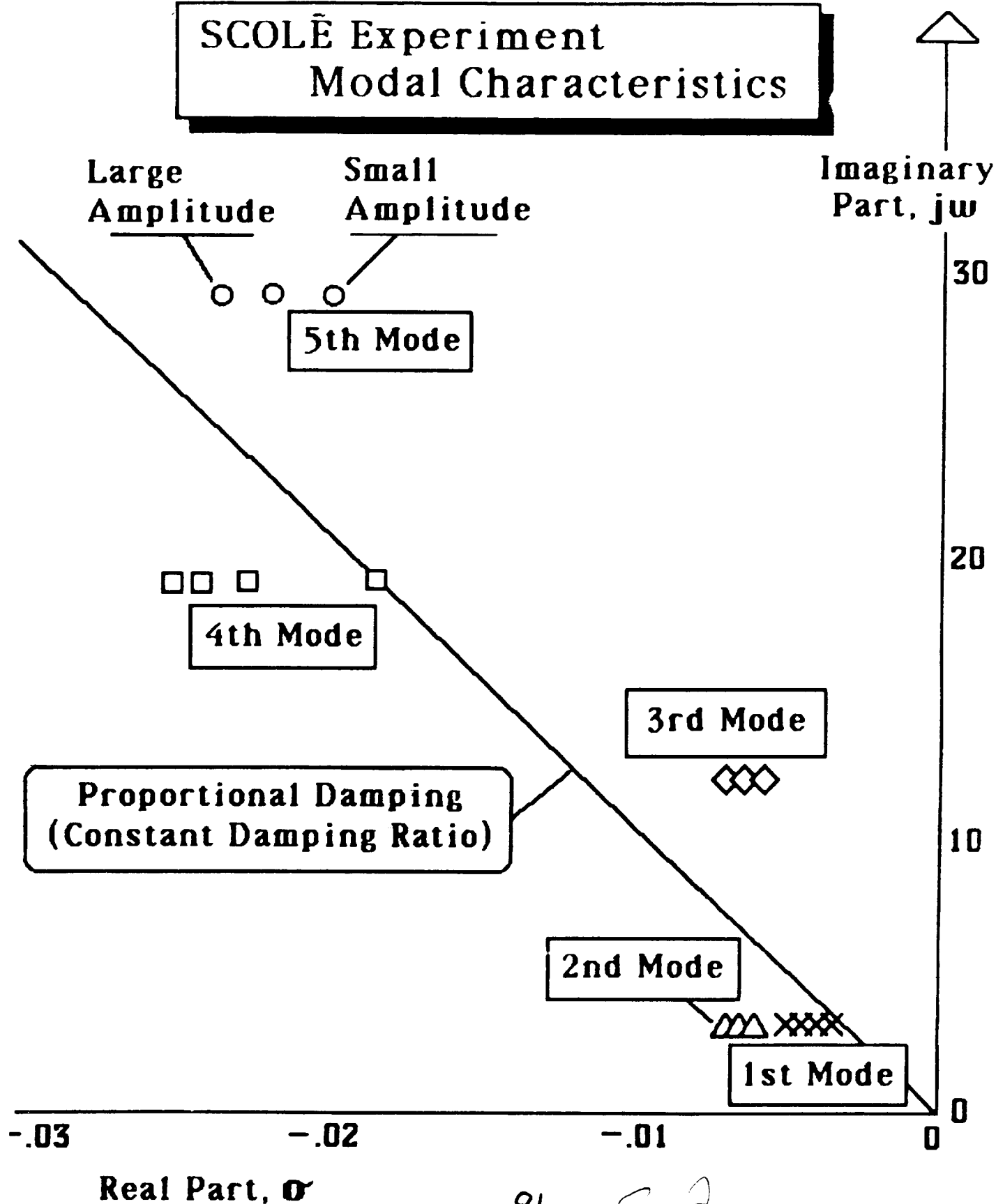|		
Smaller Mass	$\xi =$	$.0015$
Larger Mass	$\xi =$	$.0013$
General Mass	$\xi =$	$.0015\sqrt{\dfrac{m_1}{m}}$

Nonlinear Damping

$$m\ddot{x} = -c_1\dot{x} - c_2|\dot{x}|\dot{x} - kx$$

$$A_{n+1} = A_n - A_n(.00138)2\pi - (.0012)A_n^2$$

> Determined to be
> Air Damping

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SCOLĒ Experiment Modal Characteristics

SCOLE DAMPING

Viscous Damping Ratio

Mode	Configuration #1	Configuration #2
1	.0016	.0013
2	.0011	.0009
3	.00058	
4	.0011	
5	.00084	

> Nonlinear Damping is Evident for Large Amplitude Motion. Analysis is Underway.

Nonlinear Damping

Mass, Spring, Nonlinear Damper

$$m\ddot{x} = -c|x|^a|\dot{x}|^b\dot{x} - kx$$

Considering Only Light Damping ...

$$w = \sqrt{k/m}$$

For Free Decay

$$x(t) = A(t)\sin(wt)$$

$$\dot{A} = -\frac{c}{m}|x|^a|\dot{x}|^b = -\frac{c}{m}w^bA^{a+b}$$

Solving

$$dt = \frac{m\,dA}{c\,w^bA^{a+b}}$$

$$t + t_0 = \frac{m}{c(a+b-1)w^bA^{a+b-1}}$$

$$A(t) = \left[\frac{m}{c(a+b-1)w^b(t+t_0)}\right]^{\frac{1}{a+b-1}}$$

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Nonlinear Damping

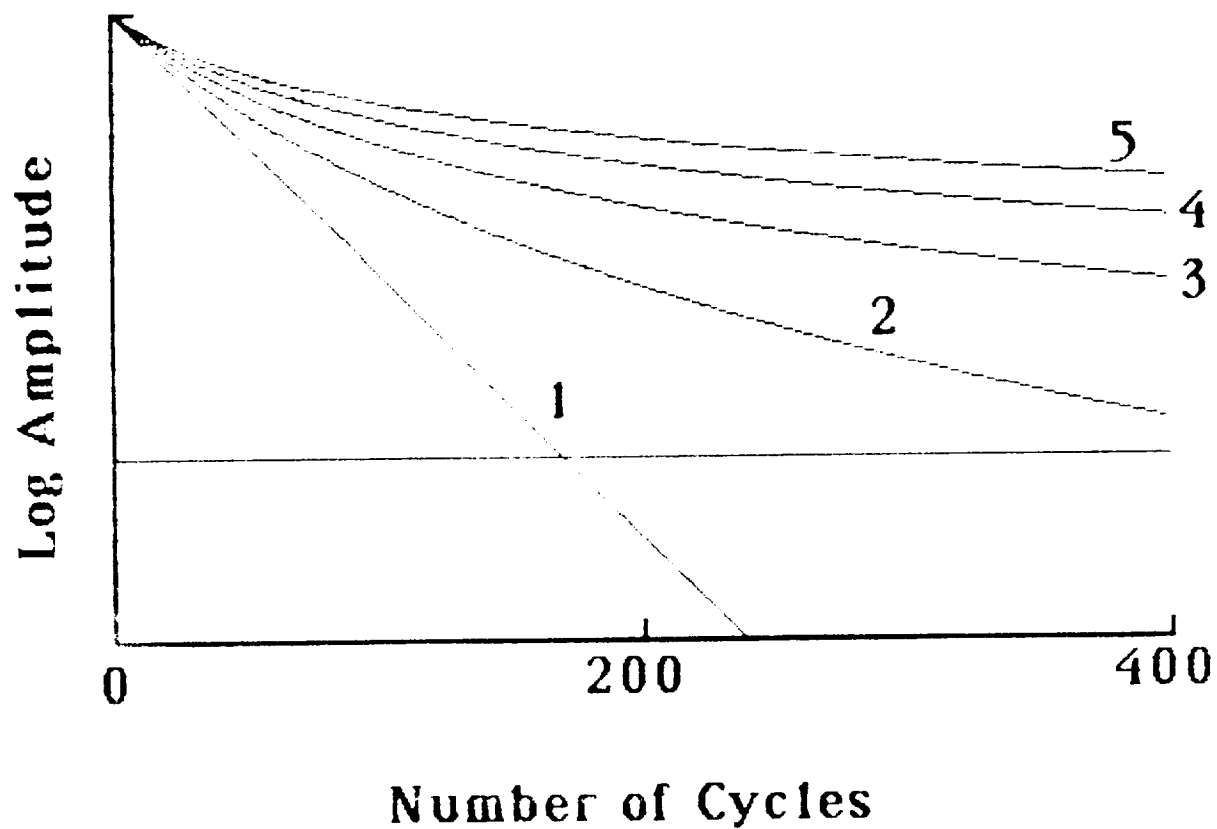$$A(n) = \left[\frac{m}{c(a+b-1)\omega^b n}\right]^{\frac{1}{a+b-1}}$$

Where $t+t_0 = n\frac{2\pi}{\omega}$, $\omega = [k/m]^{1/2}$

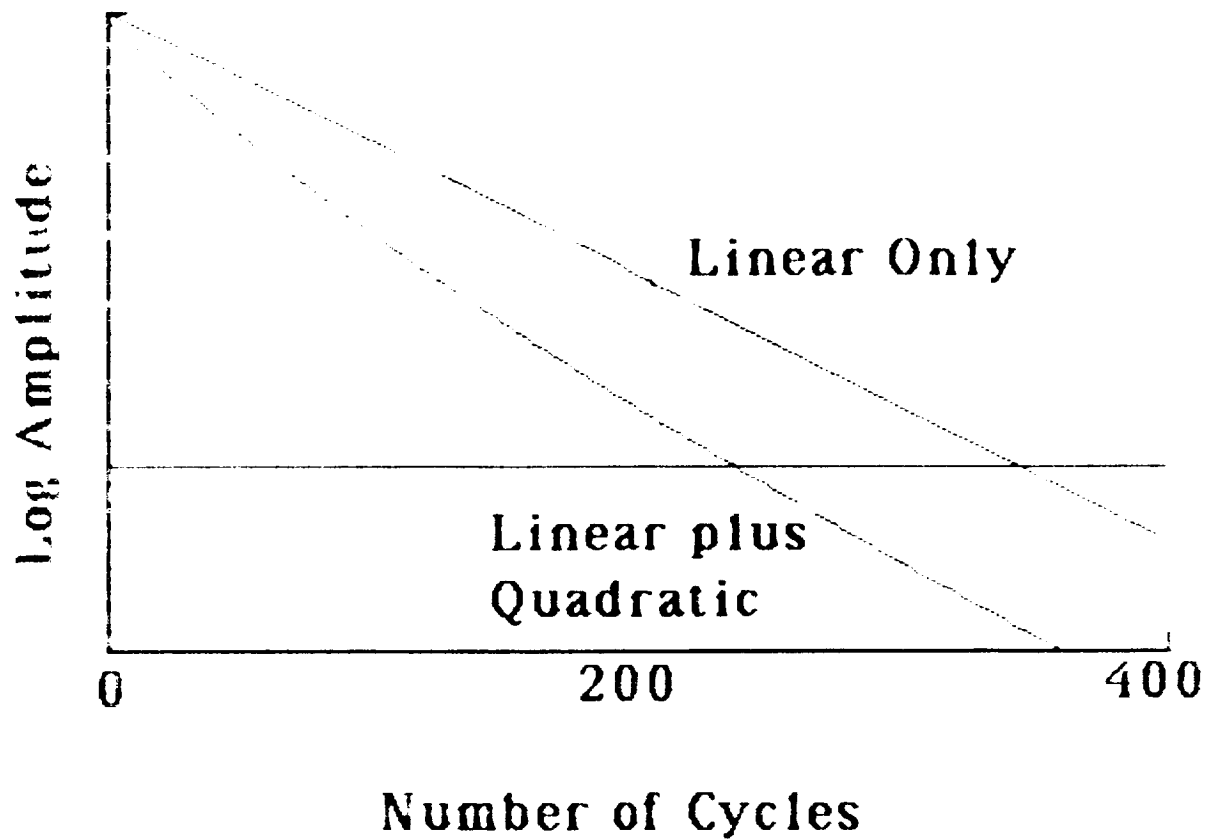$$A(n) = \left[\frac{m^{(b+1)/2}}{c(a+b-1)k^{(b-1)/2}2\pi n}\right]^{\frac{1}{a+b-1}}$$

For Example

- $\dfrac{c}{m}\dot{x}$ $a=b=0$ $\dfrac{dA}{A} = 2\pi\delta = \dfrac{c\pi}{\sqrt{km}}$

- $\dfrac{c}{m}|\dot{x}|\dot{x}$ $\begin{matrix}a=0\\b=1\end{matrix}$ $\dfrac{dA}{A} = \dfrac{4cA\omega}{3m} = \dfrac{4cAk^{1/2}}{3m^{3/2}}$

- $\dfrac{c}{m}|x|\dot{x}$ $\begin{matrix}a=1\\b=0\end{matrix}$ $\dfrac{dA}{A} = \dfrac{2cA}{3m}$

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Log Amplitude Response

Log Amplitude Response

Log Amplitude

Linear Only

Linear plus
Quadratic

0   200   400

Number of Cycles

SCOLE DAMPING

Nonlinear Damping

Linear Damping

Amplitude

Number of Cycles

97
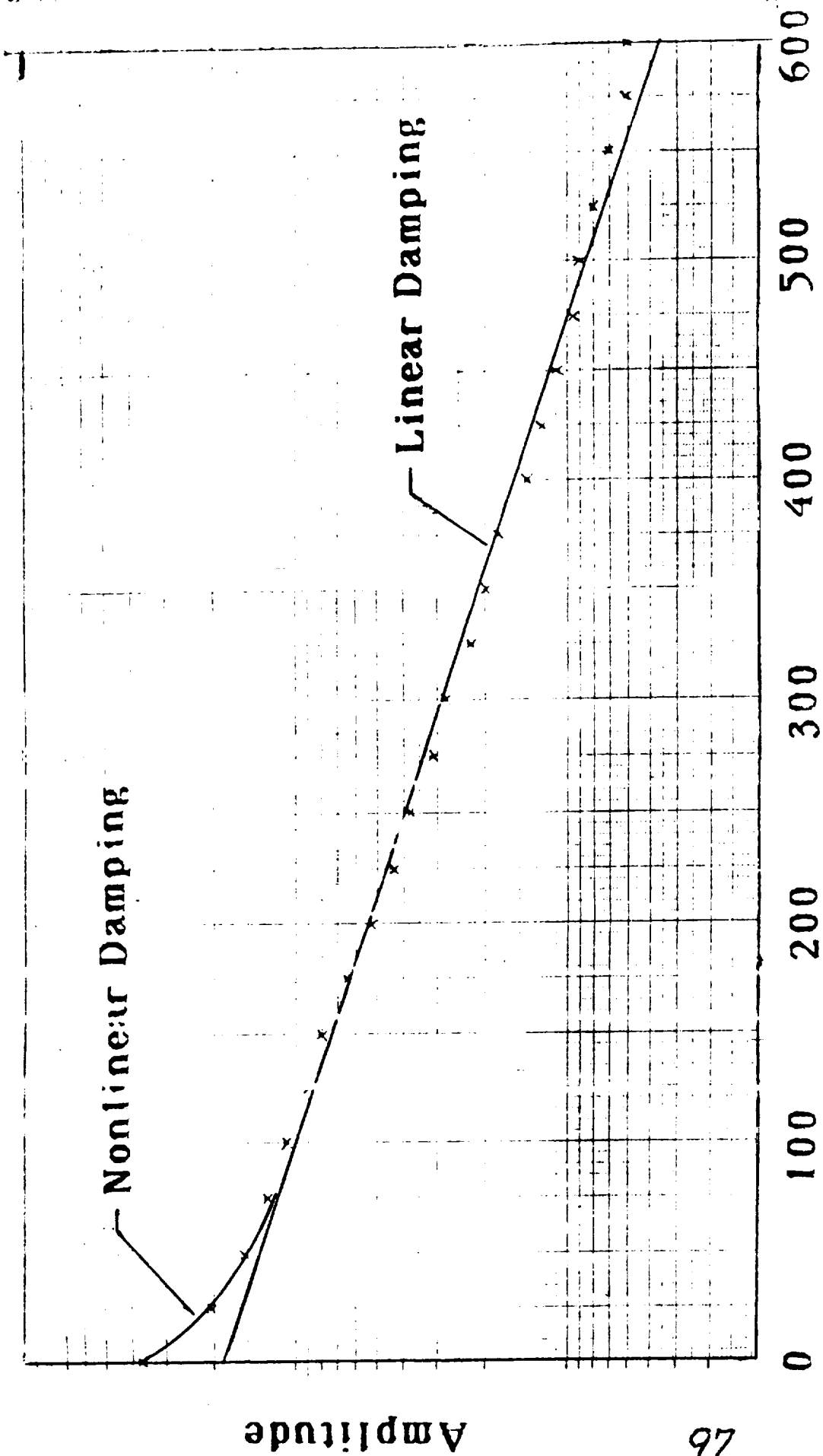
The Curse of Dimensionality

Number of Parameters (y-axis)

1000 —
800
600
400
200
0

Independent Modal Characteristics

1 Frequency
1 Damping
32 Deflections
Total = 34/Mode

Interdependent Modal Characteristics
Distributed Parameter Model

Total = 6

Number of Modes (x-axis): 1 5 10 15 20 25 30 35 40 45 50

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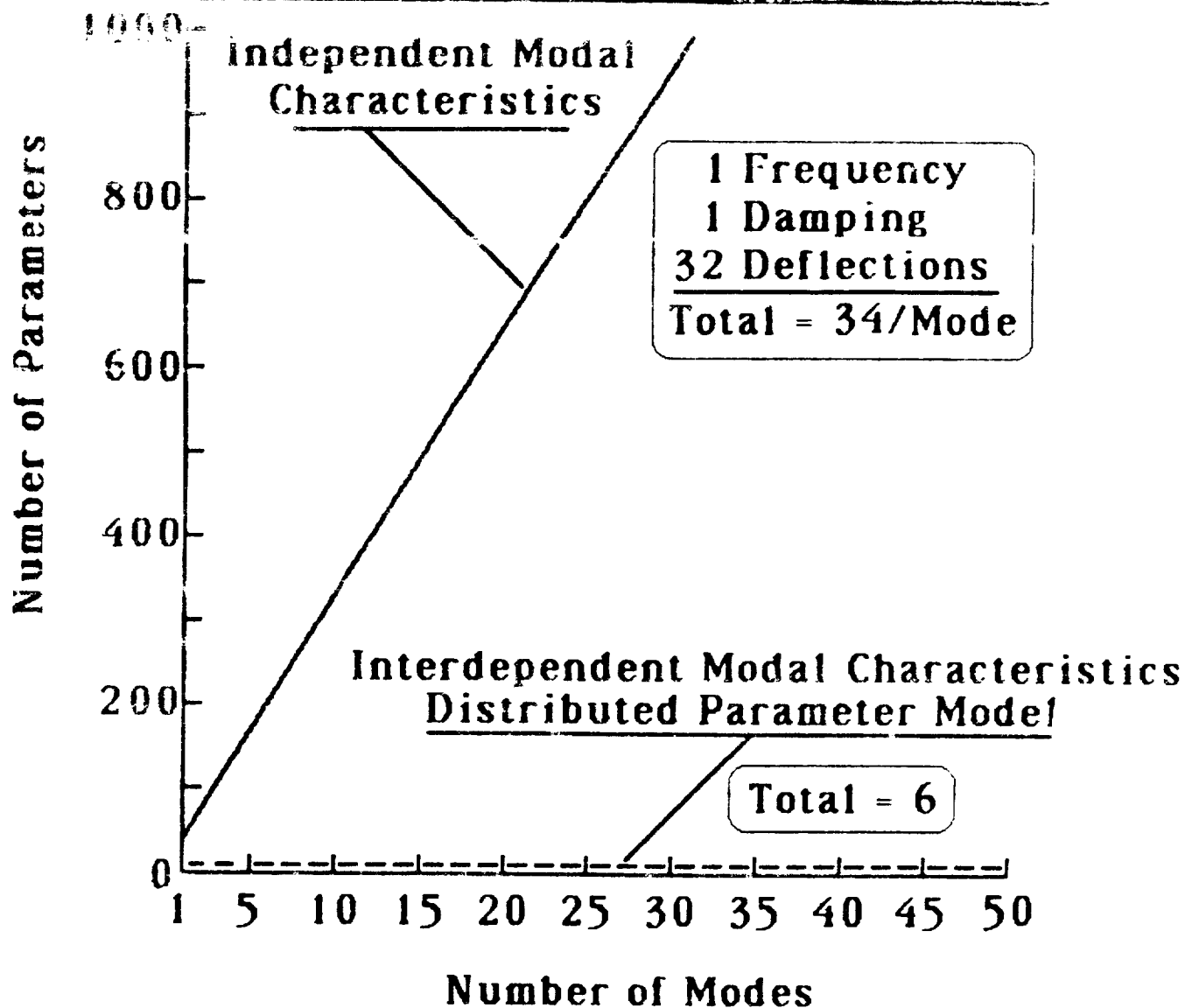
Concluding Remarks

- **The Accuracy of the Modal Characteristics of the SCOLE Configuration were Examined Using Exact and Approximate Solutions.**

- **Sixty-Seven Modes are Required for a Static Deflection Error of less than 1 %. SCOLE Model Requires Hundreds of Modes.**

- **Exact Solutions Encounter Numerical Difficulties.**

- **Asymptotic Solutions in Combination with Limited Exact Solutions Enable Generation of a Proof Model with the Required Accuracy.**

- **Damping Must be Incorporated into the Model from the Start. Proportional Damping is Not Adequate.**

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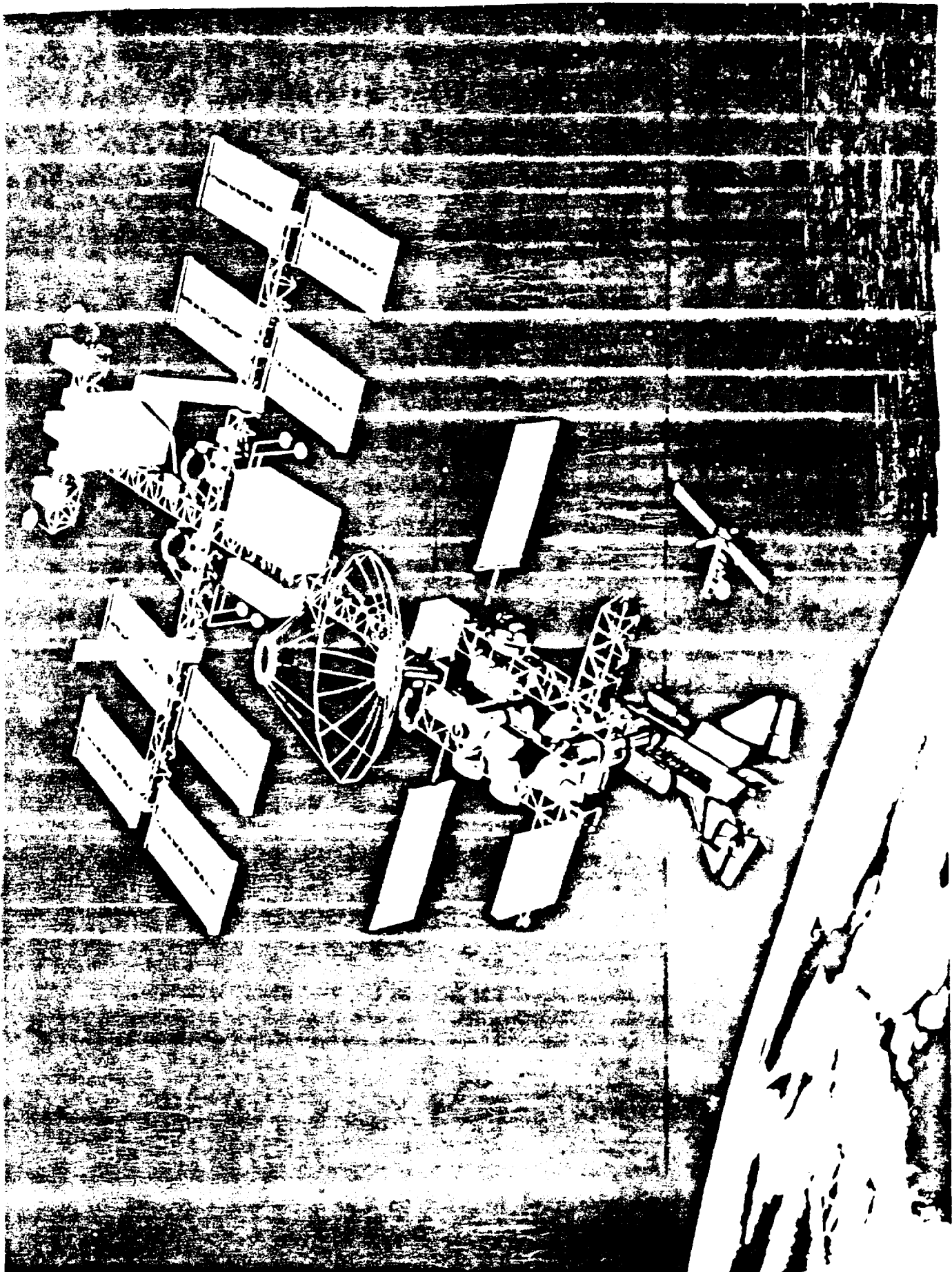
"It will never be possible to have the absolute
conviction before flight that a valid mathematical
model has been devised for a space vehicle.

....we surely must make every effort to ensure that
failures do not come from inadequate analysis of
the best models available."

<div align="right">

Peter Likins
1971

</div>

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