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# Determination of Longitudinal Aerodynamic Derivatives Using Flight Data From an Icing Research Aircraft

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DETERMINATION OF LONGITUDINAL AERODYNAMIC DERIVATIVES USING FLIGHT  
DATA FROM AN ICING RESEARCH AIRCRAFT

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Abstract

A flight test was performed with the NASA Lewis Research Center's DH-6 icing research aircraft. The purpose was to employ a flight test procedure and data analysis method, to determine the accuracy with which the effects of ice on aircraft stability and control could be measured. For simplicity, flight testing was restricted to the short period longitudinal mode.

Two flights were flown in a clean (baseline) configuration, and two flights were flown with simulated horizontal tail ice. Forty-five repeat doublet maneuvers were performed in each of four test configurations, at a given trim speed, to determine the ensemble variation of the estimated stability and control derivatives. Additional maneuvers were also performed in each configuration, to determine the variation in the longitudinal derivative estimates over a wide range of trim speeds.

Stability and control derivatives were estimated by a Modified Stepwise Regression (MSR) technique. A measure of the confidence in the derivative estimates was obtained by comparing the standard error for the ensemble of repeat maneuvers, to the average of the estimated standard errors predicted by the MSR program. A multiplicative relationship was determined between the ensemble standard error, and the averaged program standard errors. In addition, a 95 percent confidence interval analysis was performed for the elevator effectiveness estimates,  $C_{m_{\delta e}}$ . This analysis identified the speed range where changes in  $C_{m_{\delta e}}$  could be attributed to icing effects.

The magnitude of icing effects on the derivative estimates were strongly dependent on flight speed and aircraft wing flap configuration. With

wing flaps up, the estimated derivatives were degraded most at lower speeds corresponding to that configuration. With wing flaps extended to  $10^\circ$ , the estimated derivatives were degraded most at the higher corresponding speeds.

The effects of icing on the changes in longitudinal stability and control derivatives were adequately determined by the flight test procedure and the MSR analysis method discussed herein.

List of Symbols and Abbreviations

b	wing span, m
$C_l, C_m$ , and $C_n$	roll, pitch, and yaw moment coefficients, respectively
$C_x, C_y$ , and $C_z$	longitudinal, lateral, and vertical force coefficients, respectively
$\bar{c}$	mean aerodynamic chord, m
c.g.	airplane center of gravity
g	acceleration due to gravity
$I_x, I_y$ , and $I_z$	moments of inertia, $\text{kg-m}^2$
$I_{xz}$	product of inertia, $\text{kg-m}^2$
JSR	cost function for Stepwise Regression Algorithm
KIAS	knots indicated airspeed, kts
m	aircraft mass, kg
N, n	number of points
p, q, r	roll, pitch, and yaw rates, respectively, rad/sec or deg/sec

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S	wing area, m <sup>2</sup>
SHP	shaft horsepower, hp
u,v,w	longitudinal, lateral, and vertical velocity components
V	total airspeed, m/s or knots as given by $(u^2 + v^2 + w^2)^{1/2}$
$x_j(i)$	measured aircraft response or control surface input
$y_0(x_0)$	linear approximation of $C_{m\delta e}$ corresponding to some $x_0$ (velocity)
$y(i)$	computed aerodynamic coefficients
$\alpha$	corrected (free stream) angle of attack, rad or deg
$\beta$	angle of sideslip, rad or deg
$\delta e$	elevator deflection, rad or deg
$\theta$	aerodynamic stability or control parameters
$\theta_0$	constant offset term
$\theta_j$	derivative w.r.t. some response or control surface input
$\theta, \phi$	pitch and roll angles, respectively, rad or deg
$\rho$	air density, kg/m <sup>3</sup>
$\sigma_j$	variance estimate of measurement noise

### Introduction

Modern transport aircraft, certified for flight in icing conditions, normally employ ice protection systems to prevent ice from accumulating on critical lifting components. If these systems fail, or are improperly operated, ice can form on these components and pose a serious threat to safe flight characteristics. This problem, which is a continual concern for current generation aircraft, will be of even greater concern for future generation aircraft employing advanced propulsion systems and optimized aerodynamic designs. For example, high propulsive efficiency engines such as propfans, may not provide sufficient bleed air for wing and tail anti-icing. The use of energy-efficient de-icing systems, which require some small build-up of ice before activation, will be their likely replacement. Aircraft equipped with these systems will have to demonstrate acceptable flying qualities with some leading edge ice contamination, especially when flown in the take-off or landing configuration. This requirement also applies to aircraft whose aerodynamic efficiency, enhanced by advanced airfoil designs and relaxed static stability criterion, is degraded when small leading edge ice formations occur between de-icing cycles. Likewise, aircraft employing full flight control augmentation must also display an acceptable tolerance to these

small ice accretions and to those larger accretions caused by ice protection systems failures or operator errors.

Currently, no analysis methodologies exist that provide the capability to predict the effect of icing on aircraft flight characteristics. Therefore, within the scope of the NASA Lewis aircraft icing research program, a high priority has been assigned in measuring the effects of icing on aircraft performance and stability and control. Experimental data acquired from flight tests provide the basis for validating icing effects methodologies, and for establishing correlations between computational codes, wind tunnel tests, and full scale flight tests. Performance testing, which was extensively discussed in Refs. 1 and 2, provided reasonably consistent results and a high degree of confidence in measurement accuracy. As a result, these data are now being used in validating current methodologies that predict ice shape growth and their effect on aerodynamic performance. However, initial flight testing to estimate effects of icing on aircraft stability and control derivatives, showed that the results could be obscured by the inherent problem of not knowing the true accuracy of the derivative estimates. Stability and control flight testing as reported in Refs. 3 and 4 used an early version of a Modified Maximum Likelihood Estimation (MMLE) technique to estimate the derivative values. The accuracies of these derivative estimates were obtained by an approximation to the Cramer-Rao bound given by the maximum likelihood algorithm. Because it was known that these error estimates indicated higher accuracies than those which can be achieved by repeated experiments,<sup>5</sup> and because the differences between derivatives from the baseline and iced aircraft were found to be small, it was decided to conduct further flight testing to determine a realistic accuracy with which stability and control derivatives could be estimated. The purpose of this paper is to report on the results of those tests, and to present the conclusions drawn from the data analysis programs.

For simplicity, the flight test plan was structured to examine only the longitudinal short period characteristics of the research aircraft. Four data collection flights were defined: two flights, were flown clean; and two flights were flown with an artificial ice shape (described later in this report), attached to the leading edge of the horizontal tail.

Flight test maneuvers consisted of numerous repeat elevator doublets at a specified trim condition to determine the ensemble standard deviation for the parameter estimates. Additional maneuvers were also flown over the entire level flight angle of attack envelope to obtain derivative estimates for different flight conditions. Flight test maneuvers were analyzed using a modified stepwise regression (MSR) algorithm. The results are discussed relative to the accuracy of the derivative estimates, and to the differences between the estimates obtained for the uniced versus iced horizontal tail. Some conclusions are also drawn as to the physical interpretation of the derivative estimates and their relevance to aircraft handling characteristics. Following the introduction, this paper is organized in sections which describe the research airplane,

instrumentation, flight test maneuvers, data reduction and analysis techniques, accuracy of the derivative estimates, effects of tail ice on handling characteristics and conclusions.

### Research Aircraft

The NASA Lewis icing research aircraft shown in Fig. 1, is a modified DeHavilland DH-6 Twin Otter. It is powered by two 550 SHP Pratt and Whitney PT6A-20A turbine engines driving three bladed Hartzell constant speed propellers. Physical dimensions, mass, and inertial specifications for the aircraft are found in Table I.

The flight controls are mechanically operated through a system of cables and pulleys. Ailerons droop as wing flaps extend. Their degree of movement increases in proportion to the amount of wing flap extension. There are two trim tabs on the elevator. The right tab is interconnected with the wing flap system to provide automatic longitudinal compensation as the wing flaps are operated. The left trim tab is an adjustable tab, which is operated by the pilot.

For flights where an artificial ice shape was used to simulate a moderate glaze icing condition on the tail, formed aluminum caps were placed over the left and right leading edges of the horizontal tail, and held in place with straps and clamps. Artificial ice cut from styrofoam was then secured to the tail caps with double sided tape. The geometry and positioning of the artificial ice was based on an actual tail icing condition recorded on an earlier research flight in natural icing clouds. The entire arrangement is shown in Fig. 2.

### Instrumentation

The icing research aircraft was equipped with three linear control position transducers (CPT) to measure the position of the elevator, ailerons, and rudder. The elevator CPT measured elevator position at the control horn to eliminate corrections for cable stretch. Flap position, constant for a given maneuver, was manually recorded from the flap position indicator in the cockpit.

Airspeed, pressure altitude, angle of attack and angle of sideslip were measured by means of sensors located on the flight test nose boom, Fig. 3. Pitot-static pressures were sensed by a Rosemount 858 probe, which had been calibrated by the "trailing cone" method. The high frequency response angle of attack and angle of sideslip vanes, were constructed from balsa wood, and balanced by a "heavy-metal" counterweight. Angle of attack was referenced to the fuselage reference line, which is coincident with the aircraft floor line, and angle of sideslip was referenced to the aircraft longitudinal axis.

Kohlman Systems Research, Inc., Lawrence Kansas under contract to NASA, provided a stability and control data acquisition system (DAS). This system consisted of an attitude gyro, three axis orthogonal accelerometers, three axis angular rate gyros, and a digital data recording system which recorded 21 channels of flight test parameters with 12 bits per channel. All data were recorded on-board at 20 Samples per Second (SPS), and all channels were sampled within one millisecond by

the DAS. Because of buffer limitations, only 24 sec of 20 SPS data could be obtained before the buffer contents had to be written on to a cassette tape. The write-off period was nominally 60 to 90 sec. Hence, all flight test maneuvers were planned to last no more than 24 sec. During the write-off period, the aircraft would be repositioned and trimmed for the next data point. The list of recorded parameters, ranges, and sensitivities is provided in Table II.

### Flight Testing

The center of gravity of the aircraft was measured experimentally. These measurements represented the zero-fuel mass characteristics of the aircraft and were corrected in the analysis program for actual crew and fuel weights recorded during each flight test maneuver.

Longitudinal stability and control flight testing was accomplished in four airplane configurations defined according to wing flap setting and the simulated icing condition of the horizontal tail plane. They were:

- (1) Clean tail, wing flaps up ( $0^\circ$ )
- (2) Clean tail, wing flaps  $10^\circ$
- (3) Artificial horizontal tail icing, wing flaps up ( $0^\circ$ )
- (4) Artificial horizontal tail icing, wing flaps  $10^\circ$

A constant engine power setting associated with each wing flap condition, was used while executing flight test maneuvers. This eliminated engine power as a variable in the derivative estimates. Two hundred seventy-five SHP/engine was used for the  $0^\circ$  wing flap cases, and 219 SHP/engine was used in the  $10^\circ$  wing flap cases.

The standard small amplitude input was a "2-1" elevator doublet which was designed to excite the short period longitudinal mode of the airplane. Elevator control column deflection was input by the pilot to effect a planned  $\pm 0.3$  g vertical acceleration from 1.0 g level flight, however, due to an erroneous cockpit accelerometer, these maneuvers were more on the order of  $\pm 1.0$  g vertical acceleration. A typical example of one of these doublet maneuvers is shown in Fig. 4.

Forty-five sets of identical small amplitude doublets were performed at a given flight condition in each configuration to determine the accuracy of the longitudinal stability and control derivative estimates. For the  $0^\circ$  flap configurations, repeat doublets were initiated from a level, unaccelerated flight condition at a trim speed of 120 KIAS. For the  $10^\circ$  flap configurations, these doublets were initiated from a level, unaccelerated flight condition at a trim speed of 100 KIAS.

Additional small amplitude doublets were then performed in each configuration at incrementally higher angles of attack (lower trim speeds). Power settings associated with each flap configuration remained constant for these maneuvers, therefore doublet maneuvers initiated at the lower trim speeds were performed in climbs through a median test altitude.

## Data Reduction and Analysis Technique

The recorded cassette tapes from each flight were initially sent to KSR, Inc., where the data were reduced to engineering units and rewritten on nine-track magnetic tape. These data sets were then transferred to the NASA Langley Research Center, Aircraft Guidance and Control Branch, for stability and control analysis.

The first step in the analysis procedure was to correct the measured data for all known errors. All sensor data were corrected for c.g. offset, angle of attack data were corrected for upwash and boom bending effects, and several flight maneuvers were analyzed with a maximum likelihood algorithm<sup>6</sup> to estimate instrument biases and scale factors. These estimated corrections, when applied to the measured data, produced data sets which were ready for model structure determination and parameter estimation.

For the model structure determination procedure, it was assumed that,

(1) The general equations of rigid body motion adequately define the airplane motion. (See appendix)

(2) The model for the aerodynamic force-and-moment-coefficients can be represented by multi-variable polynomials in response and control variables. The parameters in these equations are the coefficients of the Taylor series expansion around the values corresponding to the initial steady-state flight.

(3) Linear terms in the Taylor series expansion make generally larger contribution to aerodynamic functions, followed by higher-order terms.

The third assumption, resulting in a constraint on the selection of significant terms in the regression equation, is what gives rise to the Modified Stepwise Regression (MSR) technique.

MSR is a modified version of linear regression, which can determine the structure of the aerodynamic model equations, and estimate the model parameters. The determination of an adequate model for the aerodynamic coefficients includes three steps: the postulation of terms that might enter the model, selection of an adequate model, and the verification of the model selected. The general form of aerodynamic model equations can be written as

$$y(t) = \theta_0 + \theta_1 x_1(t) + \dots + \theta_n x_n(t) \quad (1)$$

where  $y(t)$  represents the resultant coefficient of aerodynamic force or moment. In the polynomial representation of the aerodynamic coefficient,  $\theta_1$  to  $\theta_n$  are the stability and control derivatives,  $\theta_0$  is the value of any particular coefficient corresponding to the initial steady-flight conditions, and  $x_1$  to  $x_n$  are the regressors formed by the airplane output and control variables or their combinations.

For example, if we let  $y = C_m$ , the airplane pitching moment coefficient, we may choose the candidate variables,  $x_1, x_2, \dots, x_n$ , as:

$$\begin{array}{ccccc} \alpha & \alpha^2 & \alpha^3 & \beta & \alpha^4 \\ \frac{q\bar{c}}{2V} & \frac{\alpha q\bar{c}}{2V} & \frac{\alpha^2 q\bar{c}}{2V} & \beta^2 & \alpha^5 \\ \delta e & \alpha \delta e & \alpha^2 \delta e & \beta^3 & \end{array} \quad (2)$$

and their corresponding coefficients as the stability and control derivatives:

$$\begin{array}{ccccc} C_{m_\alpha} & C_{m_{\alpha^2}} & C_{m_{\alpha^3}} & C_{m_\beta} & C_{m_{\alpha^4}} \\ C_{m_q} & C_{m_{\alpha q}} & C_{m_{\alpha^2 q}} & C_{m_{\beta^2}} & C_{m_{\alpha^5}} \\ C_{m_{\delta e}} & C_{m_{\alpha \delta e}} & C_{m_{\alpha^2 \delta e}} & C_{m_{\beta^3}} & \end{array} \quad (3)$$

After postulating the aerodynamic model equations, the determination of significant terms among the candidate variables and estimation of the corresponding parameters follows. The variable chosen for entry into the regression equation is the one that has the largest correlation with  $y$  after adjusting for the effect on  $y$  of the variables already selected. The parameters are estimated by minimizing the cost function

$$J_{SR} = \sum_{i=1}^N \left[ y(i) - \hat{\theta}_0 - \sum_{j=1}^l \hat{\theta}_j x_j(i) \right]^2 \quad (4)$$

where  $N$  is the number of data points and  $(l + 1)$  is the number of parameters in the regression equation at any step.

At every step of the regression, the variables incorporated into the model in previous stages and a new variable entering the model are reexamined. Any variable that provides a nonsignificant contribution (due to correlation with more recently added terms) is removed from the model. The process of selecting and checking variables continues until no more variables are admitted to the equation and no more are rejected. Experience shows, however, that the model based only on the significance of individual parameters in the model in Eq. (1) can still include too many terms and, therefore, may have poor prediction capabilities. Several criteria for the selection of an adequate model are introduced in Ref. 7, and the details of the entire procedure are explained in that reference.

Applying these criteria, an adequate model equation for the airplane pitching moment coefficient was defined by a Taylor Series expansion,

$$C_m = C_{m_0} + C_{m_\alpha} \alpha + C_{m_{\delta e}} \delta e + C_{m_q} \frac{q\bar{c}}{2V} \quad (5)$$

the solution of which provided an adequate fit to the measured flight data. In a similar manner, this procedure was used to estimate the other longitudinal stability and control derivatives described in this report. The analyses for the  $C_z$  and  $C_m$  derivative estimates, and the relative effects of tail icing on these estimates is given in Figs. 5 to 8. A physical interpretation of these results will be discussed in a later section of this report.

### Accuracy of the Derivative Estimates

The 45 repeat doublet maneuvers, identically executed in each of the four configurations, provided a statistically significant data base for parameter and error estimate accuracy studies. The standard error calculated for each ensemble of parameter estimates verified the effectiveness of maneuver selection through demonstrated repeatability of the predicted parameters. In addition, a comparison of the average of the standard error values estimated by the computer program to the standard error for the ensemble, provided an evaluation of how realistic the program estimated standard error values were.

Repeat maneuvers were analyzed using the Modified Stepwise Regression and the resulting mean values of coefficients were compiled in Table III. Accompanying each parameter estimate is the average standard error associated with it as given by the estimation algorithm, the standard error for the ensemble of values from the 45 repeat maneuvers, and the ratio of the two. The ratio given for each parameter is the factor by which the program estimated standard error for that parameter should be multiplied to produce a realistic expected standard error. For example, the standard error estimated by MSR for  $C_{Z\alpha}$  should be multiplied by 2.9 to achieve a realistic approximation to an actual ensemble standard error.

A fundamental issue throughout this research is that of distinguishing between the effects of ice and the inherent variability of predicted parameters. In addition to the error analysis performed on the values obtained from the 45 repeat maneuvers, the  $C_{m\delta e}$  estimates for the  $0^\circ$  flap condition were considered (Fig. 6(c)). This set of points was chosen for this portion of the analysis because each of the two groups of values (iced and uniced) lend themselves to linear approximations. Also, determining a prediction interval in this particular case serves to quantitatively enforce what is qualitatively a very noticeable difference in parameters for the iced/uniced cases. After approximating the two groups of points by straight lines, prediction intervals about these lines were calculated as follows:

$$y_0(x_0) + t_{\alpha^*/2, n-2} s \left( 1 + n^{-1} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right)^{1/2} \quad (6)$$

where

$y_0(x_0)$	linear approximation of $C_m$ corresponding to some (velocity) value
$\alpha^*$	0.05 to achieve a 95 percent prediction interval
$n$	number of points
$t_{\alpha^*/2, n-2}$	student t-distribution
$s$	standard error (can be one of three listed below)

$sp_A(y)$  = average of standard errors obtained from the computer program

$$s_E(y) = \left( \sum \frac{(y_i - \bar{y})^2}{n-2} \right)^{1/2} ; \text{ensemble standard error for values of } C_{m\delta e}$$

$$s_R(y) = sp_A(y) \text{ *Ratio of ensemble to program average obtained from Table III.}$$

$$S_{xx} = \sum_{i=1}^N (x_i - \bar{x})^2 \text{ sum of squares of residuals}$$

The resulting interval is one within which we can expect 95 percent of all predicted elevator effectiveness values to fall. Figure 9 presents these prediction intervals and gives an illustration of both the areas in which icing effects are discernable as well as the speed range in which we cannot attribute any change in  $C_{m\delta e}$  to the artificial ice shape. A similar analysis can be done on other parameters by fitting them in a piecewise linear fashion.

### Effects of Tail Ice on Handling Characteristics

The effects of icing on the horizontal tail plane were obtained by analyzing all of the doublet maneuvers executed throughout the attainable ranges of speed and angle of attack. The MSR analyzed results of these maneuvers, in the form of the estimated stability and control derivatives, are presented in Figs. 5 to 8. Each parameter value is plotted with  $\pm 2\sigma$  error bars based on the standard error as given by the computer program. The MSR analysis indicated that no nonlinear terms were required for an adequate aerodynamic model.

Figures 5 and 6 show an analysis for the  $0^\circ$  flap cases. The  $C_Z$  derivatives indicate an apparent loss of elevator effectiveness, especially at low airspeeds that correspond with higher angles of attack. The pitching moment derivatives,  $C_{m\alpha}$ ,  $C_{mq}$ , and  $C_{m\delta e}$ , which have an important effect on airplane short period stability and control characteristics, are also more strongly affected by the ice contaminated horizontal tail at low speeds. At higher speeds, the effects of tail ice become negligible. This analysis was supported by pilot comments reporting that at low speeds, the "iced" airplane was less responsive to elevator control inputs, and more weakly damped in pitch. These comments, however, were not based on specific piloting tasks normally used in defining pilot ratings, but on the perceived relative differences (clean versus iced) in longitudinal control force, response, and pitching moment characteristics while performing parameter identification maneuvers. Further, the magnitude of these effects were not judged to be great enough to change pilot ratings if such an evaluation were to have been carried out.

In the ten degree flap cases, Figs. 7 and 8, the effects of horizontal tail icing show an opposite trend when compared with the  $0^\circ$  cases. Analysis of the stability and control derivatives show slightly degraded pitching moment coefficients up to speeds of approximately 90 kn. Above these speeds, there is a more pronounced degradation in these derivatives, and this analysis was

again verified by pilot comments. Doublet maneuvers performed at 100 kn with artificial tail ice were accompanied by elevator force lightening and a strong "burble" feedback in the controls at high negative pitch rates. This characteristic was assumed due to the high tail downwash angle induced in the pitch over portion of the doublet maneuver, causing a significant amount of separated flow to occur on the horizontal tail plane. This occurred at a low peak load factor of approximately 0 g, which as mentioned earlier in this report, was erroneously input because the cockpit g meter read incorrectly.

### Conclusions

Based on the analysis and results herein, two fundamental conclusions were drawn regarding the use of a Modified Stepwise Regression algorithm in estimating the effects of icing on aircraft stability and control characteristics: (1) The effects of the artificial ice shape attached to the tail were measurable as changes in the longitudinal stability and control derivatives for all flight conditions tested, and (2) that a multiplicative relationship was identified between the ensemble standard error and the estimated standard error from the Modified Stepwise Regression routine. The numerical value representing the ratio of the ensemble standard error, to the estimated standard error from the program when assigned to each control or stability derivative, are indicative of the level of confidence one should have in the estimated value of that parameter.

The results of this flight test program indicate that MSR is a viable technique for determining the effects of icing on the longitudinal stability and control characteristics of an aircraft. Application of this flight test and analysis technique should prove very useful for validating stability and control predictions from wind tunnels or computational methods.

### Appendix

#### Postulated Models

The six-degree-of-freedom equations of motion for the airplane are:

$$\ddot{u} = -qw + rv - g \sin \theta + \frac{\rho V^2 S}{2m} C_X \quad (A1)$$

$$\dot{v} = -ru + pw + g \cos \theta \sin \phi + \frac{\rho V^2 S}{2m} C_Y \quad (A2)$$

$$\dot{w} = -pv + qu + g \cos \theta \cos \phi + \frac{\rho V^2 S}{2m} C_Z \quad (A3)$$

$$\dot{p} = qr \left[ \frac{I_Y - I_Z}{I_X} \right] + \frac{I_{XZ}}{I_X} (pq + \dot{r}) + \frac{\rho V^2 S b}{2I_X} C_1 \quad (A4)$$

$$\dot{q} = pr \left[ \frac{I_Z - I_X}{I_Y} \right] + \frac{I_{XZ}}{I_Y} (r^2 - p^2) + \frac{\rho V^2 S \bar{c}}{2I_Y} C_m \quad (A5)$$

$$\dot{r} = pq \left[ \frac{I_X - I_Y}{I_Z} \right] + \frac{I_{XZ}}{I_Z} (p - qr) + \frac{\rho V^2 S b}{2I_Z} C_n \quad (A6)$$

along with the kinematic relations

$$\dot{\theta} = q \cos \phi - r \sin \phi \quad (A7)$$

$$\dot{\phi} = p + (q \sin \phi + r \cos \phi) \tan \theta \quad (A8)$$

where  $u$ ,  $v$ , and  $w$  are the velocity components;  $\phi$  and  $\theta$  are the roll and pitch angles;  $g$  is the acceleration due to gravity;  $\rho$  is the air density;  $m$  is the airplane mass;  $I_X$ ,  $I_Y$ ,  $I_Z$  and  $I_{XZ}$  are the moments and product of inertia;  $S$ ,  $b$ , and  $\bar{c}$  are the wing area, span and mean aerodynamic chord; and a dot over a symbol represents the derivative with respect to time. The total airspeed  $V$  is given by

$$V = (u^2 + v^2 + w^2)^{1/2}$$

Here, Eqs. (A1), (A3), (A5), and (A7) describe the longitudinal motion while Eqs. (A2), (A4), (A6), and (A8) describe the lateral motion of the airplane. The angles of attack and sideslip are given by

$$\alpha = \tan^{-1}(w/u) \quad \text{and} \quad \beta = \sin^{-1}(v/V)$$

The aerodynamic forces and moments are represented by the coefficients  $C_a$  ( $a = X, Y, Z, l, m, \text{ or } n$ ). It is postulated that these coefficients can be written as a Taylor's series polynomial expansion about an equilibrium trim condition as

$$C_a = \Theta_{a,0} + \sum_{j=1}^i \Theta_{a,j} \Delta x_j \quad (A9)$$

where  $\Theta_{a,0}$  and  $\Theta_{a,j}$  are the unknown parameter. For  $a = X, Z$ , or  $m$ , the independent variables  $x_j$  are

$$\alpha, \frac{q\bar{c}}{2V}, \text{ and } \delta e$$

and for  $a = Y, l$ , or  $n$ , they are

$$\beta, \frac{pb}{2V}, \quad \frac{rb}{2v}$$

Furthermore,

$$\Delta x_j = x_j(t) - x_j(t=0)$$

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TABLE I. - PHYSICAL CHARACTERISTICS  
OF THE DEHAVILLAND DH-6 TWIN OTTER

Characteristic	Low	High
Geometric:		
Wing area, m <sup>2</sup>	39.02	
Wing span, m	19.81	
Aspect ratio	10	
MAC, m	1.981	
Mass, kg	4150	4600
Inertias, kg m <sup>2</sup> :		
I <sub>xx</sub>	21 279	21 787
I <sub>yy</sub>	30 000	31 027
I <sub>zz</sub>	44 986	48 639
I <sub>xz</sub>	1432	1498



TABLE II. - CHARACTERISTICS OF FLIGHT TEST INSTRUMENTATION

[21 channel digital recording capability of the following parameters such that following units, ranges and resolutions are provided.]

Parameter	Range	Resolution
Longitudinal acceleration, g units	$\pm 1$	0.002
Lateral acceleration, g units	$\pm 1$	.002
Normal acceleration, g units	+3, -1	.004
Pitch rate, deg/sec	$\pm 15$	.029
Roll rate, deg/sec	$\pm 50$	.098
Yaw rate, deg/sec	$\pm 30$	.059
Pitch angle, deg	$\pm 45$	.088
Roll angle, deg	$\pm 90$	.176
Alpha, deg	+15, -10	.024
Left aileron position, deg	+19, -16	.034
Right aileron position, deg	+19, -16	.034
Elevator position, deg	$\pm 20$	.039
Rudder position, deg	$\pm 18$	.035
Beta, deg	$\pm 15$	.029
Battery A voltage, V		
Airspeed, m/sec	0 to 85	.083
Air temperature, °C	-18 to 38	.055
Voltage system reference		
Altitude, m	-150 to 3000	3.08
Reference voltage 1		
Reference voltage 2		

TABLE III. - MEAN VALUES OF STABILITY AND CONTROL  
DERIVATIVES CALCULATED FROM 45 REPEAT MANEUVERS  
USING MODIFIED STEPWISE REGRESSION.

[Flaps, 0°; 120 KIAS; baseline (no ice).]

Parameter	Mean value, $\bar{\theta}$	Standard errors <sup>a</sup>		Ratio, $s_{\bar{\theta}}(\theta)/s(\theta)$
		$s_{\bar{\theta}}(\theta)$	$s(\theta)$	
$Cz_{\alpha}$	-5.66	0.0493	0.0169	2.9
$Cz_q$	-19.97	1.047	.386	2.7
$Cz_{\dot{\delta}_e}$	-.608	.0281	.0122	2.3
$Cm_{\alpha}$	-1.31	.0147	.0191	.8
$Cm_q$	-34.2	.6444	.4314	1.5
$Cm_{\dot{\delta}_e}$	-1.74	.0248	.0131	1.9

<sup>a</sup> $s_{\bar{\theta}}(\theta)$ : Ensemble standard error;  $s(\theta)$ : average program estimated standard error.

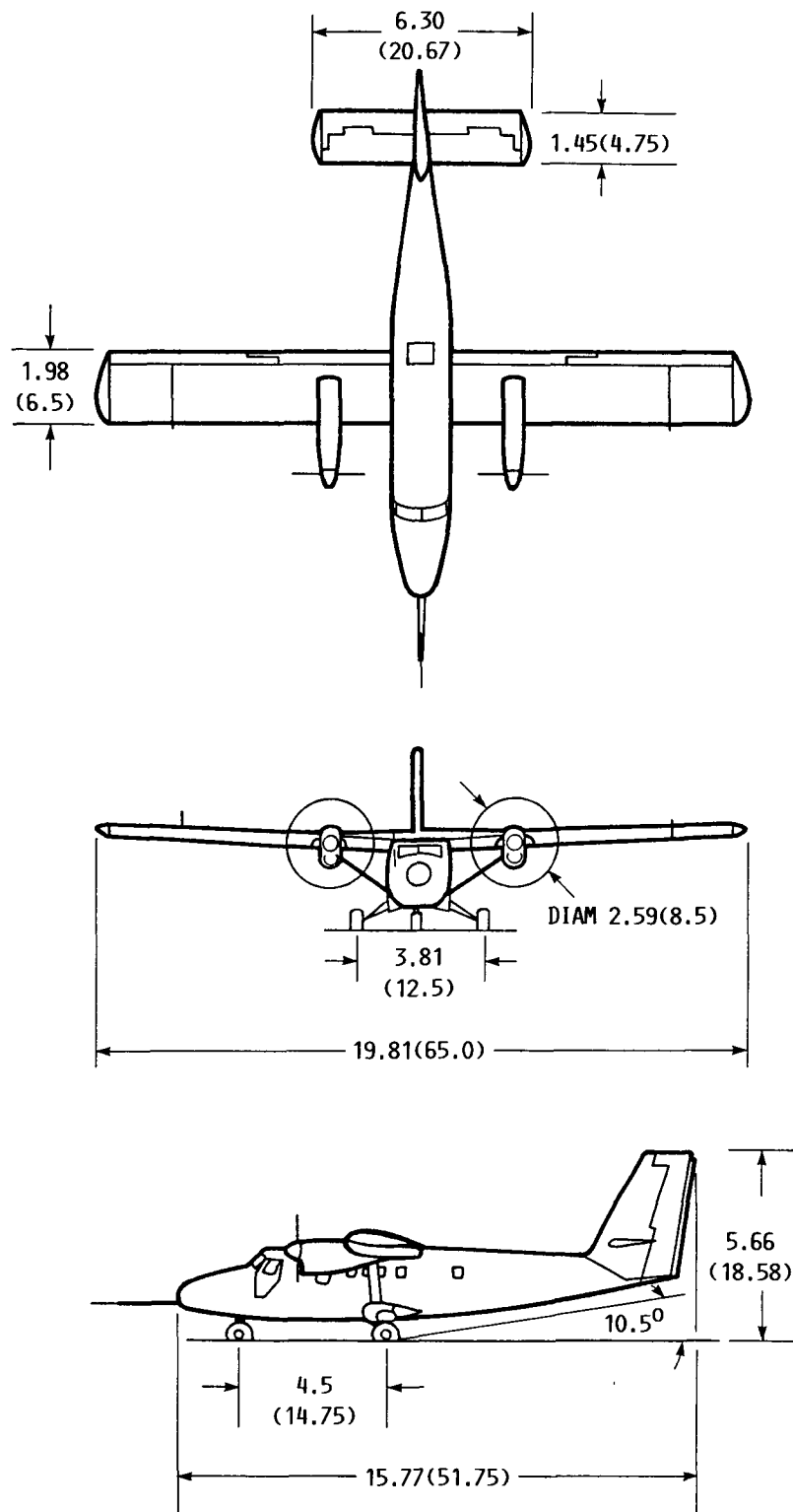
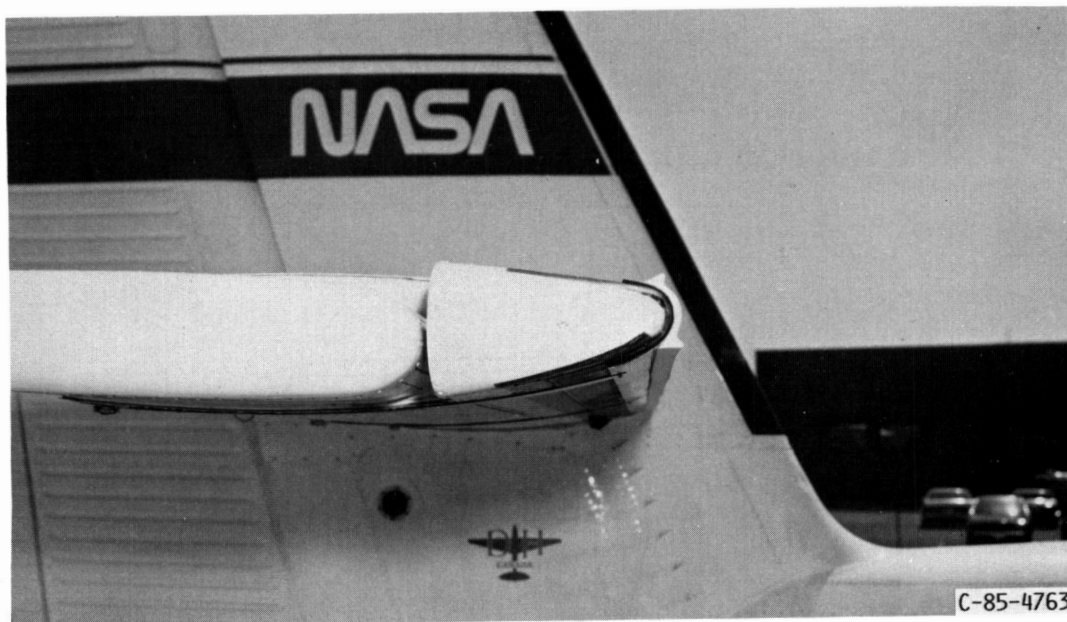
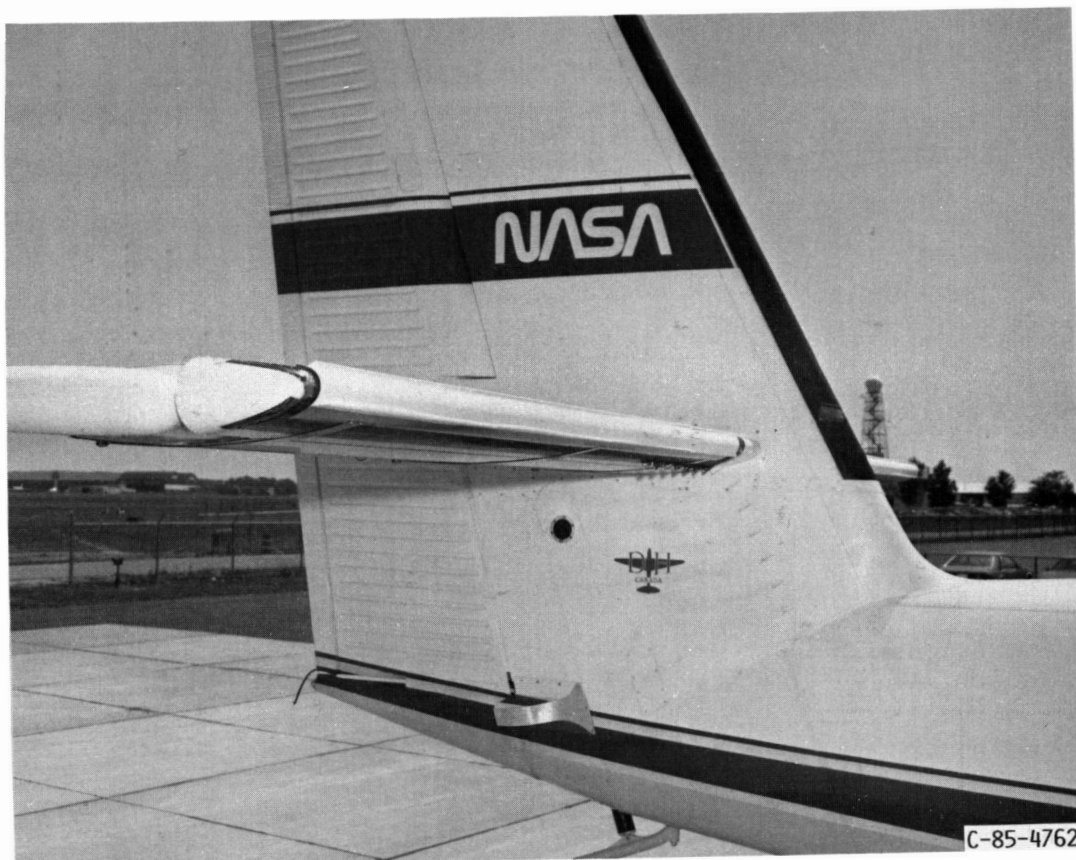


FIGURE 1. - NASA LEWIS RESEARCH CENTER ICING RESEARCH AIRCRAFT, ALL DIMENSIONS ARE IN M (FT).



(a) ARTIFICIAL TAIL ICE ATTACHMENT.



(b) FULL SPAN VIEW OF THE ARTIFICIAL ICE SHAPE.

FIGURE 2. - ARTIFICIAL MODERATE GLAZE ICE ATTACHED TO THE TAIL OF THE ICING RESEARCH AIRCRAFT.

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OF POOR QUALITY

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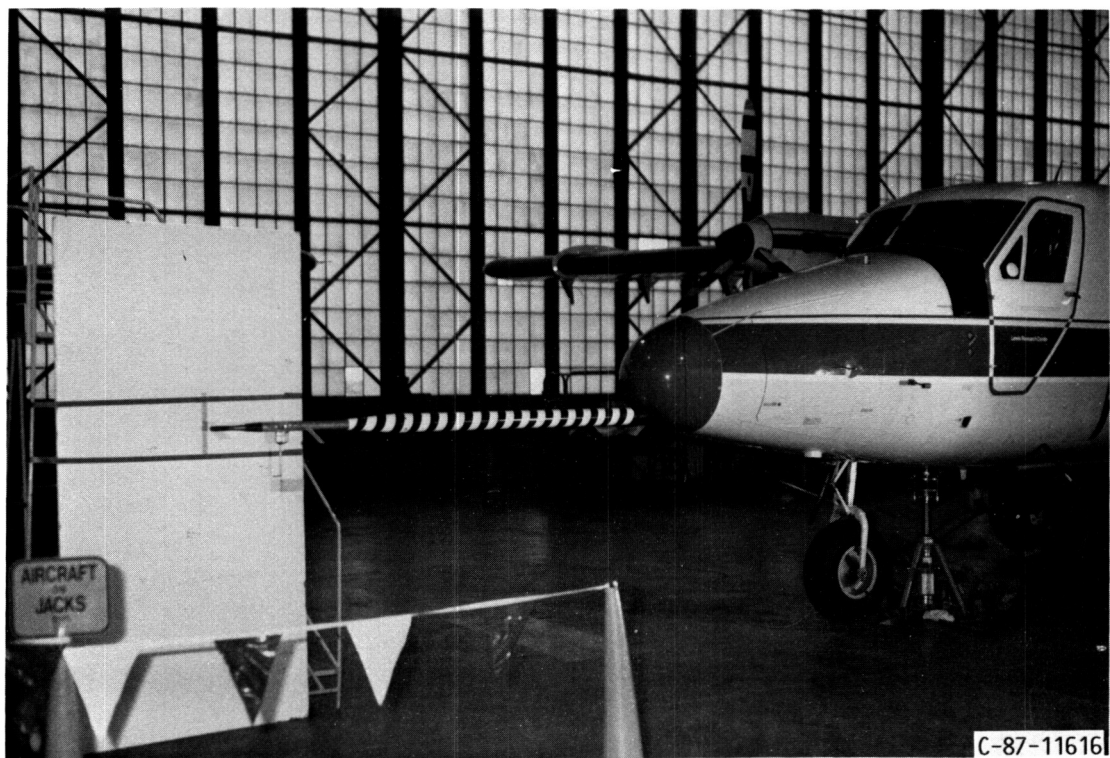


FIGURE 3. - FLIGHT TEST NOSE BOOM SHOWING  $\alpha$  AND  $\beta$  VANE INSTALLATION.

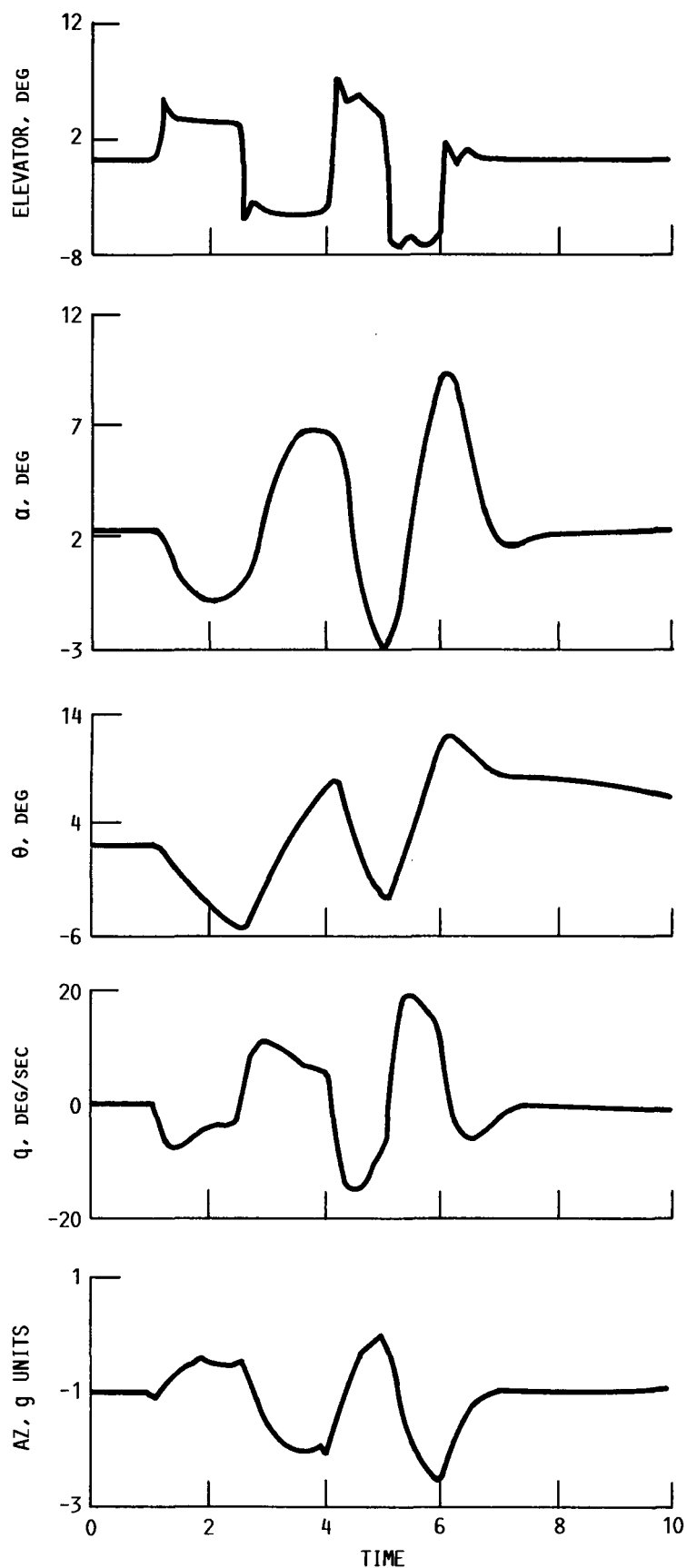


FIGURE 4. - CONTROL SURFACE INPUT AND AIRCRAFT RESPONSE FOR A TYPICAL "2-1" ELEVATOR DOUBLET.

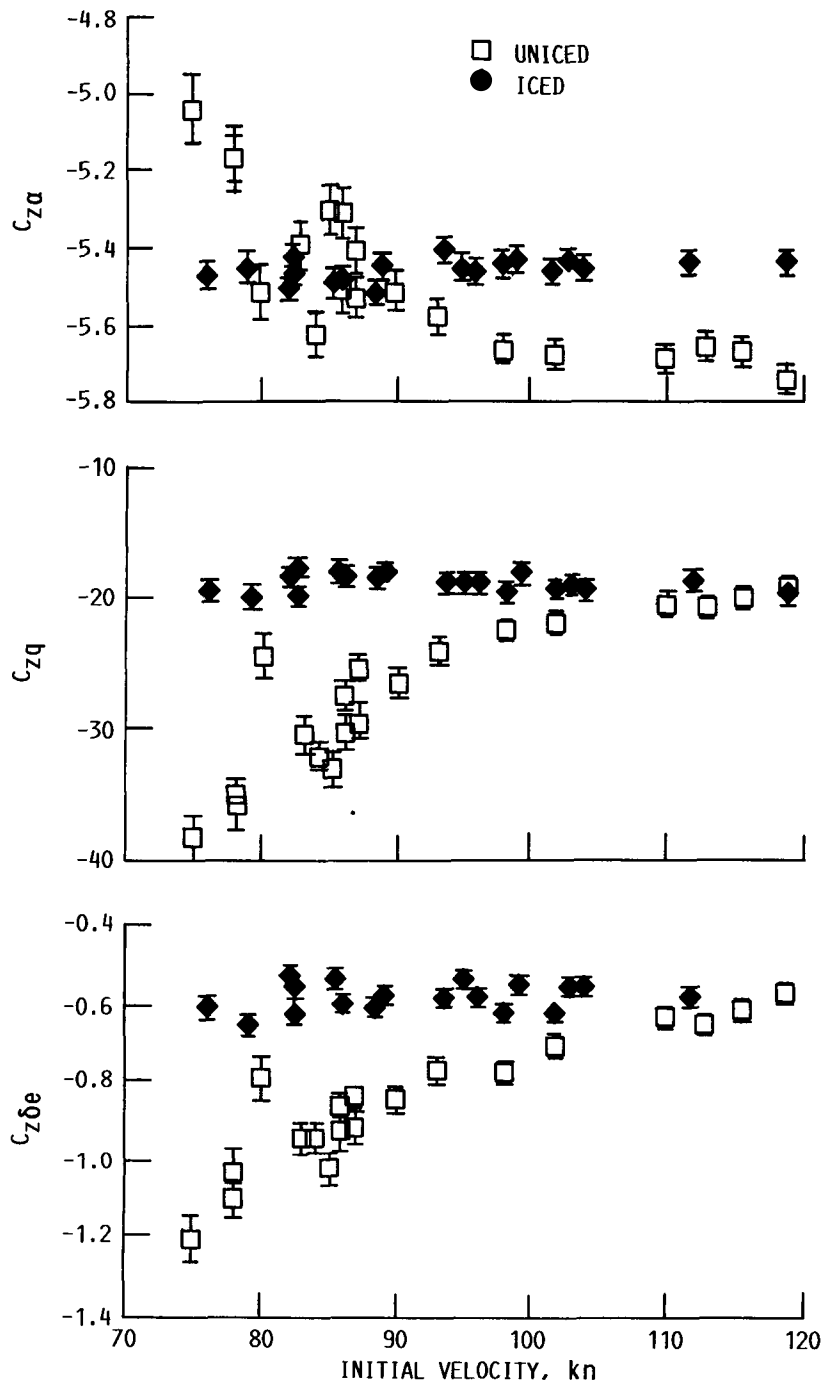


FIGURE 5. - VERTICAL FORCE COEFFICIENT DERIVATIVE ESTIMATES FOR 0° FLAP SHOWING THE EFFECTS OF TAIL ICING USING THE MSR TECHNIQUE. 2σ ERROR BARS ARE FROM THE MSR PROGRAM.

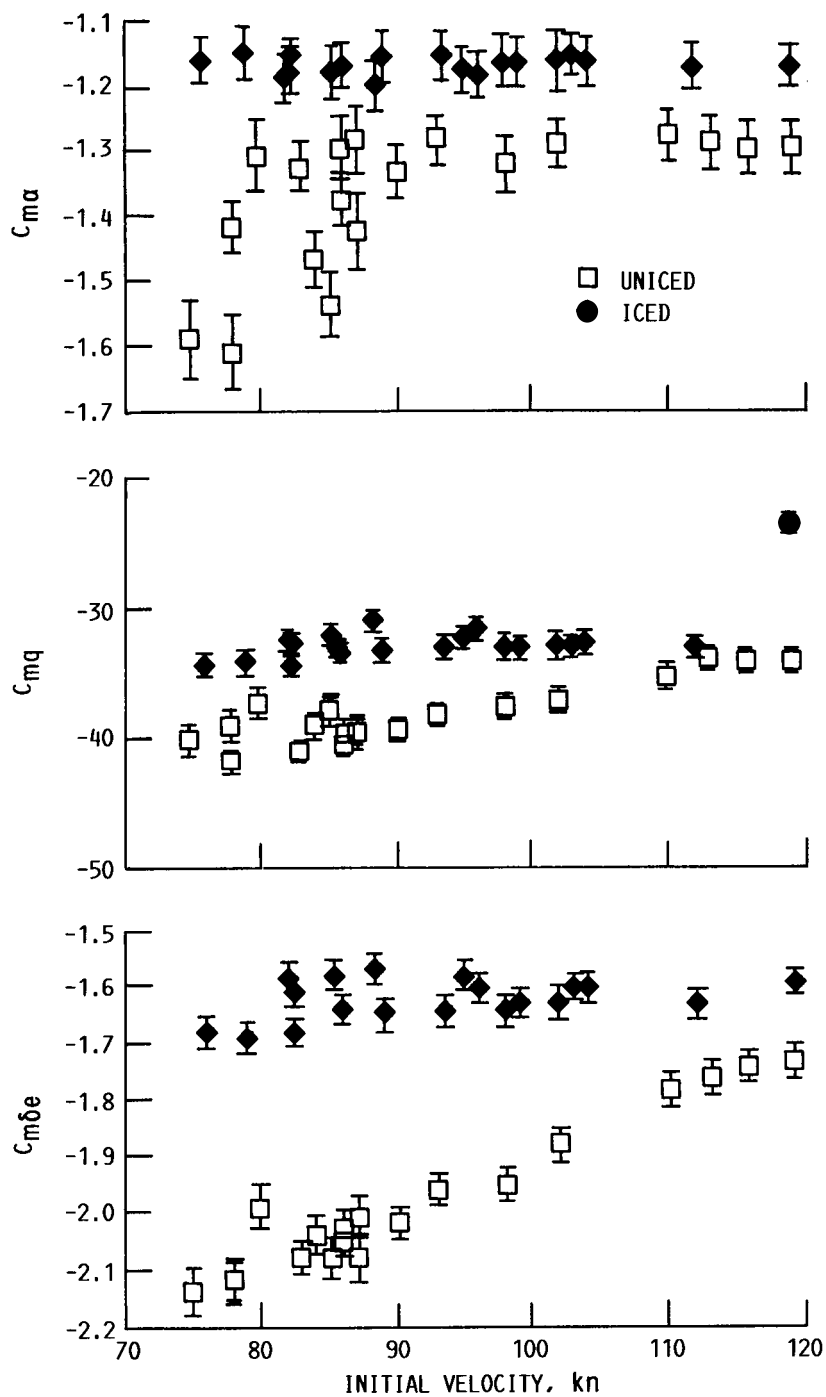


FIGURE 6. - PITCHING MOMENT COEFFICIENT DERIVATIVE ESTIMATES FOR  $0^\circ$  FLAP SHOWING THE EFFECTS OF TAIL ICING USING THE MSR TECHNIQUE.  $2\sigma$  ERROR BARS ARE FROM THE MSR PROGRAM.

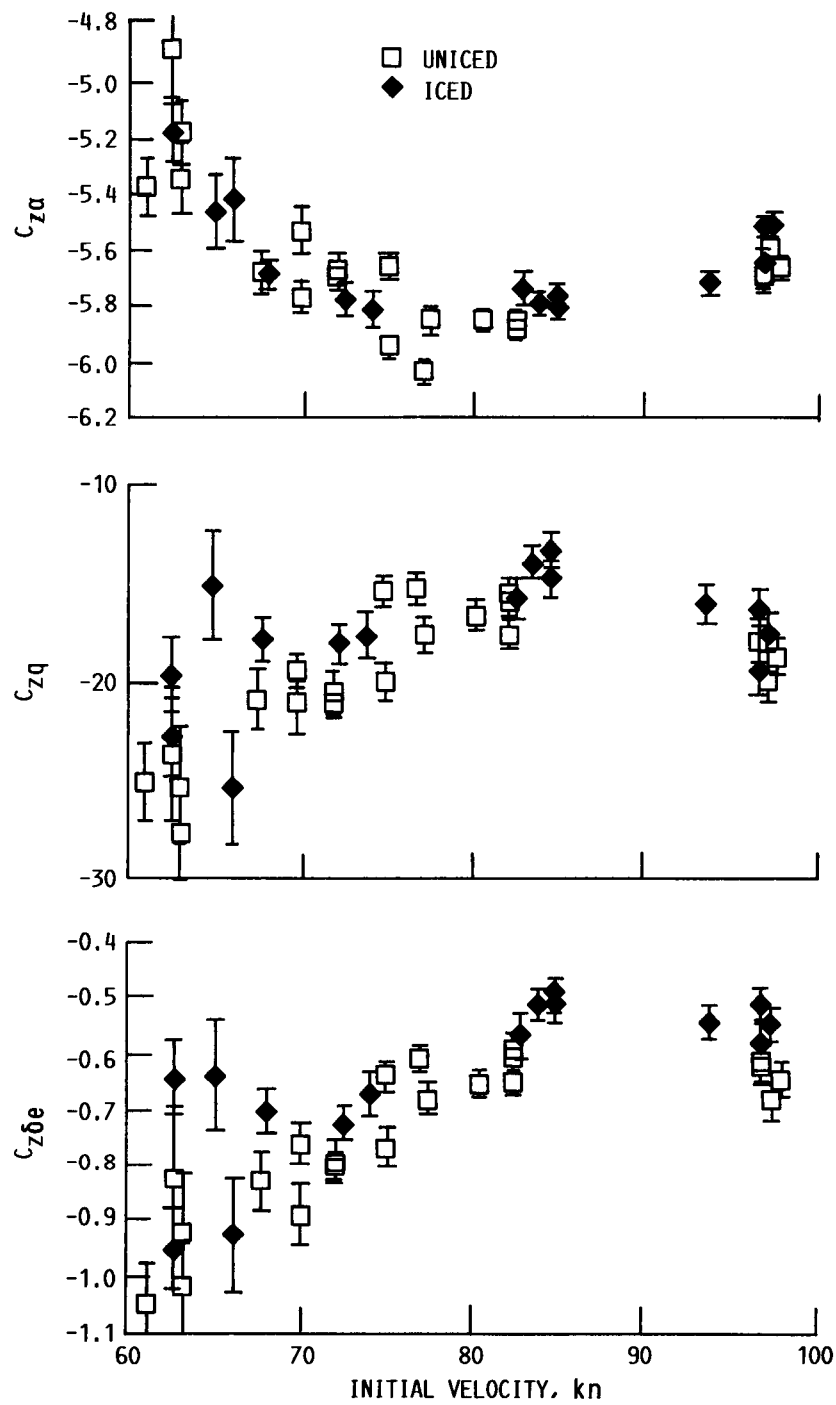


FIGURE 7. - VERTICAL FORCE COEFFICIENT DERIVATIVE ESTIMATES FOR  $10^0$  FLAP SHOWING THE EFFECTS OF TAIL ICING USING THE MSR TECHNIQUE.  $2\sigma$  ERROR BARS ARE FROM THE MSR PROGRAM.



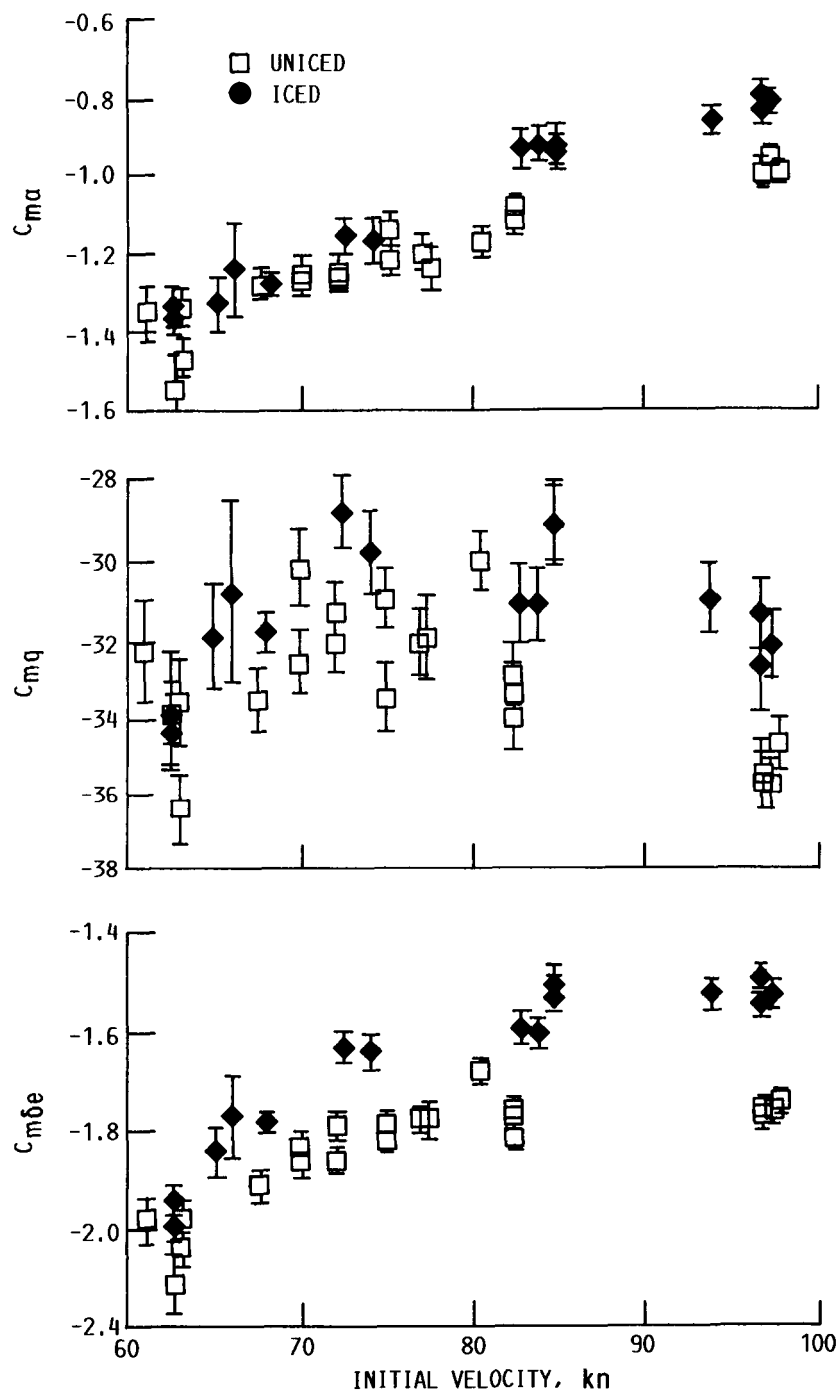


FIGURE 8. - PITCHING MOMENT COEFFICIENT DERIVATIVE ESTIMATES FOR  $10^0$  FLAP SHOWING THE EFFECTS OF TAIL ICING USING THE MSR TECHNIQUE.  $2\sigma$  ERROR BARS ARE FROM THE MSR PROGRAM.

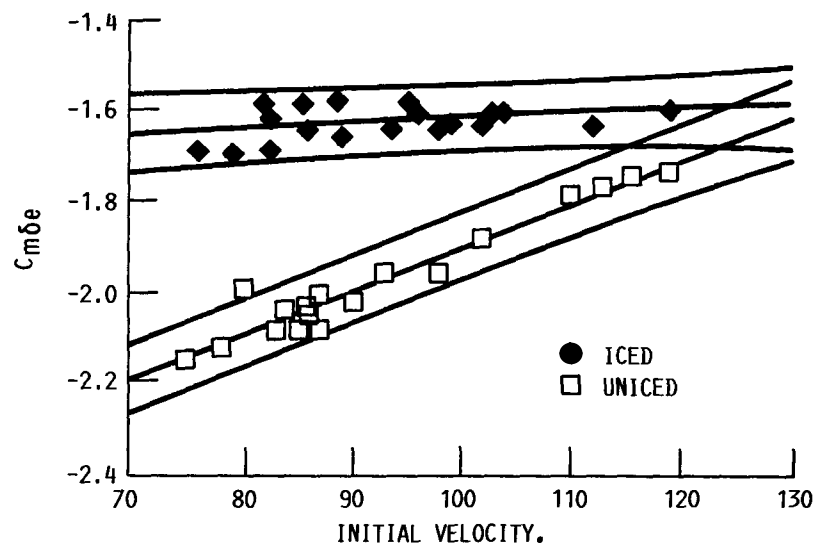


FIGURE 9. - 95 PERCENT PREDICTION INTERVALS ON ELEVATOR EFFECTIVENESS VALUES FOR THE ARTIFICIALLY ICED VERSUS UNICED AIRCRAFT TAIL. WING FLAPS AT  $0^0$ .

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16. Abstract A flight test was performed with the NASA Lewis Research Center's DH-6 icing research aircraft. The purpose was to employ a flight test procedure and data analysis method, to determine the accuracy with which the effects of ice on aircraft stability and control could be measured. For simplicity, flight testing was restricted to the short period longitudinal mode. Two flights were flown in a clean (baseline) configuration, and two flights were flown with simulated horizontal tail ice. Forty-five repeat doublet maneuvers were performed in each of four test configurations, at a given trim speed, to determine the ensemble variation of the estimated stability and control derivatives. Additional maneuvers were also performed in each configuration, to determine the variation in the longitudinal derivative estimates over a wide range of trim speeds. Stability and control derivatives were estimated by a Modified Stepwise Regression (MSR) technique. A measure of the confidence in the derivative estimates was obtained by comparing the standard error for the ensemble of repeat maneuvers, to the average of the estimated standard errors predicted by the MSR program. A multiplicative relationship was determined between the ensemble standard error, and the averaged program standard errors. In addition, a 95% confidence interval analysis was performed for the elevator effectiveness estimates, $Cm_{\delta_e}$ . This analysis identified the speed range where changes in $Cm_{\delta_e}$ could be attributed to icing effects. The magnitude of icing effects on the derivative estimates were strongly dependent on flight speed and aircraft wing flap configuration. With wing flaps up, the estimated derivatives were degraded most at lower speeds corresponding to that configuration. With wing flaps extended to 10 degrees, the estimated derivatives were degraded most at the higher corresponding speeds. The effects of icing on the changes in longitudinal stability and control derivatives were adequately determined by the flight test procedure and the MSR analysis method discussed herein.					
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