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1988

NASA/ASEE SUMMER FACULTY RESEARCH FELLOWSHIP PROGRAM

JOHN F. KENNEDY SPACE CENTER
UNIVERSITY OF CENTRAL FLORIDA

SHOCK SPECTRA APPLICATIONS TO A CLASS OF MULTIPLE DEGREE-OF-FREEDOM STRUCTURES SYSTEM

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Date: August 1988

Contract No.: University of Central Florida
NASA-NGT-60002

ACKNOWLEDGEMENT

The author wishes to thank Dr. Gary Lin for providing focus of this work, valuable discussions and support.

Sincere thanks are extended to NASA/ASEE/KSC/UCF Summer Faculty Fellowship Program for giving me the opportunity to spend two rewarding summers at the Kennedy Space Center.

Last but not least, thanks go to Ms. Carlene Bonomo for her assistance in providing computer service.

ABSTRACT

The demand on safety performance of launching structure and equipment system from impulsive excitations necessitates a study which predicts the maximum response of the system as well as the maximum stresses in the system. A method to extract higher modes and frequencies for a class of multiple degree-of -freedom (MDOF) Structure system is proposed. And, along with the shock spectra derived from a linear oscillator model, a procedure to obtain upper bound solutions for the maximum displacement and the maximum stresses in the MDOF system is presented.

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1.1 INTRODUCTION

The prevention of structure and equipment from damage by impulsive excitions neccessitates a study which will predict the maximum dynamic response of the system. Two kinds of impulsive excitations are considered in this study; a blast -pressure which acts directly on the structure or equipment, and a sudden acceleration of bases which support structure or equipment. The investigation can provide some useful informatuin which is relevant to the KSC launching equipment shock design applications.

The purpose of this study is to develop a practical method which will efficiently extract higher modes and frequencies for a class of Multiple Degree-of-Freedom (MDOF) structures. When these higher modes and frequencies are used along with the shock spectra of a linear oscillator subjected to the same excition, their contributions to the maximum stresses in the real structure could well be very significant.

2.1 ANALYSIS

At least for the purpose of estimate or in the initial design stage, a detailed dynamical analysis of a real structure system is rarely attempted. The usual practice is to choose an idealized mathematical model consisting of springs (or elastic elements), dampers, and lumped masses which closely perform in the same way as the real structure or equipment. Figure 1.1 shows how each real structure or equipment is represented by an idealized mathematical model. In this study, damping is excluded from the analysis, since only the maximum dynamical response of the system is of primary interest.

A class of structures considered in this study are beams and frames of various support conditions. These structures are the typical ones which support equipment or instruments, and in certain cases , represent the equipment itself. For the sake of simplicity and practicality, only up to three DOF structures are included in this study. Accordingly, the method is considered efficient when it is applied to these structures. In developing the method, with the exception of the first mode and frequency which require a few iterations, the solution extracts higher modes and frequencies directly from the frequency equation. The equations governing the motion of MDOF structures are written in terms of flexural modes, but they are equally applicable to the cases of torsional modes.

Table 1.1 shows the class of structures which are included in this study.

2.1.1 SINGLE DEGREE-OF-FREEDOM SYSTEM (SDOF). A brief description of the SDOF structure system is discussed first, because it can providemuch insights to the subsequent study of the MDOF structure system. The concept of SDOF model implies that a single coordinate is sufficient to describe the motion of a real structure. The equation of motion of an equivalent SDOF model is given by:

$$m_e \ddot{y}(t) + K_e y(t) = F_e(t) \qquad (1)$$

where m_e, K_e and F_e are parameters of the equivalent SDOF system, the values of which are evaluated on the basis of an assumed deflection shape of the real structure. Detailed expressions of these parameters are given is the Appendix A. The natural frequency, ω_e, of the equivalent SDOF system is simply

$$\omega_e = \left(\frac{K_e}{m_e} \right)^{\frac{1}{2}} \qquad (2)$$

whith ω_e being known, the maximum dynamic magnification factor $(DMF)_{max}$, which is defined as the ratio of the maximum dynamic deflection to the deflection which would have resulted from the static load appplication. It should be emphasized that the maximum dynamical response thus obtained for the equivalent SDOF system is identical to that in the real structure. The maximum dynamic stress is then given by:

$$\sigma_{dy} = \sigma_{st} \, (DMF)_{max} \qquad (3)$$

where σ_{st} is the maximum static stress and σ_{dy} the maximum dynamic stress, both are induced by the same impulsive excitation. An example is given in Appendix C which illustrates the application of the SDOF concept.

2.1.2 MULTIPLE DEGREE-OF-FREEDOM SYSTEM (MDOF). If a real structure system has more than one possible mode of displacement, then more than one independent coordinate is needed to describe its response. The structure system must now be rrepresented by a MDOF model. In a MDOF system; determining frequencies and modes become exceedingly cumbersome, because one must deal with a complete set of equations of motion, one equation for each degree of freedom. The complexity, however, can be reduced greatly by using the modal analysis concept in which the response in the normal modes are determined separately, and then superimposed to provide the total response.

2.1.2.1 Fundamental Mode and Frequency. In most practical problems, usually a few of the lower modes are of interest. Therefore, the Rayleigh method is convenient to use, especially in finding the fundamental frequency. By this method, the natural frequency of the fundamental mode (first mode) can be obtained with considerable accuracy and yet with relative ease. Although the mode shape obtained is less accurate, that can be improved with few iterations. In Rayleigh, the equation used to obtain the natural frequency of fundamental mode is given by:

$$\omega^2 = \frac{\sum_{r=1}^{j} F_{ri}\,\phi_r}{A \sum_{r=1}^{j} M_r\,\phi_r^2} \qquad (4)$$

where

ϕ_r = displacement coordinate of rth mass

F_{ri} = inertia force of rth mass

A = a constant

M_r = rth mass

ω = natural frequency of fundamental mode

In many practical problems, a reasonable solution of fundamental frequency is often obtained by assuming the static deflection curve as the mode shape, and the dynamic deflection curve is used in subsequent iterations if desirable.

2.1.2.2 High Modes and Frequencies. After the fundamental mode and frequency have been determined from the preceding section, the next higher modes and frequencies of a three DOF system are then directly extracted from the following frequency equation.

$$g_n^6 + C_4 g_n^4 + C_2 g_n^2 + C_0 = 0 \qquad (5)$$

where g_n relates to the frequency of higher mode and C_4, C_2, C_0 are constants which relate to masses and flexibility coefficients of the particular structure concerned. Detailed descriptions of variables and constants in Eq.(5) are given in the Appendix B.

2.1.2.3 Upper Bound Maximum Displacement of Masses. The upper bound of the maximum displacement, $y_{r,max}$, of rth mass due to all modes is given by:

$$y_{r,max} = \sum_{n=1}^{N} A_{nst} \phi_{rn} (DMF)_{max,n} \qquad (6)$$

where

A_{nst} = modal static desplacement

ϕ_{rn} = displacement coordinate of rth mass for nth mode

N = number of modes

$(DMF)_{max}$ = maximum dynamic magnification factor

The $y_{r,max}$ computed in this manner is a rather conservative estimate of the maximum displacement.

2.1.2.4 Maximum Dynamic Load. In order to find the maximum stress in the structure, the maximum relative displacement between two adjacent masses must be determined first, and which is given by the following equation.

$$\Delta_{r,max} = \sum_{r=1}^{N} A_{nst} (\phi_{rn} - \phi_{(r-1)n}) (DMF)_{max,n} \qquad (7)$$

where

$\Delta_{r,max}$ = maximum relative displacement between rth mass and (r-1) mass for all modes

The maximum dynamic force, F_r, which induces maximum dynamic stress in the real structure, is then given by:

$$F_r = K_r \Delta_{r,max}$$

Where K_r is the spring constant between the rth mass and the (r-1)th mass.

Now by replacing the static force in the real structure with one, the maximum dynamic force, in the same structure, the computation of maximum dynamic stress can be proceeded in the same way as in the static case.

3.1 APPLICATION

Eight beams and frames of various support conditions are chosen in this study. They are grouped into three catagories below and also are shown in Table 1.1

a. SDOF System

Simply Supported Beam

b. Two DOF System

Simply Supported Beam
Fixed Ends Beam
Overhanging Beam
Rigid Body on Flexible Supports

c. Three DOF System

Simply Supported Beam
Simply Supported and Fixed End Beam
Shear-Building Frame

Frequencies and modes are obtained for all eight cases. These cases, one in each catagory, are chosen in stresses computation. No attempts are made to include all possible cases, the method, however, is general enough in application that a modification on flexibility coefficients is all that required . A computer program is written for each case except the SDOF one. In the program, the flexibility coefficients are derived from the static deflection curve. Examples in Appendix D show modes and frequencies for all eight cases and stresses computation for three cases. Although programs and examples are written in flexural mode, they are equally valid in torsional mode. To obtain results in the torsional mode, simply substitute the mass, modulus of elasticity, and the area moment of inertia in the flexural mode with the mass moment of inertia, modulus of rigidity, and polar moment of inertia in the torsional mode, respectively.

4.1 RESULTS AND DISCUSSIONS

Modes and frequencies are obtained for all seven cases in the MDOF system. And stresses are computed for three cases, one in each catagory. The results are verified from some known sources. The method is general and yet efficient to extract higher modes and frequencies in a MDOF system. In application, flexibility coefficients must be obtainded first for each structure concerned. The advantage of this proposed method is that modes and frequencies obtained in the MDOF system and the shock spectra developed in the linear oscillator can each serve as an independent module. Any change in one does not affect the other. But both must act together to obtain the maximum displacements and stresses in the MDOF structure. For illustrative purpose, some sample outputs of dynamical responses and of shock spectra for a linear oscillator are given in Figures 5.1 through 5.6.

5.1 SUMMARY OF RESULTS

Results of modes, frequencies, and stresses for thje MDOF systems are summarized in Table 5.1. Verifications are made from several known sources.

6.1 FUTURE RESEARCH

Many more cases can be included in the future study. Tables and charts in each case can be gemrated for quick references in shock design applications. If enough cases are developed, most likely, one can model a real structure analogue to one of the cases.

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APPENDIX A

Governing Equation of the Equivalent SDOF System

$$m_e \ddot{y}(t) + K_e y(t) = F_e(t)$$

where

m_e = Equivalent mass

$$= \int_0^L m(x)[\phi(x)]^2 dx + \sum_{i=1}^{j} M_i [\phi(r)]^2$$

K_e = Equivalent spring constant

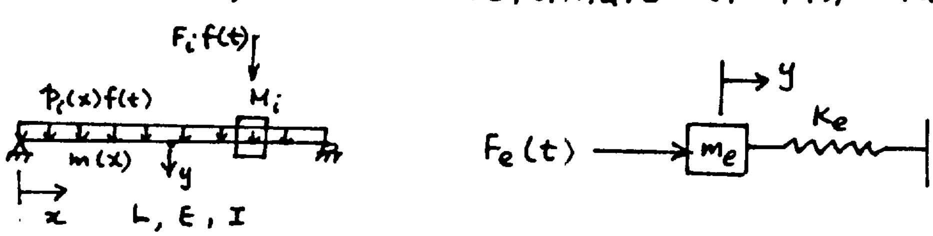
$$= \int_0^L EI(x)[\phi''(x)]^2 dx$$

$F_e(t)$ = Equivalent force

$$= \int_0^L p_i(x) f(t) \phi(x) dx + \sum_{i=1}^{j} F_i f(t) \phi(r)$$

$\phi(x)$ = Assumed mode shape curve of the real structure (normalized Deflection Curve)

$\phi(r)$ = mode Coordinate at rth mass



APPENDIX B

Governing Equations of MDOF System

I. TWO DOF SYSTEM

$$g_n^4 + c_2 g_n^2 + c_0 = 0 \qquad (1)$$

a_{ij} = flexibility coefficient

$$\psi_1 = \frac{a_{12}}{a_{11}}, \quad \psi_2 = \frac{a_{22}}{a_{11}}, \quad a_{21} = a_{12}$$

$$c_4 = -\frac{1 + \psi_2 m_2}{m_2(\psi_2 - \psi_1^2)}, \quad c_0 = \frac{1}{m_2(\psi_2 - \psi_1^2)}$$

$$m_2 = \frac{M_2}{M_1}, \quad A = \psi_2 - \psi_1^2$$

M_1 = mass 1, M_2 = mass 2

ω_1 = reference frequency $= \left(\frac{1}{a_{11}M_1}\right)^{\frac{1}{2}}$

$$g_{n2} = \left(\frac{1}{A m_2}\right)\frac{1}{g_R}$$

$$g_R = \frac{\omega_R}{\omega_1}$$

ω_R = Fundamental frequency from Rayleigh method

Second mode displacement coord. ratio

$$\frac{\phi_2}{\phi_1} = \frac{1 - g_{n2}^2}{\psi_1 m_2 g_{n2}^2}$$

APPENDIX B (CONTINUED)

II. THREE DOF SYSTEM

Governing equations of three DOF System

$$g_n^6 + c_4 g_n^4 + c_2 g_n^2 + c_0 = 0 \qquad (1)$$

a_{ij} = flexibility coefficient

$$\psi_1 = \frac{a_{12}}{a_{11}}, \quad \psi_2 = \frac{a_{22}}{a_{11}}, \quad \psi_3 = \frac{a_{23}}{a_{11}}$$

$$\psi_4 = \frac{a_{33}}{a_{11}}, \quad \psi_5 = \frac{a_{31}}{a_{11}}$$

$$a_{12} = a_{21}, \quad a_{13} = a_{31}, \quad a_{23} = a_{32}$$

M_1, M_2, M_3 = masses

$$m_2 = \frac{M_2}{M_1}, \quad m_3 = \frac{M_3}{M_1}$$

$$\psi_{ij} = \psi_i \psi_j, \quad \psi_{ijk} = \psi_i \psi_j \psi_k$$

$$A_1 = (\psi_{24} + 2\psi_{135}) - (\psi_2 \psi_5^2 + \psi_1^2 \psi_4 + \psi_3^2)$$

$$B_1 = \psi_4 - \psi_5^2, \qquad D_1 = \psi_{35} - \psi_{14}$$

$$B_2 = \psi_2 - \psi_1^2, \qquad D_2 = \psi_{13} - \psi_{25}$$

$$B_3 = \psi_{24} - \psi_3^2, \qquad D_3 = \psi_{15} - \psi_3$$

APPENDIX B (CONTINUED)

$$C_4 = -\frac{m_3 B_1 + m_2 B_2 + m_2 m_3 B_3}{m_2 m_3 A_1}$$

$$C_2 = \frac{m_2 \psi_2 + m_3 \psi_4 + 1}{m_2 m_3 A_1}$$

$$C_0 = -\frac{1}{m_2 m_3 A_1}$$

ω_1 = reference frequency $= \left(\frac{1}{a_{11} M_1}\right)^{\frac{1}{2}}$

$b = C_4 + g_R^2 , \qquad c = -C_0 \frac{1}{g_R^2}$

$$g_R = \frac{\omega_R}{\omega_1}$$

ω_R = Fundamental Frequency from Rayleigh method

$$g_{n2} = \frac{-b - (b^2 - 4c)^{\frac{1}{2}}}{2}, \quad \omega_{n2} = \omega_1 g_{n2}$$

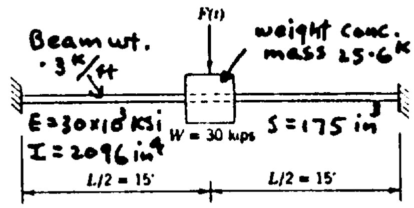
$$g_{n3} = \frac{-b + (b^2 - 4c)^{\frac{1}{2}}}{2}, \quad \omega_{n3} = \omega_1 g_{n3}$$

$$\left(\frac{\phi_2}{\phi_1}\right)_{ni} = \frac{\psi_3 + D_3 g_{ni}^2}{\psi_5 + m_2 D_2 g_{ni}^2} \qquad i = 2, \text{ seond mode}$$

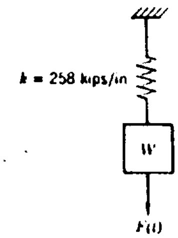
$$\left(\frac{\phi_3}{\phi_1}\right)_{ni} = \frac{\psi_3 + D_3 g_{ni}^2}{\psi_1 + m_3 D_1 g_{ni}^2} \qquad i = 3, \text{ third mode}$$

APPENDIX C

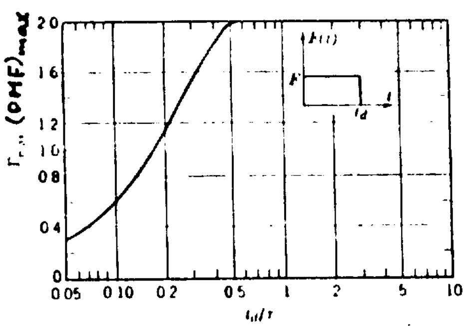
SDOF system Example



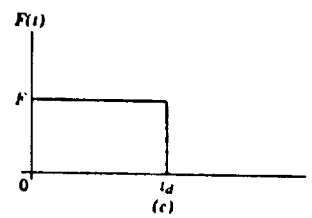
Fixed Ends Beam



Equivalent SDOF system



Shock spectra



Impulsive Force
$t_d = 0.08$ sec
$F = 50^K$

Equivalent Force $F_e = F_t \phi(\frac{L}{2}) = F_t$

Equivalent mass $m_e = \int_0^L m(\phi(x))^2 dx + M[\phi(\frac{l}{2})]^2$

Equivalent stiffness, $K_e = \int_0^L EI[\phi''(x)]^2 dx$

$$\phi(x) = \frac{4x^2(3L-4x)}{L^3}, \quad \phi(\tfrac{L}{2}) = 1, \quad M = \frac{25.6}{g}$$

$$m_e = 0.077 \frac{K\text{-sec}^2}{in}, \quad K_e = 258\ K/in$$

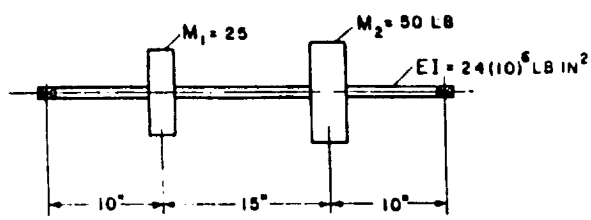
Natural Frequency $w = (\frac{K_e}{m_e})^{\frac{1}{2}} = 58$ rps, $T = 0.109$ sec

$\frac{t_d}{T} = 0.735$, From Shock spectra, $(DMF)_{max} = 2$

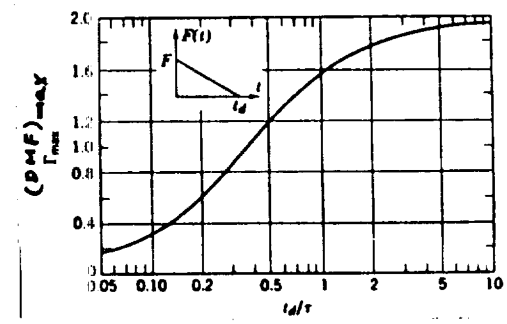
$F_{D,max} = (DMF)_{max} F = 100^K$, $\sigma_{max} = \frac{F_{D,max} L}{8S} = 25.7$ ksi

APPENDIX D

Two DOF System Example



Simply Supported Beam



Shock Spectra

$F_1 = 1^K$, $F_2 = 0.8^K$, $t_d = .02$ sec

	From proposed Method			From Shock Spectra		From Modal Analysis		
Mode	ω	ϕ_1	ϕ_2	$\frac{t_d}{T}$	$(DMF)_{max}$	M_r	F_r	Anst
1	475	1	1.07	1.51	1.7	.0647 .1295	1 0.8	.039
2	1619	1	-.465	5.16	1.9	.0647 .1295	1 0.8	.0026

Unit: ω - rps, $M_r - \frac{k-s^2}{in}$, $F_r - k$, A_{nst} — in

$$A_{nst} = \frac{\sum_{r=1}^{2} F_r \phi_{rn}}{\omega_n^2 \sum_{r=1}^{2} M_r \phi_{rn}^2}$$

Total Displacement of mass for all modes:

mass 1 : $D_{1n} = \sum_{n=1}^{2} A_{nst} \phi_{1n} (DMF)_{max,n} = .0712^{in}$

mass 2 : $D_{2n} = \sum_{n=1}^{2} A_{nst} \phi_{2n} (DMF)_{max,n} = .0685^{in}$

Maximum Dynamic Force :

$F_{d,1} = \frac{a_{22} D_{1n} - a_{12} D_{2n}}{a_{11} a_{22} - a_{12}^2}$, $F_{d,2} = \frac{a_{11} D_{2n} - \alpha_{12} D_{1n}}{a_{11} a_{22} - a_{12}^2}$

Maximum Dynamic stress $\sigma_{max} = \frac{Mc}{I} = 1.61$ ksi

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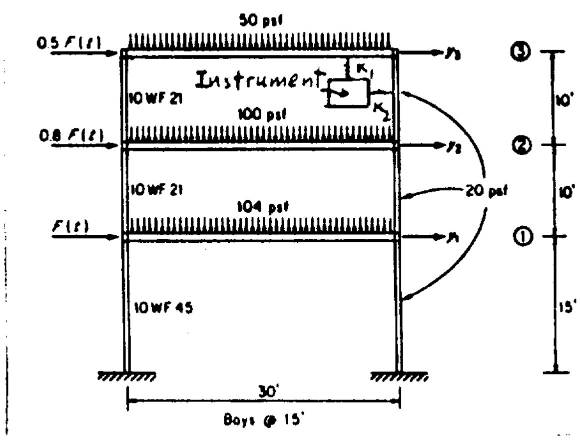
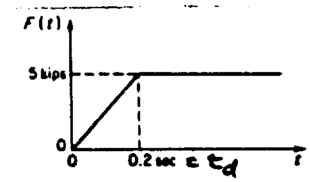
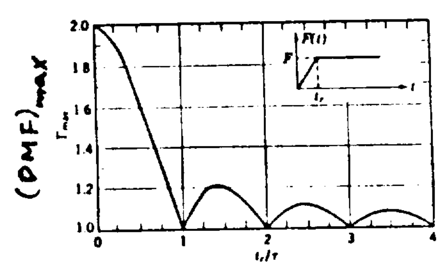
APPENDIX D

THREE DOF system Example



Three-Story Building Frame





Shock Spectra

Mode	From proposed Method				From Shock Spectra		From Modal Analysis		
	ω	ϕ_1	ϕ_2	ϕ_3	$\frac{t_d}{T}$	$(DMF)_{max}$	M_r	F_r	A_{nst}
1	8.32	1	1.471	1.639	.265	1.89	141	5	.358
							132	4	
							66	2.5	
2	24	1	-.146	-1.041	.77	1.28	141	5	.0146
							132	4	
							66	2.5	
3	35	1	-2.22	2.68	1.12	1.11	141	5	.0018
							132	4	
							66	2.5	

Units ω - rps, M_r - $\frac{K \cdot s^2}{in}$, F_r - K, A_{nst} - in

APPENDIX D

$$A_{nst} = \frac{\sum_{r=1}^{3} F_r \phi_{rn}}{w_n^2 \sum_{r=1}^{3} M_r \phi_{rn}^2}$$

The maximum roof displacement for all modes :

$$y_{3,max} = \sum_{n=1}^{3} A_{nst} \phi_{3n} (DMF)_{max,n} = 1.13^{in}$$

The maximum relative displacement between the roof and the second floor for all modes

$$\Delta_{3,max} = \sum_{n=1}^{3} A_{nst} \left[\phi_{3n} - \phi_{2n} \right] (DMF)_{max,n}$$

$$= 0.107^{in}$$

The maximum moment at column end in the roof is

$$M_{max} = \frac{6 E I \Delta_{3,max}}{L_3^2} = 142.5^{k\text{-}in}$$

The maximum dynamic stress

$$\sigma_{max} = \frac{M_{max}}{S} = 3.54 \text{ KSi}$$

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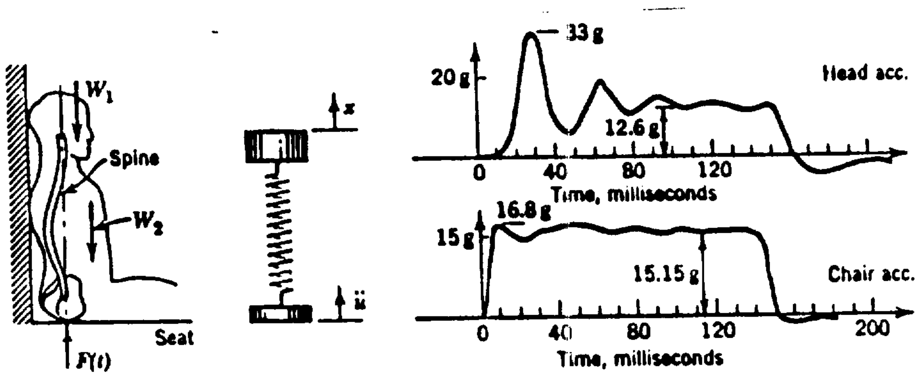
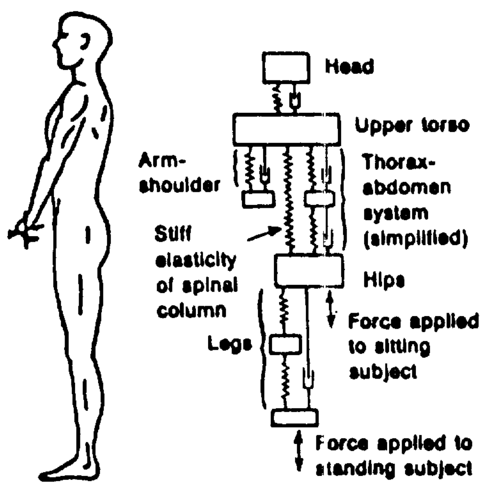
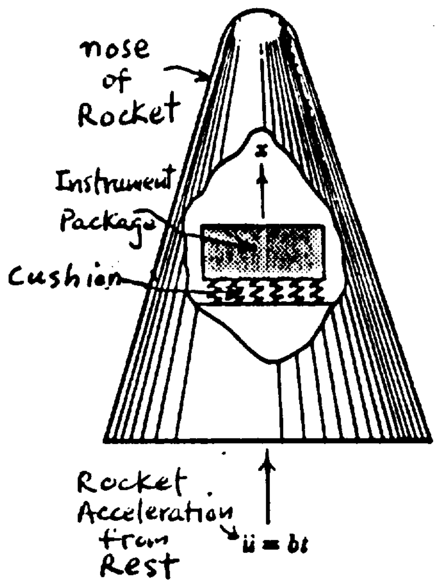


Simplifed Model Showing Human Head dynamical response due to sudden base acceleration (Jet seat Ejection)



A more complex model representing the dynamical system of a human body

FIGURE 1·1 REAL SYSTEM AND MATHEMATICAL MODEL



Simplified Model Showing an Instrment Package Inside a rocket nose Subjected to sudden lift-off

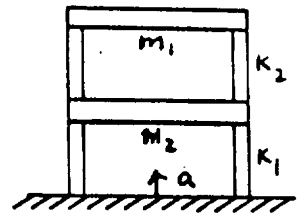
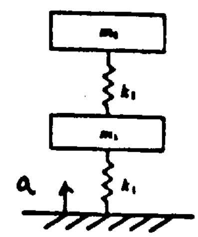
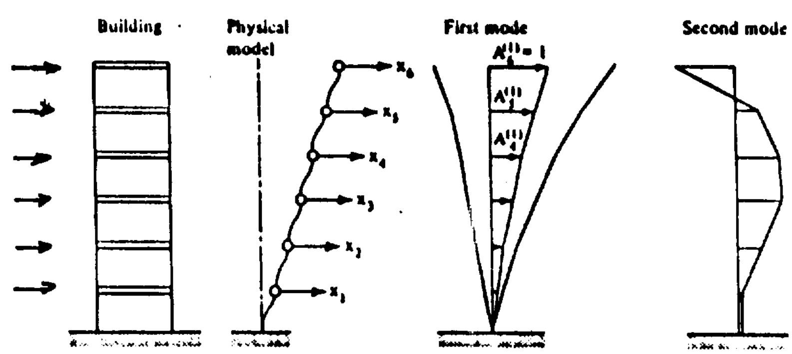


Fig. 1-9

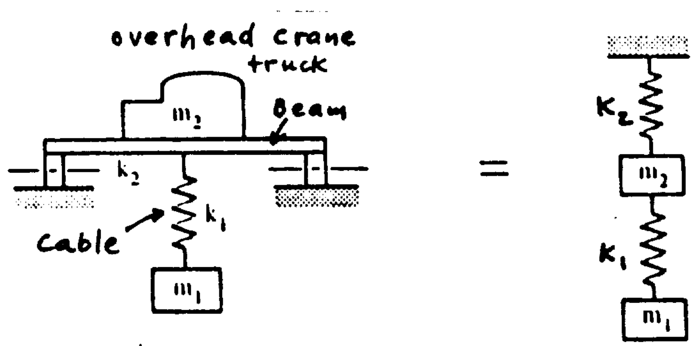


Simplified model representing a two story building due to impulsive Vertical Foundation Acceleration

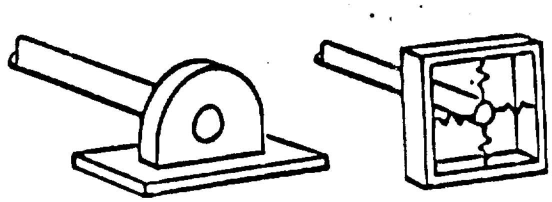


Simplified model showing a multistory building subjected to impulsive horizontal Loads

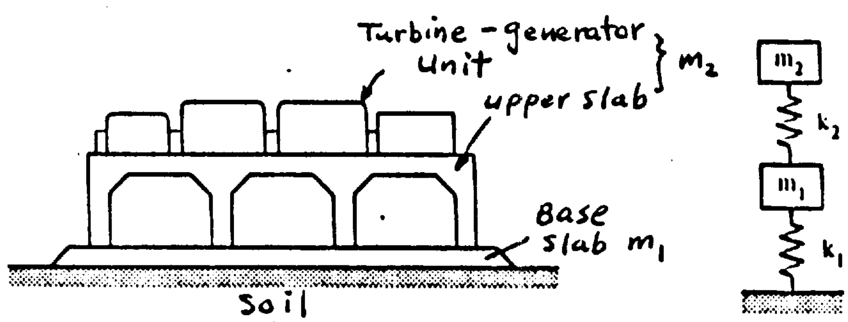
FIGURE 1.1 (CONTINUE)



Simplified model representing a overhead crane lifting a heavy object

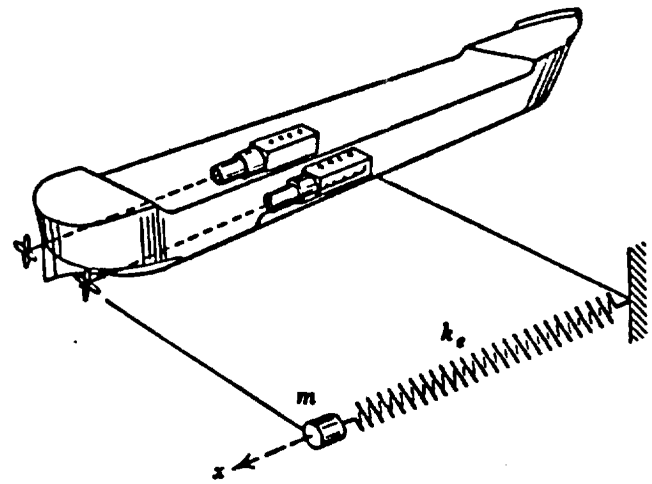


Simplified model representing a bearing support for a rotating shaft

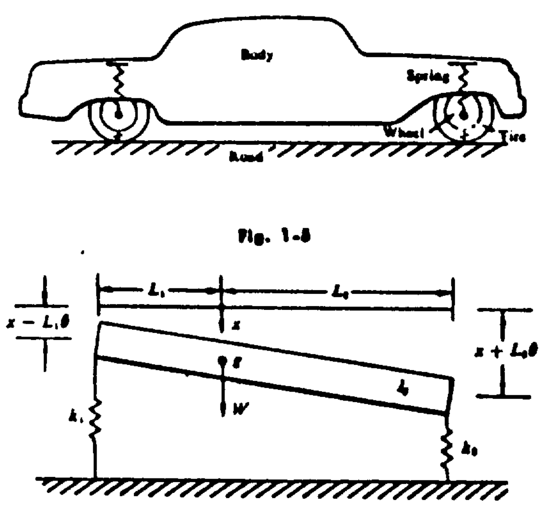


Simplified model representing a turbine-generator foundation system

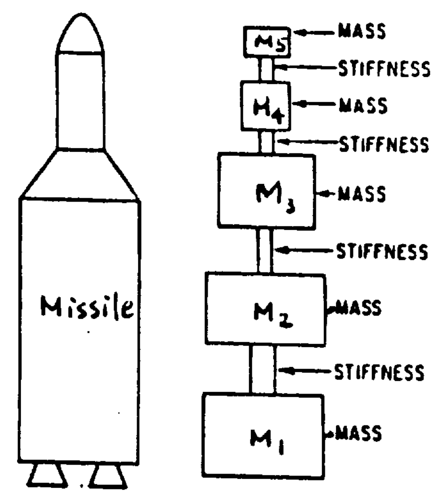
FIGURE 1·1 (CONTINUE)



Simplified Model representing propellers and Shaft of a Ship



simplified model representing an Automobile



Model of missile represented by a Simplified model of lumped masses and elastic elements with bending stiffness

FIGURE 1.1 (CONTINUE)

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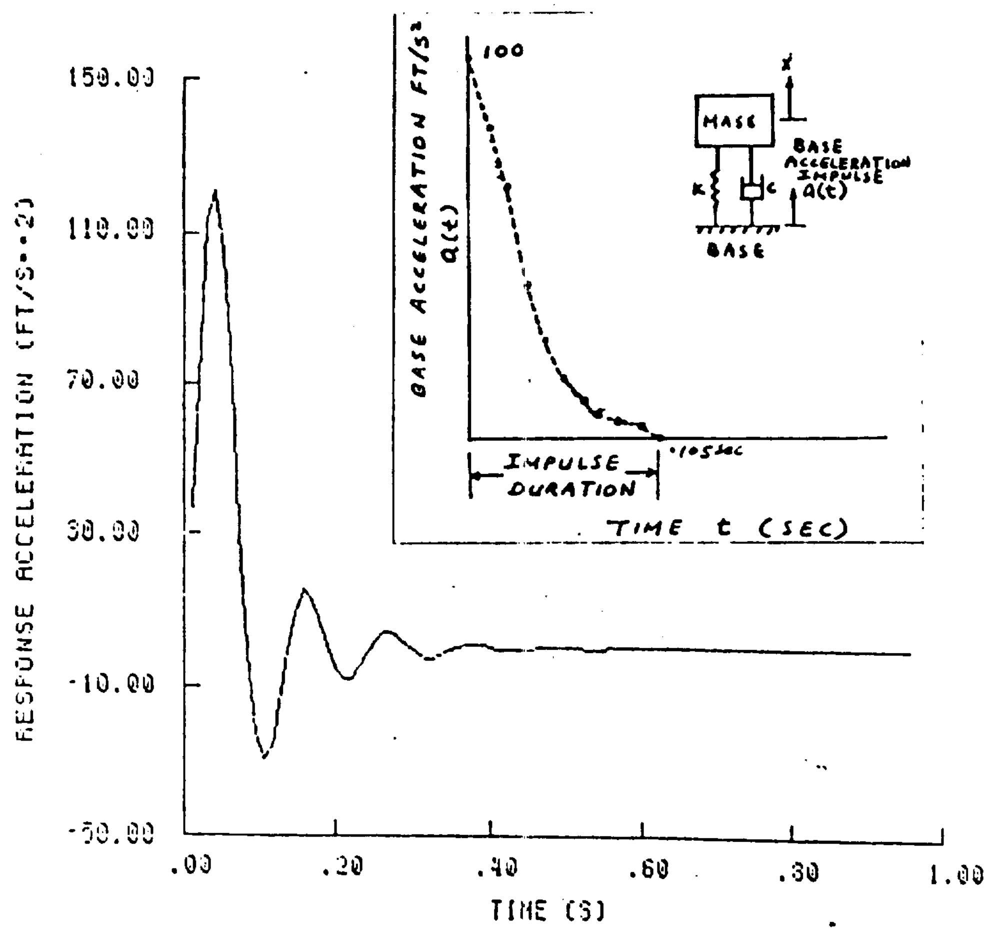

FIG. 5-1 DYNAMIC RESPONSE

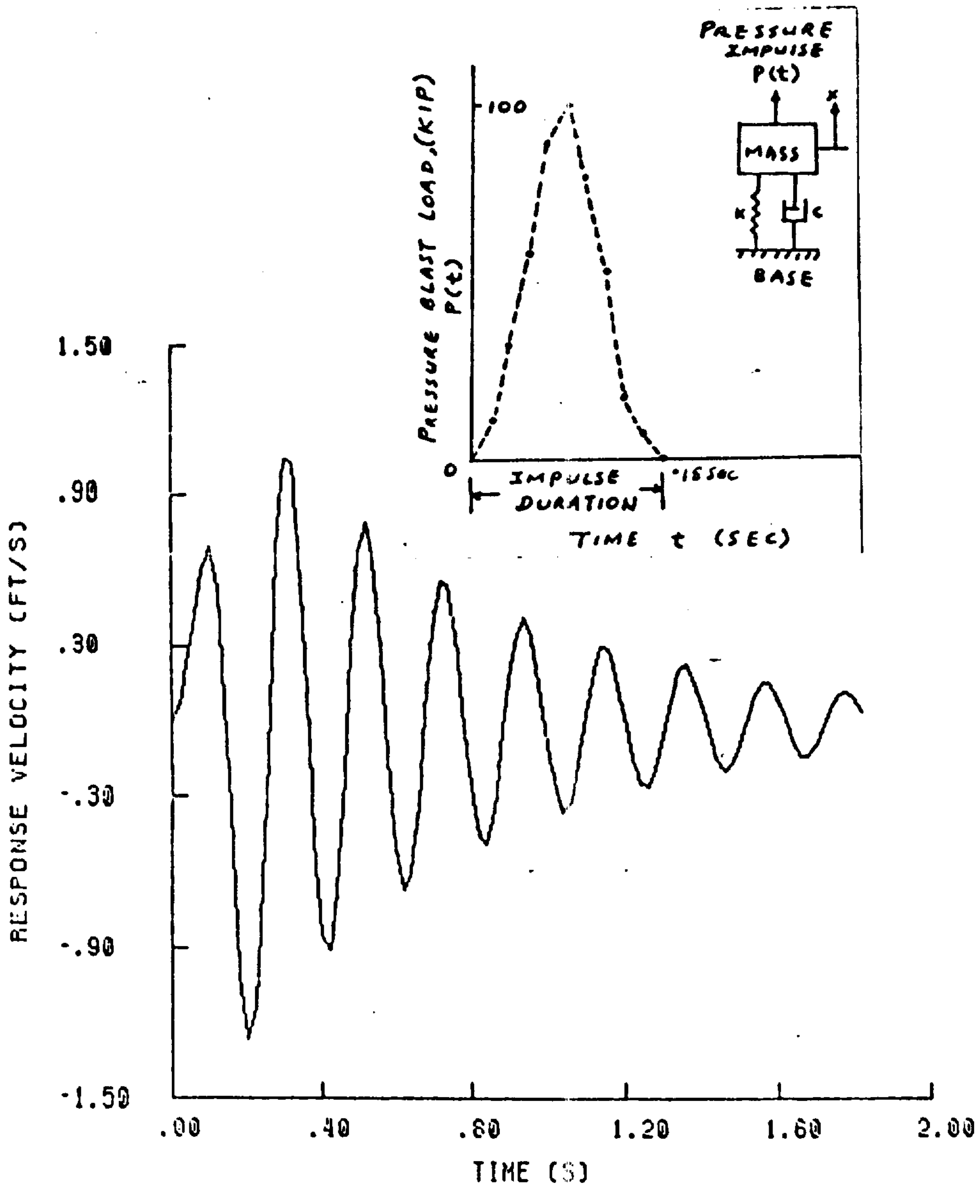

FIG. 5.2 DYNAMIC RESPONSE

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OF POOR QUALITY

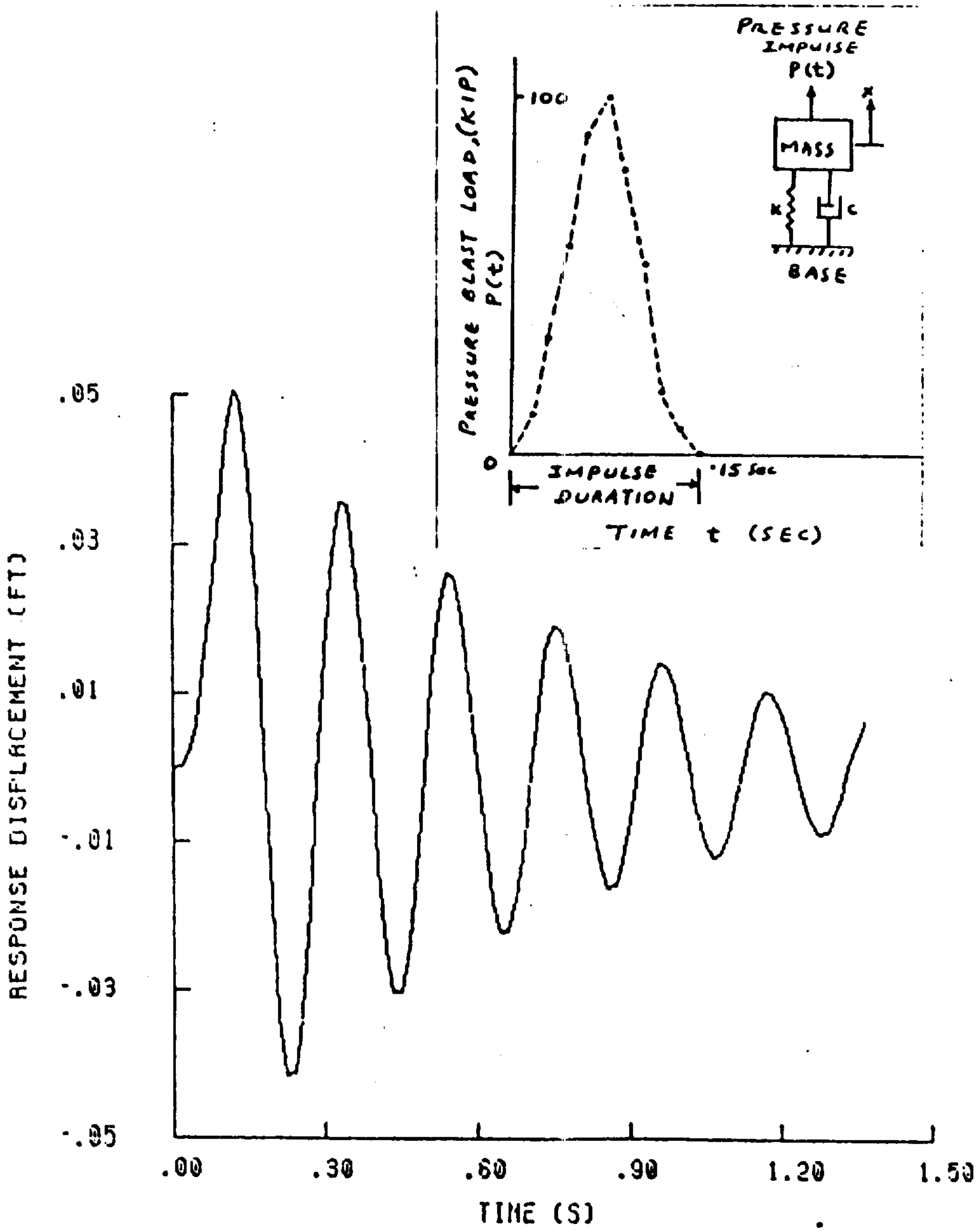

FIG. 5·3 DYNAMIC RESPONSE

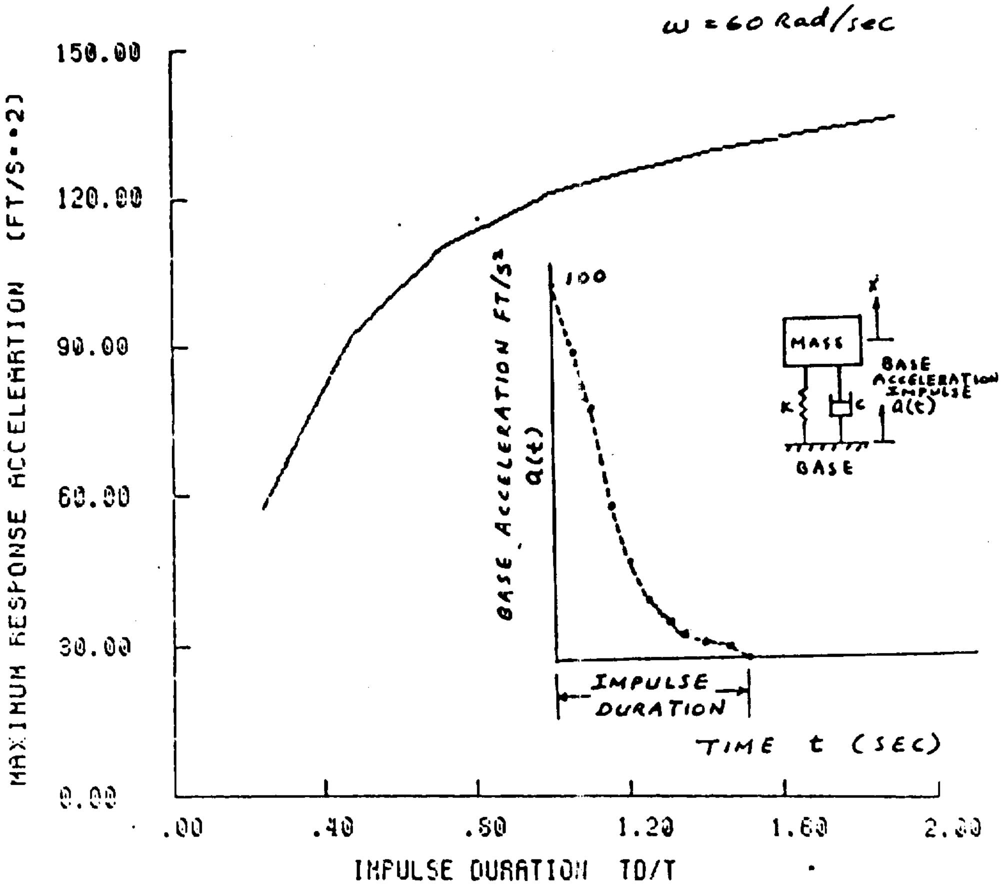

FIG. 5.4 SHOCK SPECTRA

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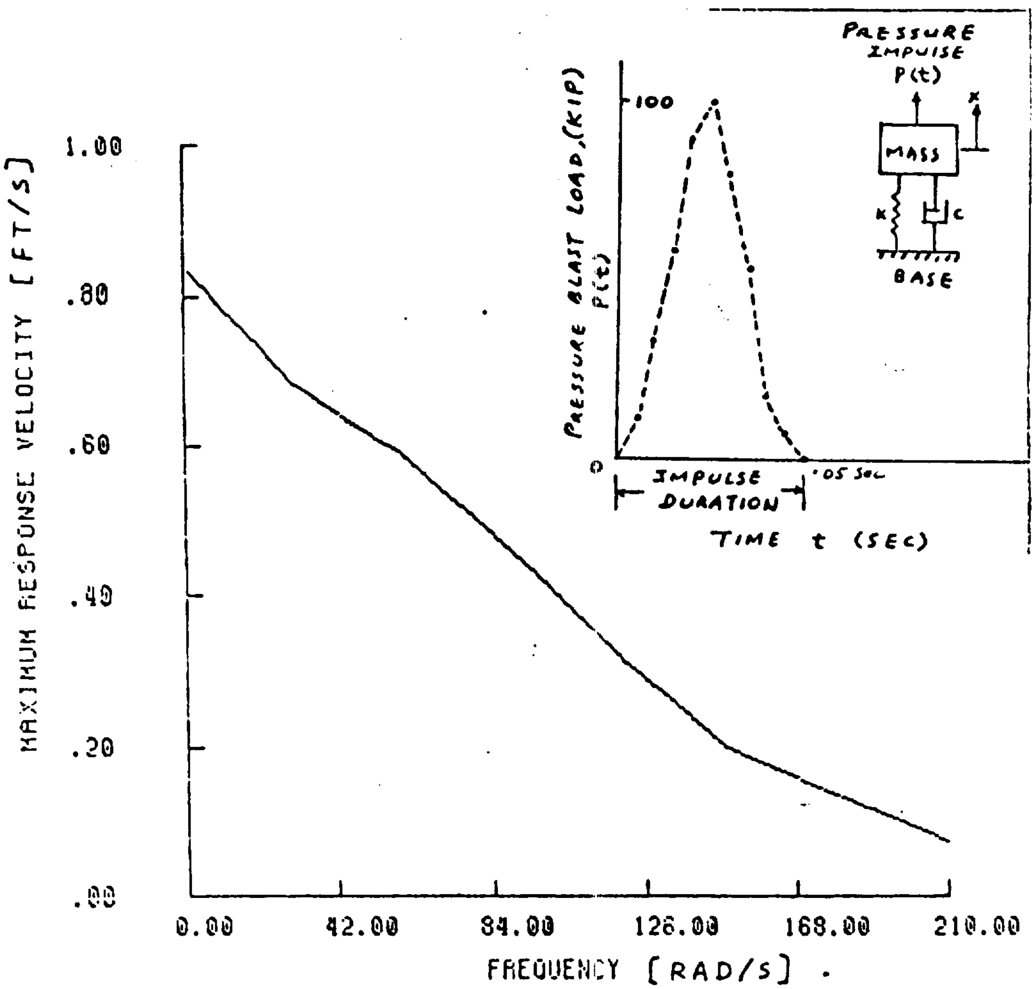


FIG. 5.5 SHOCK SPECTRA

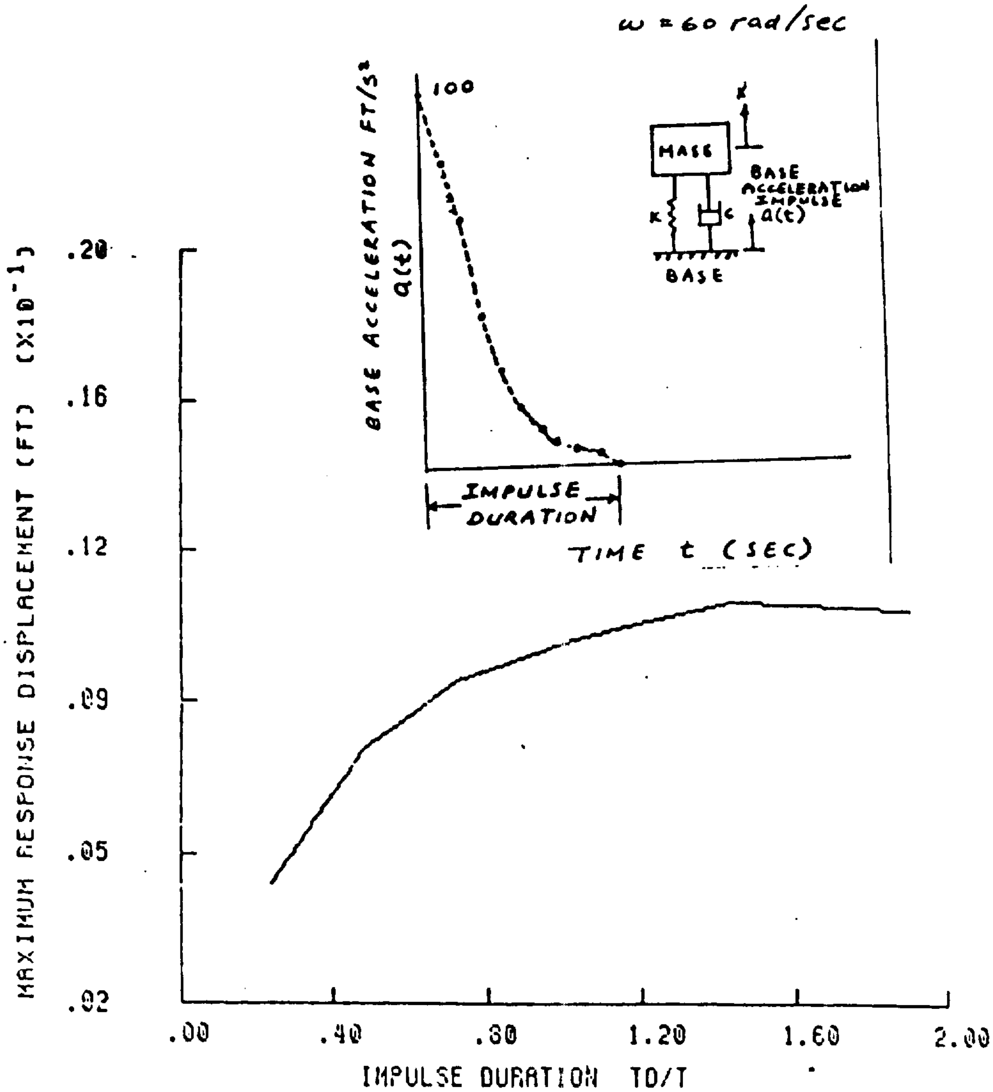
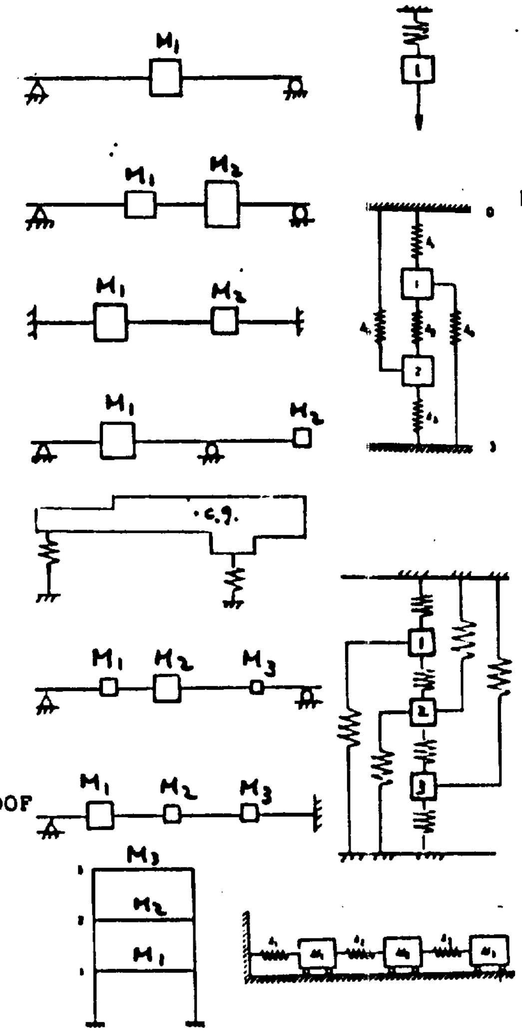


FIG. 5·6 SHOCK SPECTRA

TABLE 1.1

A CLASS OF MDOF STRUCTURE SYSTEM



SYSTEM	BEAM AND FRAME	FORTRAN FILE
SDOF		
TWO DOF		MS2.FOR
		MF2.FOR
		MOH2.FOR
		MSP2.FOR
THREE DOF		MS3.FOR
		MSF3.FOR
		MSB3.FOR

TABLE 5.1

TWO DOF SIMPLY SUPPORTED BEAM

INPUT DATA

load 1	load 2	span length	load 1 from right support	load 2 from right support	modulus of elasticity	area moment of inertia
lb	lb	inch	inch	inch	lb/in**2	in**4
25	50	35	25	10	1.E6	24

OUTPUT DATA

mode	natural frequency rad/sec	displacement coordinate mass 1	displacement coordinate mass 2
1	474.9297	1.000000	1.068182
2	1619.361	1.000000	-0.4650053

TWO DOF FIXED ENDS BEAM

INPUT DATA

load 1	load 2	span length	load 1 from right support	load 2 from right support	modulus of elasticity	area moment of inertia
lb	lb	inch	inch	inch	lb/in**2	in**4
25	50	35	25	10	1.E6	24

OUTPUT DATA

mode	natural frequency rad/sec	displacement coordinate mass 1	displacement coordinate mass 2
1	1121.801	1.000000	1.202247
2.	2336.245	1.000000	-0.3883002

TABLE 5.1 (Continue)

TWO DOF OVERHANGING BEAM

INPUT DATA

load 1	load 2	span length	load 1 from left support	load 1 from right support	load 2 from right support
lb	lb	inch	inch	inch	inch
30	7.5	24	12	12	12

mudulus of elasticity = 1.**E6 lb/in**2
Area moment of inertia = 24 in**4

OUTPUT DATA

mode	natural frequency rad/sec	displacement coordinate mass 1	displacement coordinate mass 2
1	100.3552	1.000000	-2.774851
2	211.4977	1.000000	1.441519

TWO DOF RIGID BODY BEAM
ON FLEXIBLE SUPPORTS

INPUT DATA

load 1	radius of gyration	span length	c.g. from left support	c.g. from right support	spring constant 1	spring constant 2
lb	inch	inch	inch	inch	lb/in	lb/in
5.6	3.9	10	6.93	3.07	15	5

OUTPUT DATA

mode	natural frequency rad/sec	displacement coordinate mass 1	displacement coordinate mass 2
1	23.30480	1.000000	-4.971451
2.	65.67800	1.000000	0.2011484

TABLE 5.1 (CONTINUE)

THREE DOF SHEAR BUILDING FRAME

INPUT DATA

load 1	load 2	load 3	span length	load 1 from right support	load 2 from right support	load 3 from right sopport
lb	lb	lb	inch	inch	inch	inch
650	19300	38600	466	346	173	0

modulus of elasticity = 1.E6 lb/in**2
area moment of inertia = 4320 in**4

OUTPUT DATA

mode	natural frequency rad/sec	displacement coordinate mass 1	displacement coordinate mass 2	displacement coordinate mass 3
1	6.007885	1.000000	3.906237	6.108354
2	20.01055	1.000000	2.996532	-0.9990512
3	40.79921	1.000000	-0.1600301	1.0241550E-02

THREE DOF SIMPLY SUPPORTED
AND FIXED END BEAM

INPUT DATA

load 1	load 2	load 3	span length	load 1 from right support	load 2 from right support	load 3 from right sopport
lb	lb	lb	inch	inch	inch	inch
1.5	1.0	2.0	480	390	294	168

modulus of elasticity = 1.**E6 lb/in**2
area moment of inertia = 90 in**4

OUTPUT DATA

mode	natural frequency	displacement coordinate	displacement coordinate	displacement coordinate
1	114.4588	1.000000	1.504153	1.073848
2	313.5506	1.000000	0.4624190	-1.234211
3	796.5601	1.000000	-1.574680	0.4098067

THREE DOF SIMPLY SUPPORTED BEAM

INPUT DATA

load 1	load 2	load 3	span length	load 1 from right support	load 2 from right support	load 3 from right sopport
lb	lb	lb	inch	inch	inch	inch
3	2	3	480	360	240	120

modulus of elasticity = 1.**E6 lb/in**2
area moment of inertia = 30 in**4

OUTPUT DATA

mode	natural frequency rad/sec	displacement coordinate massl1	displacement coordinate mass 2	displacement coordinate mass 3
1	32.31675	1.000000	1.400000	1.073848
2	114.0660	1.000000	3.2434639E-02	1.000000
3	275.5626	1.000000	-2.138684	1.000000

1988

NASA/ASEE SUMMER FACULTY RESEARCH FELLOWSHIP PROGRAM

JOHN F. KENNEDY SPACE CENTER
UNIVERSITY OF CENTRAL FLORIDA

STUDIES OF CRYOGENIC PROPELLANT STORAGE AND HANDLING FOR THE LUNAR LANDING AND LAUNCH FACILITY (COMPLEX 39L)

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Contract No.:	University of Central Florida NASA-NGT-60002