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A Study of the Use of Linear Programming Techniques to Improve the Performance in Design Optimization Problems

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ABSTRACT

There are two purposes of this project. One is to determine whether linear programming techniques can improve the performance in handling design optimization problems with a large number of design variables and constraints relative to the feasible directions algorithm. The second purpose is to determine whether using the Kreisselmeier-Steinhauser(KS)[1] function to replace the constraints with one constraint will reduce the cost of the total optimization. Using the software program, CONMIN[2], reference cases are run with both the linear and non-linear options. Next the same test case is run using the linear programming subroutine, LINPR1 [3], from the math library. Comparisons are then made between the solutions obtained from both subroutines, CONMIN and LINPR1.

PROCEDURE

A simple problem of a hub with 12 spokes was used as the test example (see Appendix). This problem has 12 design variables and 24 constraints. The calculations were done on a DEC MICROVAX II workstation using code written in FORTRAN 77.

Using CONMIN, results were obtained using the non-linear and linear options. Next the 24 constraints were replaced by one, a KS-based cumulative constraint, and results obtained, again for both the non-linear and linear options (figure 1).

Since the value of rho in the KS function influences the result, different values of rho were used in the KS function. The comparisons between the linear and non-linear CONMIN solutions using the 24 constraints and one constraint(when using the KS function) are in figure 2.

After obtaining the results using CONMIN, the non-liner problem was turned into a linear programming one to be solved using the linear programming subroutine LINPR1. This is a library routine from the math library and uses the simplex method. The results using this routine are compared to the CONMIN results in figure 3.

CONCLUSIONS

The optimal value of the objective function was practically the same when using CONMIN with or without linearization and with or without the KS function as the cumulative constraint (figure 1). When the KS cumulative constraint was used, the optimal objective function was influenced by the rho factor (figure 2).

The CONMIN optimized objective function was

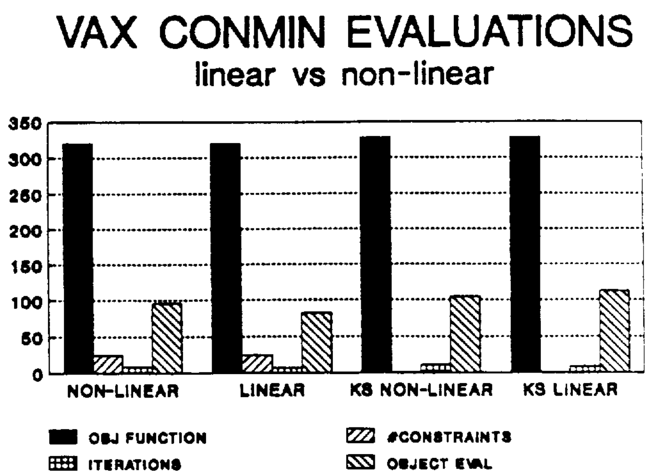


Figure 1 CONMIN, linear, non-linear for normal and KS

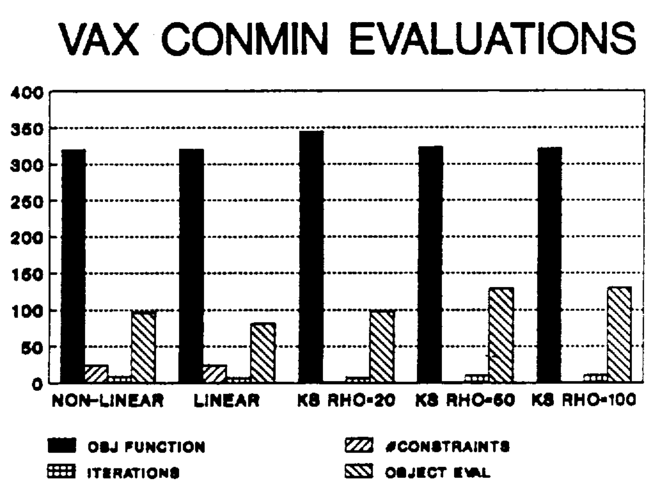


Figure 2 CONMIN, non-linear,linear, KS

consistently lower than the one obtained from the linear programming routine, regardless of the use of the KS

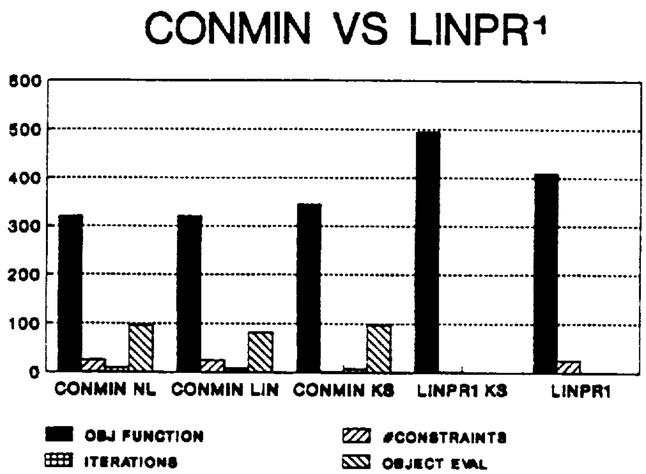


Figure 3 CONMIN vs. LINPR1

function (figure 3). This is an unexpected and important finding of this study.

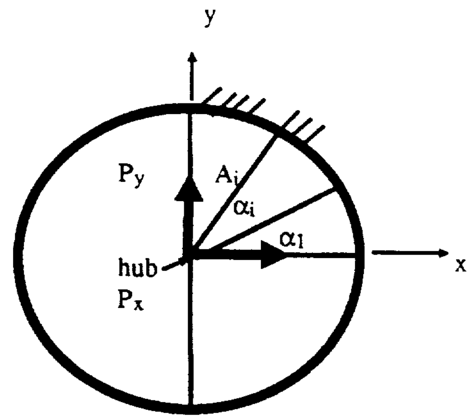
In terms of efficiency, all the runs were comparable in the number of function evaluations needed. However, there is a reduction of memory required by CONMIN when the constraints are replaced by a single cumulative constraint using the KS function.

REFERENCES

1. Kreisselmeir, G.; and Steinhauser, R.: Systematic Control Design by Optimizing a Vector Performance Index. Proceedings of IFAC Symposium on Computer Aided Design of Control Systems, Zurick, Switzerland, 1979, pp.113-117.
2. Vanderplaats, G. N.: CONMIN--A Fortran Program for Constrained Function Minimization; Users Manual. NASA TM X62282, Aug. 1973.
3. Mathematical and Statistical Software at Langley Documentation; LINPR1, Part I, Section H1.1, 03/01/87.

APPENDIX

Problem Formulation:



Nomenclature

NS	number of spokes
NLC	number of loading cases
α_{i+1}	$= \alpha_i + 2\pi/NS$
E	Youngs modulus
A_i	cross-sectional area of rod i
R	radius of the circle = length of each rod
u^j_x	displacement of the hub along x for jth loading case
u^j_y	displacement of the hub along y for jth loading case
P^j_x, P^j_y	load components along x, y for jth loading case
b_i	$=EA_i/R$
σ_{at}	allowable tension stress
σ_{ac}	allowable compression stress

Details of the Analysis

$$k_{11} = \sum_{i}^{NS} b_i cos^2 \alpha_i \quad (1)$$

$$k_{12} = \sum_{i}^{NS} \frac{1}{2} b_i sin 2\alpha_i \quad (2)$$

$$k_{22} = \sum_{i}^{NS} b_i sin 2\alpha_i \quad (3)$$

Displacements for loading casesj

$$DET = k_{11}k_{22} - k^2_{12} \quad (4)$$

$u^j_x = (P^j_x k_{22} - P^j_y k_{12})/DET$ (5)

$u^j_y = (P^j_y k_{11} - P^j_x k_{12})/DET$ (6)

Strain in rod i for loading case j

$\varepsilon^j_i = (-u^j_x \cos\alpha_i - u^j_y \sin\alpha_i)/R$ (7)

Stress in rod i for loading case j

$s^j_i = \varepsilon^j_i E$ (8)

There are NS• NLC stresses σ^j_i

Material volume

$$V = R \cdot \sum_i^{NS} A_i \quad 9)$$

DETAILS OF THE OPTIMIZATION USING CONMIN

$$\min_{A_i} \; V(A_i) \quad (10)$$

subject to the constraints

$g_m = (\sigma^j_i/\sigma_a - 1) \leq 0$ (11)

$m = 1 \rightarrow (NS \bullet NLC)$ (12)

where $\sigma_a = \sigma_{at}$. of $\sigma^j_i > 0$, otherwise $\sigma_a = \sigma_{ac}$

$A_{min} \leq A_i$

Numerical Data for Test Cast

NS = 12
NLC = 2
$A_i = 1cm^2$
R = 100cm
$E = 20 \bullet 10^6 \; N/cm^2$
loading case 1
$P^1_x = 40000$ N; $P^1_y = 0$
loading case 2
$P^2_x = 40000$N; $P^2_y = 0$
$\sigma_{at} = 80000 \; n/cm^2$; $\sigma_{ac} = -40000 \; N/cm^2$
$Amin = .1cm^2$

Using the K-S function in the Optimization.

The optimization is repeated using the cumulative K-S function in place of equation 11.

$g_m \approx KS$

where

$$KS = g_{max} + \frac{1}{\rho} ln \left[\sum_{i=1}^{NS \cdot NLC} e^{\rho(g_i - g_{max})} \right] \quad (13)$$

DETAILS OF THE OPTIMIZATION USING LINPR1

Turning the Optimization Problem into a Linear Programming One

Introduce a new variable

$X_i = 1/A_i$ (14)

Compute derivatives

$\partial V/\partial X_i$ and $\partial g^j_m/\partial X_i$ (15)

Because of 14, equation 15 becomes

$\partial V/\partial X_i = \partial V/\partial A_i \; (-1/X^2_i) \equiv \partial V/\partial A_i \; (-A^2_i)$ (16)

and $\partial g^j_m/\partial X_i = \partial g^j_m/\partial A_i(-A^2_i)$ (17)

Using equation 9 put $\partial V/\partial A_i = R$ in equation 16 then

$\partial V/\partial X_i = R(-A^2_i)$ (18)

Approximate Linear Optimization Problem

Let V^o, g^o_m be the values at the initial $X_i = X^o_i$
Approximate $V(X_i)$, $g_m(X_i)$ by extrapolation

$V = V^o + \partial V/\partial X_i(X_i - X^o_i)$ (19)

$g_m = g^o_m + \partial g_m/\partial X_i(X_i - X^o_i)$ (20)

The approximate problem is:

$$\min_{X_i}(V^o + \partial V/\partial X_i(X_i - X^o_i)) \quad (21)$$

STOC

$g^o_m + \partial g_m/\partial X_i(X_i - X^o_i) \leq 0$

where (22)

$\beta Xo \leq (X_i - X_{io}) \leq (1 + \beta)X^o_i$ (23)

equation 23 is a move limit that does not allow X_i to move too far from X^o_i. Initially $\beta = .2$.

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