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MULTIPLE POINT LEAST SQUARES EQUALISATION IN A ROOM

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ABSTRACT

Equalisation filters designed to minimise the mean square error between a delayed version of the original electrical signal and the equalised response at a point in a room, have previously been investigated by several authors. In general such a strategy degrades the response at positions in the room away from the equalisation point.

A method is presented for designing an equalisation filter by adjusting the filter coefficients to minimise the sum of the squares of the errors between the equalised responses at multiple points in the room, and delayed versions of the original electrical signal. Such an equalisation filter can give a more uniform frequency response over a greater volume of the enclosure than the single point equaliser above.

Computer simulation results are presented of equalising the frequency responses from a loudspeaker to various typical ear positions, in a 'room' with dimensions and acoustic damping typical of a car interior, using the two approaches outlined above. Adaptive filter algorithms, which can automatically adjust the coefficients of a digital equalisation filter to achieve this minimisation, will also be discussed.

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1. INTRODUCTION

In sound reproduction systems an equalisation filter is sometimes used to modify the frequency spectrum of the original source signal, before feeding it to the loudspeaker, in an attempt to compensate for unevenness in the frequency response of the loudspeaker and the listening room. Such an arrangement is illustrated in Figure 1 in which a microphone, whose response is assumed to be flat, is substituted for the human observer in order to make the net response of the reproduction chain purely electrical. Such equalisation filters can take many forms. One common form is a parallel combination of bandpass filters, the outputs of which have a manually adjustable gain and are added together to produce the output. Such filters can compensate for gross deficiencies in the frequency response of the sound reproduction chain, which includes the electroacoustic response of the loudspeaker and the acoustic response of the listening room. The transient properties of narrow bandwidth filters are, however, notoriously bad and this can lead to a degradation in the impulse response of the equalised reproduction chain.

Another approach is to design an equalisation filter by making the impulse response of the equalised sound reproduction chain as close as possible to that desired, a net impulse response of a delta function for example would mean that the sound reproduction chain had been perfectly equalised. It is, however, not possible in general to achieve such perfect inversion of the equalisation chain, since the acoustic path usually has delays and other non-minimum phase behaviour associated with it [1]. The ability of the equalisation filter to invert the response of the reproduction chain is much improved if the equalised output is compared with a delayed version of the original signal. Such a "modelling delay" is illustrated in Figure 2. In Section 2 we will formalise the design of such single channel systems, and extend the theory to the case of multiple microphones in Section 3.

2. SINGLE CHANNEL EQUALISATION

We assume that the equalisation filter to be designed is digital and has an all zero (FIR) structure with coefficients h_0 to h_{I-1}. We also assume that the response of the unequalised reproduction chain is modelled by a digital FIR filter, with coefficients c_0 to c_{J-1}. If the sampled source signal is $x(n)$, the sampled signal fed to the loudspeaker is $y(n)$ and samped output from the microphone is $\hat{d}(n)$, then:

$$y(n) = \sum_{i=0}^{I-1} h_i x(n-i) \quad , \qquad \hat{d}(n) = \sum_{j=0}^{J-1} c_j y(n-j)$$

so

$$\hat{d}(n) = \sum_{j=0}^{J-1} c_j \sum_{i=0}^{I-1} h_i x(n-i-j) = \sum_{i=0}^{I-1} h_i r(n-i) \tag{1}$$

where

$$r(n) = \sum_{j=0}^{J-1} c_j x(n-j) \tag{2}$$

The summation of equation (1) can be written in vector form as

$$\hat{d}(n) = \mathbf{r}^T(n)\mathbf{h} \tag{3}$$

where

$$\mathbf{r}^T(n) = [r(n),\ r(n-1),\ \ldots\ r(n-I+1)]$$
$$\mathbf{h}^T = [h_0,\ h_1\ \ldots\ h_{I-1}]$$

The most usual method of defining how $\hat{d}(n)$ is the 'best' approximation to $d(n)$ is to minimise the mean square difference between these two signals, i.e., to adjust the coefficients of the equalisation filter to minimise the "performance index":

$$J = E\{e^2(n)\} \tag{4}$$

where $e(n) = d(n) - \hat{d}(n)$, and E represents the expectation operator. It should be noted, however, that this performance index is not the only criterion which can be used to define the difference between the desired and equalised signals [2]. One advantage, however, of the mean square performance index, J, is that it is a quadratic function of each of the coefficients in the equalisation filter:

$$J = E\{d^2(n)\} + 2\mathbf{h}^T E\{\mathbf{r}(n)d(n)\} + \mathbf{h}^T E\{\mathbf{r}(n)\mathbf{r}^T(n)\}\mathbf{h} \tag{5}$$

which has a globally minimum value for some unique set of filter coefficients (since the matrix $E\{\mathbf{r}(n)\mathbf{r}^T(n)\}$ is positive definite). Using fairly standard optimisation methods this optimum set of filter coefficients can be shown to be given by

$$\mathbf{h}_{opt} = -\,[E\{\mathbf{r}(n)\mathbf{r}^T(n)\}\,]^{-1}E\{\mathbf{r}(n)d(n)\} \tag{6}$$

In practice, adaptive algorithms can be used to automatically adjust the coefficients of $\mathbf{h}$ to be a close approximation to $\mathbf{h}_{opt}$, and these will be discussed in a later section. We are concerned here with the physical consequences of designing an equalisation filter according to this criterion.

Previous studies of such equalising filters [3,4] have demonstrated that it is possible to obtain good equalisation at the equalisation microphone position, but that the equalised response away from this point can be worse than the unequalised response. In order to illustrate this point, and to introduce the acoustic model which will be used in later sections, we consider using the equalisation strategy above in an enclosure with dimensions and acoustic damping typical of a car interior.

The acoustic response from an acoustic source in one position in the enclosure to a microphone in another, was modelled as the sum of the contributions of a finite number of acoustic modes in the enclosure [5]. The size of the enclosure was 1.9 m long by 1.1 m high by 1.0 m wide, and all modes with a natural frequency below 1200 Hz were included in the modal summation (about 500 modes), even though the response was only calculated for frequencies up to 512 Hz, the sample rate being 1024 Hz. The damping ratio of all the modes was set to 0.1. This purely acoustic response was then convolved with a filter which had a zero at d.c., and a zero at half the sample rate, which represented the high pass filtering action of a loudspeaker and the low pass filtering action of the anti-aliasing and reconstruction filters which would be used in any practical system. In the coordinate system used for the computer simulation the origin was in the front bottom right hand corner of the enclosure (as seen from the interior) and the coordinates (x1, x2, x3) represent the distance back, across and up, respectively, from this origin. The loudspeaker was represented by a point acoustic source at (0.0, 0.9, 0.7) which is approximately the position of a front dashboard loudspeaker on the left hand side of a real car. The frequency response was calculated from this loudspeaker to a microphone at (0.1, 0.1, 0.9) which corresponds approximately to the position of the driver's right hand ear in a real car. This frequency response, and a truncated version of the corresponding impulse response, obtained by inverse Fourier transformation from the frequency response, are shown in Figure 3. These responses do have the gross characteristics of those actually measured in car interiors. Of particular note is the delay of about 3-4 ms before the dominant point in the impulse response, which corresponds to the acoustic delay time for the distance from the loudspeaker to the microphone (0.92 m).

A 50 coefficient FIR filter has been used to equalise the frequency response of Figure 3 using a modelling delay of 15 samples. This filter was adapted to minimise the mean square modelling error, using an algorithm discussed in Section 4 below. The frequency response and impulse response of the equalisation filter after convergence are shown in Figure 4. It can be seen that the frequency response in Figure 4 is a good approximation to the inverse of that in Figure 3. This is further illustrated in Figure 5 which shows the original (solid) and equalised (dashed) response at the equalisation microphone (microphone 0), and also the original response and the effect of this equalisation filter at three other microphones: microphone 1, at position (0.9,0.9,0.9), corresponding approximately to the front passenger's left hand ear; microphone 2, at (1.9,0.1,0.9), corresponding approximately to the right hand rear passenger's position; and microphone 3, at (1.9,0.9,0.9), corresponding approximately to the left hand rear passenger's position.

It is clear that although the frequency response has been significantly improved at microphone 0, and somewhat improved at microphone 1, this equalistion filter makes the responses more peaky at the rear microphone positions. This is lagely due to the presence of the first longitudinal acoustic mode in the enclosure, with a natural frequency of about 90 Hz. This has little effect at microphones 0 and 1, since they are close to the nodal plane of this mode, and these microphones have a relatively low response at about 90 Hz, which is boosted by the equalisation filter. The microphones in the rear of the enclosure (2 and 3) pick up this mode strongly, however, even before boosting by the equalisation filter, so the effect of the equalistion filter is to produce a peak of some 15 dB above the average response at 90 Hz.

Figure 5 illustrates a point made by Mourjopoulos [3] and Farnsworth *et al* [4], that equalisation at one point can significantly disturb the response at other points in the enclosure. The results presented here are in a rather lower frequency range than those presented in [3] and [4], however, and may still represent a practical equalisation strategy for, say, the front two seats if it were applied only to the low frequency ("woofer") unit of an in-car entertainment system.

For completeness, Figure 6 also shows the time domain response from the loudspeaker to each microphone before and after equalisation. An extra delay equal to the modelling delay has been added to the unequalised responses for clarity. It is again clear that although the equalised impulse response at microphone 0 is very close to that desired (a delayed delta function), a considerable amount of low frequency ringing has been added to the responses in the rear of the car by the equalisation filter.

3. MULTIPLE POINT EQUALISATION

The failure of single point equalisation schemes to control the response at points away from the equalisation microphone within the enclosure suggests that the problem of equalisation at a number of points might be cast as a more general least squares problem. This is illustrated in Figure 7, in which the output of a single equalisation filter is coupled to multiple microphones via multiple room impulse responses, and each microphone output is subtracted from a desired signal, formed by passing the source signal through an individual modelling delay (of Δ_ℓ samples for the ℓ'th microphone), to obtain an error signal at each microphone.

The vector of output signals can now be represented [6,7] as:

$$\mathbf{e}(n) = \mathbf{d}(n) + \mathbf{R}(n)\mathbf{h} \tag{7}$$

where

$$\mathbf{e}^T(n) = [e_1(n), e_2(n) \ \ldots \ \ e_L(n)]$$
$$\mathbf{d}^T(n) = [d_1(n), d_2(n) \ \ldots \ \ d_L(n)]$$
$$\mathbf{R}^T(n) = [\mathbf{r}_1(n), \mathbf{r}_2(n) \ \ldots \ \ \mathbf{r}_L(n)]$$

and $\mathbf{r}_\ell(n)$ and $\mathbf{h}$ are defined similarly to the vectors in the previous section. The object of the equalising filter is now to minimise the sum of the squares of each of the errors, and this new performance index may be written as:

$$J = E\{\mathbf{e}^T(n)\mathbf{e}(n)\},$$

so that

$$J = E\{\mathbf{d}^T(n)\mathbf{d}(n)\} + 2\mathbf{h}^T E\{\mathbf{R}^T(n)\mathbf{d}(n)\} + \mathbf{h}^T E\{\mathbf{R}^T(n)\mathbf{R}(n)\}\mathbf{h} \tag{8}$$

This performance index again has a globally minimum value for a unique set of equalisation filter coefficients given by

$$\mathbf{h}_{opt} = -[E\{\mathbf{R}^T(n)\mathbf{R}(n)\}]^{-1}E\{\mathbf{R}^T(n)\mathbf{d}(n)\} \tag{9}$$

An adaptive algorithm is presented below for automatically adjusting the coefficients of $\mathbf{h}$ to be a close approximation to $\mathbf{h}_{opt}$, and this has been used to obtain an equalising filter for the enclosure described in the last section. This equalising filter, however, now attempts to do the best job of equalising at all four microphone positions by minimising the sum of the squares of the differences between the microphone outputs and delayed source signals.

The frequency response and impulse response of this new equalisation filter are shown in Figure 8, and Figure 9 shows the equalised response at all microphone positions, compared to the original responses. It is clear that the peaks which are common to all four microphone responses, for example that at about 200 Hz, have been largely removed. However, the equalising filter has to cope with conflicting requirements at about 90 Hz: of increasing the response in the front of the enclosure and of suppressing the response in the rear. In fact the equalisation filter does suppress the peak in the rear at the expense of creating a dip in the front at this frequency, since this strategy generates a smaller total residual error than boosting the response in the front and having the response in the back rise even further. Apart from the dips in the equalised responses at about 90 Hz in the front, and at about 180 Hz at microphone 2, the equalisation filter can be seen to be doing a reasonable job of equalisation at all points. The variation in the frequency response function from 2 Hz to 500 Hz, averaged across microphones, is about 15 dB when using this equalisation filter, compared to the original average variation in the frequency response function of some 28 dB over this frequency range. Figure 10 shows the original and equalised impulse responses at the four microphones, again with a shift equal to the individual modelling delay added to the original responses for clarity. The equalised responses at each of the microphones tend to be more "compact" in time than the unequalised responses, and a significant pair of "reflections" at about 25 and 30 ms have been largely removed from the responses at the front microphones.

The modelling delays used to generate the desired signals, d(n), at each microphone for these results were chosen for microphones 0, 1, 2 and 3 to be 15, 14, 18 and 17 samples respectively. It was found that if all the modelling delays were set to be equal, a significantly poorer equalised response was obtained overall. It is interesting to note that the differences in the modelling delays used above are approximately equal to the differences in the propagation times of a direct acoustic wave from the loudspeaker to each of the microphones. This suggests that the equalisation filter can equalise the response at each microphone best by simulating a plane propagating wave in the enclosure.

4. ADAPTIVE ALGORITHMS

4.1 Single Channel FIR Algorithms

All of the algorithms used here for adaptively adjusting the coefficients of the equalisation filters are based on the instantaneous gradient descent methods introduced widely by Widrow [8]. The simplest and most common form of this algorithm is the LMS algorithm in which the single error signal e(n) is given by

$$e_1(n) = d(n) - \mathbf{x}^T(n)\mathbf{h}(n) \tag{10}$$

where

$$\mathbf{x}^T(n) = [x(n), x(n-1), \quad \ldots, \quad x(n-I+1)]$$
$$\mathbf{h}^T(n) = [h_0(n), h_1(n), \quad \ldots, \quad h_{I-1}(n)]$$

The LMS algorithm adjusts each of the filter coefficients in $\mathbf{h}(n)$ at each sample time by an amount proportional to the gradient of the instantaneous mean square error [8]:

$$\mathbf{h}(n+1) = \mathbf{h}(n) - \alpha \frac{\partial e_1^2(n)}{\partial \mathbf{h}(n)}$$

Therefore

$$\mathbf{h}(n+1) = \mathbf{h}(n) - \alpha \mathbf{x}(n) e_1(n) \tag{11}$$

α is the convergence coefficient of the algorithm which determines the speed of convergence of the algorithm. If α is too large, however, the algorithm will become unstable and an estimate of the largest stable value of α (α_{max}) has been given by Haykin [9], from a consideration of the second order statistics of $h(n)$, as

$$\alpha_{max} \approx \frac{2}{\overline{x^2}\; I} \tag{12}$$

The use of an adaptive filter using the LMS algorithm for single channel equalisation is illustrated in Figure 11(a). One disadvantage of the algorithm, however, is that the coefficients will be biased by any measurement noise at the input to the adaptive filter, i.e., the output of the system to be inverted [8]. For this reason, Widrow introduced the "filtered x" LMS algorithm, illustrated in Figure 11(b), which also has the advantage that it may be used on-line, and was later found to be generalisable to multiple channels, as we shall see below. The output error for this arrangement may be written, as in Section 2 above, as:

$$e_2(n) = d(n) - \mathbf{r}^T(n)\mathbf{h}(n) \tag{13}$$

Applying the same philosophy, of minimising the instantaneous squared error, leads to the algorithm:

$$\mathbf{h}(n+1) = \mathbf{h}(n) - \alpha\, \mathbf{r}(n) e_2(n) \tag{14}$$

This is called the "filtered x" algorithm because $\mathbf{r}(n)$ consists of the reference signal, $x(n)$, filtered by the impulse response of the system to be inverted as in equation (2) above. In practice this impulse response can only be imperfectly estimated and an approximation to $r(n)$ must be used in the algorithm, although it is found that the algorithm is very robust to errors made in the generation of this reference signal.

α is once again a convergence coefficient which has a certain maximum value before the algorithm becomes unstable. A consideration of the first order statistics of h(n) suggests that α_{max} is inversely proportional to $\overline{r^2}$ in this case rather than $\overline{x^2}$ above. In this case, however, the maximum convergence coefficient is found to depend not only on the length of the adaptive filter and the spectrum of the filtered reference signal, but also on the delays inherent in the error path (i.e., in c(n)). Computer simulations, using a white noise reference signal and a pure delay (of δ samples) in the error path, have been used to establish the maximum convergence coefficient under a variety of conditions. The results of these simulations are shown in Figure 12 and they suggest that α_{max} in 'his case is of the form

$$\alpha_{max} \approx \frac{2}{\overline{r^2}\,(I+\delta)} \tag{15}$$

If the filtered reference signal is not white, however, which was the case when this algorithm was used in the arrangement of Section 2, in which a white reference signal, x(n), was passed through a filter with a non-uniform frequency response to give r(n), this simple formula no longer holds. In particular, $1/\alpha_{max}$ becomes approximately proportional to 1.2I, rather than 0.5I as in equation (15) above. This more complicated behaviour is probably due to the correlation between the samples of the filtered reference signals, which was not present in the simulations used to obtain the results presented in Figure 10. This may be similar to the "eigenvalue spread" problem discussed in [8] and [9], and certainly the convergence of the squared error, in the simulations of the filtered x algorithm used to obtain the results in Section 2, shows evidence of several "modes" of convergence.

4.2 Multiple Channel FIR Algorithm

The generalisation of the single channel filtered x algorithm to multiple channels has been discussed [6] and may be expressed, using the notation of Section 3, as

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \alpha\mathbf{R}^T(n)\mathbf{e}(n) \tag{16}$$

The expression for the maximum convergence coefficient in the single channel case (equation (15)) suggests a generalisation for α_{max}, for the case of L microphones, of the form

$$\alpha_{max} \approx \frac{2}{\sum_{\ell=1}^{L} \overline{r_\ell^2}\,(I + \delta_\ell)} \tag{17}$$

where $\overline{r_\ell^2}$ is the mean square value of the ℓ'th reference signal and the ℓ'th error path has a delay of δ_ℓ. In the simulations of the 4 channel filtered x algorithm performed for Section 3 of this paper, the delays in the error path were relatively small ($\delta_\ell \approx 4$ to 8 samples) so their effect was difficult to determine, but the variation of α_{max} with filter length (I) conformed reasonably well to the expression

$$\alpha_{max} \approx \frac{1}{1.2 \sum_{\ell=1}^{4} \overline{r_\ell^2}\, I} \tag{18}$$

4.3 Adaptive IIR Algorithm

One other algorithm was investigated for use in this application, which differed from those above in that the equalisation filter had a recursive form, i.e., its output y(n) was related to its input x(n) by

$$y(n) = \sum_{i=0}^{I/2} a_i x(n - i) + \sum_{j=1}^{I/2} b_j y(n - j) \tag{19}$$

where the total number of filter coefficients (a_i and b_j) is now $I + 1$. The motivation for such recursive (IIR) filters is that they are potentially more efficient than non-recursive (FIR) ones, i.e., they can achieve the same performance with a smaller number of coefficients. Unfortunately the recursive nature of equation (19) potentially complicates any adaptive algorithm designed to adjust the coefficients. This is partly due to the fact that for some combinations of the coefficients b_j, the filter may be inherently unstable and an adaptive algorithm may find it difficult to recover from such an instability. Another difficulty is that the mean square error is now no longer a simple quadratic function of each of the filter coefficients, so gradient descent methods such as those used above are no longer guaranteed to converge.

Despite these potential disadvantages, single channel adaptive recursive filters have been used by Eriksson *et al* [10] in active noise control applications, which are formally very similar to the equalisation problem considered here, and these authors have reported that the algorithm they used was reliable and robust. A multiple-channel generalisation of the Eriksson algorithm was reported in [11] and this has been used in a simple comparison with the results of using a non-recursive adaptive filter reported in Section 3.

The results suggest that the algorithm reported in [11] when used to minimise the modelling error at 4 microphones in Section 3 with 26 non-recursive and 25 recursive coefficients, does converge on a timescale comparable with that of the convergence of the FIR algorithm. The final residual error is also very close to that found with the 50 coefficient FIR equalisation filter, and if the responses at each of the microphones are calculated when equalised with the IIR filter, they are barely distinguishable from those using the FIR filter reported in Figure 9. The converged IIR filter coefficients have been used to calculate the first 100 samples of the (infinite) impulse response of this equaliser which is shown in Figure 13, together with the 50 coefficients which comprise the complete impulse response of the converged FIR filter from Figure 8. It is clear that the impulse responses of the two equalisation filters are very similar and although the recursive property of the IIR filter allows it to have some response beyond 50 samples, this response is small and has little effect on the frequency domain behaviour.

This preliminary investigation suggests that although the multiple channel IIR adaptive algorithm does converge, in the application considered here, it gives no better results than an FIR adaptive filter with a comparable number of coefficients.

5. CONCLUSIONS

The equalisation of the acoustic response of rooms using adaptive digital filters has been considered. The problem of equalising the response at one position only in the room is first discussed, and this is formulated in the time domain by minimising the mean square error between the equalised response and a delayed version of the original signal.

The acoustic response of a small enclosure, with dimensions and acoustic damping typical of a car interior, has been modelled, using a modal summation, up to a frequency of about 500 Hz. Using an adaptive FIR digital filter with 50 coefficients, and a modelling delay of 15 samples, it is found that the response at one position can be very effectively equalised. The consequence of equalisation at this one point, on the response at other points in the enclosure, has also been calculated. It is found that at some other points in the enclosure the response is significantly degraded by the equalisation filter, as has previously been suggested by Mourjopoulos *et al* [3] and Farnsworth *et al* [4].

An alternative equalisation strategy has been presented in order to try to overcome this problem; that of minimising the sum of the mean square errors between the equalised responses at several points in the enclosure and individually delayed versions of the original signal. Results from applying this approach at four rather widely spaced positions in the enclosure above have been presented, which demonstrate that some improvements can be effected, although there inevitably remain differences in the equalised responses at the four positions. The modelling delays, used to form the error signals at the four equalisation positions, which gave the most convincing overall equalisation suggest that the equalisation filter does best by attempting to simulate a plane progressive wave in the enclosure.

Various algorithms have been presented for practically adapting the coefficients of an FIR equalisation filter to achieve a good approximation to the single or multiple point least squares solution. The maximum convergence coefficients of these algorithms have been considered in relation to the delays in the path to be equalised and the number of coefficients in the adapting filter. An algorithm for adapting the coefficients for an IIR digital filter for multiple point equalisation has also been investigated. This filter converges to a solution very similar to that found by an adaptive FIR of comparable length, and there would appear to be no advantage to using such a filter over an FIR filter in this particular application.

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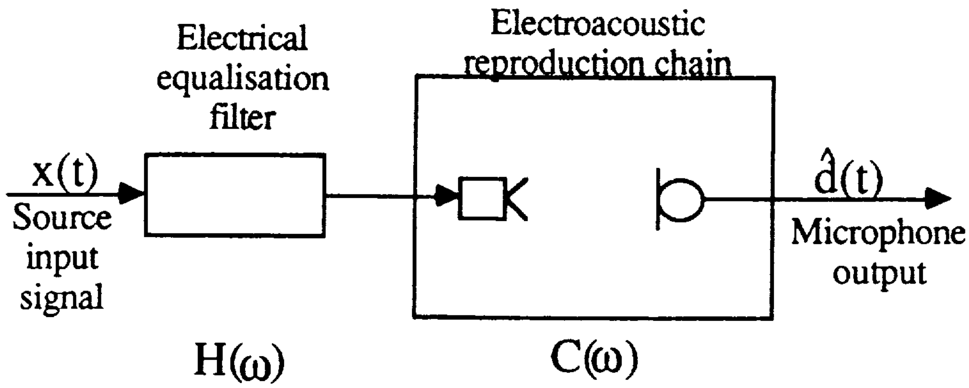

Figure 1: The use of an equalisation filter in a sound reproduction system.

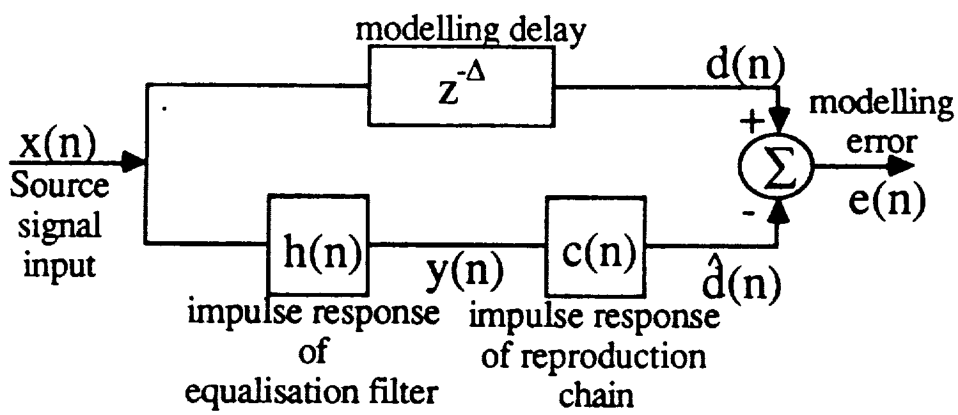

Figure 2: Block diagram of single point equalisation problem with sampled signals.

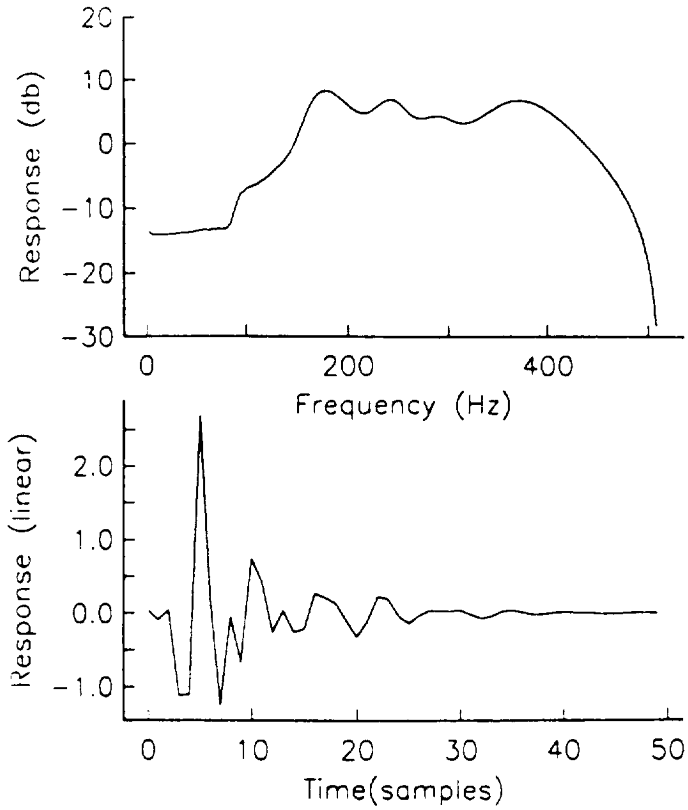

Figure 3:

A typical frequency and impulse response in the enclosure used for the computer simulations.

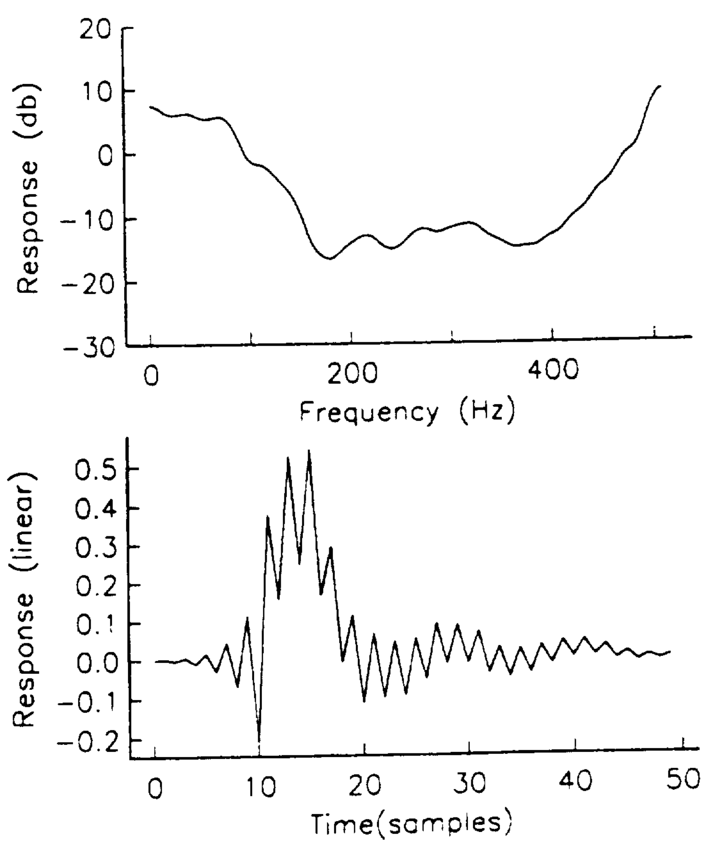

Figure 4:

The frequency and impulse responses of the filter which equalises the enclosure response of Figure 3 with a modelling delay of 15 samples.

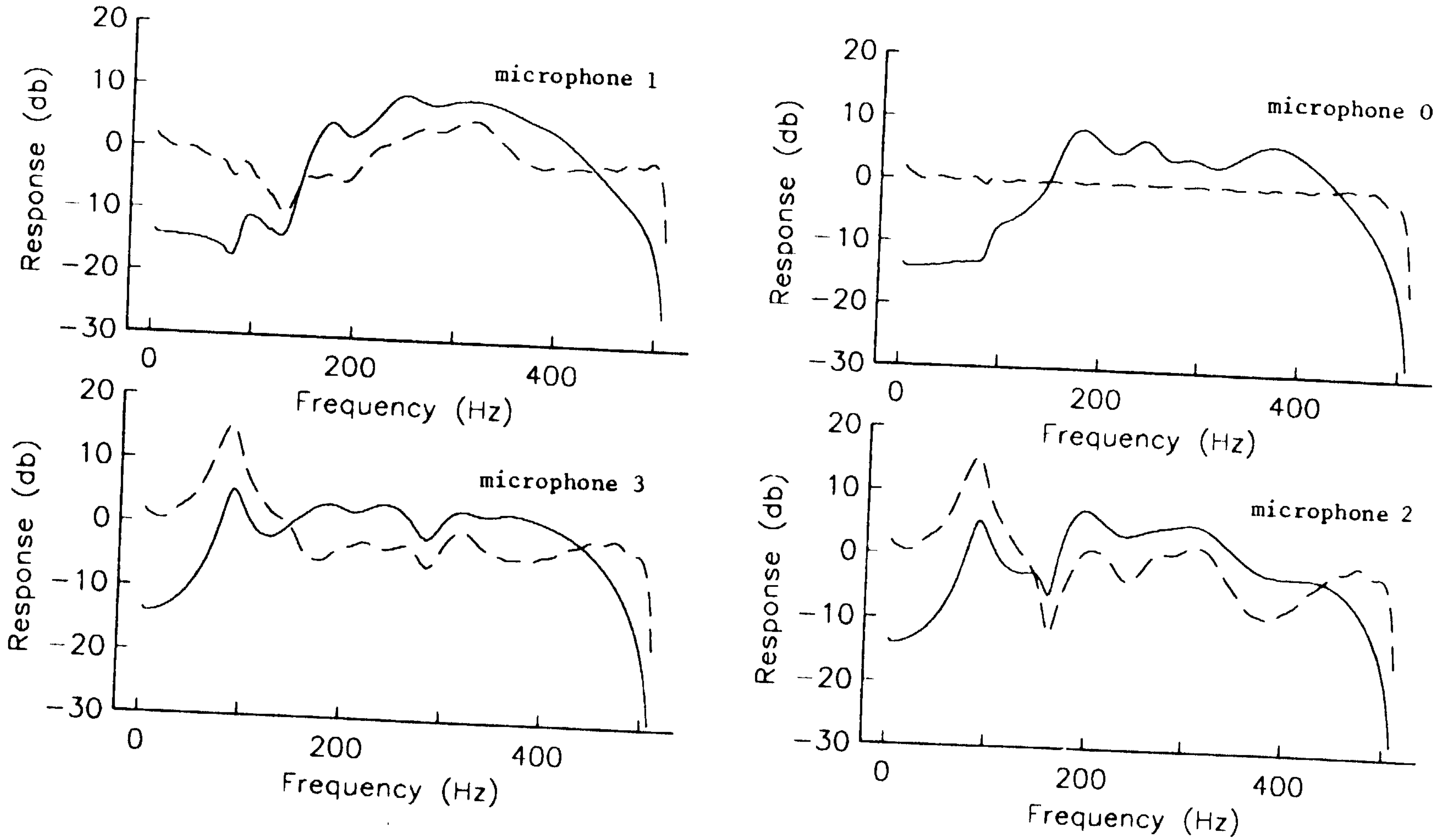

Figure 5: The original frequency response from the loudspeaker to four microphone position in the enclosure (solid line) and the response after the introduction of an equalisation filter designed to equalise the response at microphone 0 (dashed line).

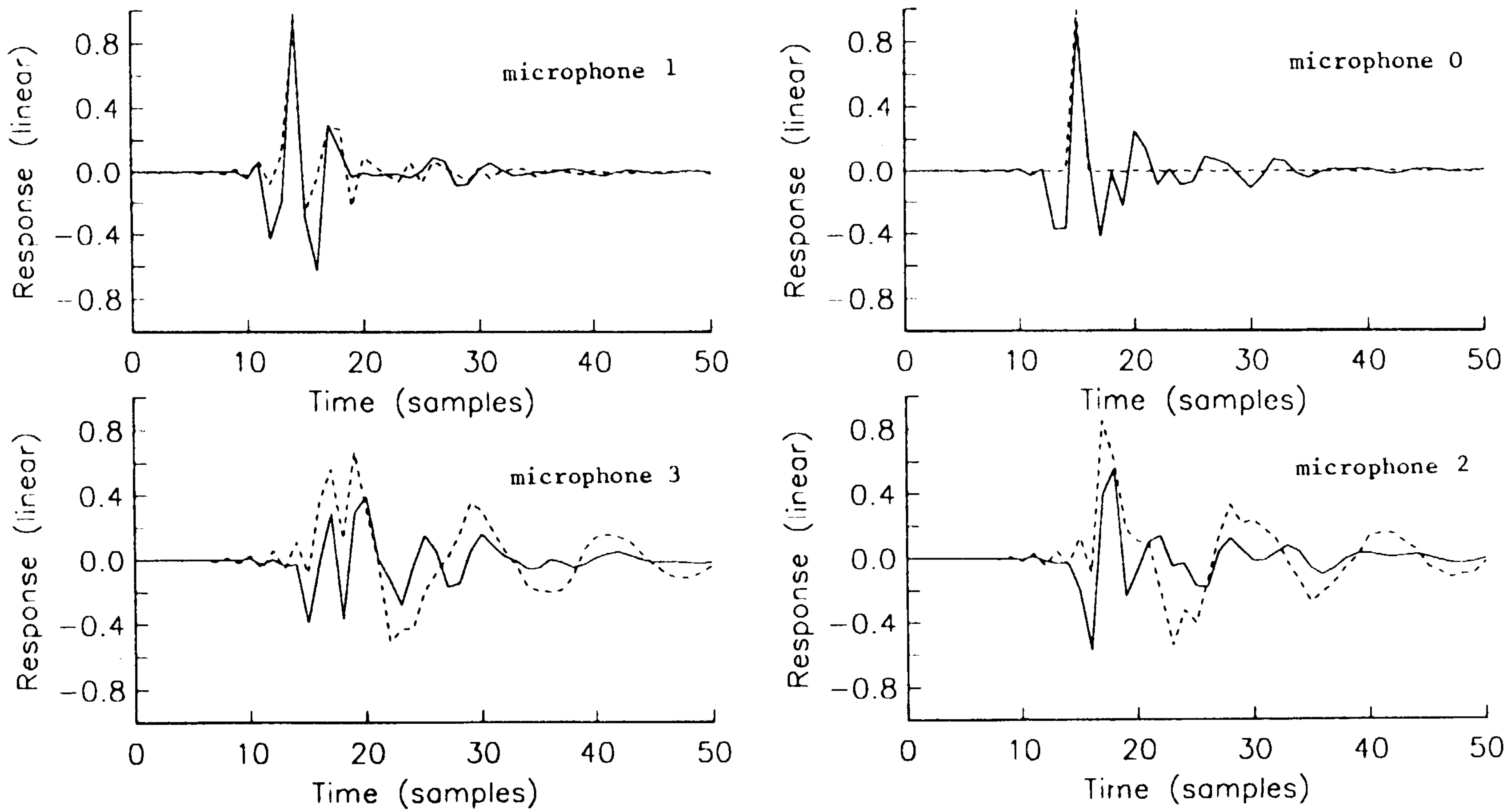

Figure 6: The original impulse response from the loudspeaker to four microphone positions in the enclosure (solid line) and the net impulse responses after the introduction of an equalising filter designed to equalise the response at microphone 0 (dashed line).

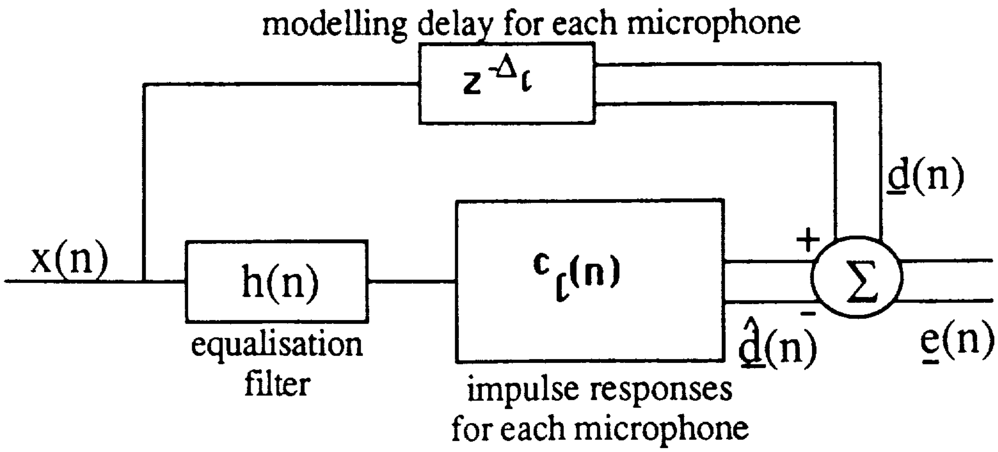

Figure 7: Block diagram of the multiple point equalisation problem with sampled signals.

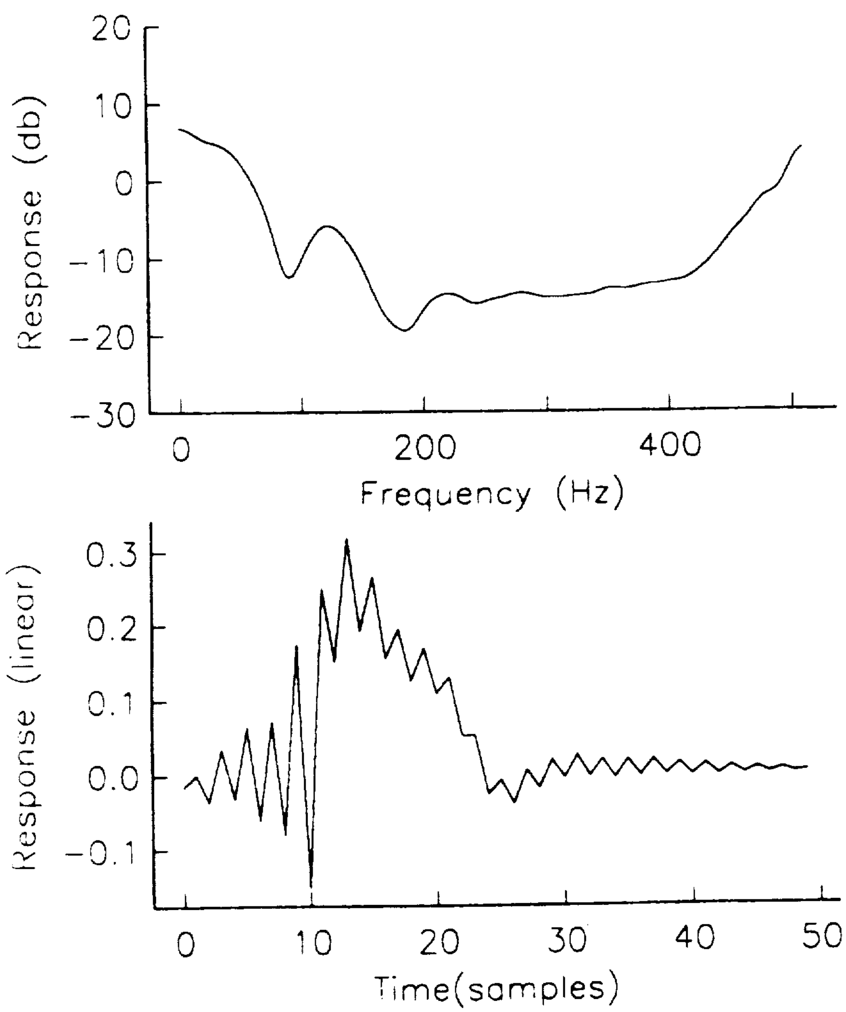

Figure 8:

The frequency and impulse response of the equalisation filter which minimises the sum of the mean squared modelling error at all four microphones.

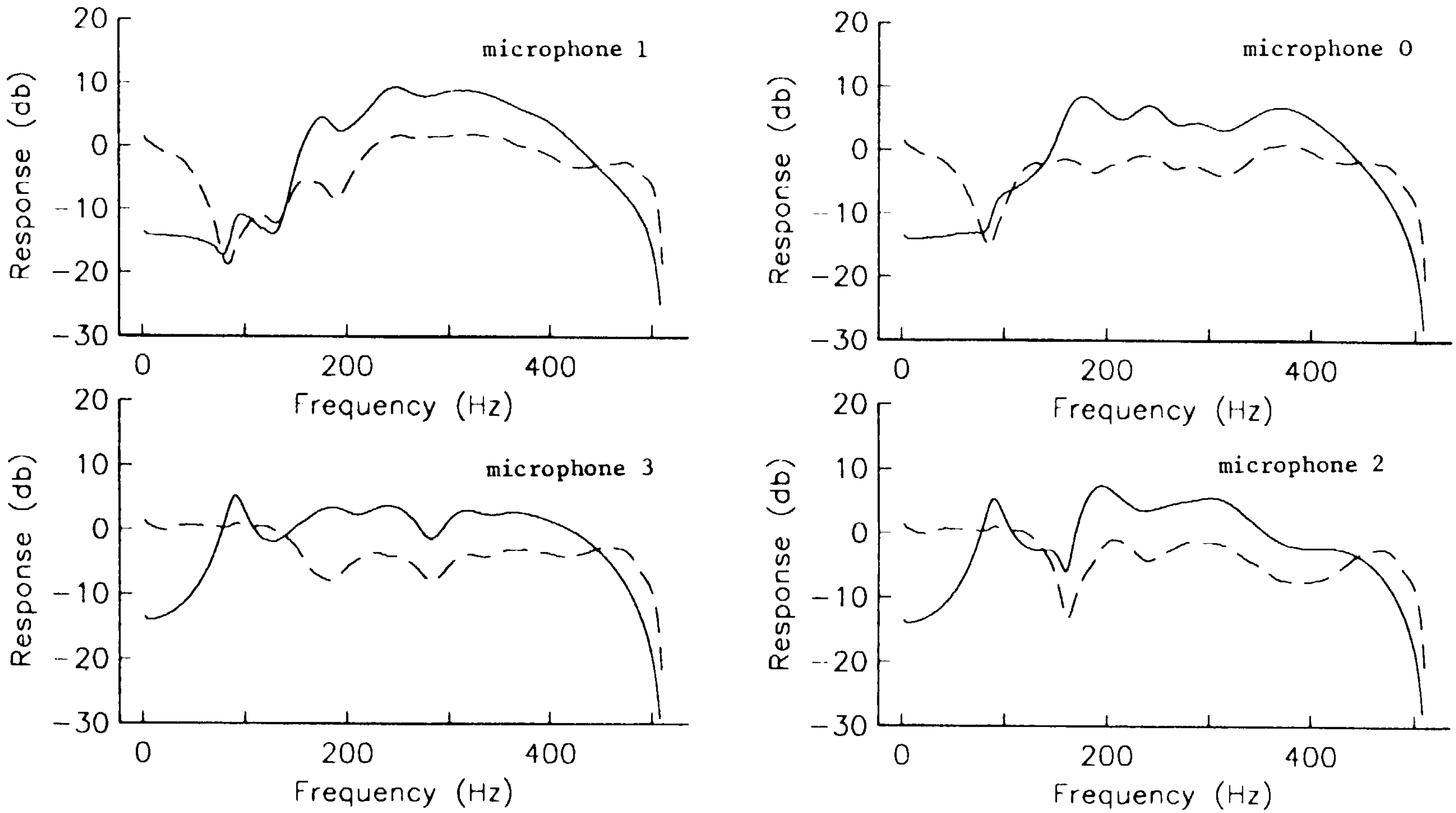

Figure 9: The original response from the loudspeaker to four microphone positions in the enclosure (solid line) and the response after the introduction of an equalisation filter which minimises the mean square modelling error at all four microphones (dashed line).

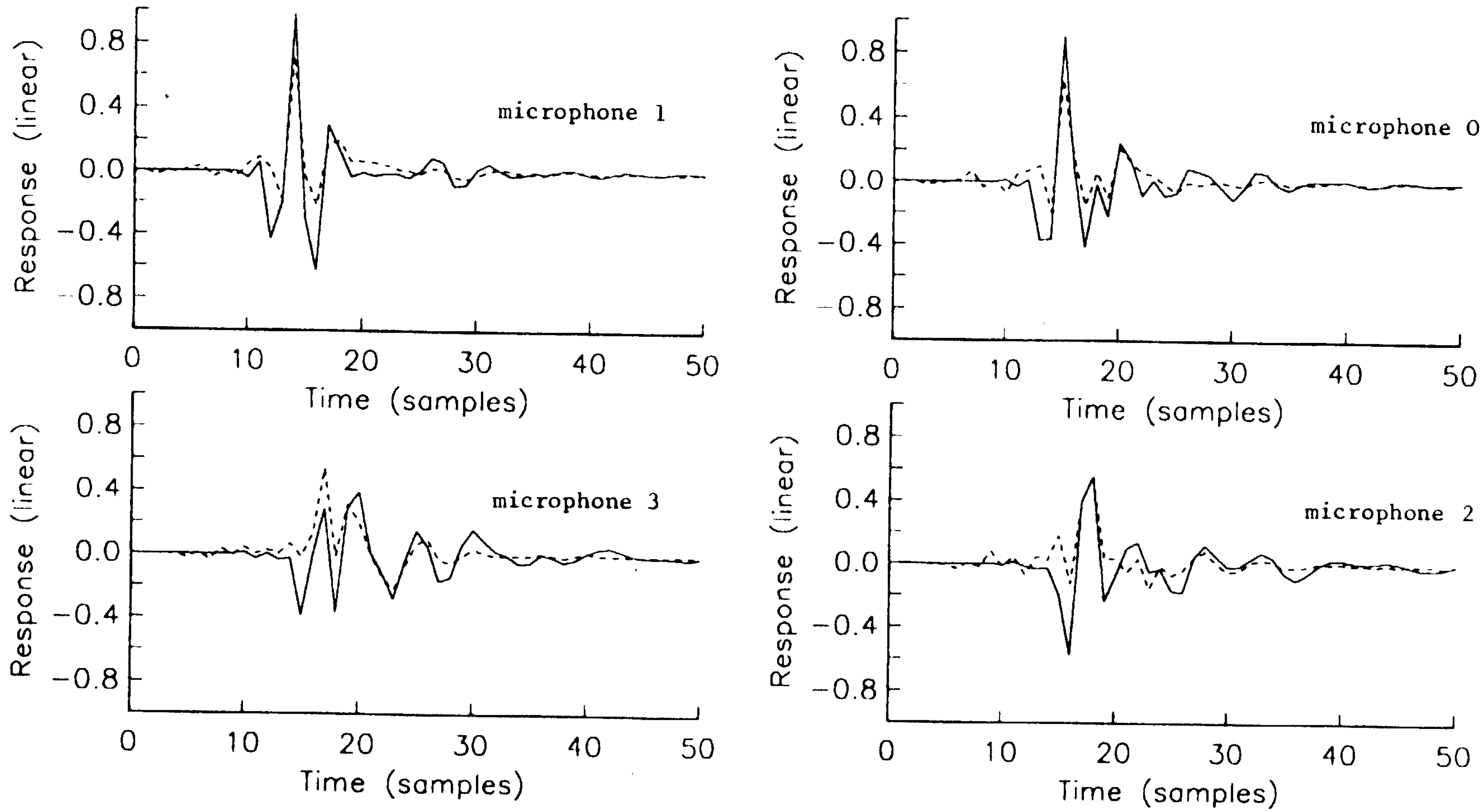

Figure 10: ***The original impulse response from the loudspeaker to four microphone positions in the enclosure (solid line), and the net impulse response after the introduction of an equalisation filter which minimises the mean square modelling error at all four microphones (dashed line).***

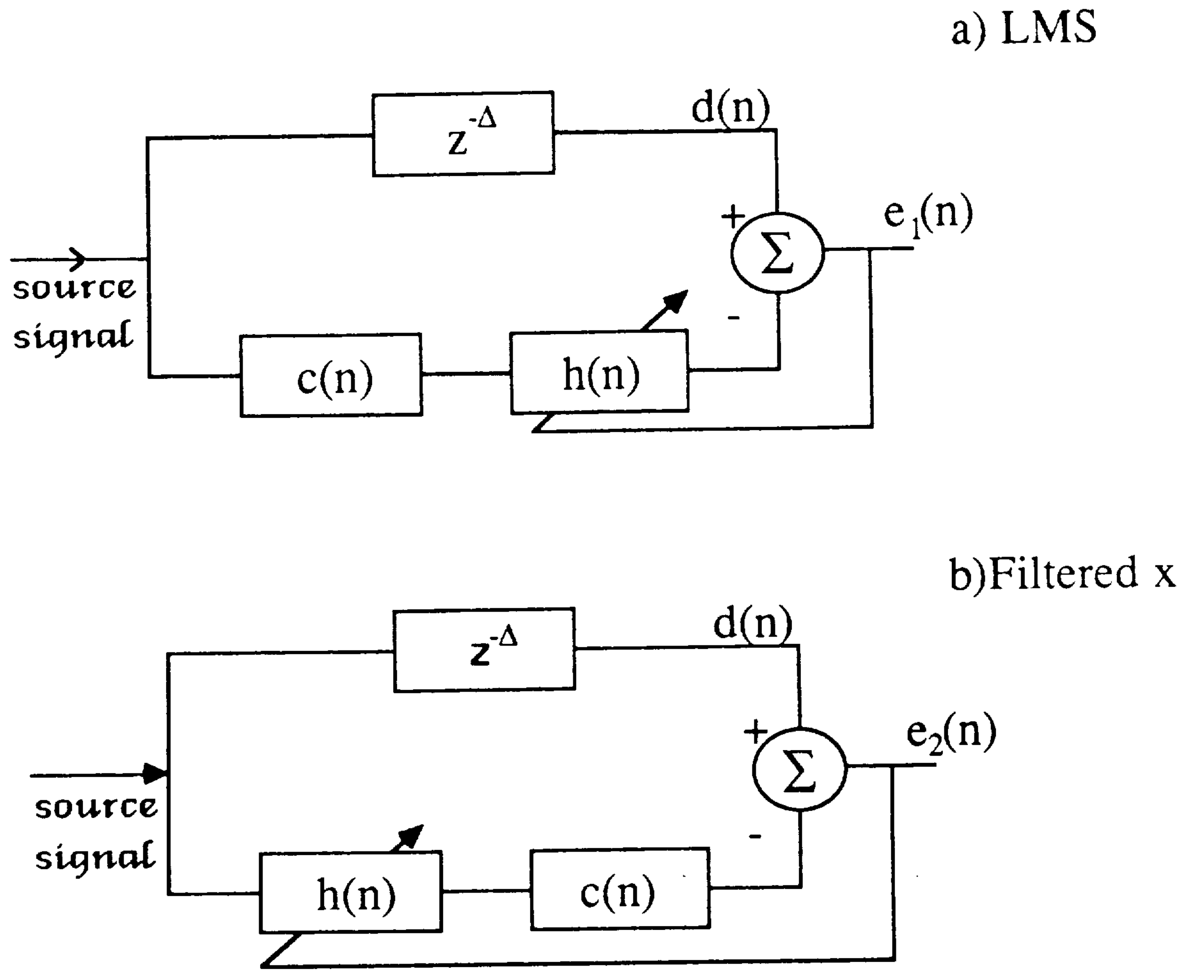

Figure 11: Block diagrams showing the difference between the conventional LMS algorithm used for equalisation of c(n), (a) and the filtered x algorithm (b).

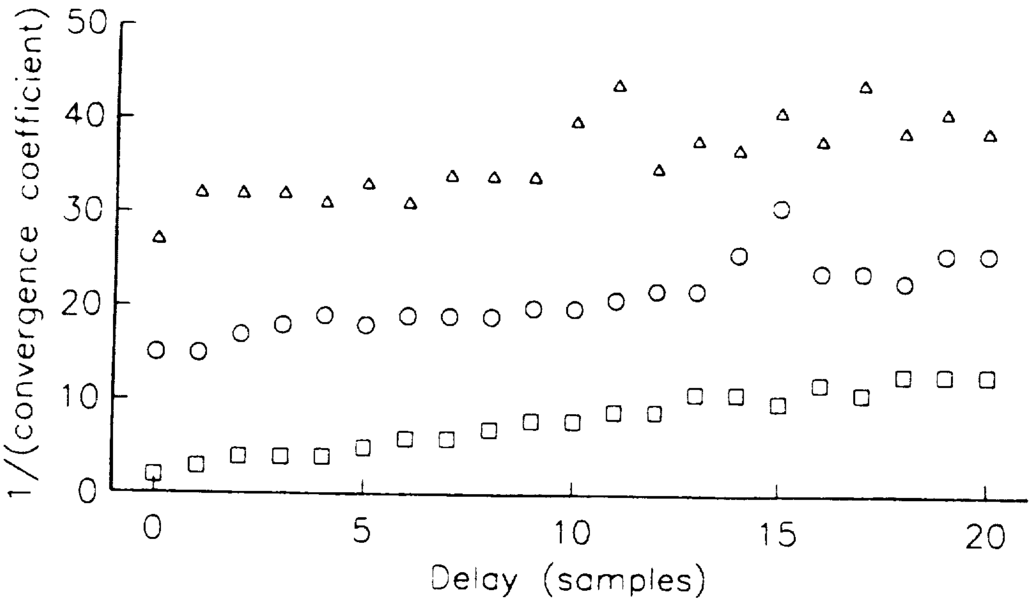

Figure 12 The reciprocal of the largest convergence coefficient before instability for simulations of the filtered x algorithm with a white noise reference signal, various pure delays in the error path and for filter lengths of : 1 coefficient (˙), 25 coefficients (o) and 50 coefficients (Δ).

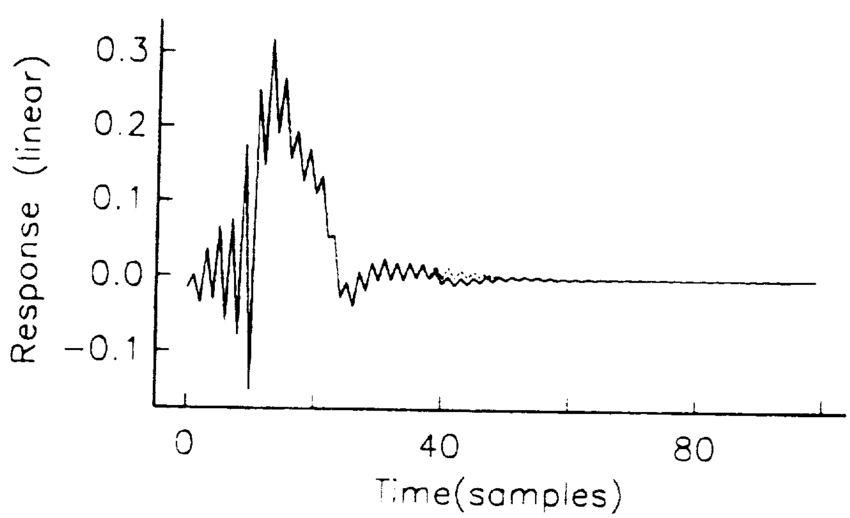

Figure 13: The impulse response computed from the coefficients of the adapted IIR filter used for equalisation at four microphones (solid line), and the coefficients of the adapted FIR filter (dotted line), which extends only for the first 50 samples.

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16. Abstract
Equalisation filters designed to minimise the mean square error between a delayed version of the original electrical signal and the equalised response at a point in a room have previously been investigated by several authors. In general, such a strategy degrades the response at positions in the room away from the equalisation point.

A method is presented for designing an equalisation filter by adjusting the filter coefficients to minimise the sum of the squares of the errors between the equalised responses at multiple points in the room and delayed versions of the original electrical signal. Such an equalisation filter can give a more uniform frequency response over a greater volume of the enclosure than the single point equaliser above.

Computer simulation results are presented of equalising the frequency responses from a loudspeaker to various typical ear positions, in a 'room' with dimensions and acoustic damping typical of a car interior, using the two approaches outlined above. Adaptive filter algorithms, which can automatically adjust the coefficients of a digital equalisation filter to achieve this minimisation, will also be discussed.

17. Key Words (Suggested by Author(s))
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