A Study on the Thermal and Moisture Influences on the Free-Edge Delamination of Laminated Composites

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Index

List of Figures

List of Tables

Abstract

Laminated composite structures exhibit a number of different failure modes. These include fiber-matrix debonding within individual layers, delamination or separation of the layers, transverse cracks through one or more layers and fiber fracture. These failures are influenced by enviromental conditions. Thermal and moisture conditions are significant factors in interlaminar delamination as well as the other modes of failures.

A simple delamination analysis method is presented here. It is based on a shear-type deformation theory and includes hygrothermal effects. These enviromental conditions are applied to the strain energy release rate and interlaminar shear stresses.

The method is applied to mixed mode edge delamination specimens made of T300/5208 graphite/epoxy material. Residual thermal and moisture stresses significantly influenced the strain energy release rate and interlaminar stresses. Both experienced large increases when thermal conditions were added to the mechanical strains. These effects were alleviated when moisture stresses were included. Thermal effects on the interlaminar shear stress and total energy release rate were totally alleviated for the same specific moisture content. Moreover, this value of moisture content was not significantly affected by the stacking sequence for the laminates considered.

Introduction

Composites have been used in the aircraft industry since 1969. One aspect of concern for using composites in structures is separation of plies or delamination. This occurs in regions of stress raisers such as holes, ply terminations, cut-outs and free edges. Delamination along the free edge of laminates have been observed during testing and service. The presence of delamination, initiated by interlaminar stresses, causes redistribution of the stresses among the plies in a laminate. Thus, it usually results in a reduction of stiffness and strength.

Figure 1 shows delamination in a rotor hub made of S2/SP250 glass/epoxy. The specimen has delaminated in two places that can be depicted by the dark lines. Figure 2 shows delamination in a single cracked-lap-shear test specimen made of AS4/3502 graphite/epoxy[1]. The specimen layup is $[\pm 45/0/90]_{6S}$ quasi-isotropic with 8 plies in the strap and 40 plies in the lap. The tests were performed on a displacement controlled machine. Fiber glass tabs were attached to the specimen ends. Multiple, isolated free edge delaminations occur in the neighborhood of the lap/strap junction and the tab. Figure 3 illustrates an I-beam section made of C3000/5225 graphite/epoxy woven cloth in the post-buckled regime. Free edge delamination depicted in the flanges precipitated the final failure in the specimen. Figure 4 shows how delamination can take place in single cracked-lap-shear specimens subjected to compressive and tensile loading. Specimens A and B delaminated under compressive loading while C experienced a tensile loading. These examples illustrate the importance of investigating delamination problems in composites.

Thermal residual and moisture effects on a composite are practical enviromental conditions. Determining the response of these conditions on interlaminar stresses and energy release rates in laminated composites is the primary objective of this work.

Delamination analysis can be based on two approaches. They are the strain energy release rate and interlaminar stresses. The interlaminar stresses are due to Poisson's ratio mismatch and difference in the coefficients of thermal and moisture expansion between plies. Delamination occurs when these stresses reach the interlaminar strength of the material. An alternative approach is based on the actual process of fracture rather than the strength concept. Delamination can propagate when the strain energy release rate at the crack front is sufficient to overcome the material's fracture resistance or toughness.

The strain energy release rate can be obtained for three particular modes of crack action. These three modes are known as Mode I or opening mode, Mode II or forward shearing mode and Mode III or tearing mode. Several failure laws are formulated in terms of these modes [2].

A simple analysis predicting interlaminar shear stresses and total energy release rate with the influence of thermal and moisture effects is developed. This simple approach is useful in understanding the basic mechanics of the problem and predicting the factors controlling the behavior. The method is directed to the needs of a practical design environment. It is not intended to compete with large-scale numerical approaches, but rather to serve as the means for selecting and screening candidate configurations and providing trend information. Simple codes for a desktop computer have been created to analyze laminate configurations.

Literature Summary

A historical discussion of previously developed work for predicting interlaminar stresses and energy release rates is presented to establish a basis for the proposed model and to permit the present work to be placed in proper perspective.

Earlier analyses have reflected the prediction of interlaminar stress and energy release rate without hygrothermal conditions. O'Brien[3,4] investigated delamination onset and growth in graphite/epoxy laminates under uniform extension. A simple expression was developed for the total energy release rate based on classical lamination theory. Whitney and Knight[5] used classical laminated plate theory to develop an edge delamination specimen analysis. This work was limited to Mode I behavior.

An analysis based on a shear deformation theory and a sublaminate formulation [6] was developed by Armanios and Rehfield[7,8]. This method provides good estimates for the interlaminar shear stresses. Energy release rate components are estimated based on these stresses. However, this method does not provide reliable estimates of peel stress since thickness strain is neglected. This analysis was limited to mechanical strain only.

O'Brien[9] modified his analysis to include thermal and moisture conditions. The influence of thermal effects was considered by Whitney[10].

The work of Reference 9 was based on a classical laminated plate theory. It was applied to mixed-mode edge delamination specimens.

The results were limited to strain energy release rate. Finite element modeling was used to determine the strain energy release rate components. O'Brien's results reflected an increase in the strain energy release rate due to thermal effects. It decreased with the addition of moisture considerations. For a T300/5208 graphite/epoxy laminate with $[\pm 30/\pm 30/90/\overline{90}]_s$ layup, the thermal effect increased the total energy release rate by 170 percent when compared to mechanical loading alone. However, a specific moisture level of 0.75 percent completely alleviated this increase. In calculating the total strain energy he showed that bending and coupling effects became important at high levels of moisture content.

In Reference 10 a higher-order plate theory with transverse normal strain effects was developed. Peel as well as interlaminar shear stresses could be predicted by this method. The thermal influence on total energy release rate and interlaminar stresses was investigated using a Mode I specimen. Residual thermal effects showed a significant influence on the stresses and release rates. For a graphite/epoxy laminate of $[0_3, 90_3]_s$ layup, thermal effects increased the maximum peel stress by a factor of 2.7 over that of pure mechanical strains.

In the present work both thermal and moisture influences are studied in a mixed-mode delamination specimen. The analysis includes total energy release rate as well as interlaminar stresses. Similarities between the interlaminar stesses and total energy release predictions with hygrothermal effects is investigated.

In the subsequent sections, the analytical approach is developed.

The method is then applied to six graphite/epoxy laminates. A discussion of the hygrothermal effects on interlaminar shear stress and total energy release rate predictions is provided. Recommendations for further investigations are proposed. Appendices are included for completeness. The first provides detailed expressions of the governing equations. Appendix II defines the hygrothermal expressions and their use in the analysis. The last appendix shows a listing of the program used and sample output.

Analytical Approach

Overview

The sublaminate modeling approach describes the essential features of the laminate behavior in a simple way. A free edge delamination specimen is shown in Figure 5. A uniform strain , ϵ, is applied in the axial direction. From symmetry, only one quarter of the specimen is considered. In Figure 6, the specimen is modeled as if it were composed of four distinct sublaminates. Sublaminates 2 and 3 represent the group of plies above and below the crack, respectively in the cracked portion of the laminate, while sublaminates 1 and 0 denote the same group of plies in the uncracked portion of the laminate.

The use of sublaminates -- groups of plies that are conveniently treated as laminated units -- simplifies the analysis considerably. This approach is applied with confidence when the characteristic length of the response is large compared to the individual sublaminate thickness[6]. This sublaminate modeling approach has been verified in Reference 7 by comparison with a ply-by-ply finite element solution. These sublaminates are connected by enforcing the proper continuity conditions on stresses and displacements at their interfaces.

Displacement fields within each sublaminate are defined as:

$$u = x\epsilon + U(y) + z\beta_x(y)$$

$$v = V(y) + z\beta_y(y) \qquad (1)$$

$$w = W(y)$$

where u,v, and w denote the displacements relative to the x, y, and z axes, respectively. Shear deformation is recognized through the rotations β_x and β_y. The governing equations for each sublaminate are derived using a virtual work approach. The derivation of the governing equations used in the development appears in Appendix I. The derivation is an extension of the work of Reference 8 with hygrothermal effects included.

The constituitive relationships in terms of these force and moment resultants can be written as

$$N_i = A_{ij}\,\epsilon_j + B_{ik}\,\kappa_k - N_i^{NM} \qquad (i,j,k = 1,2,6)$$

$$M_i = B_{ij}\,\epsilon_j + D_{ik}\,\kappa_k - M_i^{NM} \qquad (i,j,k = 1,2,6) \qquad (2)$$

$$Q_i = A_{ij}\,\epsilon_j \qquad (i,j = 4,5)$$

where the subscripts x, y, z, yz, xz, and xy are replaced by the subscripts 1-6 respectively. The force and moment resultants are denoted by N_i and M_i, respectively. Non-mechanical forces and moments resulting from the hygrothermal effects are labeled with a superscript NM. They are defined as:

$$(N_i^{NM},\ M_i^{NM}) = \int_{-h/2}^{h/2} Q_{ij}(1,z)\ (\alpha_j\ \Delta T + b_j\,C)\ dz \qquad (3)$$

The swelling coefficient is denoted by b_j in Equation (3), the thermal coefficient by α_j. The change in temperature is denoted by ΔT and moisture weight gain by C.

The elastic stiffnesses A_{ij} , B_{ij} , and D_{ij} are defined in terms of the sublaminate reduced stiffness Q_{ij} for a plane stress situation. These bear the classical definition.

$$(A_{ij}\ ,\ B_{ij}\ ,\ D_{ij}) = \int_{-h/2}^{h/2} (1, z, z^2)\ Q_{ij}\ dz \tag{4}$$

The equilibrium equations are:

$$\begin{aligned}
&N_{xy,y} + (t_{2x} - t_{1x}) = 0 \\
&N_{y,y} + (t_{2y} - t_{1y}) = 0 \\
&Q_{y,y} + (p_2 - p_1) = 0 \\
&M_{xy,y} - Q_x + h/2\ (t_{2x} + t_{1x}) = 0 \\
&M_{y,y} - Q_y + h/2\ (t_{2x} + t_{1x}) = 0
\end{aligned} \tag{5}$$

where t_{2x}, t_{2y}, p_2 and t_{1x}, t_{1y}, p_1 denote the interlaminar stress components in the x-z, y-z and z directions at the sublaminate upper and lower surfaces, respectively. These stress components appear in Figure 7.

The displacements, resultant forces and moments, and interlaminar shear stresses in each sublaminate is governed by the displacement distribution (1), constituitive (2), and equilibrium (5) equations. These equations are applied to each sublaminate. The variables associated with each sublaminate are coupled through the continuity requirements at their interfaces. Enforcement of the boundary conditions lead to a solution for these variables. This procedure discussed in general terms above is applied to the analysis of the edge delamination specimen shown in Figure 2 in the following sections.

The response associated with sublaminates 1 and 0 shown in Figure 2 is coupled through the continuity conditions at their common interface. The situation is different with sublaminate 2 and 3 where the continuity conditions are relaxed due to the presence of the crack.

Uncracked Region: Sublaminates 0 and 1

From symmetry conditions at the sublaminate bottom surface the shear stresses are zero. Interlaminar stresses at the top surface of sublaminate 0 are equal to those on the bottom of sublaminate 1. Substituting these conditions into the equilibrium and constitutive relations and enforcing continuity of displacements at their common interface yields a homogeneous system of ordinary differential equations. These can be expressed in terms of the sublaminate rotations β_x and β_y. Assuming an exponential solution of the form

$$(\beta_{1x}, \beta_{0x}, \beta_{1y}, \beta_{0y}) = (\beta^*_{1x}, \beta^*_{0x}, \beta^*_{1y}, \beta^*_{0y})\, e^{sy} \quad (6)$$

results in a characteristic equation of the form

$$E_8 s^8 + E_6 s^6 + E_4 s^4 + E_2 s^2 + E_0 = 0 \quad (7)$$

Parameter E_0 depends only on the stiffness coefficients A_{44}, A_{55} and A_{45} for both sublaminates while E_8 is predominantly influenced by the bending and coupling coefficients D_{ij} and B_{ij}. Thus, its numerical value can be orders of magnitude smaller than the other coefficients. This results in the presence of a boundary zone in the response.

The characteristic roots controlling the behavior of the laminate are determined from Equation 7 which has a closed-form solution.

Crack Region of the Laminate: Sublaminates 2 and 3

With this group of laminates, there are free surfaces at both the top and bottom of sublaminates 0 and 1 respectively. This is due to the presence of the crack. With the crack dividing the sublaminates, continuity conditions are not enforced at the boundary interface. This results in zero shear stresses at the surfaces of each sublaminate. Thus, the equilibrium and constitutive relations combine to produce a second order differential equation in terms of the sublaminate rotations β_{2X} for sublaminate 2 and β_{3X} for sublaminate 3.

Interlaminar Stresses

The arbitrary constants that are obtained from the eighth degree polynomial are determined by enforcing the stress free boundary conditions at the free edges of sublaminates 2 and 3, and the continuity of force, moment, displacement and rotations between sublaminates 0 and 3, as well as between 1 and 2. This yields the following expression for the interlaminar shear stresses.

$$t_X = N_{XY_1,Y} = G_J \; e^{-s_j y} \tag{8}$$

$$t_Y = N_{Y_1,Y} = T_J \; e^{-s_j y} \qquad (\, j = 1 - 4 \,) \tag{9}$$

Parameters T_j and G_j represent the amplitude of the response.

Energy Release Rate

A complete formulation of the strain energy release rate appears in Appendix II. The total energy release rate can be determined by considering the work done by external forces.

$$G = - 1/2 * dW/da \tag{10}$$

The total energy release rate is denoted by G and the crack length by a. The work done is defined as

$$W = L/2\int_0^b N_{xi}\epsilon_{mi}\, dy \tag{11}$$

where subscript i denotes the sublaminate being referenced. The term ϵ_{mi} represents the mechanical strain in each ply. This is defined as the difference between the total strain and the strain corresponding to free expansion for each ply. This strain is estimated by using a procedure similar to Reference 5. However, in Reference 5 the free expansion strain was determined by considering groups of plies in the cracked and uncracked regions of a Mode I edge delamination specimen. This approach is valid for a limited class of laminates. A ply-by-ply analysis rather than a sublaminate modeling should be used. In the following analysis, free expansion strains are determined on a ply-by-ply basis.

From the symmetric conditions that exist in the uncracked section of the laminate, there exist no curvature. In the cracked portion, the moment about the y-axis is assumed to be zero. Using both of these boundary conditions in Equation (2) yields the following.

$$N_{x1} = A_{11}\epsilon + A_{12}\,\epsilon_y + A_{16}\,\epsilon_z - N_{x1}^{NM} \qquad (12)$$

$$N_{x1} = A_{11}\,\epsilon + A_{12}\,\epsilon_y + A_{16}\,\epsilon_z + B_{12}\,\kappa_y - N_{x2}^{NM}$$

Strain components ϵ_y and ϵ_z in Equation 12 are expressed in terms of the applied strain by

$$\epsilon_y = C_v\ \epsilon + C_v^{NM} \qquad (13)$$

$$\epsilon_z = C_u\ \epsilon + C_u^{NM}$$

The terms C_v, C_u, C_v^{NM} and C_u^{NM} are functions of the extensional stiffness components A_{ij} of sublaminates 1 and 0.

Using these expressions, Equation 12 can be re-written in the form.

$$N_{xc}^k = (\ E_c^k\,\epsilon_c^k + T_c^k\ ) \qquad (14)$$

$$N_{xu}^k = (\ E_u^k\,\epsilon_u^k + T_u^k\ )$$

Parameters E_c^k, T_c^k, E_u^k and T_u^k are defined in Appendix II. Superscript k denotes the individual ply. Subscripts u and c represent the uncracked and cracked portions, respectively. The non-mechanical strain in each ply corresponding to a state of free expansion is obtained by allowing the stress in each ply to vanish. This yields the following.

$$\epsilon_u^k = -T_u^k \,/\, E_u^k \tag{15}$$

$$\epsilon_c^k = -T_c^k \,/\, E_c^k$$

The strain corresponding to free expansion of the entire laminate is obtained by letting the resultant force vanish. The non-mechanical strain is

$$\epsilon^{NM} = \left\{ T_u^* - (T_u^* - T_c^*)\, 2a/b \right\} / \left\{ E_u^* - (E_c^* - E_u^*)\, 2a/b \right\} \tag{16}$$

The terms T_u^*, T_c^*, E_u^* and E_c^* represent the summation of T_u^k, T_c^k, E_u^k and E_c^k over their respective sublaminates. These strain definitions for the effects of moisture and temperature can now be used in the general expression for the strain energy. The strain field is altered to represent the hygrothermal effects. The total strain for a sublaminate is expressed as:

$$\epsilon^T = \epsilon + \epsilon^{NM} \tag{17}$$

The strain energy expression is given below showing the use of Equations (13) and (17).

$$W = \frac{L}{2}\left[\Sigma (E_c^k \epsilon^T + T_c^k)(\epsilon^T - \epsilon_c^k) + (E_u^k \epsilon^T + T_u^k)(\epsilon^T - \epsilon_u^k) \right] \tag{18}$$

Substituting this into Equation (11) yields the total strain energy release rate per crack.

Results and Discussion

Benchmark Study

The analytical model is applied to the edge delamination specimen shown in Figure 5. The material considered is T300/5208 graphite/epoxy. Its mechanical properties are listed in Table I. The coefficients of swelling and thermal expansions are also stated. The geometry of the specimen is given in Table II. Cure temperature for this material is 350°F. The ambient operating temperature is 70°F. The moisture level for all cases was allowed to vary from 0 to 1.2 percent of the laminate weight. This moisture level reflects feasible conditions. The mechanical strain is taken as 0.00254. This particular value of strain was taken from test experiments[9]. It is considered a practical value for the material.

Six laminates have been analyzed. They can be divided into two groups. The first group is composed of laminates $[35/-35/0/90]_s$, $[35/0/-35/90]_s$, and $[0/35/-35/90]_s$. These laminates are prone to delaminate at the interfaces indicated by the arrow[9]. The Mode III in these laminates is negligible. This is due to the fact that relative sliding at the crack front and the interlaminar shear stress in the x-z direction, τ_{xz} , is neglegible. The second group of laminates is $[30/-60/75/15]_s$, $[-35/55/10/-80]_s$, and $[35/80/-55/-10]_s$. In these laminates Mode I, Mode II, and Mode III are finite. The Mode III strain energy release rate component due to mechanical loading in these laminates are significantly large, ranging from 60 to 90 percent[7].

Interlaminar Shear Stresses

The interlaminar stress τ_{yz} at the delamination interface appear in Figure 8 - 10, for the first group of laminates. The interlaminar shear stresses τ_{xz} and τ_{yz} for the second group of laminates appear in Figures 11 - 13. The labels M, M+T, and M+T+H stand for mechanical, mechanical and thermal, and mechanical, thermal and moisture respectively.

The boundary layer of decay for all laminates ranged from 0.85 to 0.93 of the laminate semi-width. In this context the boundary layer decay length is defined as the distance where the stress decays to 5 percent of its maximum value. The stress boundary zone is not significantly influenced by the environmental conditions.

The magnitude of shear stress however showed a strong dependency on thermal and moisture conditions. At the delamination front, the ratio of stress with thermal effects as compared to pure mechanical loading ranged from 3.22 to 3.36 for the first group of laminates. This maximum was experienced at the crack tip. For the laminates where Mode III was present, this ratio ranged from 4.16 to 5.23 for τ_{yz}. The shear in the x-z direction showed a ratio of 1.4 to 2.16 for the maximum stresses. The maximum τ_{yz} stress for the second group of laminates was experienced at the crack front. However, the maximum τ_{xz} stress occured slightly ahead of the crack. This can be seen in Figures 11 - 13.

The addition of moisture alleviated the thermal effect. A moisture content of 0.4 has reduced the stress of thermal influence by approximatly 40 percent as compared to thermal influences alone. This trend is similar to the results of Reference 9.

Numerical values of interlaminar shear stress at various moisture levels are provided in the sample output of Appendix III. Interlaminar shear stresses show numerical decrease with increase of moisture levels.

Energy Release Rate

The hygrothermal effect on total energy release rate appears in Figures 14 -15. The hygrothermal effects on total energy release rate show a similar trend to that of interlaminar shear stresses. Residual thermal stress tends to increase total energy release rate while residual moisture alleviates this effect. The figures show that for total alleviation of the thermal effect, the specific moisture content ranges between 0.70 and 0.77 for all laminates. This indicates there exist a weak dependency on the stacking sequence.

The effects of thermal conditions alone on the energy release rate does not correspond to the same numerical value as the interlaminar shear stresses. The total energy release rate of layups where Mode III was negligible showed a ratio of 5.1 for mechanical and thermal compared to mechanical conditions only. For the laminates where Mode III is finite, this ratio varied from 1.6 in the $[30/-60/75/-15]_s$ layup to 3.37 in the $[35/80/-55/-10]_s$ layup.

The total energy release rate in the first group of laminates is approximately the same for mechanical loading as shown in Figure 14. The influence of thermal and moisture does not appear to alter this trend. The energy release rate for the $[35/-35/0/90]_s$ laminate is indistinguishable from the $[0/35/-35/90]_s$ layup. The rate of alleviation due to moisture is the same for the three laminates. This is in contrast with the alleviation rate of the laminates where Mode III is finite as shown in Figure 15. For this class of laminates, the rate of alleviation due to moisture is different for each laminate.

In some of the laminates, the rate of alleviation is not constant. There is a steep gradient in the rate of alleviation until the moisture content approaches the totally alleviated state. After such moisture content, the decrease in total energy release rate with respect to moisture addition is not as significant.

It is worth noting there is a similarity between the strain energy release rate prediction and the interlaminar stresses for the totally alleviated state. This is shown in Figure 16 for a $[-35/55/10/-80]_s$ layup. The specific moisture percent producing complete alleviation of the total energy release rate from the thermal effect is 0.76 as seen in Figure 15. The interlaminar shear stress distribution corresponding to this level of moisture is indistinguishable from the mechanical loading alone. The same conclusion was reached studying the other laminates.

Conclusions

A simple analysis was developed that predicted the influence of thermal and moisture effects on the interlaminar shear stresses and strain energy release rate. The analysis was applied to six mixed-mode edge delamination specimens. The results provide several significant findings.

1. Residual thermal strain has a significant influence on the interlaminar shear stress and total energy release rate. The interlaminar stress and total energy release rate increased by 330 and 510 percent respectively over that of pure mechanical loading.

2. Moisture tends to alleviate the thermal effect for both the interlaminar stress and energy release rate. At a specific moisture content of approximately 0.75 percent, the thermal influence is totally alleviated.

3. The moisture content for total alleviation found from the total energy release rate analysis also produced an interlaminar stress distribution similar to pure mechanical loading conditions.

The first two findings are in agreement with the results of previous investigators. The third finding is new. It establishes a similarity in behavior between a delamination analysis expressed in terms of the energy release rate and the strength approach expressed by the interlaminar stresses.

These findings point to new directions for further inquiry. These are discussed in the following section.

Recommendations

The thermal effects on the laminates showed a large increase in both the interlaminar shear stresses and strain energy release rate. The analysis should be supplemented with experimental tests to verify the result. Several fracture laws are expressed in terms of the strain energy release components, as well as the total strain energy. Further analysis should include predictions of these components in the presence of hygrothermal conditions.

Throughout this work the temperature is assumed to be uniform through the thickness of the laminate. The same is true with the moisture. An approach corresponding to a practical environment method should account for temperature and moisture gradients in the laminate. In this situation, the hygrothermal gradients through the thickness may create an unbalance effect in an originally balanced construction. This consideration is of significant importance in aerospace structural components subjected to a large temperature difference between the upper and lower surface.

The loading considered here is uniaxial extension. However, it is known that the load transfer points are not always in the plane of the laminate. Therefore, investigating laminate response under combined loads is of great practical importance. It is recommended that bending, torsion as well as their combined effect be addressed.

Findings by previous investigators suggested that delamination behavior in laminates subjected to fatigue loading follows static loading conditions. Further work is needed to investigate the influence of hygrothermal conditions on the delamination of laminates under fatigue loading.

Finally, the present analysis is applied to the mixed-mode edge delamination specimen. Extension of this work to other specimens such as the single- and double-crack-lap shear and the Mode II edge notch flexure specimen is recommended.

When accomplished, these recommendations, together with the present research will provide a better understanding of the delamination problem in composites. Consequently, this will enable predicting, managing and ultimately preventing interlaminar fracture in laminated composites.

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TABLE I - T300/5208 GRAPHITE/EPOXY PROPERTIES

E_{11} = 18.7 MSI

E_{22} = 1.23 MSI

G_{12} = 0.832 MSI

Poisson Ratio = 0.292

Swelling Coefficients of the Material direction:

b(1-direction) = 0

b(2-direction) = 5560 $\mu\varepsilon$/ %weight

Thermal Coefficients of the Material direction:

α(1-direction) = -.23 $\mu\varepsilon/°F$

α(2-direction) = 14.9 $\mu\varepsilon/°F$

TABLE II - GEOMETRIC DIMENSIONS OF SPECIMEN

Ply thickness = 0.0054 inch

Width = 1.5 inch

Crack length = 6 x ply thickness = 0.0324 inch

FIGURES 1-16

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OF POOR QUALITY

FIGURE 1 - GLASS/EPOXY ROTOR HUB AFTER DELAMINATION

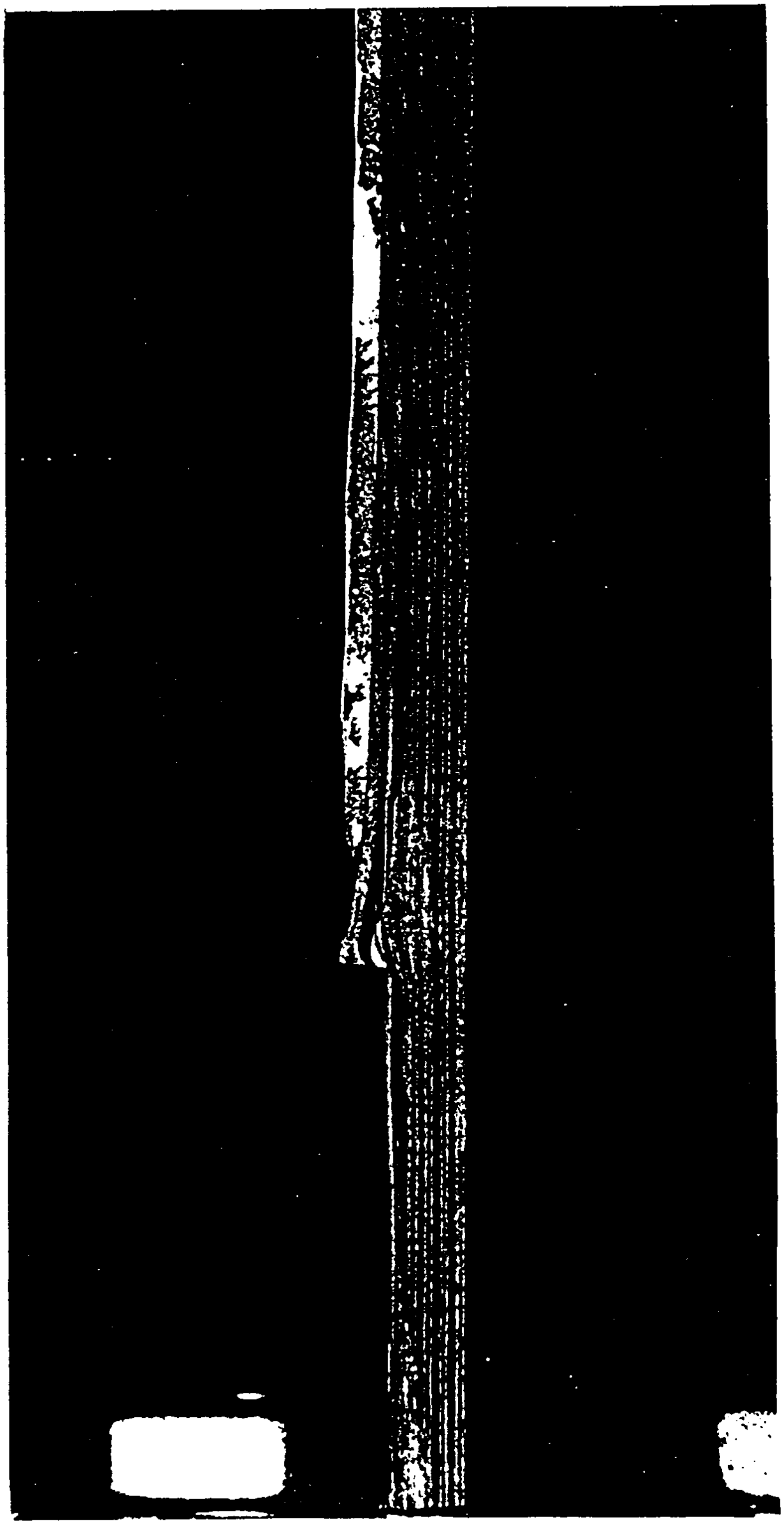

FIGURE 2 - GRAPHITE/EPOXY SINGLE CRACK-LAP-SHEAR SPECIMEN

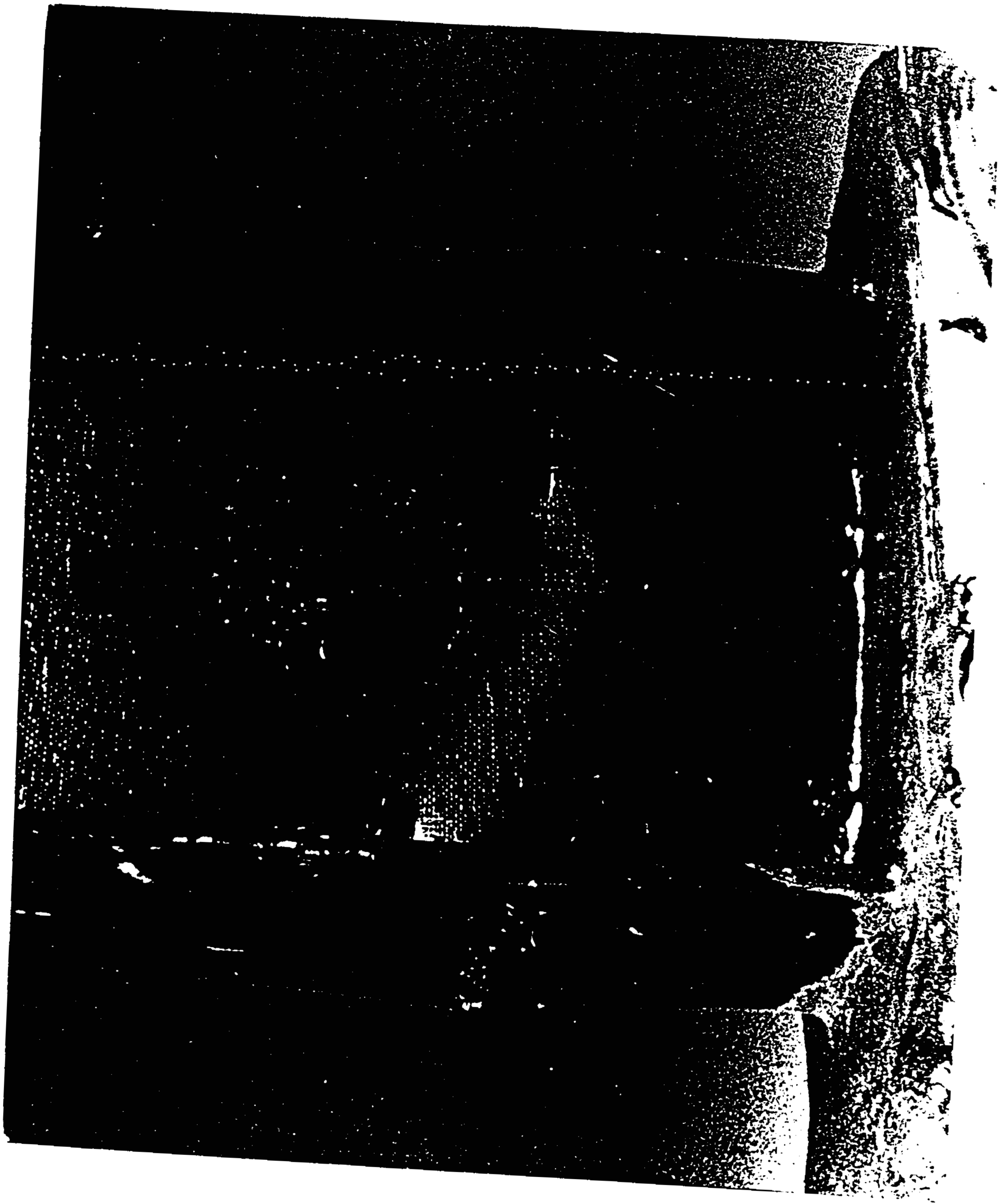

FIGURE 3 - GRAPHITE/EPOXY POST-BUCKLED I-BEAM

FIGURE 4 - COMPRESSIVE AND TENSILE DELAMINATION SPECIMENS

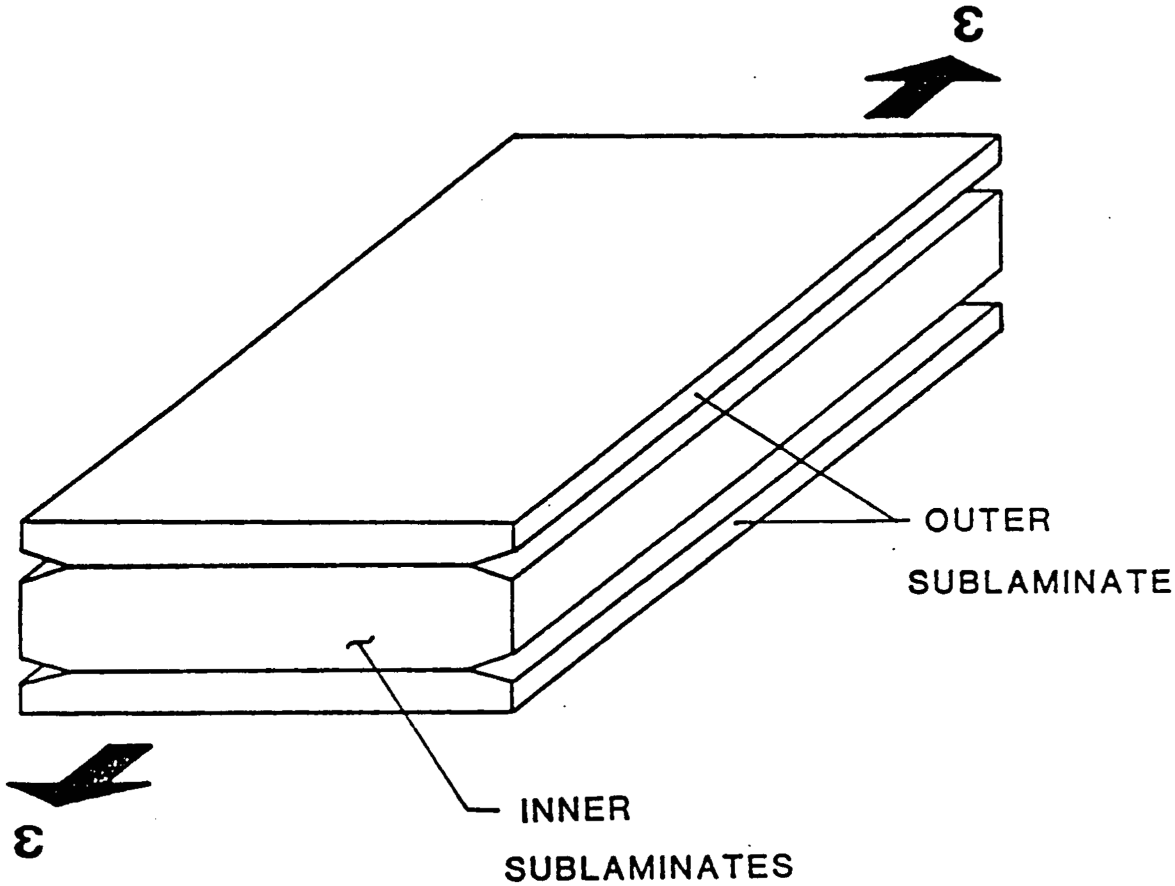

FIGURE 5 - FREE EDGE DELAMINATION SPECIMEN

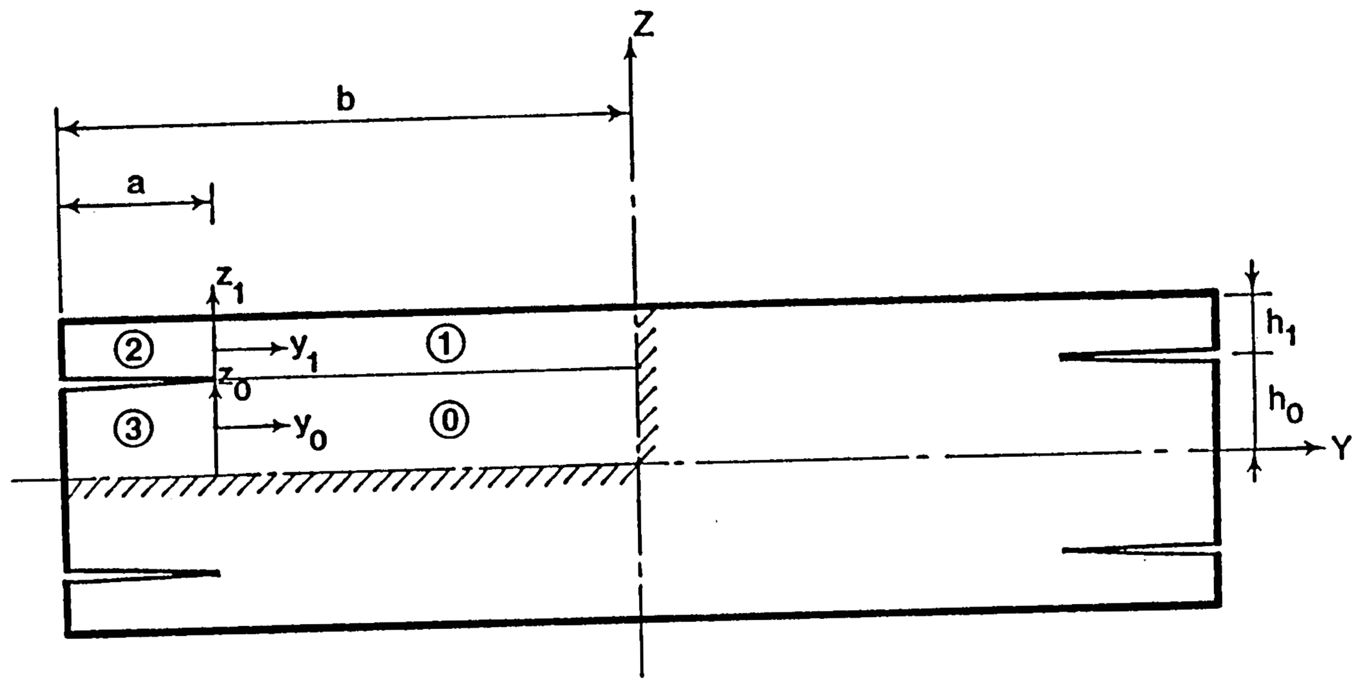

FIGURE 6 - SUBLAMINATE DESCRIPTION AND COORDINATE SYSTEM

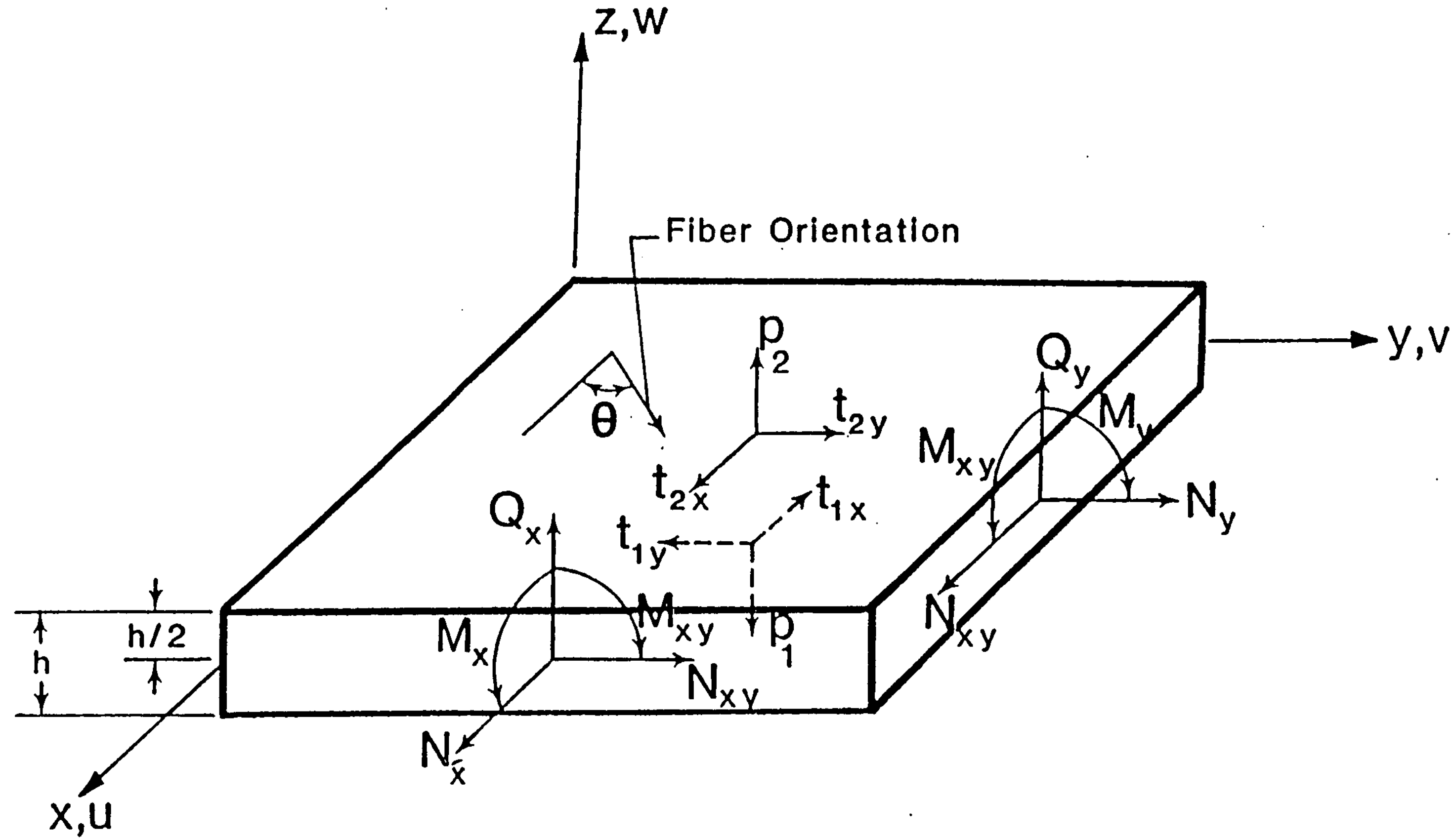

FIGURE 7 - NOTATIONS AND SIGN CONVENTION FOR GENERIC SUBLAMINATE

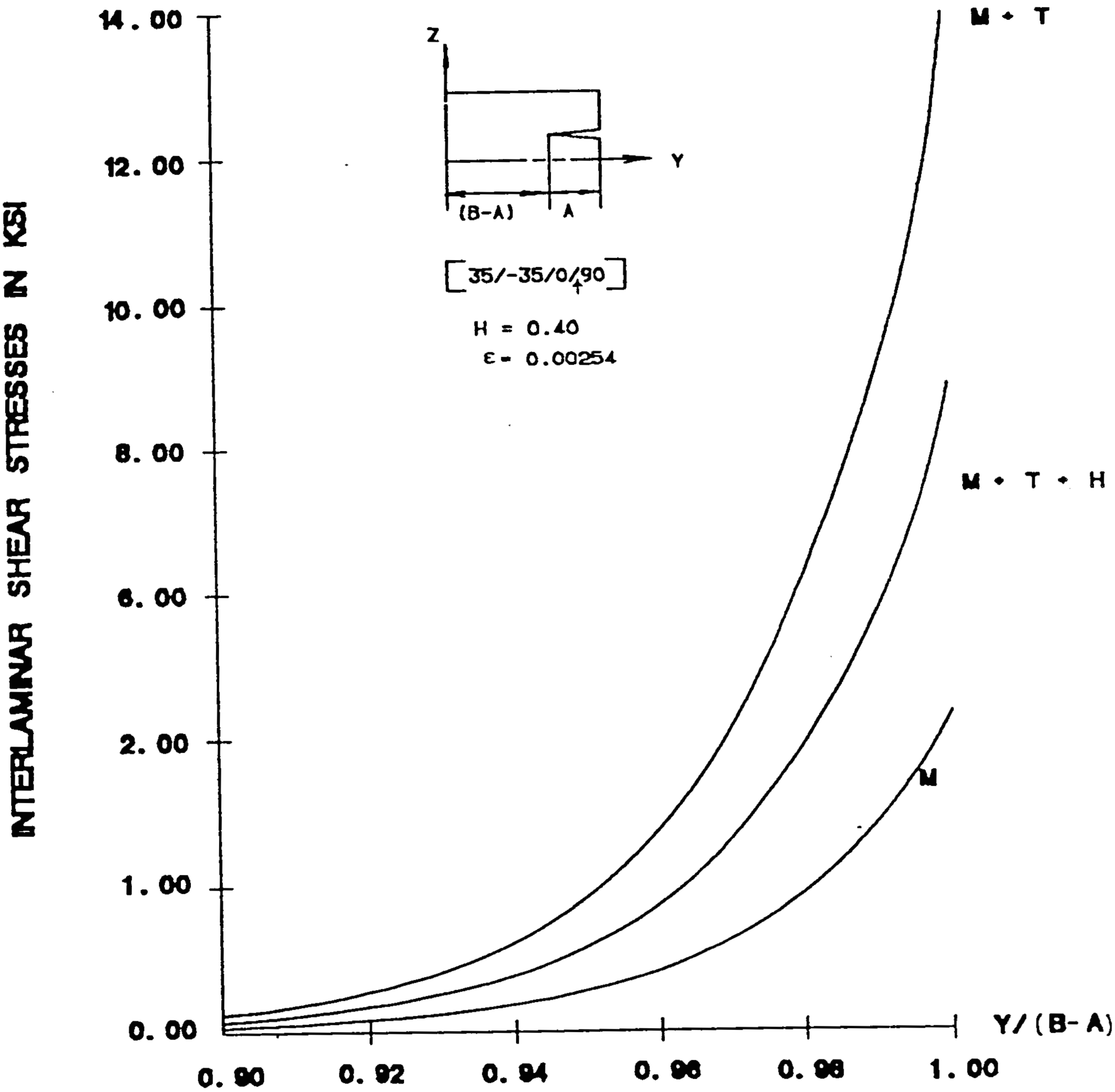

FIGURE 8 - COMPARISON OF INTERLAMINAR STRESS

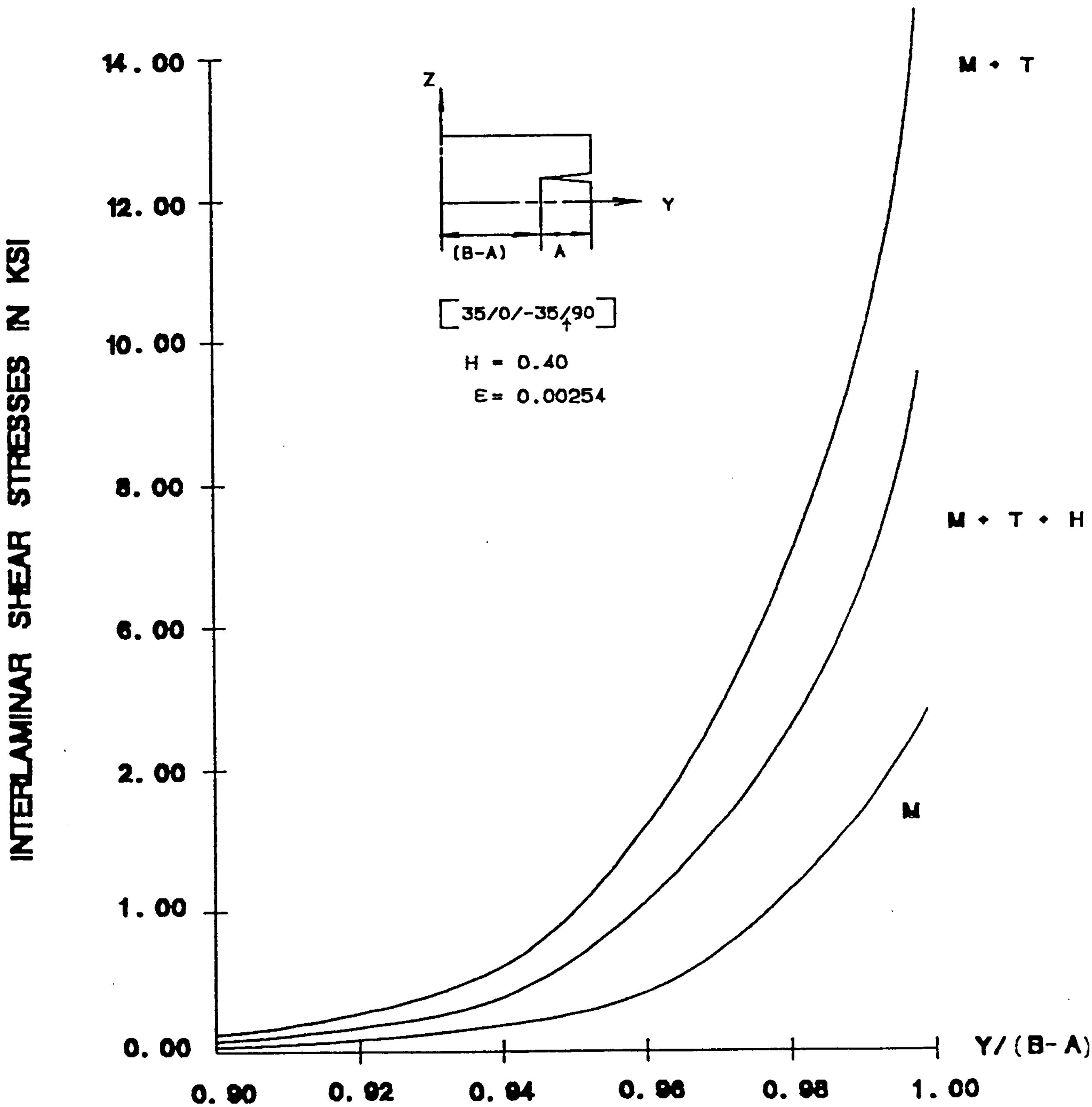

FIGURE 9 - COMPARISON OF INTERLAMINAR STRESS

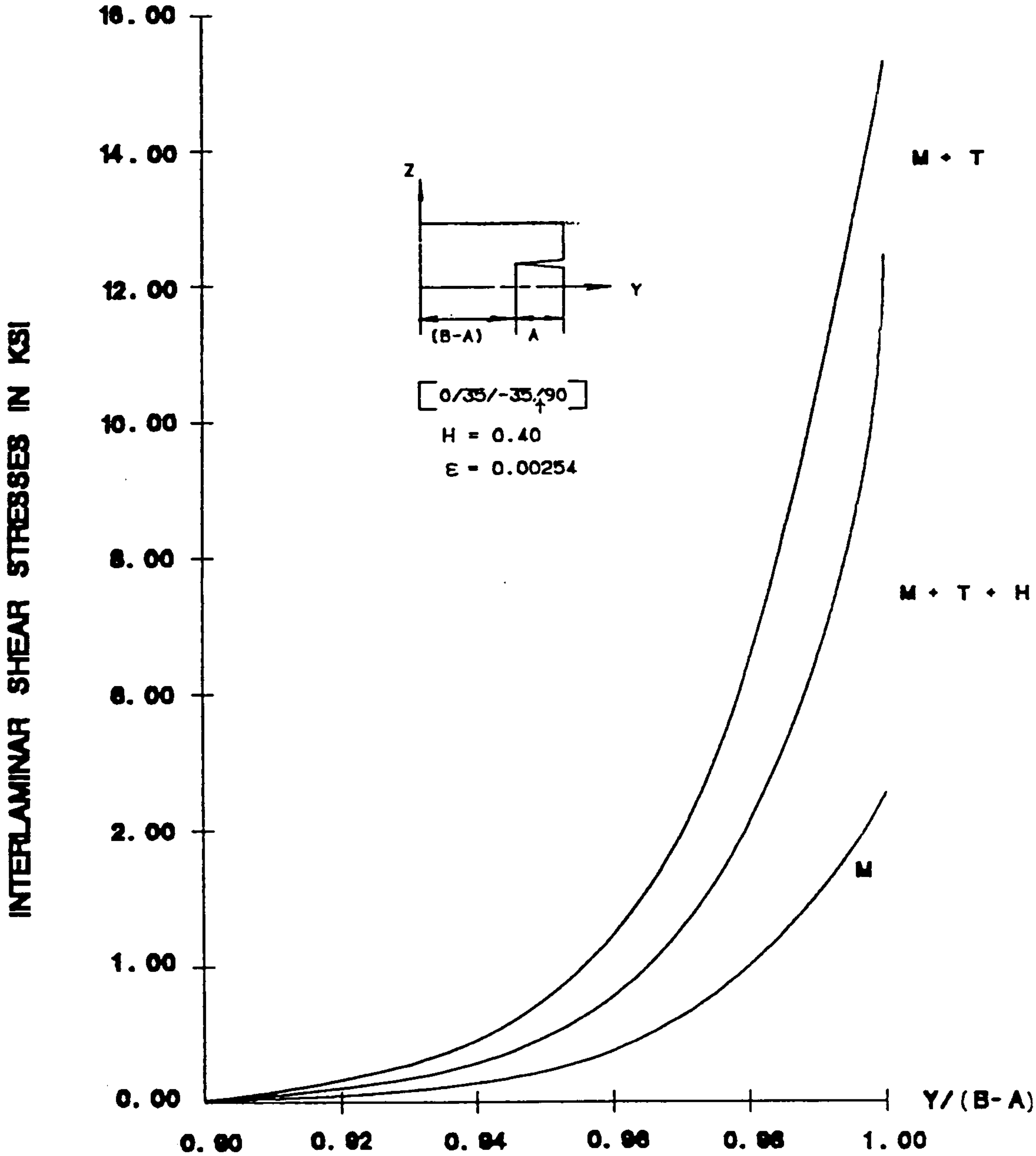

FIGURE 10 - COMPARISON OF INTERLAMINAR STRESS

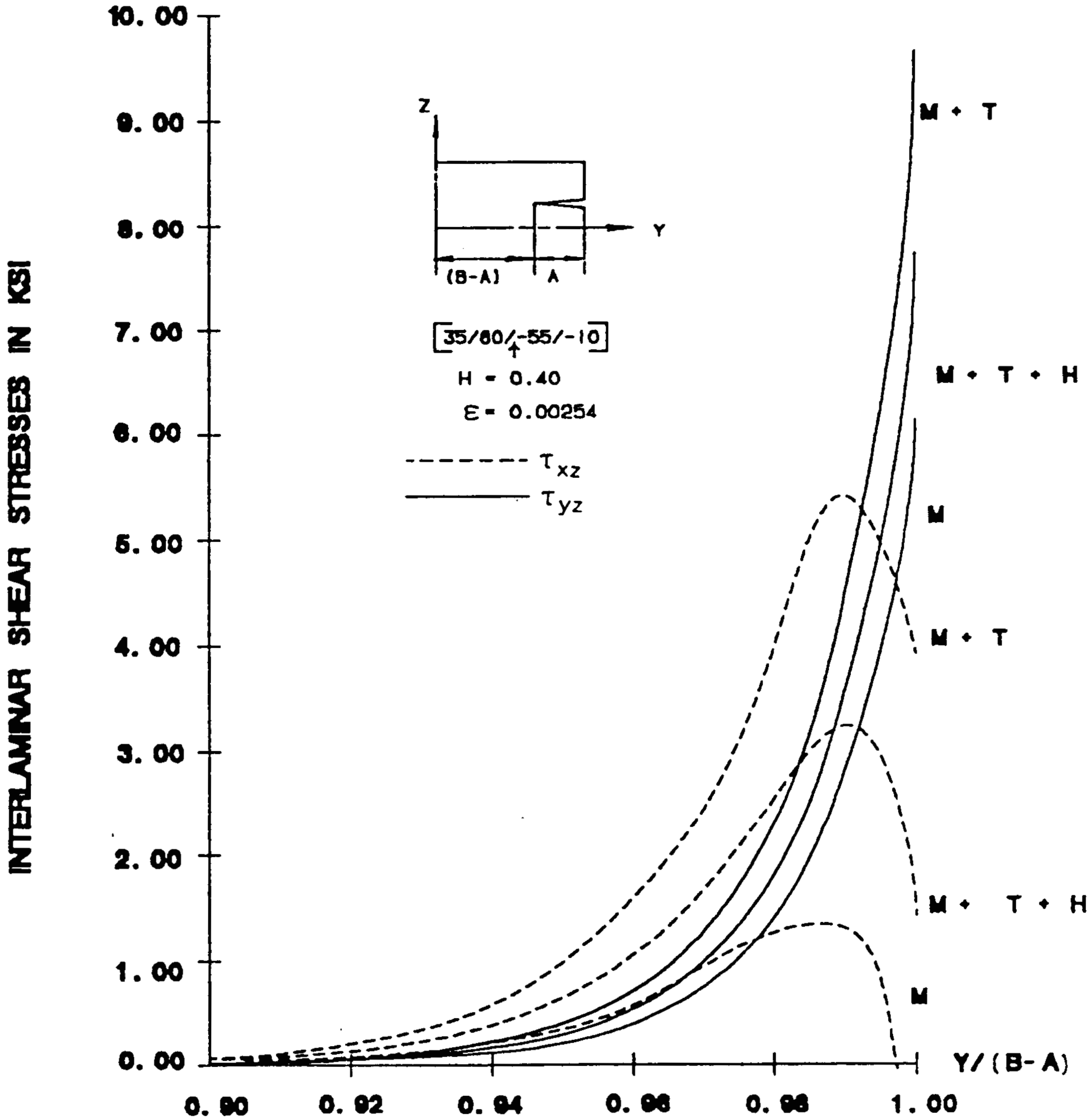

FIGURE 11 - COMPARISON OF INTERLAMINAR STRESSES

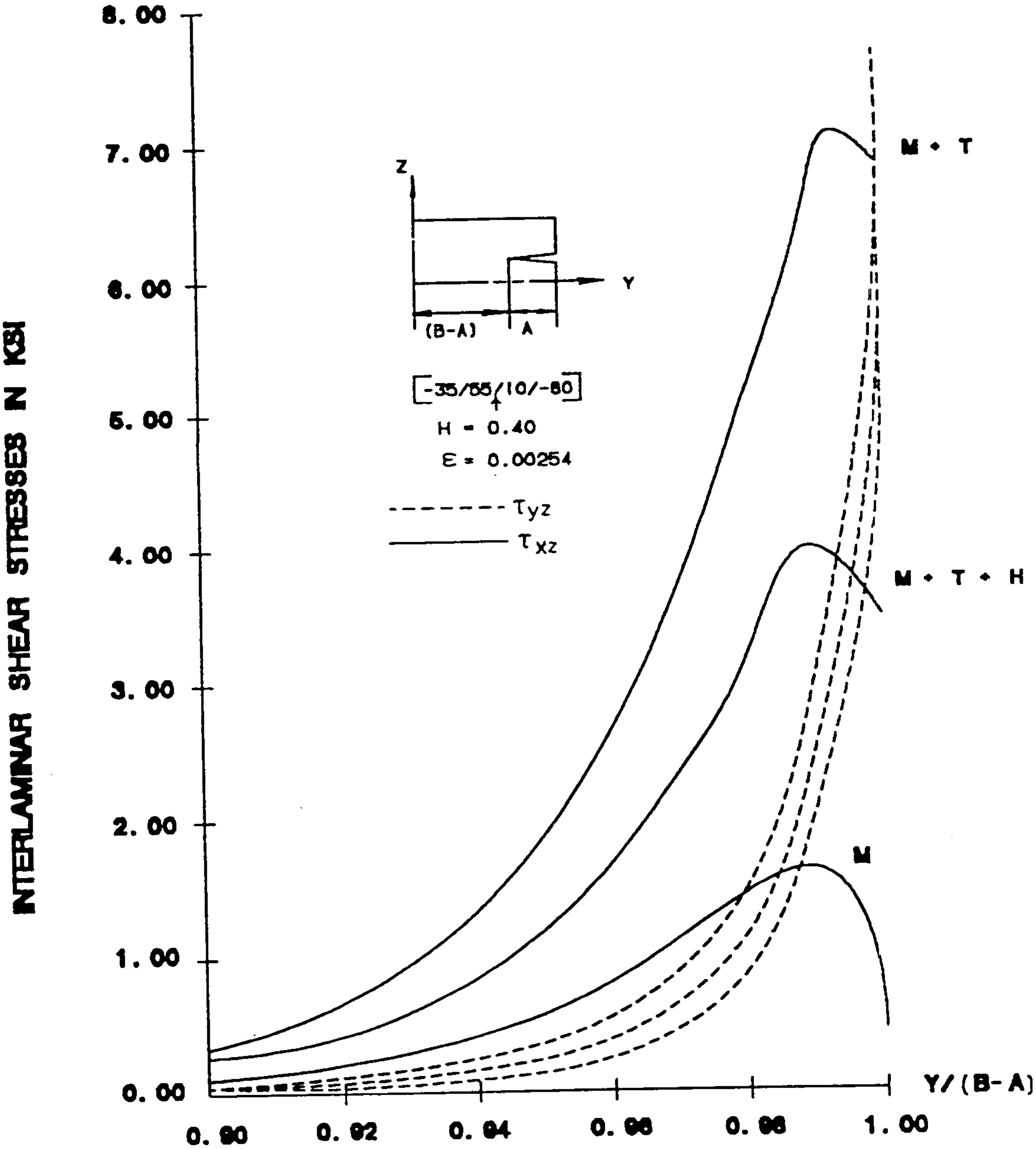

FIGURE 12 - COMPARISON OF INTERLAMINAR STRESSES

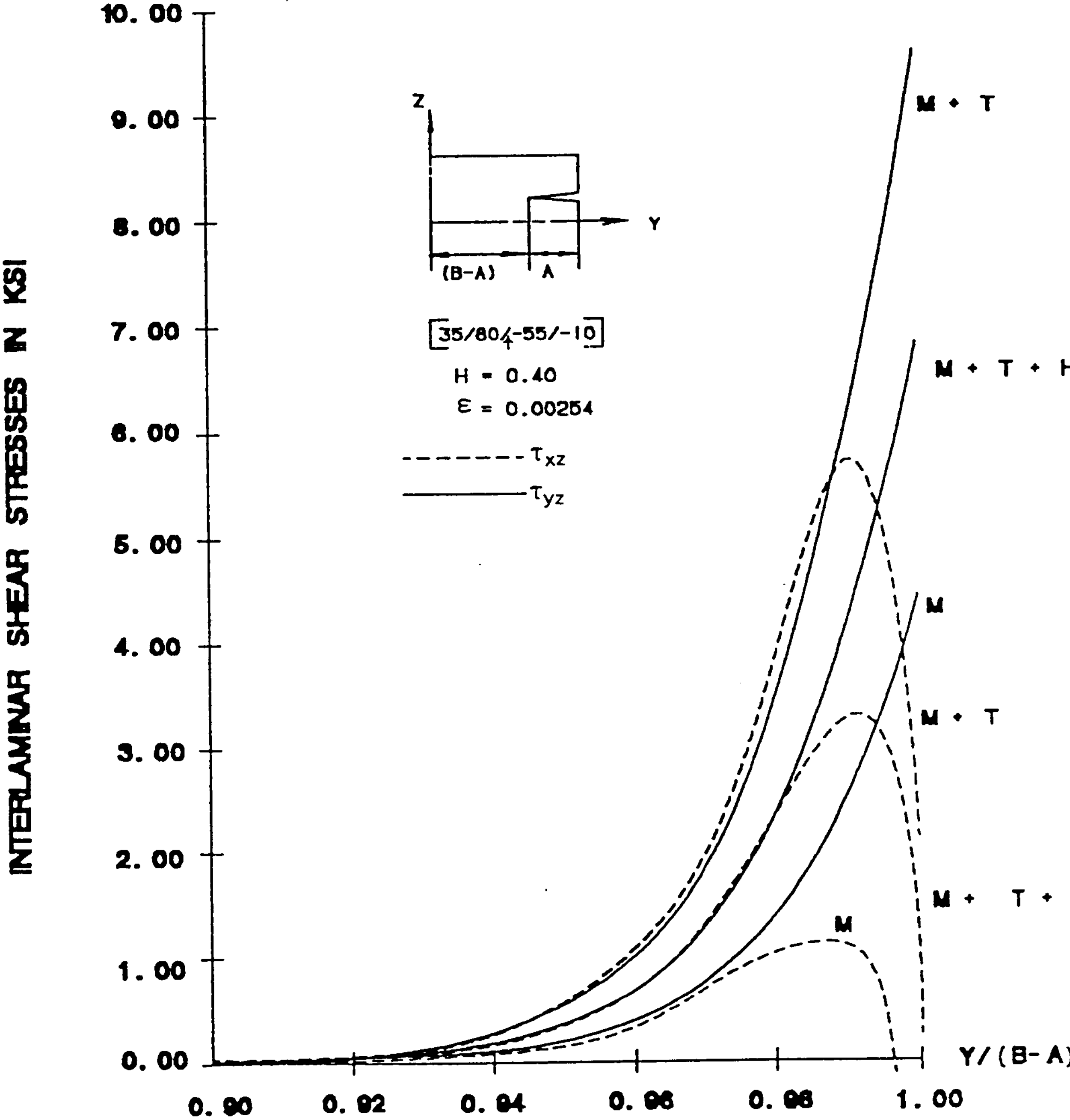

FIGURE 13 - COMPARISON OF INTERLAMINAR STRESSE

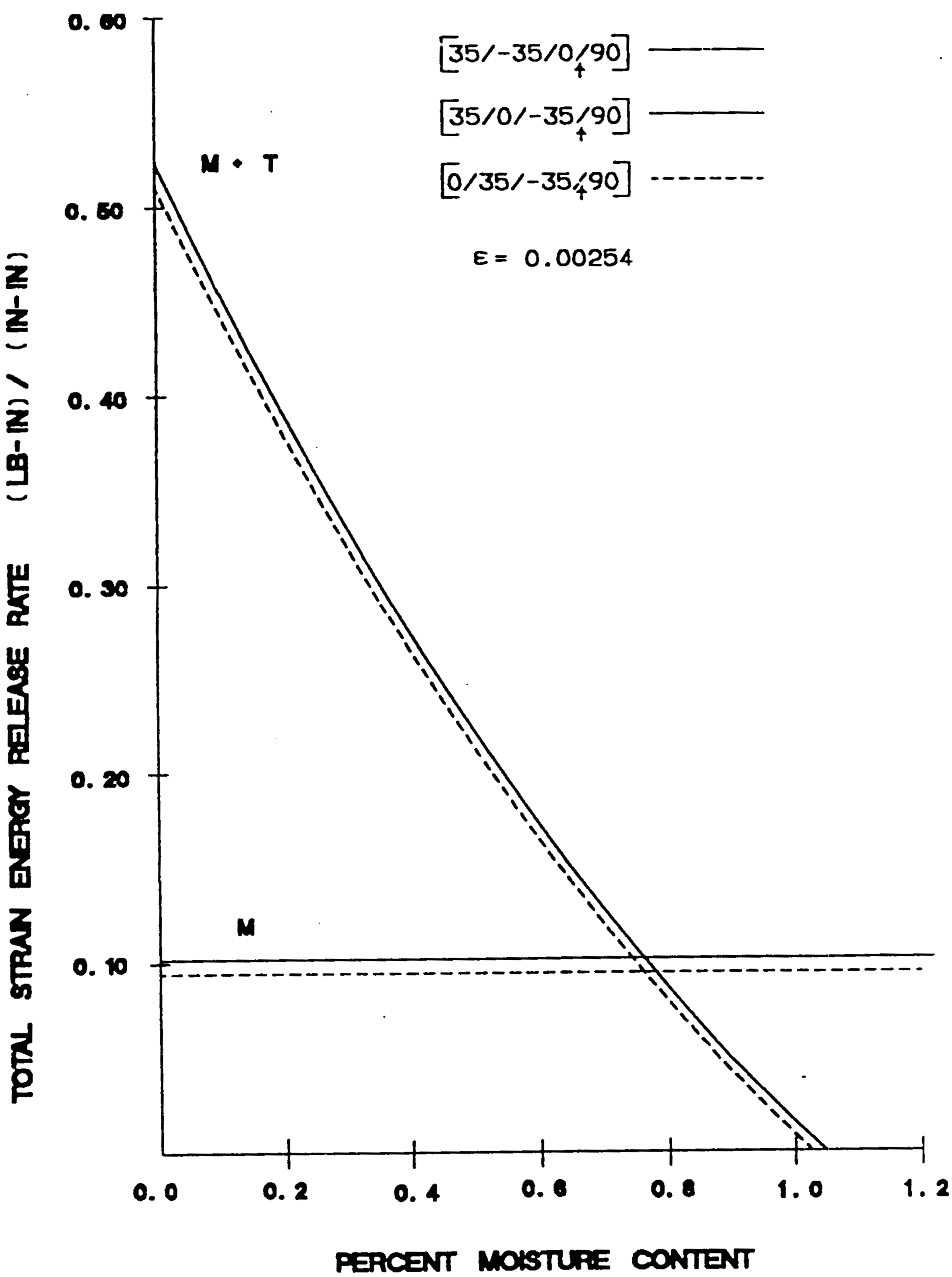

FIGURE 14 - ENERGY RELEASE RATE DISTRIBUTION FOR LAMINATES WITHOUT MODE III

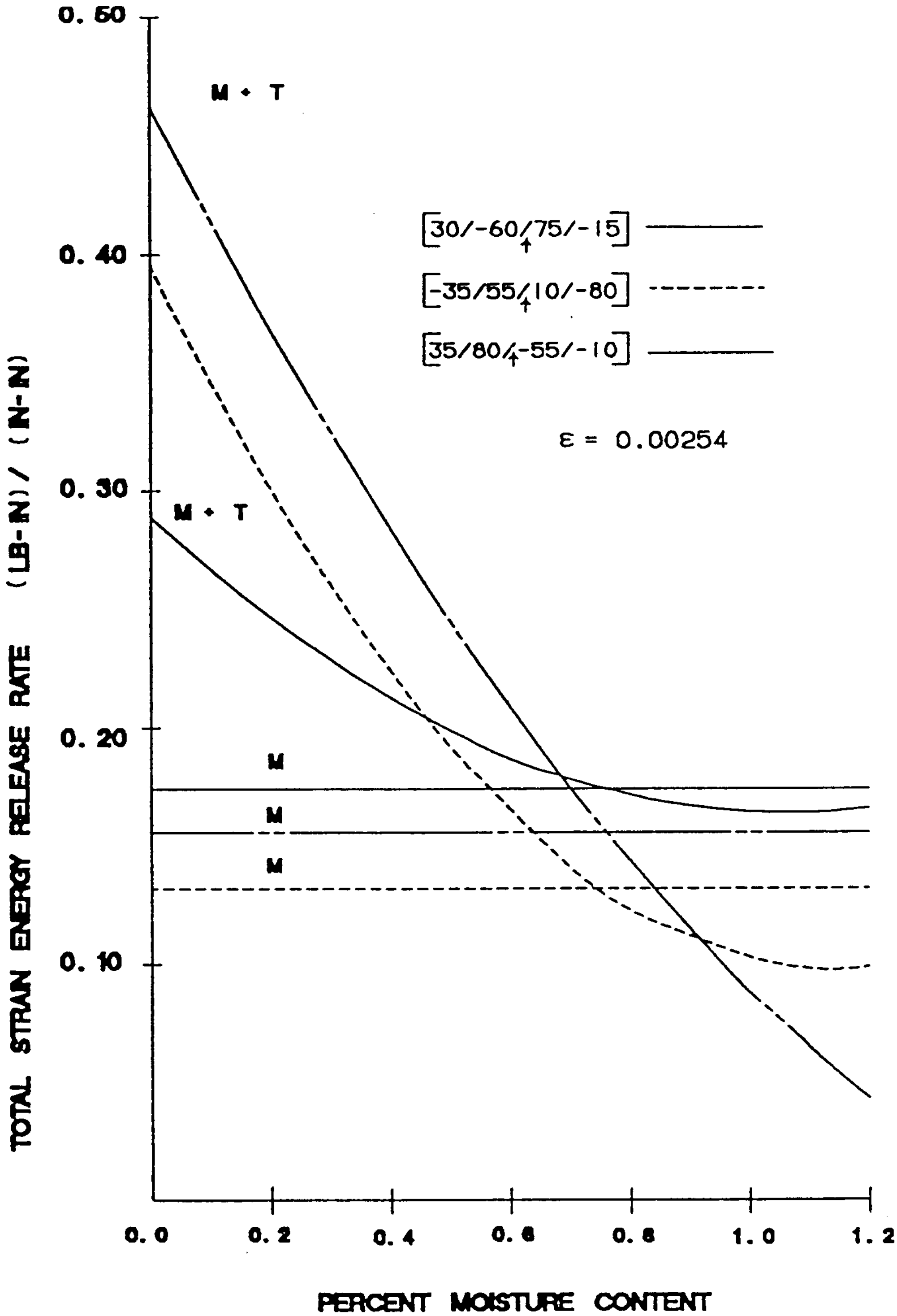

FIGURE 15 - ENERGY RELEASE RATE DISTRIBUTION FOR LAMINATES WITH MODE III

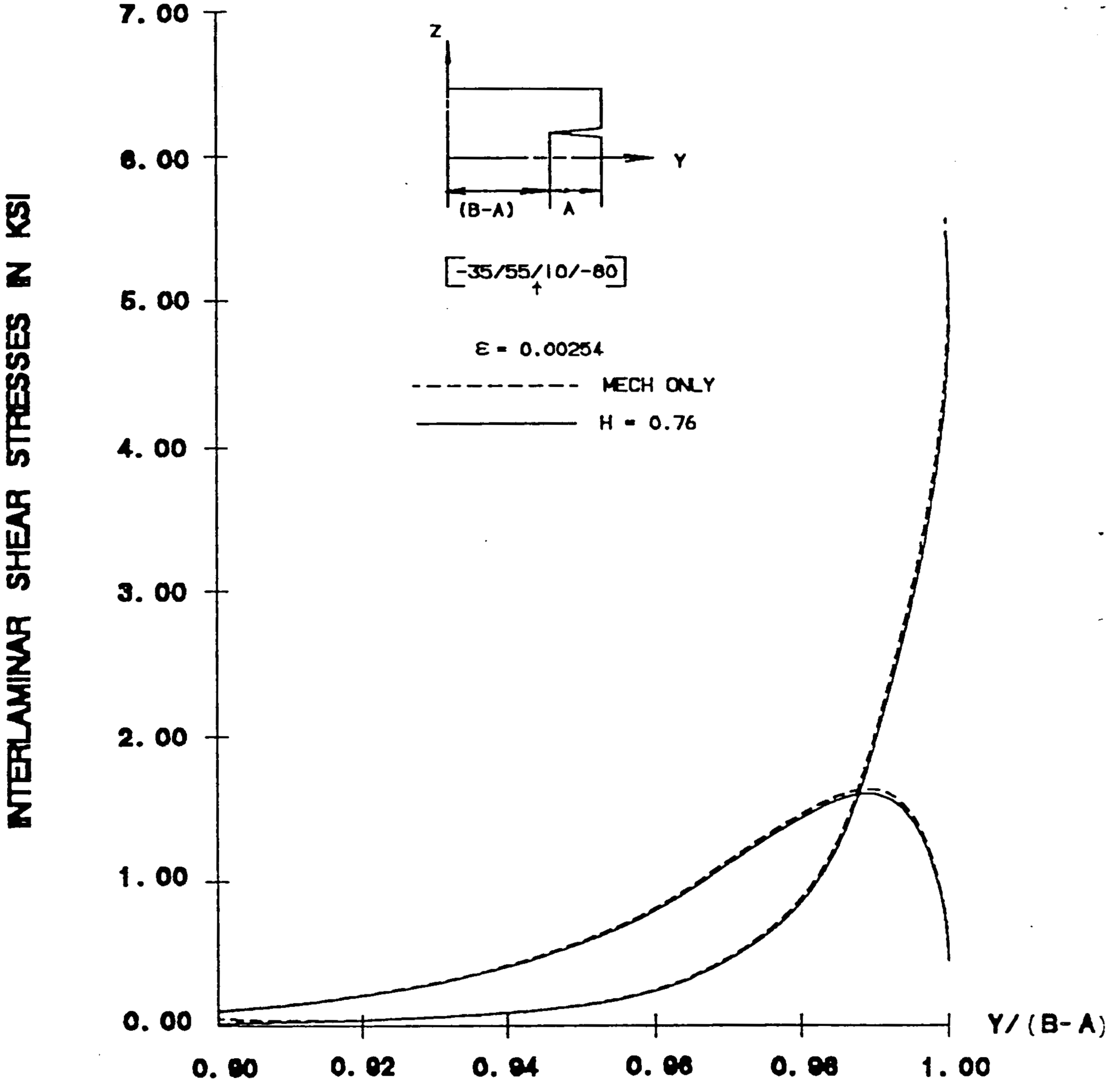

FIGURE 16 - TOTAL ALLEVIATED STATE STRESS DISTRIBUTION

APPENDIX I

Derivation of the Governing Equations

In this Appendix the governing equations for the sublaminate shown in Figure 7 are derived using the principle of virtual work.

Consider a sublaminate of thickness h. The origin of a cartesian coordinate system is located within the central plane (x-y) with the z-axis being normal to this plane. The material of each ply is assumed to possess a plane of elastic symmetry parallel to xy as shown in Figure 6.

Stress and moment resultants are given below.

$$(N_x, N_y, N_{xy}, Q_x, Q_y) = \int_{h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}, \tau_{xz}, \tau_{yz})\, dz$$

$$(M_x, M_y, M_{xy}) = \int_{h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) z dz \tag{I-1}$$

Because of the existence of a plane of elastic symmetry, the constitutive relations are given by

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & & & \\ C_{12} & C_{22} & \text{SYM} & \\ C_{13} & C_{23} & C_{33} & \\ C_{16} & C_{26} & C_{36} & C_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \end{bmatrix}_M$$

$$\begin{bmatrix} \tau_{yz} \\ \tau_{xz} \end{bmatrix} = \begin{bmatrix} C_{44} & \text{SYM} \\ C_{45} & C_{55} \end{bmatrix} \begin{bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{bmatrix}_M \tag{I-2}$$

where C_{ij} are components of the anisotropic stiffness matrix and γ_{xy}, γ_{yz} and γ_{xz} are engineering shear strains.

The displacements are assumed to be of the form

$$\begin{aligned} u &= U(x,y) + z\beta_x(x,y) \\ v &= V(y) + z\beta_y(x,y) \\ w &= W(x,y) \end{aligned} \tag{I-3}$$

where u,v and w are the displacement components in the x, y and z directions, respectively. Equation (I-3) in conjunction with the strain-displacement relations of classical theory of elasticity leads to the following kinematic relations

$$
\begin{aligned}
\epsilon_{xx} &= U_{,x} + z\beta_{x,x} \\
\epsilon_{yy} &= V_{,y} + z\beta_{y,y} \\
\epsilon_{zz} &= 0 \\
\gamma_{xy} &= U_{,y} + V_{,x} + z(\beta_{x,y} + \beta_{y,x}) \\
\gamma_{xz} &= \beta_x + W_{,x} \\
\gamma_{yz} &= \beta_y + W_{,y}
\end{aligned}
\qquad \text{(I-4)}
$$

Substitute Equation (I-4) into Equation (I-2) and put the results into Equation (I-1). This yields the following constitutive relations:

$$
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{12} & B_{26} & D_{11} & D_{12} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} U_{,x} \\ V_{,y} \\ U_{,y}+V_{,x} \\ \beta_{x,x} \\ \beta_{y,y} \\ \beta_{x,y}+\beta_{y,x} \end{bmatrix} - \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix}^{NM}
$$

$$
\begin{bmatrix} Q_y \\ Q_x \end{bmatrix} = \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{bmatrix} \beta_y + W_{,y} \\ \beta_x + W_{,x} \end{bmatrix}
$$

where

$$
(A_{ij},\ B_{ij},\ D_{ij}) = \int_{h/2}^{h/2} C_{ij}\ (1,\ z,\ z^2)dz \qquad \text{(I-5)}
$$

and the non-mechanical terms are defined in Appendix II.

The variation of the strain energy due to virtual displacements δu, δv and δw is

$$\delta\bar{V} = \int_{V} (\sigma_x\ \delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \sigma_z\delta\varepsilon_z + \tau_{xy}\ \delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz})dV \qquad (I\text{-}6)$$

where $\delta\varepsilon_x$, $\delta\varepsilon_y$, $\delta\varepsilon_z$. $\delta\gamma_{xy}$, $\delta\gamma_{xz}$ are the strains associated with the virtual displacements. Using Equations (I-3) and (I-1) then integrating through the thickness gives

$$\delta\bar{V} = \int_{A} [N_x\ \delta U,_x + N_y\delta V,_y + N_{xy}\ \delta U,_y + Q_x\ \delta\beta_x + Q_y\ (\delta\beta_y + \delta W,_y) + M_x\delta\beta_{x,x} + M_y\ \delta\beta_{y,y} + M_{xy}\ \delta\beta_{x,y}]\ dA \qquad (I\text{-}7)$$

The variation of the work done by the external forces and by the surface tractions is

$$\delta\bar{W} = \int_{A} (n_x\ \delta U + n_y\ \delta V + q\delta W + m_x\delta\beta_x + m_y\ \delta\beta_y)\ dA + \int_{S} (\bar{N}_n\ \delta\bar{U}_n + \bar{N}_{ns}\ \delta\bar{U}_s + \bar{M}_n\ \delta\bar{\beta}_n + \bar{M}_{ns}\delta\bar{\beta}_s)ds \qquad (I\text{-}8)$$

where a bar denotes values on the boundary. Variables n and s are coordinates normal and tangential to the edge, and

$$n_x = t_{2x} - t_{1x}$$
$$n_y = t_{2y} - t_{1y}$$
$$q = p_2 - p_1$$
$$m_x = \frac{h}{2}\ (t_{2x} + t_{1x})$$
$$m_y = \frac{h}{2}\ (t_{2y} + t_{1y}) \qquad (I\text{-}9)$$

where n_x and n_y can be regarded as effective distributed axial forces. Terms m_x and m_y are effective distributed moments and q is an effective lateral pressure.

From the principle of virtual work the equations of equilibrium and boundary conditions are determined from the Euler equations and boundary conditions of the variational equation.

$$\delta\bar{V} = \bar{\delta}W \qquad (I\text{-}10)$$

Substitution of Equations (I-7) and (I-8) into Equation (I-10) leads the following equations of equilibrium:

$$N_{x,x} + N_{xy,y} + n_x = 0$$

$$N_{xy,x} + N_{y,y} + n_y = 0$$

$$Q_{x,x} + Q_{y,y} + q = 0$$

$$M_{x,x} + M_{xy,y} - Q_x + m_x = 0$$

$$M_{xy,x} + M_{y,y} - Q_y + m_y = 0 \qquad \text{(I-11)}$$

and one member of the following five products must be prescribed on the sublaminate edges

$$N_n\, U_n,\ N_{ns} U_s,\ M_n \beta_n,\ M_{ns}\, \beta_s \text{ and } Q_n\, W \qquad \text{(I-12)}$$

For the ED specimen under uniform extension, U(x,y) in Equation (I-3) is given by

$$U(x,y) = U^*(y) + x\epsilon \qquad \text{(I-13)}$$

and the response is a function of y and z coordinates only. For this case the equilibrium equations (I-11) take the form

$$N_{xy,y} + n_x = 0$$

$$N_{y,y} + n_y = 0$$

$$Q_{y,y} + q = 0$$

$$M_{xy,y} - Q_x + m_x = 0$$

$$M_{y,y} - Q_y + m_y = 0 \qquad \text{(I-14)}$$

Substitution of the constitutive relations in Equation (I-5) into Equation (I-14) yields the following equilibrium equations in terms of kinematic variables.

$$\begin{bmatrix} A_{66}L_{yy} & & & & \\ A_{26}L_{yy} & A_{22}L_{yy} & & \text{SYM} & \\ 0 & 0 & -A_{44}L_{yy} & & \\ B_{66}L_{yy} & B_{26}L_{yy} & -A_{45}L_{yy} & (D_{66}L_{yy} - A_{55}) & \\ B_{26}L_{yy} & B_{22}L_{yy} & -A_{44}L_{y} & (D_{26}L_{yy} - A_{45} & (D_{22}L_{yy} - A_{44}) \end{bmatrix} \begin{bmatrix} U^* \\ V \\ W \\ \beta_x \\ \beta_y \end{bmatrix} = - \begin{bmatrix} n_x \\ n_y \\ -q \\ m_x \\ m_y \end{bmatrix} \qquad \text{(I-15)}$$

where the operators

$$L_{yy} = d^2/dy^2$$

$$L_y = d/dy$$

From these governing equations the basis of the work presented in this paper is formed. Appendix II gives a detailed formulation of the hygro thermal terms and the formulation of the total strain energy release rate and interlaminar stresses.

APPENDIX II

Hygrothermal Effects on Edge Delamination

The displacement field and constitutive relations governing the free edge ply separation were presented in Appendix I. The hygrothermal expressions, represented with the superscript NM for non-mechanical, are defined as follows

$$(N_i^{NM}, M_i^{NM}) = \int_{h/2}^{h/2} (1,z)\, \bar{Q}_{ij} \left\{ \bar{\alpha}_j\ (T-T_r) + \bar{\beta}_j\ C \right\} dz \qquad \text{(II-1)}$$

where

$\bar{\alpha}_j$ - Coefficient of thermal expansion

$\bar{\beta}_j$ - Swelling coefficient

T - Local temperature

T_r - Reference temperature

C - Specific moisture concentration

$\bar{Q}_{ij}$ - Reduced stiffness coefficient

The terms $\bar{\alpha}_j$ and $\bar{\beta}_j$ are transformed as second order tensors with the assumption of no thermal or swelling shear strain.

The concept of sublaminates is used when enforcing the boundary conditions.

Cracked Sublaminates

Sublaminate 2:

The boundary conditions for this sublaminate are expressed as:

$$N_{y2} = N_{xy2} = M_{y2} = Q_{y2} = 0 \qquad \text{(II-2)}$$

$$M_{xy2,y} - Q_{x2} = 0$$

Using the first three conditions in the governing equations, one can express V_2, U_2 and β_{2y} in terms of β_{2x} to obtain:

$$\begin{bmatrix} A^1_{12} & B^1_{26} \\ A^1_{16} & B^1_{66} \\ B^1_{12} & D^1_{26} \end{bmatrix} \begin{Bmatrix} \epsilon \\ \beta_{2x,y} \end{Bmatrix} + \begin{bmatrix} A^1_{22} & A^1_{26} & B^1_{22} \\ A^1_{26} & A^1_{66} & A^1_{26} \\ B^1_{22} & B^1_{26} & D^1_{22} \end{bmatrix} \begin{Bmatrix} V_{2,y} \\ V_{2,y} \\ \beta_{2y,y} \end{Bmatrix} - \begin{Bmatrix} N_{y1} \\ N_{xy1} \\ M_{Y1} \end{Bmatrix}^{NM} = 0 \qquad \text{(II-3)}$$

Sublaminate 3:

The boundary conditions for this sublaminate are given as:

$N_{y3} = N_{xy3} = 0$

$M_{y3,y} - Q_{y3} = 0$

$M_{xy3,y} - Q_{x3} = 0$ (II-4)

These are used in a similar manner (as in sublaminate 2) to obtain

$$\begin{bmatrix} A^o_{22} \\ A^o_{26} \end{bmatrix} \begin{Bmatrix} V_{3,Y} \\ U_{3,Y} \end{Bmatrix} + \begin{bmatrix} A^o_{12} & B^o_{22} & B^o_{26} \\ A^o_{16} & B^o_{26} & B^o_{66} \end{bmatrix} \begin{Bmatrix} \epsilon \\ \beta_{3,y} \\ \beta_{3x,Y} \end{Bmatrix} \begin{Bmatrix} N_{yo} \\ N_{xyo} \end{Bmatrix} = 0 \qquad \text{(II-5)}$$

These equations are then substituted back into the governing equations to obtain expressions for the force and moment resultants. They can be expressed in terms of the strain plus non-mechanical effects.

UNCRACKED SUBLAMINATES

Sublaminates 0 and 1:

The boundary conditions of continuity at the interfaces must be satisfied.

$N_{y1}(0) = N_{xy1}(0) = N_{yo}(0) = N_{xyo}(0) = 0$ (II-6)

$M_{xy1}(0) = M_{xy2}(0)$

$M_{yo}(0) = M_{y3}(0)$

$$M_{xy0}(0) = M_{xy3}(0) \qquad \text{(II-7)}$$

and

$$\begin{aligned}
M_{xy1}\ (0) &= M_{xy2}\ (0) \\
\beta_{1x}\ (0) &= \beta_{2x}(0) \\
M_{y0}(0) &= M_{y3}(0) \\
\beta_{0y}(0) &= \beta_{3y}(0) \\
M_{xy0}(0) &= M_{xy3}(0) \\
\beta_{0x}(0) &= \beta_{3x}(0)
\end{aligned} \qquad \text{(II-8)}$$

Enforcing equations (II-6) and (II-7) in the governing equations yields the following:

$$\begin{bmatrix} A^1_{12} & A^1_{22} & A^1_{26} \\ A^1_{16} & A^1_{26} & A^1_{66} \\ A^o_{12} & A^o_{22} & A^o_{26} \\ A^o_{16} & A^o_{26} & A^o_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} - \begin{Bmatrix} N_{y1} \\ N_{xy1} \\ N_{yo} \\ N_{xyo} \end{Bmatrix}^{NM} + \begin{Bmatrix} -N_{y1j} \\ -N_{xy1j} \\ N_{y1j} \\ N_{xy1j} \end{Bmatrix} s_j G_j = 0 \qquad \text{(II-9)}$$

$$\begin{bmatrix} B^1_{12} & B^1_{22} & B^1_{26} \\ -A_1 & B^1_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} - \begin{Bmatrix} M_{y1}^{NM} \\ A_1^{NM} \end{Bmatrix} - \begin{Bmatrix} M_{y1j} s_j \\ M_{xy1j} s_j + B_1 s_c \alpha_j \end{Bmatrix} \qquad \text{(II-10)}$$

$(j = 1 - 4)$

The expressions in (II-9) and (II-10) are defined below

$$\begin{bmatrix}\epsilon_y \\ \gamma_{xy}\end{bmatrix} = \begin{bmatrix}C_v \\ C_u\end{bmatrix}\epsilon + \begin{bmatrix}C_v \\ C_u\end{bmatrix}^{NM} \qquad \text{(II-11)}$$

$$\Delta = \overset{*}{A}_{22}\ \overset{*}{A}_{66} - (\overset{*}{A}_{26})^2 \qquad \text{(II-12)}$$

$$\begin{bmatrix}C_v \\ C_u\end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix}\overset{*}{A}_{26}\ \overset{*}{A}_{16} - \overset{*}{A}_{66}\ \overset{*}{A}_{12} \\ \overset{*}{A}_{26}\ \overset{*}{A}_{12} - \overset{*}{A}_{22}\ \overset{*}{A}_{16}\end{bmatrix} \qquad \text{(II-13)}$$

$$\begin{bmatrix}C_v \\ C_u\end{bmatrix}^{NM} = \frac{1}{\Delta}\begin{bmatrix}\overset{*}{A}_{66}\ (N_{y1} + N_{yo})^{NM} - \overset{*}{A}_{26}\ (N_{xy1} + N_{xyo})^{NM} \\ -\overset{*}{A}_{26}\ (N_{y1} + N_{yo})^{NM} + \overset{*}{A}_{22}\ (N_{xy1} + N_{xyo})^{NM}\end{bmatrix} \qquad \text{(II-14)}$$

A_1 and A_1^{NM} are functions of A_{ij}, B_{ij} and D_{ij}. The superscripts * implies a summation of the upper and lower sublaminates.

Continuing with the derivation one can substitute the expressions set forth into equation (I-7) as well as 10 and 11 in the report. This gives the following expression for the total energy release rate.

$$G = \frac{1}{2}\ \frac{d}{da}\int_0^b \left[\int_{h/2}^{h/2} N_x\ \epsilon_m\ dz\right] DY \qquad \text{(II-15)}$$

The concept of free-expansion in the x-direction is implemented to find the strain induced by the non-mechanical effects on the structure. Setting $N_x = 0$ for each ply in Equation (I-5) and using the boundary conditions of (II-2), (II-4) and (II-6) allows the following.

$$\epsilon_u^k = -T_u^k / E_u^k$$

$$\epsilon_c^k = -T_c^k / E_c^k \qquad \text{(II-16)}$$

where

$$T_u^k = h^k\, C_v^{NM}\, \bar{Q}_{12}^k + h^k\, C_u^{NM}\, \bar{Q}_{16}^k - (N_x^{NM})^k$$

$$E_u^k = h^k\, (\bar{Q}_{11}^k + C_v\, \bar{Q}_{12}^k + C_u\, \bar{Q}_{16}^k)$$

$$T_c^k = \bar{Q}_{12}^k\, F_1^{NM}\, h^k + \bar{Q}_{16}^k\, F_2^{NM}\, h^k + B_{12}^k\, F_3^{NM} - (N_x^{NM})^k$$

$$E_c^k = \bar{Q}_{11}^k\, h^k + \bar{Q}_{12}^k\, h^k\, Cd_{11} + \bar{Q}_{16}^k\, h^k\, Cd_{12}^k + B_{12}^k\, Cd_{31} \qquad \text{(II-17)}$$

Superscript k represents the ply. Expressions Cd_{ij} and F_i^{NM} are found by substituting the conditions of (II-2) into (II-3).

$$\begin{bmatrix} Cd \end{bmatrix}_{3x2} = - \begin{bmatrix} A_{22}^1 & A_{26}^1 & B_{22}^1 \\ A_{26}^1 & A_{66}^1 & B_{26}^1 \\ B_{22}^1 & B_{26}^1 & D_{22}^1 \end{bmatrix}^{-1} \begin{bmatrix} A_{12}^1 & B_{26}^1 \\ A_{16}^1 & B_{66}^1 \\ B_{12}^1 & D_{26} \end{bmatrix} \qquad \text{(II-18)}$$

$$\left\{ F^{NM} \right\}_{3x1} = \begin{bmatrix} A_{22}^1 & A_{26}^1 & B_{22}^1 \\ A_{26}^1 & A_{66}^1 & D_{22}^1 \\ B_{22}^1 & B_{26}^1 & D_{22}^1 \end{bmatrix}^{-1} \begin{Bmatrix} N_y \\ N_{xy1} \\ M_{y1} \end{Bmatrix}^{NM} \qquad \text{(II-19)}$$

Sublaminate 3 has $B_{ij} = 0$ due to symmetry of the structure. When considering these plies, the term F_1^{NM} and F_2^{NM} are found by substituting the boundary conditions (II-4) into (II-5).

This gives a second expression of F^{NM} for this sublaminate

$$\begin{Bmatrix} F^{NM} \end{Bmatrix}_{2x1} = \begin{bmatrix} A^o_{22} & A^o_{26} \\ A^o_{26} & A^o_{66} \end{bmatrix}^{-1} \begin{Bmatrix} N_{y0} \\ N_{xy0} \end{Bmatrix}^{NM} \qquad \text{(II-20)}$$

To find the total strain associated with the non-mechanical effects, it is necessary to sum the force over the entire structure and set it to zero. These are used in order to obtain Equation (16), (17) and (18) in the report on page 17. Substituting this in Equation (II-15) gives the total strain energy release rate expression per unit length

$$G = \frac{1}{2} \sum_{K} - (E_c^k \epsilon^T + T_c^k)(\epsilon^T - \epsilon_c^k) + (E_u^k \epsilon^T + T_u^k)(\epsilon^T - \epsilon_u^k)$$

The expression $\epsilon^T - \epsilon^k_{c,u}$ is in essence the total mechanical strain of that ply.

INTERLAMINAR STRESSES

The interlaminar stresses of the structure are defined in Equations (8) and (9)

$$t_x = N^1_{xy,y} - N_{xy1j}\, s_j^2\, G_j\, e^{-s_j y}$$

$$t_y = N^1_{y,y} - N_{y1j}\, s_j^2\, G_j\, e^{-s_j y} \qquad (j = 1\text{-}4) \qquad \text{(II-21)}$$

While s_j, the positive roots reulsting from the ploynominal

$$E_8 s^8 + E_6 s^6 + E_4 s^4 + E_2 s^2 + E_o = 0, \qquad \text{(II-22)}$$

are independent of the hygrothermal effects, the rest of the terms are not.

Solving Equations (II-9) and (II-10) gives the term G_j while N_{xy1j} and N_{y1j} are found from

$$\begin{Bmatrix} N_{yj} \\ N_{xyj} \end{Bmatrix} = \begin{bmatrix} A^1_{22} & A^1_{26} & B^1_{22} & B^1_{26} \\ A^1_{26} & A^1_{66} & B^1_{26} & B^1_{66} \end{bmatrix} \begin{Bmatrix} v_j \\ u_j \\ \alpha_j \end{Bmatrix} \qquad (j = 1,2,3,4) \qquad \text{(II-23)}$$

where v_j, U_j and α_j are found by imposing the boundary conditions on the mode shapes. They are dependent on the four values of s_j as well as the sublaminate stiffness matrices.

APPENDIX III

```
      PROGRAM START (INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT)

     THIS  PROGRAM IS FOR THE FINAL PAPER   8-16-87
   DIMENSION STATEMENTS

      REAL BG(4),E(9),GG(4,4), MATR32(3,2), MATR33(3,3), MATR3(3),
     C STRAIN(25), SAVE3(3), SAVE33(3,3), WKAREA(99), ZR
      COMPLEX SJ(8)
      DOUBLE PRECISION BNEG, A, C, DIFF1, DIFF2, UNSY(2), UNSX(2),
     C  SSSS, SSSC, SSCC, SCCC, ZZZO(0:50),J22,J26,FNM(3),SINM,S2NM,
     C  MEMSY, MEMSX, F11M, F22M, SS1, SS2, SSY, SSX, ZZZ1(0:50),
     C  THICK(40),THETA(50), E1(50), E2(50), CCCC, HSS(5),HSN1,HSN2,
     C  Q(6,6,50), ZO(0:40), AO(6,6), A1(6,6), U13, U14, BG1, BG2,
     C  NXO(4), E15,E16,E17, E18, E19, ZTT(0:40),
     C  NY1(4), X(2), Y(3), CV, CU, W(2), ZZ(3), VV11, VV12
      DOUBLE PRECISION  ALPHA(4), PHI(4), GAMMA(4), NX1(4),
     C  B1(6,6), BO(6,6), DO(6,6), D1(6,6), F(4,4), VV13, VV14, J66,
     C  RDLT, RTA1, RTA2, RSB1, RSB2, U12, U11, A1NM,
     C  NXY1(4), MY1(4), MXY1(4), WD(2,3), CD(3,2), WIDTH,
     C  V12(50), V21(50),  SS, CC, K66, K26, K22,
     C  Z1(0:40), FX, FY, GI(2,35), K16,K12, H66
      DOUBLE PRECISION  SV(5), SU(5), AL, SC, S(8), DY,
     C  G12(50), G31(50), C2, C1, THETV, THETU, GIII(2,35), CS,
     C  DEL, HO, H1, H22, HE, HG, HNY(50), HNXY(50), HM3,CVNM,
     C  C11, C12, C22, C26, C44, D, C55, C66, H26, SQ, DUM,CUNM,
     C  CONY, CONXY, SMNY, SMNXY,
     C  SB1,SB2,TA1,TA2,ATHM1(40),ATHM2(40),ATHM6(40),BSW2(40),
     C  NMNYO,NMNXYO,GII(2,35),
     C  DVV11, DVV12, DU13, DU14, DF(4,4), DX, ATH,CCONY,CCONXY,
     C  BSW6(40), DELTEMP, BSW1(40), CMOIST(35),
     C  SIGX(0:40,79:120),
     C  NMNY1,NMNXY1,NMMX1,NMMY1,SIGY(0:40,79:120)
       DOUBLE PRECISION   NMSTO(50), NMST2(50),NMST3(50), TNC,UNCL,
     C  T1(50), T12(50), T13(50),EX(50),EXX(50),EX3(50), JY,
     C  EXNC,ESTAR, TSTAR, TNMST, GLC(0:50), NXNM(50),B12(50)
      DATA  Q/1800*0.0/,   ZR/0.00/
*******************************************************************************
*******************************************************************************

      DATE OF PROGRAM : SEPT. 1, 1987

     THIS IS THE FINAL PROGRAM FOR PREDICTING THE ENERGY RELEASE RATE OF
      COMPOSITE LAMINATES INCLUDING HYGROTHERMAL EFFECTS ..........
      HOWEVER, IT ONLY CONSIDERS EXTENSION EFFECTS OF STRAIN WHEN DEALING
      WITH HYGRALTHERMAL EFFECTS.... LIKE WHITTNEY'S PAPER.....
      EXCEPT ON A PLY BY PLY ANALYSIS BASIS OF THE HYGRALTHERMAL EFFECTS


       THE INPUT ALLOWS FOR: THE LAMINATE LAY-UP TO CHANGE AND POSITION
                 OF THE CRACK, DIFFERENT STRAIN VALUES TO BE EVALUATED,
                 (UP TO 40 LAMINATES AND 25 DIFFERENT STRAIN VALUES)
                 AND FOR THE EVALUATION OF ONE MOISTURE CONSTANT OR A
                 RANGE OF THE MOISTURE CONSTANT FROM 0 TO 1.2.

      THIS PROGRAM IS FOR THE GIVEN DATA TO BE IN ENGLISH UNITS.

       ALL LAMINATES ARE EVALUATED WITHOUT THRMAL EFFECTS AUTOMATICALLY

*******************************************************************************
*******************************************************************************
         LZQ IS THE NUMBER OF DIFFERENT LAMINATE (OR CRACK POSITIONS)
           TO BE EVALUATED.

       READ(5,*)   LZQ
        DO 400     LZZ = 1,LZQ
```

```
      READ(5,*)  WIDTH, NPLYO, NPLY1, AL
        NEXTPL = NPLYO + 1.
        TPLY = NPLYO + NPLY1

 FOR EACH PLY IN THE SUBLAMINATE, THE MATERIAL CHARACTERISTICS
 MUST BE READ IN.

          PI = 4. * ATAN(1.)
          HO = 0.0
          ATH = 0.0
          H1=0.0
          DO 3  LK = 1, TPLY
          ZO(LK)  = 0.0
 3        Z1(LK)  = 0.0


       DO 5   I = 1,NPLYO
          READ(5,*) THICK(I), THETA(I), E1(I), E2(I)
          READ(5,*) V12(I)
          READ(5,*) G12(I), G31(I)
          THETA(I) = THETA(I) * PI / 180.
5      HO = HO + THICK(I)

       DO 10  I = NEXTPL, TPLY
          READ(5,*) THICK(I), THETA(I), E1(I), E2(I)
          READ(5,*) V12(I)
          READ(5,*) G12(I), G31(I)
          THETA(I) = THETA(I) * PI / 180.
10     H1 = H1 + THICK(I)

********************************************************************
********************************************************************
   THESE ARE WRITE STATEMENTS TO CHECK THE INITIAL CONDITIONS OF THE
                 SUBLAMINATE  AND VALUES READ IN
********************************************************************
********************************************************************

    EACH PLY MAY HAVE DIFFERNT PROPERTIES SO THE PROPERTY OF EACH

      WRITE(6,289)
CC
      WRITE(6,201)  WIDTH
      WRITE(6,202) NPLY1, NPLYO

          WRITE(6,204)
            DO 15 J = 1, TPLY
            JJ = TPLY + 1 - J
            WRITE(6,206) J
            WRITE(6,207) THICK(JJ), THETA(JJ)*180/PI
            WRITE(6,208) E1(JJ)/1E+06 , E2(JJ)/1E+06
            WRITE(6,209) V12(JJ)
15          WRITE(6,214) G12(JJ)/1E+06, G31(JJ)/1E+06


 *******************************************************************
 *******************************************************************
      DETERMINE THE Z COMPONENT OF ALL LAMINATES

       CHECK = 0.00000001
       ZTT(0)  = 0.0
       ZO(0)  = -HO / 2.0
        DO 20 I = 1, NPLYO
          ZTT(I)  = THICK(I) + ZTT(I-1)
20        ZO(I)  = THICK(I) +  ZO(I-1)

       Z1(NPLYO)  = -H1 / 2.0
```

```
      DO 25 I = NEXTPL,TPLY
        ZTT(I) = THICK(I) + ZTT(I-1)
25      Z1(I) = THICK(I) +  Z1(I-1)

************************************************************************
FIRST READ IN THE NUMBER OF STRAINS TO BE EVALUATED AND THEIR VALUE
THEN READ IN IF THE MOISTURE CONTENT SHOULD VARY OVER 0 TO 1.2 OR
 BE A CONSTANT.

           NSTRA =  ..... NUMBER OF VARIOUS STRAIN VALUES
        IF MOISTV = 1 ... CMOIST VARIES OVER 0 TO 1.2
        IF MOISTV = 0 ... CMOIST IS A SPECIFIC VALUE
************************************************************************
       READ(5,*) NSTRA
        DO 27  J=1,NSTRA
27     READ(5,*) STRAIN(J)

         DO 400 LST = 1,NSTRA
          READ(5,*) RDLT,RSB1,RSB2,RTA1,RTA2
          WRITE(6,231) STRAIN(LST),RDLT, RSB1, RSB2, RTA1, RTA2

         READ(5,*) MOISTV
         IF (MOISTV.EQ.0) READ(5,*) CM
         IF (MOISTV.EQ.0)  MMC = 1
         IF (MOISTV.EQ.0) WRITE(6,232) CM
         IF (MOISTV.EQ.1)  MMC = 25
         IF (MOISTV.EQ.1)  WRITE(6,233)
************************************************************************
         DO 300  JM = 1,MMC +1

 FIND Q'S AS WELL AS Q-BAR ,  SAVING Q-BAR
 AND READ AND CALCULATE THE HYGRO THERMO EFFECTS

************************************************************************
      DO 200  IZZ = 1,2
        LIL = 0
        IF (IZZ.EQ.1) JMM = JM
        IF (IZZ.EQ.2) JMM = 0
        IF(JM.EQ.1. AND .IZZ.EQ.1) LIL = 1
    IF (JM.GT.1 .AND. IZZ.EQ.2) GO TO 200

          NMNY1 = ZR
          NMNXY1 = ZR
          NMMX1 = ZR
          NMMY1 = ZR

          NMNYO = ZR
          NMNXYO = ZR
          HM3 = ZR
          SMNY = ZR
          SMNXY = ZR

       DO 24 I=1,5
          E(I)  = ZR
          E(I+4) = ZR
24        HSS(I) = ZR
        DO 26 I=1,6
        DO 26 J= 1,6
           NXNM(I)= ZR
         DF(I,J) = ZR
         AO(I,J) = ZR
         BO(I,J) = ZR
         DO(I,J) = ZR
         A1(I,J) = ZR
         B1(I,J) = ZR
26       D1(I,J) = ZR
```

```
          DO 28  MM = 1,TPLY
28        IF( THICK(MM)  .GT. ATH ) ATH = THICK(MM)

    IZZ = 2 IS FOR LAMINATE WITHOUT ANY HYGROTHERMAL EFFECTS
    IZZ = 1 IS FOR HYGROTHERMAL EFFECTS CONSIDERED

        DO 30  I = 1, TPLY

  READ THE HYGROTHERMO EFFECTS, BOTTOM PLY IS FIRST AND UPWARD

          IF  (IZZ.EQ.2) GO TO 35
              IF  (MOISTV.EQ.0) CMOIST(JM) = CM
              IF  (MOISTV.EQ.1) CMOIST(JM) = 0.05 * (JM-1)
            DELTEMP = RDLT
             SB1 = RSB1
             SB2 = RSB2
             TA1 = RTA1
             TA2 = RTA2
             GO TO 40

35          DELTEMP = ZR
            CMOIST(JM) = ZR
             SB1 = ZR
             SB2 = ZR
             TA1 = ZR
             TA2 = ZR

40           V21(I) = V12(I) * E2(I) / E1(I)
             C11 = E1(I) / (1 - V12(I) * V21(I) )
             C12 = E2(I) * V12(I) / ( 1 - V12(I) * V21(I) )
             C22 = E2(I) / ( 1 - V12(I) * V21(I) )
             C44 = G31(I)
             C55 = G31(I)
             C66 = G12(I)

      SS = DSIN(THETA(I)) * DSIN(THETA(I))
      CC = 1 - SS
      CS = 0.5 * DSIN(2*THETA(I))


      SSSS = SS * SS
      SSSC = SS * CS
      SSCC = SS * CC
      SCCC = CC * CS
      CCCC = CC * CC

      Q(1,1,I) = C11 * CCCC + 2 * (C12 + 2 * C66 ) * SSCC
   C             + C22 * SSSS
      Q(1,2,I) = (C11 + C22 - 4 * C66) * SSCC + C12 * ( SSSS
   C           + CCCC )
      Q(2,2,I) = C11 *  SSSS + 2 * ( C12 + 2 * C66) * SSCC
   C         + C22 * CCCC
      Q(1,6,I) = (C11 - C12 - 2 * C66) * SCCC
   C               + (C12 - C22 + 2 * C66 ) * SSSC
      Q(2,6,I) = (C11 - C12 - 2 * C66 ) * SSSC
   C                 + (C12 - C22 + 2 * C66 ) * SCCC
      Q(6,6,I) = (C11 + C22  - 2 * C12 - 2 * C66) * SSCC
   C             + C66 * ( SSSS + CCCC )
      Q(4,4,I)= C44 * CC+ C55 * SS
      Q(5,5,I) = C44 * SS + C55 * CC
      Q(4,5,I) = CS *  (C44 - C55)
       Q(6,2,I) = Q(2,6,I)
       Q(6,1,I) = Q(1,6,I)
       Q(2,1,I) = Q(1,2,I)

        HSS(1) = HSS(1) + Q(1,2,I)
```

```
      HSS(2) = HSS(2) + Q(2,2,I)
      HSS(3) = HSS(3) + Q(2,6,I)
      HSS(4) = HSS(4) + Q(1,6,I)
      HSS(5) = HSS(5) + Q(6,6,I)


      ATHM1(I) = TA1 * CC + TA2 * SS
      ATHM2(I) = TA1 * SS + TA2 * CC
      ATHM6(I) = CS * ( TA2 - TA1)
      BSW1(I) = SB1 * CC + SB2 * SS
      BSW2(I) = SB1 * SS + SB2 * CC
      BSW6(I) = CS * ( SB2 - SB1 )

30    CONTINUE

  FIND THE A, B, AND D MATRICES FOR THE LOWER AND UPPER SUBLAMINATE.
  ALSO FINDS HYGRALTHERMAL EXPRESSIONS ON A PER LAMINA AND PER
  SUBLAMINATE BASIS.

               ZZZO(0) = ZO(0) * ZO(0) * ZO(0)
      DO 45  I = 1,NPLYO

     NXNM(I) = (Q(1,1,I)*( ATHM1(I)*DELTEMP + BSW1(I)*CMOIST(JM))
    C               + Q(1,2,I)*( ATHM2(I)*DELTEMP + BSW2(I)*CMOIST(JM))
    C + Q(1,6,I)*( ATHM6(I)*DELTEMP + BSW6(I)*CMOIST(JM)) ) * THICK(I)

     NMNYO= NMNYO+(Q(1,2,I)*( ATHM1(I)*DELTEMP + BSW1(I)*CMOIST(JM) )
    C              + Q(2,2,I)*( ATHM2(I)*DELTEMP + BSW2(I)*CMOIST(JM) )
    C + Q(2,6,I)*( ATHM6(I)*DELTEMP + BSW6(I)*CMOIST(JM)) ) * THICK(I)

     NMNXYO= NMNXYO+(Q(1,6,I)*( ATHM1(I)*DELTEMP + BSW1(I)*CMOIST(JM) )
    C               + Q(2,6,I)*( ATHM2(I)*DELTEMP + BSW2(I)*CMOIST(JM) )
    C  + Q(6,6,I)*( ATHM6(I)*DELTEMP + BSW6(I)*CMOIST(JM)) ) * THICK(I)
C


        ZZZO(I) = ZO(I) * ZO(I) * ZO(I)
     B12(I) = Q(1,2,I)*0.5*((ZO(I)*ZO(I))-(ZO(I-1)*ZO(I-1)))
          DO 45 L =1,6
          DO 45  J = 1,6
                IF( REAL( Q(J,L,I) ).EQ.ZR) GO TO 45
     AO(J,L) = AO(J,L) + Q(J,L,I) * THICK(I)
     BO(J,L) = BO(J,L)+Q(J,L,I)*0.5*((ZO(I)*ZO(I))-(ZO(I-1)*ZO(I-1)))
     DO(J,L) = DO(J,L)+Q(J,L,I)/3.0*( ZZZO(I) -  ZZZO(I-1)  )
45      CONTINUE

          ZZZ1(NPLYO) = Z1(NPLYO) * Z1(NPLYO) * Z1(NPLYO)


         DO 50  I = NEXTPL,TPLY
         ZZZ1(I) = Z1(I) * Z1(I) * Z1(I)


     NXNM(I) = ( Q(1,1,I)*( ATHM1(I)*DELTEMP + BSW1(I)*CMOIST(JM) )
    C               + Q(1,2,I)*( ATHM2(I)*DELTEMP + BSW2(I)*CMOIST(JM) )
    C   + Q(1,6,I)*( ATHM6(I)*DELTEMP + BSW6(I)*CMOIST(JM) ))* THICK(I)


     NMNY1= NMNY1+ ( Q(1,2,I)*( ATHM1(I)*DELTEMP + BSW1(I)*CMOIST(JM) )
    C               + Q(2,2,I)*( ATHM2(I)*DELTEMP + BSW2(I)*CMOIST(JM) )
    C   + Q(2,6,I)*( ATHM6(I)*DELTEMP + BSW6(I)*CMOIST(JM)) )* THICK(I)
```

```
C

      NMNXY1= NMNXY1+(Q(1,6,I)*( ATHM1(I)*DELTEMP + BSW1(I)*CMOIST(JM) )
     C              + Q(2,6,I)*( ATHM2(I)*DELTEMP + BSW2(I)*CMOIST(JM) )
     C   + Q(6,6,I)*( ATHM6(I)*DELTEMP + BSW6(I)*CMOIST(JM)) )* THICK(I)

      NMMX1= NMMX1 + 0.5 * ( Z1(I)*Z1(I) - Z1(I-1)*Z1(I-1) ) *
     C        ( Q(1,1,I)*( ATHM1(I)*DELTEMP + BSW1(I)*CMOIST(JM) )
     C        + Q(1,2,I)*( ATHM2(I)*DELTEMP + BSW2(I)*CMOIST(JM) )
     C        + Q(1,6,I)*( ATHM6(I)*DELTEMP + BSW6(I)*CMOIST(JM) ) )

      NMMY1= NMMY1 + 0.5 * ( Z1(I)*Z1(I) - Z1(I-1)*Z1(I-1) ) *
     C        ( Q(1,2,I)*( ATHM1(I)*DELTEMP + BSW1(I)*CMOIST(JM) )
     C        + Q(2,2,I)*( ATHM2(I)*DELTEMP + BSW2(I)*CMOIST(JM) )
     C        + Q(2,6,I)*( ATHM6(I)*DELTEMP + BSW6(I)*CMOIST(JM) ) )

      B12(I) = Q(1,2,I)*0.5*((Z1(I)*Z1(I))-(Z1(I-1)*Z1(I-1)))
              DO  50   L=1,6
              DO  50   J=1,6
                    IF ( REAL( Q(J,L,I) ).EQ.0 ) GO TO 50
      A1(J,L) = A1(J,L) + Q(J,L,I) * THICK(I)
      B1(J,L) = B1(J,L)+Q(J,L,I)*0.5*((Z1(I)*Z1(I))-(Z1(I-1)*Z1(I-1)))
      D1(J,L) = D1(J,L)+Q(J,L,I)/3.0*( ZZZ1(I) -  ZZZ1(I-1)  )

50    CONTINUE

 ***********************************************************************

 SEE IF COUPLING IS TAKING PLACE ...........................

 ***********************************************************************
              COUPL = 2
          DO 60  I=1,6
            DO 60 J=1,6
          IF  ( REAL(B0(I,J)).GT.CHECK ) COUPL=1
60        IF  ( REAL(B1(I,J)).GT.CHECK ) COUPL=1
          IF  ( REAL  (D1(2,6)).GT. CHECK ) COUPL=1
          IF  ( REAL  (D0(2,6))  .GT. CHECK ) COUPL=1

          IF  ( COUPL.EQ.1 .AND. LIL.EQ.1 ) WRITE(6,205)
          IF  ( COUPL.EQ.2 .AND. LIL.EQ.1 ) WRITE(6,210)
***********************************************************************

      CHECK THE SIGN OF THE PEEL STRESS ***************************

           HSN1 = NMNY1 + NMNY0
           HSN2 = NMNXY1 + NMNXY0

 ***********************************************************************
         HDD= HSS(2) * HSS(5) - HSS(3) * HSS(3)
         HE =    HSS(3) * HSS(4) - HSS(1) * HSS(5)
         HE = HE /HDD
         HG =    HSS(1) * HSS(3) - HSS(2) * HSS(4)
         HG = HG / HDD
        DO 65 I=1,TPLY
            HNY(I) = ATH * STRAIN(LST) * ( Q(1,2,I) + Q(2,2,I) * HE +
     C                Q(2,6,I) * HG )
            HNXY(I) = ATH * STRAIN(LST) * ( Q(1,6,I) + Q(2,6,I) * HE +
     C                 Q(6,6,I) * HG)
65      CONTINUE


         DO 70 I = 1,NPLY1
```

```
      HM3 = HM3 + ATH * HNY(I) * ( NPLY1 - I + .5 )
      SMNY = SMNY + HNY(I)
70    SMNXY = SMNXY  + HNXY(I)


      IF  ( HM3.GT.ZR) GO TO 85
C
C     IF  (LIL.EQ.1) WRITE(6,*)' CASE OF COMPRESSIVE PEEL STRESS
C     WRITE(6,218)
C       DO 75 I=1,TPLY
75    WRITE(6,220)THETA(I),HNY(I),HNXY(I)
C     WRITE(6,*)' THE MOMENT CALCULATED WAS = ',HM3
C
85     DO 80 I=1,6
       DO 80 J=1,6
        IF( ABS( REAL(BO(I,J)) ).LT.CHECK)  BO(I,J)=ZR
80      IF( ABS( REAL(B1(I,J)) ).LT.CHECK)  B1(I,J) = ZR
*************************************************************************
 ***********************************************************

     DEFINE SOME PARAMETERS NEEDED IN THE PROGRAM

  ***********************************************************
        H22 = B1(2,2) + H1 / 2.00 * A1(2,2)
        H26 = B1(2,6) + H1 / 2.00 * A1(2,6)
        H66 = B1(6,6) + H1 / 2.00 * A1(6,6)

         C22 = BO(2,2) + HO / 2.00 * A1(2,2)
        C26 = BO(2,6) + HO / 2.00 * A1(2,6)
        C66 = BO(6,6) + HO / 2.00 * A1(6,6)

        K22 = A1(2,2) +  AO(2,2)
        K26 = A1(2,6) +  AO(2,6)
        K66 = A1(6,6) +  AO(6,6)
        K12 = A1(1,2) +  AO(1,2)
        K16 = A1(1,6) +  AO(1,6)

        D = K22 * K66 -( K26 * K26)

        E15 = DO(2,2) - HO/2*BO(2,2)
        E16 = DO(2,6) - HO/2*BO(2,6)
        E17 = DO(6,6) - HO/2*BO(6,6)
        E18 = BO(1,2) - HO/2*AO(1,2)
        E19 = BO(1,6) - HO/2*AO(1,6)

          VV11 = ( K26 * H26 - K66 * H22 ) / D  + ( H1 / 2.00 )
          VV12 = ( K26 * C26 - K66 * C22 ) / D  + ( HO / 2.00 )
          VV13 = ( K26 * H66 - K66 * H26 ) / D
          VV14 = ( K26 * C66 - K66 * C26 ) / D

           U11 = ( K26 * H22 - K22 * H26 ) / D
           U12 = ( K26 * C22 - K22 * C26 ) / D
           U13 = ( K26 * H26 - K22 * H66 ) / D  + ( H1 / 2.00 )
           U14 = ( K26 * C26 - K22 * C66 ) / D  + ( HO / 2.00 )

       F(1,1) = D1(2,2) + B1(2,2) * H1 / 2.0 + H22*VV11 + H26 * U11
       F(2,1) = H22 * VV12  +  H26 * U12
       F(3,1) = D1(2,6) + B1(2,6) * H1 / 2.0 + H22*VV13 + H26 * U13
       F(4,1) = H22 * VV14 + H26 * U14

       F(2,2) = DO(2,2) - BO(2,2) * HO / 2.0 + C22 * VV12 + C26*U12
       F(3,2) = H26 * VV12 + H66 * U12
       F(4,3) = H26 * VV14 + H66 * U14

       F(3,3) = D1(6,6) + B1(6,6) * H1 / 2.0 + H26*VV13 + H66 * U13
       F(4,2) = DO(2,6) - BO(2,6) * HO / 2.0 + C22*VV14 + C26 * U14
```

C-2

```
      F(4,4)  = DO(6,6)  - BO(6,6)  * HO / 2.0 + C26*VV14 + C66 * U14

      DX   = K22 * K66

        DVV11 =   - K66 * H22  / DX   + ( H1 / 2.00 )
        DVV12 =   - K66 * C22  / DX   + ( HO / 2.00 )
         DU13 =   - K22 * H66  / DX   + ( H1 / 2.00 )
         DU14 =   - K22 * C66  / DX   + ( HO / 2.00 )

      DF(1,1)  = D1(2,2)  + B1(2,2)  * H1 / 2.0 + H22*DVV11
      DF(2,1)  = H22 * DVV12
      DF(2,2)  = DO(2,2)  - BO(2,2)  * HO / 2.0 + C22 * DVV12
      DF(4,3)  = H66 * DU14

      DF(3,3)  = D1(6,6)  + B1(6,6)  * H1 / 2.0 +  H66 * DU13
      DF(4,4)  = DO(6,6)  - BO(6,6)  * HO / 2.0 + C66 * DU14


      W(1)  = F(3,3)*( F(2,2)*F(4,4)-F(4,2)*F(4,2)  )- F(3,2)*F(3,2)*
C              F(4,4)  + 2*F(4,3)*F(4,2)*F(3,2)  - F(2,2)*F(4,3)*F(4,3)

      W(2)  = - F(3,3)*(F(2,2)*AO(5,5)  + F(4,4)*AO(4,4)  - 2*F(4,2)
C               * AO(4,5) )  -  A1(5,5)  *  ( F(2,2)  * F(4,4)  -
C              F(4,2)  * F(4,2)) + F(3,2)  * F(3,2)  * AO(5,5)  -
C         2.0 *F(4,3)*F(3,2)*AO(4,5)  + F(4,3)  * F(4,3)  * AO(4,4)


     X(1)=F(3,1)*(F(2,2)*F(4,4)-F(4,2)*F(4,2))-F(3,2)*(F(2,1)*F(4,4)
C           - F(4,1)*F(4,2))  + F(4,3)*(F(2,1)*F(4,2)  - F(4,1)*F(2,2))

      X(2)  = - F(3,1)  *  ( F(2,2)  * AO(5,5)  + F(4,4)  * AO(4,4)
C            - 2 * F(4,2)  * AO(4,5)  ) - A1(4,5)  *   ( F(2,2)  * F(4,4)
C             - F(4,2)  * F(4,2)  ) + F(3,2)  *  ( AO(5,5)  * F(2,1)  -
C        F(4,1)*AO(4,5))  - F(4,3)*( F(2,1)*AO(4,5)  - F(4,1)*AO(4,4))


      Y(1)  = F(3,1)*(F(3,2)*F(4,4)  - F(4,3)*F(4,2))- F(3,3)*(F(2,1)*
C       F(4,4)  - F(4,1)*F(4,2))+F(4,3)*(F(2,1)*F(4,3)-F(4,1)*F(3,2))

      Y(2)  = O - A1(4,5)*(F(3,2)*F(4,4)  - F(4,2)*F(4,3))
C            - F(3,1)  *  (  F(3,2)*AO(5,5)  - F(4,3)  * AO(4,5)  ) +
C            A1(5,5)  *  ( F(2,1)  * F(4,4)  - F(4,1)  * F(4,2)  ) +
C            F(3,3)  *  ( F(2,1)  * AO(5,5)  - F(4,1)  * AO(4,5)  )
      Y(3)  = A1(4,5)*(F(3,2)*AO(5,5)  - F(4,3)*AO(4,5))  - A1(5,5)  *
C            ( F(2,1)  * AO(5,5)  - F(4,1)  * AO(4,5)  )


      ZZ(1)  = F(3,1)*(F(3,2)*F(4,2)- F(4,3)*F(2,2))  - F(3,3)*(F(2,1)*
C           F(4,2)-F(4,1)*F(2,2))+F(3,2)*(F(2,1)*F(4,3)-F(4,1)*F(3,2))

      ZZ(2)  = F(3,1)*(F(4,3)*AO(4,4)- F(3,2)*AO(4,5))  - A1(4,5)*
C           (F(3,2)*F(4,2)  - F(4,3)*F(2,2))+ A1(5,5)*(F(2,1)* F(4,2)-
C           F(4,1)*F(2,2))  - F(3,3)*(F(4,1)*AO(4,4)- F(2,1)*AO(4,5))
      ZZ(3)  = O - A1(4,5)*(F(4,3)*AO(4,4)-F(3,2)*AO(4,5))+A1(5,5)*
C              ( F(4,1)  * AO(4,4)  - F(2,1)  * AO(4,5)  )
****************************************************************************

  NOW OBTAIN THE VALUES OF E SO THAT THE 8TH ORDER POLYNOMIAL MAY BE SOLVED

  **************************************************************************
      E(1)  = F(1,1)*W(1)  - F(3,1)*X(1)  + F(2,1)*Y(1)  - F(4,1)*ZZ(1)
      E(3)  = F(1,1)*W(2)  - A1(4,4)*W(1)  - F(3,1)*X(2)  + A1(4,5)*X(1)
C               + F(2,1)  * Y(2)  - F(4,1)  * ZZ(2)
      E(5)  =  (AO(4,4)*AO(5,5)  - AO(4,5)*AO(4,5))*(F(1,1)*F(3,3)
C           - F(3,1)*F(3,1))  +  (F(2,2)*AO(5,5)  + F(4,4)*AO(4,4)  -
C            2*F(4,2)*AO(4,5))*(F(1,1)*A1(5,5)  - F(3,1)*A1(4,5)  )
```

```
C         - A1(4,4)*W(2) + A1(4,5)*X(2)+ F(2,1)*Y(3) - F(4,1)*ZZ(3)

      E(7) = - (AO(4,4)*AO(5,5) - AO(4,5)*AO(4,5))*(F(1,1)*A1(5,5)
C         + F(3,3)*A1(4,4)- 2*F(3,1)*A1(4,5)) - (A1(4,4)*A1(5,5)-
C         A1(4,5)*A1(4,5))*(F(2,2)*AO(5,5) + F(4,4)*AO(4,4)
C          - 2 * AO(4,5) * F(4,2) )

      E(9) = (AO(4,4)*AO(5,5) - AO(4,5)*AO(4,5) ) *
C            (A1(4,4) * A1(5,5) - A1(4,5) * A1(4,5) )

       CALL UP SUBROUTINE TO SOLVE 8TH ORDER POLYNOMIAL

        NDEG = 8
        IER = 0
        CALL ZPOLR (E,NDEG,SJ,IER)

        KK = 0
*******************************************************************
*******************************************************************


        IF (LIL.EQ.1) WRITE(6,217)
        DO 90  L = 1, 8
        S(L) = REAL(SJ(L))
        IF (REAL(SJ(L)).GT.0) KK = KK + 1
90      IF (REAL(SJ(L)).GT.0) S(KK) = S(L)
           DO 95 KK = 1,4
95         IF (LIL.EQ.1) WRITE(6,221) KK, S(KK)

*******************************************************************

*******************************************************************
 NOW FIND THE UNCOUPLED S VALUES AND THOSE OF THE MEMBRANE

      BNEG= DF(1,1) * AO(4,4) + DF(2,2) * A1(4,4)
      A = DF(1,1) * DF(2,2) - DF(2,1) * DF(2,1)
      C = AO(4,4) * A1(4,4)
       SQ = DSQRT (BNEG * BNEG - ( 4.0 * A * C))
       DIFF1 = DABS ( BNEG - SQ )
       DIFF2 = DABS ( BNEG + SQ )
        IF (DIFF1.GT.DIFF2) GO TO 100
        UNSY(1) = DSQRT ( (BNEG+SQ) / 2.0 / A  )
        GO TO 105
100     UNSY(1) = DSQRT ( (BNEG-SQ) / 2.0 / A  )
105     UNSY(2) = DSQRT( (BNEG/A) - UNSY(1)* UNSY(1) )

      BNEG= DF(3,3) * AO(5,5) + DF(4,4) * A1(5,5)
      A = DF(3,3) * DF(4,4) - DF(4,3) * DF(4,3)
      C = AO(5,5) * A1(5,5)
       SQ = DSQRT (BNEG * BNEG - ( 4.0 * A * C) )
       DIFF1 = DABS ( BNEG - SQ )
       DIFF2 = DABS ( BNEG + SQ )
        IF (DIFF1.GT.DIFF2) GO TO 110
        UNSX(1) = DSQRT ( (BNEG+SQ) / 2.0 / A )
        GO TO 115
110     UNSX(1) = DSQRT ( (BNEG-SQ) / 2.0 / A )
115     UNSX(2) = DSQRT ( (BNEG/A) - UNSX(1) * UNSX(1) )

*******************************************************************
      IF (LIL.EQ.1) WRITE(6,224)  UNSX(1), UNSX(2)
      IF (LIL.EQ.1) WRITE(6,223)  UNSY(1), UNSY(2)

*******************************************************************

      NOW THE S FOR THE MEMBRANE ONLY
```

```
      F11M = H1/2. * ( A1(2,2) * (DVV11 + DVV12*HO/H1*A1(4,4)/AO(4,4))
     C      + B1(2,2) )

      F22M = H1/2. * ( A1(6,6) * (U13 + U14*HO/H1*A1(5,5)/AO(5,5))
     C       + B1(6,6) )

        MEMSY = DSQRT ( A1(4,4) / F11M )
        MEMSX = DSQRT ( A1(5,5) / F22M )

*********************************************************************

        IF (LIL.EQ.1) WRITE(6,219) MEMSX, MEMSY

*********************************************************************

        DUM = AO(2,2) * AO(6,6) - AO(2,6) * AO(2,6)
        S1NM = ( AO(6,6)*NMNYO - AO(2,6)*NMNXYO ) / DUM
        S2NM = ( AO(2,2)*NMNXYO - AO(2,6)*NMNYO ) / DUM
          IF (LIL.EQ.1) WRITE(6,288) S1NM,S2NM

          IF ( COUPL.EQ.2)GO TO 130

C     SOLVE FOR WD, CD, CU AND CV

        WD(1,1) = (AO(2,6) * AO(1,6) - AO(6,6) * AO(1,2) ) / DUM
        WD(1,2) = (AO(2,6) * BO(2,6) - AO(6,6)*BO(2,2) ) / DUM
        WD(1,3) = (AO(2,6)*BO(6,6) - AO(6,6)*BO(2,6) ) / DUM
        WD(2,1) = (AO(2,6) * AO(1,2) - AO(2,2) * AO(1,6) )/DUM
        WD(2,2) = (AO(2,6)*BO(2,2) - AO(2,2)* BO(2,6) ) / DUM
        WD(2,3) = (AO(2,6) * BO(2,6) - AO(2,2)*BO(6,6) ) / DUM


          MATR33(1,1) = A1(2,2)
          MATR33(1,2) = A1(2,6)
          MATR33(1,3) = B1(2,2)

          MATR33(2,1) = MATR33(1,2)
          MATR33(2,2) = A1(6,6)
          MATR33(2,3) = B1(2,6)

          MATR33(3,1) = MATR33(1,3)
          MATR33(3,2) = MATR33(2,3)
          MATR33(3,3) = D1(2,2)


          MATR32(1,1) =  - A1(1,2)
          MATR32(1,2) =  - B1(2,6)

          MATR32(2,1) =  - A1(1,6)
          MATR32(2,2) =  - B1(6,6)

          MATR32(3,1) =  - B1(1,2)
          MATR32(3,2) =  - D1(2,6)
          IF (IZZ.EQ.2) GO TO 122

           DO 120 I=1,3
           DO 120 K=1,3
 120       SAVE33(I,K) = MATR33(I,K)
            SAVE3(1) = NMNY1
            SAVE3(2) = NMNXY1
            SAVE3(3) = NMMY1
               IRR = 0
         CALL LEQT2F(SAVE33,1,3,3,SAVE3,0,WKAREA,IRR)

 122       M=2
```

```
          N=3
          IA=3
          IRR=0
          IDD=0

        CALL LEQT2F(MATR33,M,N,IA,MATR32,IDD,WKAREA,IRR)

            DO  125 I=1,3
           IF  (IZZ.EQ.2)  SAVE3(I) = ZR
            FNM(I)  = SAVE3(I)
              DO 125  L=1,2
125         CD(I,L)  = MATR32(I,L)


        SC = DSQRT(   ( A1(5,5)  - A1(4,5)  * A1(4,5)  / A1(4,4)  )
   C         / ( D1(6,6) + B1(2,6)*CD(1,2) + B1(6,6)*CD(2,2)
   C                 + D1(2,6)  * CD(3,2) ) )
        GO TO 135
*********************************************************************

          IN CASE THE LAYERS ARE UNCOUPLED

*********************************************************************
130     DY = -1/ ( A1(2,2)*A1(6,6) -  A1(2,6) * A1(2,6) )
        CD(1,1) = ( A1(6,6)*A1(1,2) - A1(2,6)*A1(1,6) ) / DY
        CD(2,1) = ( A1(2,2)*A1(1,6) - A1(2,6)* A1(1,2) ) / DY
        CD(3,2) = ( A1(2,2)*A1(6,6) + A1(2,6)*A1(2,6) ) *
   C                D1(2,6) / D1(2,2)/ DY
        CD(1,2) = ZR
        CD(2,2) = ZR
        CD(3,1) = ZR

        DR = -1 / (  AO(2,2)*AO(6,6) - AO(2,6)*AO(2,6) )
        WD(1,1) = ( AO(6,6)*AO(1,2) - AO(2,6)*AO(1,6) ) / DR
        WD(2,1) = ( AO(2,2)*AO(1,6) - AO(2,6)*AO(1,2) ) / DR
        WD(1,2) = ZR
        WD(1,3) = ZR
        WD(2,2) = ZR
        WD(2,3) = ZR
        SSY = DSQRT( AO(5,5) / DO(6,6) )
        SSX = DSQRT( AO(4,4) / DO(2,2) )

          FNM(1) = (A1(2,6)*NMNXY1 - A1(6,6)*NMNY1) / DY
          FNM(2) = (A1(2,6)*NMNY1 - A1(2,2)*NMNXY1 ) / DY
      FNM(3)=(A1(2,6)*A1(2,6) - A1(2,2)*A1(6,6))/DY * NMMY1/D1(2,2)

           SC = DSQRT(  (A1(5,5)* A1(6,6)) / (A1(6,6)*D1(6,6)
   C              - (B1(6,6)*B1(6,6)) ) )


*******************************************************************
 *******************************************************************


135       C1 = B1(1,6) + CD(1,1)*B1(2,6) + CD(2,1)*B1(6,6)
   C                      + CD(3,1) * D1(2,6)
         C2 = D1(6,6) + CD(1,2)*B1(2,6) + CD(2,2)*B1(6,6) +
   C                       CD(3,2) * D1(2,6)


          J22 = DO(2,2) + BO(2,2) * WD(1,2) + BO(2,6) * WD(2,2)
          J66 = DO(6,6) + BO(2,6) * WD(1,3) + BO(6,6) * WD(2,3)
          J26 = DO(2,6) + BO(2,2) * WD(1,3) + BO(2,6) * WD(2,3)

         BNEG = J22 * AO(5,5) + J66 * AO(4,4) - 2. * J26 * AO(4,5)
         A = J22 * J66 - J26 * J26
```

```
      C = AO(4,4) * AO(5,5) - AO(4,5) * AO(4,5)

      SQ = DSQRT ( (BNEG * BNEG) - 4.0 * C * A )
      DIFF1 = DABS( BNEG + SQ )
      DIFF2 = DABS( BNEG - SQ )
      IF (DIFF1.GT.DIFF2) GO TO 140
      SS1 = DSQRT ((BNEG + SQ) / 2. / A )
      GO TO 145
140   SS1 = DSQRT ( (BNEG-SQ) / 2. / A )
145   SS2 = DSQRT ( (BNEG/A) - SS1 * SS1 )

       SSY = DSQRT ( AO(4,4) * J22 )
       SSX = DSQRT ( AO(5,5) * J66 )

******************************************************************

     IF (LIL.EQ.1) WRITE(6,*)'     S1 AND S2 = ',SS1, SS2
     IF (LIL.EQ.1) WRITE(6,*)'     SX AND SY = ',SSX, SSY

******************************************************************

    CVNM = ( K66 * (NMNY1 + NMNYO) - K26 * (NMNXY1 + NMNXYO) ) / D
    CUNM = ( K22 * (NMNXY1 + NMNXYO) - K26 * (NMNY1 + NMNYO) ) / D

     CV = STRAIN(LST) / D * ( K26 * K16 - K66 * K12 ) + CVNM
     CU = STRAIN(LST) / D * ( K26 * K12 - K22 * K16 ) + CUNM

*****************************************************************

     NOW FIND SOME OF THE NEEDED CONSTANTS.....
      FIRST DO LOOP IS TO VARY THE VALUES OF S

         DO 150  I = 1,4

 FORM THE A MATRIX (MATR32) AND ITS B (MATR3)

        MATR33(1,1) =  - ( F(3,1) * S(I) * S(I) - A1(4,5) )
        MATR33(1,2) =  - ( F(2,1) * S(I) * S(I) )
        MATR33(1,3) =  - ( F(4,1) * S(I) * S(I) )
        MATR33(2,1) =  - ( F(3,3) * S(I) * S(I) - A1(5,5) )
        MATR33(2,2) =  - ( F(3,2) * S(I) * S(I) )
        MATR33(2,3) =  - ( F(4,3) * S(I) * S(I) )
        MATR33(3,1) =  - ( F(3,2) * S(I) * S(I) )
        MATR33(3,2) =  - ( F(2,2) * S(I) * S(I) - AO(4,4) )
        MATR33(3,3) =  - ( F(4,2) * S(I) * S(I) - AO(4,5) )

        MATR3(1) = (F(1,1) * S(I) * S(I) ) - A1(4,4)
        MATR3(2) = F(3,1) * S(I) * S(I)  - A1(4,5)
        MATR3(3) = F(2,1) * S(I) * S(I)


    CALL UP  ROUTINE  TO FIND THE VALUES OF ALPHA, PHI AND GAMMA

         M=1
         N=3
         IRR=0
         IDD=0
         IA=3
         CALL LEQT2F(MATR33,M,N,IA,MATR3,IDD,WKAREA,IRR)
          ALPHA(I) = MATR3(1)
          PHI(I) = MATR3(2)
          GAMMA(I) = MATR3(3)

    SV(I) = VV11 + ALPHA(I)*VV13 + PHI(I)*VV12 + GAMMA(I)*VV14
150 SU(I) = U11 + ALPHA(I)*U13 + PHI(I)*U12 + GAMMA(I)*U14
```

```
      DO 155 I = 1,4

   NX1(I)=A1(1,2)*SV(I) + A1(1,6)*SU(I) + B1(1,2) + B1(1,6)*ALPHA(I)
   NY1(I)=A1(2,2)*SV(I) + A1(2,6)*SU(I) + B1(2,2) + B1(2,6)*ALPHA(I)
  NXY1(I)=A1(2,6)*SV(I) + A1(6,6)*SU(I) + B1(2,6) + B1(6,6)*ALPHA(I)
   MY1(I)=B1(2,2)*SV(I) + B1(2,6)*SU(I) + D1(2,2) + D1(2,6)*ALPHA(I)
  MXY1(I)=B1(2,6)*SV(I) + B1(6,6)*SU(I) + D1(2,6) + D1(6,6)*ALPHA(I)
  NXO(I) =AO(1,2)*SV(I) + AO(1,6)*SU(I) + E18*PHI(I) + E19*GAMMA(I)
        GG(1,I) = NY1(I)
        GG(2,I) = NXY1(I)
        GG(3,I) = MY1(I)
            FTH = C2 * SC
155     GG(4,I) = MXY1(I) + FTH * ALPHA(I)/ S(I)


    A1NM = B1(2,6)* FNM(1) + B1(6,6)*FNM(2) + D1(2,6) * FNM(3)

                                                                 **
   BG(1) = A1(1,2)*STRAIN(LST) + CV * A1(2,2) + A1(2,6) * CU-NMNY1
   BG(2) = A1(1,6)*STRAIN(LST) + CV * A1(2,6) + A1(6,6) * CU-NMNXY1
   BG(3) = B1(1,2)*STRAIN(LST) + B1(2,2) * CV + B1(2,6) * CU-NMMY1
   BG(4) = B1(1,6)*STRAIN(LST) + B1(2,6) * CV + B1(6,6) * CU-A1NM
 C              - C1 * STRAIN(LST)

        BG1= BG(1)
        BG2 = BG(2)

     M=1
     N=4
     IA=4
     IDD=0
     IRR=0
         CALL LEQT2F(GG,M,N,IA,BG,IDD,WKAREA,IRR)

**********************************************************************

     TVNM =   S1NM - FNM(1) + H1 / 2.0 * FNM(3)
     TUNM =   S2NM - FNM(2)

     THETV =  -CD(1,1) + WD(1,1) + H1/2.0*CD(3,1)
     THETU =  -CD(2,1) + WD(2,1)
      THETV = THETV  + TVNM
      THETU = THETU  + TUNM

   IF (LIL.EQ.1) WRITE(6,215) THETV, THETU
   IF (LIL.EQ.1) WRITE(6,216)  SMNY, SMNXY

**********************************************************************
THE STEPS USED TO FIND THE TOTAL ENERGY RELEASE FROM USING A PURE
EXTENSION ANALYSIS FOR THE HYGRALTHERMAL EFFECTS.  (SIMILAR TO
WHITNEY'S). ANALYSIS IS CARRIED OUT ON A PLY BY PLY BASIS


    ZV = ( K26 * K16 - K66 * K12 ) / D
    ZU = ( K26 * K12 - K22 * K16 ) /D
C
     DO 162  LL = 1,TPLY
C
C
     EX(LL) = THICK(LL) * ( Q(1,1,LL) + ZV*Q(1,2,LL) +
 C                ZU*Q(1,6,LL) )
     T1(LL) = NXNM(LL) - CVNM*THICK(LL)*Q(1,2,LL) -
 C                 CUNM * THICK(LL) * Q(1,6,LL)
     EXX(LL) = Q(1,1,LL)*THICK(LL) + Q(1,2,LL)*THICK(LL)*CD(1,1)
```

```
C             + Q(1,6,LL)*THICK(LL)*CD(2,1) + B12(LL)*CD(3,1)
      T12(LL) = NXNM(LL) - FNM(1)*Q(1,2,LL)*THICK(LL) -
C             FNM(2)*Q(1,6,LL)*THICK(LL) - FNM(3)*B12(LL)
      EX3(LL) = Q(1,1,LL)*THICK(LL) + WD(1,1)*Q(1,2,LL)*THICK(LL)
C              + WD(2,1)*Q(1,6,LL)*THICK(LL)
      T13(LL) = NXNM(LL) - Q(1,2,LL)*THICK(LL)*S1NM - Q(1,6,LL)
C              * THICK(LL) * S2NM

          IF (IZZ.EQ.2) NMSTO(LL) = 0.0
          IF (IZZ.EQ.2) NMST2(LL) = 0.0
          IF (IZZ.EQ.2) NMST3(LL) = 0.0
          IF (IZZ.EQ.2) GO TO 162


       NMSTO(LL) =  T1(LL)  / EX(LL)
       NMST2(LL) =  T12(LL) / EXX(LL)
       NMST3(LL) =  T13(LL) / EX3(LL)

162   CONTINUE

         WRITE(6,*)' JMM, NMSTO,2,3 OF ALL PLYS ',( NMSTO(JP),
C       NMST2(JP),NMST3(JP),' ---  ',JP=1,TPLY)

         WRITE(6,*)' EX, EXX EX3  OF ALL PLYS ',( EX(JP),
C       EXX(JP),EX3(JP),' ---  ',JP=1,TPLY)

          TNC = 0.0
          EXNC = 0.0
          TSTAR = 0.0
          ESTAR = 0.0
          DO 163  LK = 1,TPLY
           TNC =  TNC  + T1(LK)
           EXNC = EXNC + EX(LK)
           IF (LK.LE.NPLYO) TSTAR = TSTAR + T13(LK)
           IF (LK.GT.NPLYO) TSTAR = TSTAR + T12(LK)

           IF (LK.LE.NPLYO) ESTAR = ESTAR + EX3(LK)
163        IF (LK.GT.NPLYO) ESTAR = ESTAR + EXX(LK)

          IF(IZZ.EQ.2) TNMST = 0.0
          IF (IZZ.EQ.2) GO TO 89

     TNMST = (  TNC -(TNC-TSTAR)*2*AL/WIDTH ) / ( EXNC -
C             (EXNC-ESTAR)*2*AL/WIDTH )
89   WRITE(6,*)'  TNMST EQUALS ',TNMST


     DO 164   LL = 1,TPLY

       IF (LL.LE.NPLYO) WWC =  (EX3(LL) *
C         (STRAIN(LST) + TNMST) - T13(LL) )
C *     ( STRAIN(LST) - NMST3(LL) + TNMST )

       IF (LL.GT.NPLYO) WWC =  (EXX(LL) *
C         (STRAIN(LST) + TNMST) - T12(LL) )
C *     ( STRAIN(LST) - NMST2(LL) + TNMST )

     WWO = ( EX(LL)* (STRAIN(LST) + TNMST) - T1(LL) )
C          *  ( STRAIN(LST) - NMSTO(LL) + TNMST )


164   GLC(JMM) =  GLC(JMM) + WWO - WWC

         GLC(JMM) = GLC(JMM) / 2.0
*******************************************************************
```

```
      THIS IS TO CALCULATE THE INTERLAMINAR SHEAR STRESSES.
    ******************************************************************
              UNCL =(WIDTH / 2.0)- AL
                DO 180  JX = 80,100
                 JY =(  1.0 -  JX /100.0)   * UNCL
                 SIGX(JMM,JX)  = 0
                 SIGY(JMM,JX)  = 0
                    DO 180  JS = 1,4
                    SIGX(JMM,JX) =  BG(JS)  * S(JS)  * DEXP  ( -S(JS)* JY )
       C                          * NXY1(JS)  + SIGX(JMM,JX)

                    SIGY(JMM,JX)  =  BG(JS)  * S(JS)  * DEXP  ( -S(JS)* JY )
       C                          * NY1(JS)  + SIGY(JMM,JX)

     180         CONTINUE

        THE PROGRAM CONTINUES AND FINDS THE VARIOUS STRAIN ENERGY RELEASE
        COMPONENTS .....


              IF  (COUPL.EQ.1)  GO TO 165

              THIS  IS  FOR A SYSTEM THAT IS COUPLED, THE  CRACK LENGTH IS

              DEL = S(4)  * S(2)  * ATH * ATH
              DEL = DEL * DEL * 0.6144
              GO TO 170

          THIS IS FOR AN UNCOUPLED SYSTEM...................

     165     SSW = .65 *  ( S(1)  + S(2)  + S(3)  )
             DEL = 18.7 * S(4)  * SSW * ATH * ATH
             DEL = DEL * DEL / 571.00
     170     DEL = 135.7 * DEL * ATH

    C        IF  (LIL.EQ.1)  WRITE(6,211)  DEL

             FY = ZR
             FX = ZR


              DO 175   JP=1,4

               CONY= NY1(JP)*BG(JP)*( DEXP  ( -S(JP)  * DEL )  -  1 )
               CONXY= NXY1(JP)* BG(JP)*( DEXP( -S(JP)  *DEL)-  1 )
               CCONXY = CCONXY + CONXY/S(JP)
    175        CCONY = CCONY + CONY/S(JP)

    ***************************************************************
    ***************************************************************

               FY = BG1 + CCONY / DEL
               FX = BG2 + CCONXY / DEL

              GII(IZZ,JM)  =    FY / 2.0  * THETV *STRAIN(LST)
              GIII(IZZ,JM)  =    FX / 2.0 * THETU * STRAIN(LST)

             DIFFG= GII(IZZ,JM)  - GIII(IZZ,JM)
             CON = 2
               IF  (DIFFG.GT.REAL(GLC(JMM)))  CON=1
               IF  (DIFFG.GT.REAL(GLC(JMM)))  DEL = DEL * .9
               IF  (DIFFG.GT.REAL(GLC(JMM)))  WRITE(6,*) ' IT EXPLODES

                GI(IZZ,JM)  = GLC(JMM)-GII(IZZ,JM)-GIII(IZZ,JM)
```

```
200      CONTINUE
   RESULTS ARE PRINTED FOR EACH RUN.

 300          CONTINUE
         WRITE(6,266)  STRAIN(LST)
         WRITE(6,267)
         WRITE(6,269)  GLC(0)  , GI(2,1), GII(2,1), GIII(2,1),
    C     GI(2,1)/GLC(0)

            DO 350  I=1,MMC +1
        WRITE(6,268)  CMOIST(I),GLC(I), GI(1,I),GII(1,I),
    C     GIII(1,I), GI(1,I)/GLC(I)
 350      CONTINUE

       WRITE(6,287)

            DO 360   NS = 80,90,2
 360         WRITE(6,285)NS/100., SIGX(0,NS), ( SIGX(KL,NS),KL=1,22,4)

            DO 365   NS = 91,100
 365         WRITE(6,285)NS/100.,SIGX(0,NS),  ( SIGX(KL,NS),KL=1,22,4)

       WRITE  (6,286)
            DO 370   NS = 80,90,2
 370         WRITE(6,285)  NS/100.,SIGY(0,NS), ( SIGY(KL,NS),KL=1,22,4)

            DO 375   NS = 91,100
 375         WRITE(6,285)  NS/100.,SIGY(0,NS), ( SIGY(KL,NS), KL=1,22,4)


 400          CONTINUE

 201  FORMAT(//,'  THE WIDTH OF THE LAMINATE IS ',F8.5)
 287  FORMAT(/////,' THESE ARE THE IN-PLANE  INTERLAMINAR SHEAR ',
    C 'STRESSES -- SIGMA XY  ',/,'  THEY ARE FOUND AT VARIOUS',
    C '  MOISTURE CONTENTS ',//,'  Y LOCATION',7X,' MECH ONLY',8X,
    C   'H=0.0',10X,'H=0.2',10X,'H=0.4',10X,
    C     'H=0.6',10X,'H=0.8',10X,'H=1.0',//)
 285  FORMAT(3X,F7.2,4X,7F15.8)
 202  FORMAT(' THE NUMBER OF LAMINATES ABOVE AND BELOW THE CRACK IS'
    C      ,I3,5X,I3)
 204      FORMAT(///,'  THE PLYS ARE INPUTTED FROM BOTTOM TO TOP',/,
    C  ' BUT THE PLY CHARACTERISTICS FROM TOP TO BOTTOM ARE ')
 206  FORMAT(//,' FOR PLY',I5,' THE SUBLAMINATE HAS THESE PROPERTIES')
 205   FORMAT(//,'  WITH THIS LAYUP, THE PLYS ARE COUPLED ',//)
 210   FORMAT(//,'  WITH THIS LAYUP, THE PLYS ARE DECOUPLED ',//)
 286  FORMAT(//,' THESE ARE THE OUT-OF-PLANE  INTERLAMINAR SHEAR ',
    C 'STRESSES -- SIGMA YZ  ',/,'  THEY ARE FOUND AT VARIOUS',
    C ' MOISTURE CONTENTS ',//,' Y LOCATION',8X,' MECH ONLY',8X,
    C   'H=0.0',10X,'H=0.2',10X,'H=0.4',10X,
    C     'H=0.6',10X,'H=0.8',10X,'H=1.0',//)
 208  FORMAT(' E1 AND E2 ARE  (MSI)                 ',F8.4,10X,F8.4)
 289  FORMAT(//,'  THE LAMINA PLY CHARACTERISTICS INITIALLY ARE ',/)
 288  FORMAT(/,'  S1NM AND S2NM ARE EQUAL TO ',F14.10,4X,F14.10)
 211  FORMAT(//,'  THE CRACK LENGTH STEP SIZE IS ',F12.8)
 266    FORMAT('0','1','   THE STRAIN IS EQUAL TO ',F12.7,/,
    C '  THE VALUES OF GT,  GI,  GII,  AND  GIII  ARE  IN  IN-LB/IN/IN
    C  ')
 267    FORMAT  (/,3X,'% CMOIST',8X,'GGG(WHITNEY)',6X,'GI',9X,
    C    ' GII',8X,'GIII',6X,' GI/G(W-T)',//)
 269  FORMAT(/,'  MECH. ONLY ',3X,F12.9,4(2X,F11.7),/)
 268  FORMAT(5X,F8.3,3X,F12.9,4(2X,F11.7) )
 215  FORMAT(////,'  THETA V IS ',F15.10,' THETA U IS ',F24.19)
 216  FORMAT(//,'  NY IS ',F23.11,'  NXY IS ',F23.18)
```

```
207  FORMAT(/,' THE THICKNESS AND THETA VALUES ARE ',F9.6,5X,F8.3)
209  FORMAT(' THE POISSON RATIO (1,2) IS    ',F10.5)
214  FORMAT(' G OF (1-2),  AND (3-1) ARE -MSI ', 2(F9.4,2X))
217  FORMAT('0',////,8X,'THE FOUR CHARACTERISTIC VALUES ASSOCIATED'
   C ,/,8X,' WITH THE 8 DEGREE POLYNOMIAL FOR THE COUPLED CASE ARE')
218  FORMAT(//,5X,' THETA ',6X,' NY ',8X,' NXY ',//)
219  FORMAT(///,' THE S VALUES OF THE MEMBRANE ARE ',F15.5,3X,F15.5)
220  FORMAT(/,4X,F9.4,3X,F9.2,3X,F9.2)
221  FORMAT(/,'  S OF ',I2,' IS EQUAL TO ',F20.10)
223  FORMAT(/,'  THE UNCOUPLED SY (1,2) VALUES ARE ',F15.5,3X,F15.5)
224  FORMAT(/,'  THE UNCOUPLED SX (1,2) VALUES ARE ',F15.5,3X,F15.5)
231  FORMAT(//,'  THE STRAIN IS EQUAL TO ',F12.8,/,
   C '  THE CHANGE IN TEMPERATURE IS ',F12.5,/,
   C '  THE COEFFICIENTS SWELLING DUE TO MOISTURE ARE ',2(2X,F12.8)
   C ,/,'  THE COEFFICIENTS OF THERMAL EXPANSION ARE ',2(2X,F15.9))

232  FORMAT(/,'  THE MOISTURE COEFFICENT IS ',F15.8)
233  FORMAT(/,'  THE MOISTURE COEFFICIENT VARIES FROM 0 TO 1.2 ')

      STOP
      END
```

```
THE LAMINA PLY CHARACTERISTICS INITIALLY ARE

THE WIDTH OF THE LAMINATE IS  1.51200
THE NUMBER OF LAMINATES ABOVE AND BELOW THE CRACK IS  3        1


FOR PLY    1 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE   .005400       35.000
E1 AND E2 ARE (MSI)                 18.7000           1.2300
THE POISSON RATIO (1,2) IS         .29200
G OF (1-2),  AND (3-1) ARE -MSI     .8320      .8320

FOR PLY    2 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE   .005400      -35.000
E1 AND E2 ARE (MSI)                 18.7000           1.2300
THE POISSON RATIO (1,2) IS         .29200
G OF (1-2),  AND (3-1) ARE -MSI     .8320      .8320

FOR PLY    3 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE   .005400          .000
E1 AND E2 ARE (MSI)                 18.7000           1.2300
THE POISSON RATIO (1,2) IS         .29200
G OF (1-2),  AND (3-1) ARE -MSI     .8320      .8320

FOR PLY    4 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE   .005400       90.000
E1 AND E2 ARE (MSI)                 18.7000           1.2300
THE POISSON RATIO (1,2) IS         .29200
G OF (1-2),  AND (3-1) ARE -MSI     .8320      .8320

THE STRAIN IS EQUAL TO    .00254000
THE CHANGE IN TEMPERATURE IS   -280.00000
THE COEFFICIENTS SWELLING DUE TO MOISTURE ARE      .00000000      .00556000
THE COEFFICIENTS OF THERMAL EXPANSION ARE      -.000000230      .000014900

THE MOISTURE COEFFICIENT VARIES FROM 0 TO 1.2

WITH THIS LAYUP, THE PLYS ARE COUPLED


      THE FOUR CHARACTERISTIC VALUES ASSOCIATED
       WITH THE 8 DEGREE POLYNOMIAL FOR THE COUPLED CASE ARE

S OF  1 IS EQUAL TO       407.0573682744

S OF  2 IS EQUAL TO       141.1197780418

S OF  3 IS EQUAL TO       116.7332723860

S OF  4 IS EQUAL TO        55.5544207729

THE UNCOUPLED SX (1,2) VALUES ARE      392.22478        106.21737
```

THE UNCOUPLED SY (1,2) VALUES ARE 134.68589 56.25754

THE S VALUES OF THE MEMBRANE ARE 177.38945 66.26466

S1NM AND S2NM ARE EQUAL TO -.0000157292 .0000000000
S1 AND S2 = 134.9324711639578039431 20257 641.50029909958418278794 3089
SX AND SY = 7.0035820814209067139240 9983 33.296655439895957255714 3052

THETA V IS .3932559441 THETA U IS -.09810177423355 29623

NY IS -38.32041690358 NXY IS -61.959633961712929972

THE STRAIN IS EQUAL TO .0025400
THE VALUES OF GT, GI, GII, AND GIII ARE IN IN-LB/IN/IN

% CMOIST	GGG(WHITNEY)	GI	GII	GIII	GI/G(W-T)
MECH. ONLY	.101653935	.0670191	.0346174	.0000174	.6592868
.000	.522939955	.4084740	.1144151	.0000509	.7811106
.050	.488576635	.3794622	.1090658	.0000486	.7766688
.100	.455156902	.3513864	.1037241	.0000464	.7720115
.150	.422680755	.3242463	.0983903	.0000442	.7671187
.200	.391148194	.2980421	.0930641	.0000420	.7619672
.250	.360559220	.2727737	.0877458	.0000397	.7565296
.300	.330913832	.2484412	.0824351	.0000375	.7507731
.350	.302212031	.2250445	.0771323	.0000353	.7446576
.400	.274453816	.2025836	.0718371	.0000331	.7381338
.450	.247639187	.1810586	.0665497	.0000308	.7311387
.500	.221768145	.1604694	.0612701	.0000286	.7235909
.550	.196840689	.1408161	.0559982	.0000264	.7153809
.600	.172856820	.1220986	.0507341	.0000242	.7063566
.650	.149816537	.1043169	.0454777	.0000220	.6962975
.700	.127719840	.0874710	.0402291	.0000197	.6848665
.750	.106566730	.0715610	.0349882	.0000175	.6715139
.800	.086357206	.0565869	.0297550	.0000153	.6552653
.850	.067091269	.0425486	.0245296	.0000131	.6341891
.900	.048768918	.0294461	.0193120	.0000109	.6037875
.950	.031390154	.0172794	.0141021	.0000087	.5504724
1.000	.014954976	.0060486	.0088999	.0000064	.4044538
1.050	-.000536616	-.0042464	.0037055	.0000042	7.9132595
1.100	-.015084621	-.0136055	-.0014811	.0000020	.9019465
1.150	-.028689040	-.0220288	-.0066600	-.0000002	.7678481
1.200	-.041349872	-.0295163	-.0118312	-.0000024	.7138182

THESE ARE THE IN-PLANE INTERLAMINAR SHEAR STRESSES -- SIGMA XZ
THEY ARE FOUND AT VARIOUS MOISTURE CONTENTS

Y LOCATION	MECH ONLY	H=0.0	H=0.2	H=0.4	H=0.6	H=0.8	H=1.0
.80	.06513504	.21416031	.17460962	.13505894	.09550826	.05595758	.01640689
.82	.14545604	.47814372	.38984949	.30155526	.21326102	.12496679	.03667255
.84	.32456173	1.06633353	.86946804	.67260256	.47573707	.27887159	.08200610
.86	.72279559	2.37175315	1.93411286	1.49647257	1.05883228	.62119200	.18355171
.88	1.60209840	5.24186753	4.27581313	3.30975873	2.34370433	1.37764993	.41159552
.90	3.51076178	11.41070230	9.31370571	7.21670913	5.11971255	3.02271596	.92571938
.91	5.15145827	16.64239220	13.59188733	10.54138246	7.49087759	4.44037273	1.38986786
.92	7.47883378	23.94089980	19.57004819	15.19919658	10.82834496	6.45749335	2.08664173
.93	10.67162145	33.68618367	27.57413829	21.46209291	15.35004753	9.23800215	3.12595677
.94	14.79223282	45.68179330	37.47523970	29.26868609	21.06213248	12.85557888	4.64902527
.95	19.47341229	58.01265161	47.76700252	37.52135343	27.27570433	17.03005524	6.78440615
.96	23.13934536	64.48280475	53.47668684	42.47056893	31.46445102	20.45833311	9.45221520
.97	21.18134698	49.34714597	41.81302367	34.27890137	26.74477908	19.21065678	11.67653448
.98	2.06849051	-21.49262058	-15.32190098	-9.15118139	-2.98046180	3.19025779	9.36097738
.99	-56.33255229	-192.21483366	-156.17259550	-120.13035734	-84.08811918	-48.04588102	-12.00364285
1.00	-101.55665215	-68.23277131	-76.30397933	-84.37518734	-92.44639536	-100.51760338	-108.58881140

THESE ARE THE OUT-OF-PLANE INTERLAMINAR SHEAR STRESSES -- SIGMA YZ
THEY ARE FOUND AT VARIOUS MOISTURE CONTENTS

Y LOCATION	MECH ONLY	H=0.0	H=0.2	H=0.4	H=0.6	H=0.8	H=1.0
.80	1.32976655	4.37292262	3.56528382	2.75764502	1.95000623	1.14236743	.33472863
.82	2.97130973	9.77117861	7.96652670	6.16187479	4.35722288	2.55257097	.74791906
.84	6.63941979	21.83410057	17.80150758	13.76891460	9.73632161	5.70372862	1.67113564
.86	14.83663802	48.79268635	39.78092397	30.76916159	21.75739921	12.74563683	3.73387445
.88	33.15858378	109.05518007	88.91263246	68.77008485	48.62753723	28.48498962	8.34244201
.90	74.12869062	243.83823352	198.79834182	153.75845013	108.71855844	63.67866674	18.63877505
.91	110.86070837	364.70855904	297.33899561	229.96943219	162.59986876	95.23030534	27.86074191
.92	165.83595666	545.65327849	444.85249399	344.05170949	243.25092499	142.45014049	41.64935600
.93	248.16757475	816.71124591	665.82431294	514.93737997	364.05044699	213.16351402	62.27658105
.94	371.58510425	1223.12780736	997.13593640	771.14406544	545.15219448	319.16032352	93.16845256
.95	556.84776170	1833.20386401	1494.47103417	1155.73820433	817.00537449	478.27254465	139.53971481
.96	835.49958063	2750.22755443	2242.07591738	1733.92428033	1225.77264328	717.62100623	209.46936917
.97	1255.78433442	4130.35826748	3367.46245574	2604.56664400	1841.67083225	1078.77502051	315.87920877
.98	1892.06540014	6208.29180153	5062.74636672	3917.20093190	2771.65549709	1626.11006228	480.56462747
.99	2859.77789575	9329.28302912	7612.08957918	5894.89612924	4177.70267929	2460.50922935	743.31577941
1.00	4336.01416454	13960.55352851	11405.37993197	8850.20633544	6295.03273890	3739.85914236	1184.68554582

THE LAMINA PLY CHARACTERISTICS INITIALLY ARE

THE WIDTH OF THE LAMINATE IS 1.51200
THE NUMBER OF LAMINATES ABOVE AND BELOW THE CRACK IS 3 1

THE PLYS ARE INPUTTED FROM BOTTOM TO TOP
BUT THE PLY CHARACTERISTICS FROM TOP TO BOTTOM ARE

FOR PLY 1 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE .005400 35.000
E1 AND E2 ARE (MSI) 18.7000 1.2300
THE POISSON RATIO (1,2) IS .29200
G OF (1-2), AND (3-1) ARE -MSI .8320 .8320

FOR PLY 2 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE .005400 .000
E1 AND E2 ARE (MSI) 18.7000 1.2300
THE POISSON RATIO (1,2) IS .29200
G OF (1-2), AND (3-1) ARE -MSI .8320 .8320

FOR PLY 3 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE .005400 -35.000
E1 AND E2 ARE (MSI) 18.7000 1.2300
THE POISSON RATIO (1,2) IS .29200
G OF (1-2), AND (3-1) ARE -MSI .8320 .8320

FOR PLY 4 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE .005400 90.000
E1 AND E2 ARE (MSI) 18.7000 1.2300
THE POISSON RATIO (1,2) IS .29200
G OF (1-2), AND (3-1) ARE -MSI .8320 .8320

THE STRAIN IS EQUAL TO .00254000
THE CHANGE IN TEMPERATURE IS -280.00000
THE COEFFICIENTS SWELLING DUE TO MOISTURE ARE .00000000 .00556000
THE COEFFICIENTS OF THERMAL EXPANSION ARE -.000000230 .000014900

THE MOISTURE COEFFICIENT VARIES FROM 0 TO 1.2

WITH THIS LAYUP, THE PLYS ARE COUPLED

THE FOUR CHARACTERISTIC VALUES ASSOCIATED
WITH THE 8 DEGREE POLYNOMIAL FOR THE COUPLED CASE ARE

S OF 1 IS EQUAL TO 360.7162423543

S OF 2 IS EQUAL TO 136.3961604492

S OF 3 IS EQUAL TO 113.9856584772

S OF 4 IS EQUAL TO 56.0801738692

THE UNCOUPLED SX (1,2) VALUES ARE 352.62874 86.89277

THE UNCOUPLED SY (1,2) VALUES ARE 126.45615 59.53405

THE S VALUES OF THE MEMBRANE ARE 193.07807 70.65819

S1NM AND S2NM ARE EQUAL TO -.0000157292 .0000000000
S1 AND S2 = 134.93247116395780394312O257 641.50029909958418278794 3089
SX AND SY = 7.00358208142090671392409983 33.29665543989595725571 43052

THETA V IS .9784024562 THETA U IS .0000000000000009718

NY IS -38.32041690358 NXY IS -61.95963396171 2929972

THE STRAIN IS EQUAL TO .0025400
THE VALUES OF GT, GI, GII, AND GIII ARE IN IN-LB/IN/IN

% CMOIST	GGG (WHITNEY)	GI	GII	GIII	GI/G (W-T)
MECH. ONLY	.094207177	.0078275	.0863797	.0000000	.0830882
.000	.510289070	.2262277	.2840614	.0000000	.4433324
.050	.476405835	.2055316	.2708742	.0000000	.4314212
.100	.443446720	.1857497	.2576970	.0000000	.4188771
.150	.411411727	.1668819	.2445298	.0000000	.4056324
.200	.380300856	.1489284	.2313725	.0000000	.3916067
.250	.350114106	.1318890	.2182251	.0000000	.3767028
.300	.320851477	.1157637	.2050878	.0000000	.3608016
.350	.292512969	.1005526	.1919603	.0000000	.3437545
.400	.265098582	.0862557	.1788428	.0000000	.3253723
.450	.238608317	.0728730	.1657353	.0000000	.3054085
.500	.213042173	.0604044	.1526377	.0000000	.2835328
.550	.188400151	.0488500	.1395501	.0000000	.2592888
.600	.164682250	.0382098	.1264724	.0000000	.2320215
.650	.141888470	.0284838	.1134047	.0000000	.2007475
.700	.120018811	.0196719	.1003469	.0000000	.1639065
.750	.099073274	.0117741	.0872991	.0000000	.1188427
.800	.079051858	.0047906	.0742613	.0000000	.0606005
.850	.059954563	-.0012788	.0612334	.0000000	-.0213295
.900	.041781389	-.0064340	.0482154	.0000000	-.1539925
.950	.024532337	-.0106751	.0352074	.0000000	-.4351429
1.000	.008207406	-.0140020	.0222094	.0000000	-1.7060144
1.050	-.007193403	-.0164147	.0092213	.0000000	2.2819054
1.100	-.021670092	-.0179132	-.0037569	.0000000	.8266329
1.150	-.035222659	-.0184976	-.0167251	.0000000	.5251615
1.200	-.047851104	-.0181678	-.0296833	.0000000	.3796734

THESE ARE THE IN-PLANE INTERLAMINAR SHEAR STRESSES -- SIGMA XZ
THEY ARE FOUND AT VARIOUS MOISTURE CONTENTS

Y LOCATION	MECH ONLY	H=0.0	H=0.2	H=0.4	H=0.6	H=0.8	H=1.0
.80	.07771543	.25410395	.20728704	.16047014	.11365323	.06683632	.02001942
.82	.17995738	.58840154	.47999260	.37158366	.26317472	.15476578	.04635684
.84	.41625242	1.36100874	1.11025224	.85949573	.60873922	.35798272	.10722621
.86	.96042737	3.14028217	2.56170677	1.98313136	1.40455595	.82598055	.24740514
.88	2.20343709	7.20451590	5.87713337	4.54975083	3.22236829	1.89498576	.56760322
.90	4.98862540	16.31116726	13.30594680	10.30072635	7.29550589	4.29028544	1.28506498
.91	7.43148427	24.29851376	19.82167961	15.34484547	10.86801132	6.39117717	1.91434302
.92	10.93853873	35.76543047	29.17589574	22.58636100	15.99682627	9.40729153	2.81775680
.93	15.78922990	51.62559809	42.11393649	32.60227488	23.09061327	13.57895167	4.06729006
.94	22.04642211	72.08456237	58.80347722	45.52239206	32.24130691	18.96022176	5.67913661
.95	28.96545539	94.70752965	77.25831827	59.80910689	42.35989552	24.91068414	7.46147276
.96	33.46881767	109.43204584	89.26994356	69.10784128	48.94573900	28.78363672	8.62153445
.97	26.48582400	86.59994907	70.64450369	54.68905831	38.73361292	22.77816754	6.82272216
.98	-14.02438527	-45.85513558	-37.40664203	-28.95814848	-20.50965493	-12.06116138	-3.61266782
.99	-113.31362224	-370.49834340	-302.23657021	-233.97479701	-165.71302381	-97.45125061	-29.18947741
1.00	85.42214908	279.30238304	227.84283872	176.38329439	124.92375006	73.46420574	22.00466141

THESE ARE THE OUT-OF-PLANE INTERLAMINAR SHEAR STRESSES -- SIGMA YZ
THEY ARE FOUND AT VARIOUS MOISTURE CONTENTS

Y LOCATION	MECH ONLY	H=0.0	H=0.2	H=0.4	H=0.6	H=0.8	H=1.0
.80	.94069588	3.07576670	2.50907783	1.94238895	1.37570008	.80901121	.24232233
.82	2.18029521	7.12884953	5.81540800	4.50196647	3.18852495	1.87508342	.56164190
.84	5.05374241	16.52407852	13.47963063	10.43518274	7.39073485	4.34628695	1.30183906
.86	11.71605439	38.30765144	31.24972996	24.19180849	17.13388701	10.07596554	3.01804406
.88	27.17098279	88.84019338	72.47199838	56.10380339	39.73560839	23.36741339	6.99921839
.90	63.06243899	206.19347184	168.20374189	130.21401195	92.22428201	54.23455207	16.24482213
.91	96.12643272	314.30187627	256.39391588	198.48595549	140.57799510	82.67003472	24.76207433
.92	146.61442086	479.38102208	391.05836372	302.73570537	214.41304702	126.09038866	37.76773031
.93	223.81654951	731.80663700	596.97629411	462.14595122	327.31560833	192.48526545	57.65492256
.94	342.10566992	1118.57322594	912.48379743	706.39436892	500.30494041	294.21551190	88.12608339
.95	523.86709173	1712.87340224	1397.28825109	1081.70309995	766.11794880	450.53279765	134.94764651
.96	804.27369732	2629.71094384	2145.20477742	1660.69861100	1176.19244458	691.68627816	207.18011174
.97	1239.16420856	4051.66013925	3305.16960716	2558.67907507	1812.18854298	1065.69801089	319.20747880
.98	1917.53432739	6269.70771612	5114.55716346	3959.40661080	2804.25605814	1649.10550548	493.95495282
.99	2971.17774803	9714.77578612	7924.89512080	6135.01445547	4345.13379015	2555.25312483	765.37245950
1.00	4429.35526207	14482.53753133	11814.22953505	9145.92153878	6477.61354250	3809.30554622	1140.99754994

THE LAMINA PLY CHARACTERISTICS INITIALLY ARE

THE WIDTH OF THE LAMINATE IS 1.51200
THE NUMBER OF LAMINATES ABOVE AND BELOW THE CRACK IS 2 2

FOR PLY 1 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE .005400 30.000
E1 AND E2 ARE (MSI) 18.7000 1.2300
THE POISSON RATIO (1,2) IS .29200
G OF (1-2), AND (3-1) ARE -MSI .8320 .8320

FOR PLY 2 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE .005400 -60.000
E1 AND E2 ARE (MSI) 18.7000 1.2300
THE POISSON RATIO (1,2) IS .29200
G OF (1-2), AND (3-1) ARE -MSI .8320 .8320

FOR PLY 3 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE .005400 75.000
E1 AND E2 ARE (MSI) 18.7000 1.2300
THE POISSON RATIO (1,2) IS .29200
G OF (1-2), AND (3-1) ARE -MSI .8320 .8320

FOR PLY 4 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE .005400 -15.000
E1 AND E2 ARE (MSI) 18.7000 1.2300
THE POISSON RATIO (1,2) IS .29200
G OF (1-2), AND (3-1) ARE -MSI .8320 .8320

THE STRAIN IS EQUAL TO .00254000
THE CHANGE IN TEMPERATURE IS -280.00000
THE COEFFICIENTS SWELLING DUE TO MOISTURE ARE .00000000 .00556000
THE COEFFICIENTS OF THERMAL EXPANSION ARE -.000000230 .000014900

THE MOISTURE COEFFICIENT VARIES FROM 0 TO 1.2

WITH THIS LAYUP, THE PLYS ARE COUPLED

THE FOUR CHARACTERISTIC VALUES ASSOCIATED
WITH THE 8 DEGREE POLYNOMIAL FOR THE COUPLED CASE ARE

S OF 1 IS EQUAL TO 202.3666962066

S OF 2 IS EQUAL TO 150.2447234209

S OF 3 IS EQUAL TO 96.3990023366

S OF 4 IS EQUAL TO 72.1855545293

THE UNCOUPLED SX (1,2) VALUES ARE 178.08581 90.73315

THE UNCOUPLED SY (1,2) VALUES ARE 137.79324 77.57481

THE S VALUES OF THE MEMBRANE ARE 107.30164 75.55664

S1NM AND S2NM ARE EQUAL TO -.0013126127 .0012407192
S1 AND S2 = 145.712738394363145201234304 261.712596265395612237542175
SX AND SY = 35.575685116743911519014787 9 60.958554967731431231955331 2

THETA V IS .2402517909 THETA U IS 2.07343844702180178 61

NY IS -35.48501616832 NXY IS -61.461850910940726086

THE STRAIN IS EQUAL TO .0025400
THE VALUES OF GT, GI, GII, AND GIII ARE IN IN-LB/IN/IN

% CMOIST	GGG(WHITNEY)	GI	GII	GIII	GI/G(W-T)
MECH. ONLY	.174065036	.0136734	.0093651	.1510265	.0785535
.000	.288599244	.0062276	.0365562	.2458155	.0215787
.050	.277169512	.0030909	.0347433	.2393353	.0111516
.100	.266289514	.0005010	.0329323	.2328563	.0018813
.150	.255959252	-.0015422	.0311230	.2263784	-.0060250
.200	.246178724	-.0030385	.0293156	.2199016	-.0123427
.250	.236947932	-.0039880	.0275100	.2134260	-.0168309
.300	.228266874	-.0043908	.0257062	.2069515	-.0192354
.350	.220135552	-.0042468	.0239042	.2004781	-.0192916
.400	.212553964	-.0035559	.0221040	.1940059	-.0167296
.450	.205522112	-.0023183	.0203057	.1875347	-.0112801
.500	.199039994	-.0005339	.0185092	.1810647	-.0026824
.550	.193107612	.0017973	.0167144	.1745959	.0093072
.600	.187724965	.0046753	.0149215	.1681281	.0249050
.650	.182892052	.0081001	.0131304	.1616615	.0442888
.700	.178608875	.0120716	.0113412	.1551961	.0675870
.750	.174875432	.0165900	.0095537	.1487317	.0948676
.800	.171691725	.0216552	.0077681	.1422685	.1261282
.850	.169057753	.0272671	.0059842	.1358064	.1612888
.900	.166973515	.0334259	.0042022	.1293454	.2001866
.950	.165439013	.0401314	.0024220	.1228856	.2425751
1.000	.164454245	.0473837	.0006436	.1164269	.2881270
1.050	.164019213	.0551828	-.0011329	.1099693	.3364412
1.100	.164133916	.0635287	-.0029077	.1035129	.3870542
1.150	.164798353	.0724214	-.0046806	.0970576	.4394548
1.200	.166012526	.0818609	-.0064517	.0906034	.4931008

THESE ARE THE IN-PLANE INTERLAMINAR SHEAR STRESSES -- SIGMA XZ
THEY ARE FOUND AT VARIOUS MOISTURE CONTENTS

Y LOCATION	MECH ONLY	H=0.0	H=0.2	H=0.4	H=0.6	H=0.8	H=1.0
.80	.05676172	.13383978	.11323003	.09262028	.07201053	.05140078	.03079103
.82	.16404039	.38262997	.32416132	.26569268	.20722403	.14875538	.09028674
.84	.47716558	1.09647535	.93073896	.76500256	.59926616	.43352976	.26779336
.86	1.40027761	3.15249905	2.68324457	2.21399008	1.74473560	1.27548112	.80622663
.88	4.15778238	9.10572208	7.77928135	6.45284061	5.12639987	3.79995914	2.47351840
.90	12.53662711	26.47078249	22.72981406	18.98884564	15.24787721	11.50690879	7.76594036
.91	21.92391761	45.27705638	39.00125296	32.72544955	26.44964613	20.17384271	13.89803930
.92	38.55372890	77.65399817	67.13410301	56.61420784	46.09431268	35.57441752	25.05452235
.93	68.22669529	133.62809212	116.00701342	98.38593472	80.76485602	63.14377733	45.52269863
.94	121.62386648	230.94006045	201.43654971	171.93303897	142.42952823	112.92601749	83.42250675
.95	218.74381651	401.50598451	352.07750811	302.64903171	253.22055531	203.79207891	154.36360251
.96	398.03248718	704.47704597	621.39034887	538.30365177	455.21695468	372.13025758	289.04356048
.97	736.82244009	1255.70230253	1114.59938720	973.49647187	832.39355653	691.29064120	550.18772587
.98	1403.24313766	2305.35974007	2059.22700449	1813.09426892	1566.96153334	1320.82879776	1074.69606219
.99	2809.86570653	4480.13776195	4023.01286260	3565.88796325	3108.76306390	2651.63816455	2194.51326520
1.00	6138.30786283	9650.30926292	8687.60293079	7724.89659866	6762.19026652	5799.48393439	4836.77760226

THESE ARE THE OUT-OF-PLANE INTERLAMINAR SHEAR STRESSES -- SIGMA YZ
THEY ARE FOUND AT VARIOUS MOISTURE CONTENTS

Y LOCATION	MECH ONLY	H=0.0	H=0.2	H=0.4	H=0.6	H=0.8	H=1.0
.80	.16585980	.40442354	.34069983	.27697611	.21325240	.14952869	.08580498
.82	.46943498	1.14791593	.96669931	.78548269	.60426608	.42304946	.24183285
.84	1.32621331	3.25626779	2.74082736	2.22538692	1.70994649	1.19450605	.67906562
.86	3.73681543	9.22891300	7.76243950	6.29596600	4.82949250	3.36301900	1.89654550
.88	10.48846496	26.12319742	21.94948048	17.77576354	13.60204660	9.42832966	5.25461272
.90	29.26971559	73.80117525	61.91744992	50.03372460	38.14999927	26.26627395	14.38254862
.91	48.74474315	123.91308072	103.85796283	83.80284494	63.74772705	43.69260916	23.63749127
.92	80.94539936	207.83462309	173.98914281	140.14366253	106.29818225	72.45270198	38.60722170
.93	133.89723658	348.06735925	290.95937200	233.85138474	176.74339749	119.63541023	62.52742298
.94	220.24445378	581.52733260	485.22950028	388.93166795	292.63383563	196.33600331	100.03817099
.95	359.01194536	967.47546061	805.37239730	643.26933399	481.16627068	319.06320737	156.96014406
.96	575.60439987	1596.12349454	1324.41972889	1052.71596324	781.01219759	509.30843195	237.60466630
.97	890.96357501	2584.87680021	2134.31704487	1683.75728954	1233.19753421	782.63777888	332.07802355
.98	1261.01507117	3998.83788332	3271.80265391	2544.76742450	1817.73219508	1090.69696567	363.66173626
.99	1302.72815835	5415.47562712	4327.26291739	3239.05020766	2150.83749794	1062.62478821	-25.58792151
1.00	-883.85251960	3927.98882475	2670.82514260	1413.66146045	156.49777830	-1100.66590385	-2357.82958600

THE LAMINA PLY CHARACTERISTICS INITIALLY ARE

THE WIDTH OF THE LAMINATE IS 1.51200
THE NUMBER OF LAMINATES ABOVE AND BELOW THE CRACK IS 2 2

FOR PLY 1 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE .005400 -35.000
E1 AND E2 ARE (MSI) 18.7000 1.2300
THE POISSON RATIO (1,2) IS .29200
G OF (1-2), AND (3-1) ARE -MSI .8320 .8320

FOR PLY 2 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE .005400 55.000
E1 AND E2 ARE (MSI) 18.7000 1.2300
THE POISSON RATIO (1,2) IS .29200
G OF (1-2), AND (3-1) ARE -MSI .8320 .8320

FOR PLY 3 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE .005400 10.000
E1 AND E2 ARE (MSI) 18.7000 1.2300
THE POISSON RATIO (1,2) IS .29200
G OF (1-2), AND (3-1) ARE -MSI .8320 .8320

FOR PLY 4 THE SUBLAMINATE HAS THESE PROPERTIES
THE THICKNESS AND THETA VALUES ARE .005400 -80.000
E1 AND E2 ARE (MSI) 18.7000 1.2300
THE POISSON RATIO (1,2) IS .29200
G OF (1-2), AND (3-1) ARE -MSI .8320 .8320

THE STRAIN IS EQUAL TO .00254000
THE CHANGE IN TEMPERATURE IS -280.00000
THE COEFFICIENTS SWELLING DUE TO MOISTURE ARE .00000000 .00556000
THE COEFFICIENTS OF THERMAL EXPANSION ARE -.000000230 .000014900

THE MOISTURE COEFFICIENT VARIES FROM 0 TO 1.2

WITH THIS LAYUP, THE PLYS ARE COUPLED

THE FOUR CHARACTERISTIC VALUES ASSOCIATED
WITH THE 8 DEGREE POLYNOMIAL FOR THE COUPLED CASE ARE

S OF 1 IS EQUAL TO 233.9388572236

S OF 2 IS EQUAL TO 156.9014619788

S OF 3 IS EQUAL TO 115.2992565645

S OF 4 IS EQUAL TO 48.0575100718

THE UNCOUPLED SX (1.2) VALUES ARE 186.41344 96.42124

THE UNCOUPLED SY (1.2) VALUES ARE 128.78572 49.37034

THE S VALUES OF THE MEMBRANE ARE 119.36060 57.91987

S1NM AND S2NM ARE EQUAL TO -.0007221149 -.0007139234
S1 AND S2 = 143.691863396728751259452089 285.264127104852281159045526
SX AND SY = 32.1436979546791447820272102 62.2049978939766992050711339

THETA V IS .4347536621 THETA U IS -1.7344415408201789576

NY IS -54.36619889946 NXY IS 45.618657445050692111

THE STRAIN IS EQUAL TO .0025400
THE VALUES OF GT, GI, GII, AND GIII ARE IN IN-LB/IN/IN

% CMOIST	GGG (WHITNEY)	GI	GII	GIII	GI/G (W-T)
MECH. ONLY	.131525889	.0207693	.0219689	.0887877	.1579102
.000	.394637005	.1683076	.0866210	.1397084	.4264871
.050	.369347077	.1508205	.0823114	.1362152	.4083435
.100	.345184223	.1344556	.0780061	.1327225	.3895185
.150	.322148443	.1192131	.0737050	.1292304	.3700564
.200	.300239735	.1050929	.0694082	.1257387	.3500298
.250	.279458102	.0920949	.0651156	.1222475	.3295483
.300	.259803542	.0802193	.0608274	.1187569	.3087691
.350	.241276055	.0694660	.0565434	.1152667	.2879108
.400	.223875642	.0598350	.0522636	.1117771	.2672688
.450	.207602302	.0513262	.0479882	.1082879	.2472335
.500	.192456036	.0439398	.0437170	.1047992	.2283110
.550	.178436843	.0376757	.0394500	.1013111	.2111431
.600	.165544724	.0325339	.0351874	.0978235	.1965264
.650	.153779678	.0285144	.0309290	.0943363	.1854237
.700	.143141706	.0256172	.0266748	.0908497	.1789638
.750	.133630807	.0238423	.0224250	.0873635	.1784190
.800	.125246982	.0231897	.0181794	.0838779	.1851516
.850	.117990230	.0236594	.0139381	.0803928	.2005198
.900	.111860552	.0252514	.0097010	.0769081	.2257398
.950	.106857947	.0279657	.0054683	.0734240	.2617089
1.000	.102982416	.0318023	.0012397	.0699404	.3088128
1.050	.100233958	.0367612	-.0029845	.0664573	.3667539
1.100	.098612573	.0428424	-.0072045	.0629747	.4344517
1.150	.098118262	.0500459	-.0114202	.0594925	.5100571
1.200	.098751025	.0583717	-.0156316	.0560109	.5910999

THESE ARE THE IN-PLANE INTERLAMINAR SHEAR STRESSES -- SIGMA XZ
THEY ARE FOUND AT VARIOUS MOISTURE CONTENTS

Y LOCATION	MECH ONLY	H=0.0	H=0.2	H=0.4	H=0.6	H=0.8	H=1.0
.80	-.57590259	-1.81324378	-1.48462505	-1.15600631	-.82738757	-.49876884	-.17015010
.82	-1.15500766	-3.63542168	-2.97665842	-2.31789515	-1.65913188	-1.00036862	-.34160535
.84	-2.31809701	-7.29018842	-5.96965162	-4.64911482	-3.32857801	-2.00804121	-.68750441
.86	-4.66121070	-14.62677009	-11.97991892	-9.33306775	-6.68621658	-4.03936541	-1.39251424
.88	-9.41933953	-29.38693329	-24.08301174	-18.77909019	-13.47516863	-8.17124708	-2.86732552
.90	-19.28114635	-59.25567119	-48.63455785	-38.01344451	-27.39233116	-16.77121782	-6.15010448
.91	-27.86803356	-84.38953087	-69.36790478	-54.34627868	-39.32465259	-24.30302649	-9.28140039
.92	-40.76807828	-120.61801802	-99.38712650	-78.15623498	-56.92534347	-35.69445195	-14.46356043
.93	-60.74791755	-173.40082769	-143.42661283	-113.45239797	-83.47818312	-53.50396826	-23.52975340
.94	-93.01797598	-251.60892083	-209.36180513	-167.11468943	-124.86757373	-82.62045803	-40.37334233
.95	-148.04567478	-370.59922298	-311.19653492	-251.79384686	-192.39115881	-132.98847075	-73.58578269
.96	-248.37373255	-559.47456937	-476.16166609	-392.84876280	-309.53585952	-226.22295624	-142.91005296
.97	-446.98922674	-881.36980878	-764.37575049	-647.38169221	-530.38763392	-413.39357564	-296.39951735
.98	-885.27484698	-1503.21141125	-1335.11941983	-1167.02742841	-998.93543699	-830.84344557	-662.75145415
.99	-2013.12212234	-2986.72662307	-2717.77038626	-2448.81414945	-2179.85791263	-1910.90167582	-1641.94543901
1.00	-5568.98832566	-7674.18805110	-7084.53781995	-6494.88758880	-5905.23735765	-5315.58712650	-4725.93689534

THESE ARE THE OUT-OF-PLANE INTERLAMINAR SHEAR STRESSES -- SIGMA YZ
THEY ARE FOUND AT VARIOUS MOISTURE CONTENTS

Y LOCATION	MECH ONLY	H=0.0	H=0.2	H=0.4	H=0.6	H=0.8	H=1.0
.80	3.27617888	10.31723571	8.44724614	6.57725656	4.70726699	2.83727741	.96728784
.82	6.56753777	20.68268192	16.93393167	13.18518142	9.43643117	5.68768092	1.93893067
.84	13.16491904	41.46150260	33.94640184	26.43130108	18.91620032	11.40109956	3.88599881
.86	26.38654493	83.11303161	68.04745804	52.98188448	37.91631091	22.85073735	7.78516379
.88	52.87019183	166.59261716	136.39009616	106.18757515	75.98505414	45.78253314	15.58001213
.90	105.84680471	333.84310700	273.29260785	212.74210869	152.19160953	91.64111038	31.09061122
.91	149.66289213	472.50148909	386.76451483	301.02754057	215.29056630	129.55359204	43.81661778
.92	211.43474532	668.58894580	547.18476643	425.78058706	304.37640769	182.97222833	61.56804896
.93	298.27565716	945.66979161	773.75215728	601.83452295	429.91688862	257.99925428	86.08161995
.94	419.76910709	1336.64857623	1093.18597910	849.72338197	606.26078483	362.79818770	119.33559056
.95	588.25330316	1886.88626457	1542.09703881	1197.30781306	852.51858731	507.72936156	162.94013580
.96	817.80756656	2656.91555150	2168.73092336	1680.54629522	1192.36166707	704.17703893	215.99241079
.97	1117.61798402	3719.40898556	3029.03467417	2338.66036279	1648.28605140	957.91174002	267.53742864
.98	1460.19459243	5123.16377041	4151.96405479	3180.76433917	2209.56462355	1238.36490793	267.16519231
.99	1628.54905698	6683.48801754	5345.79387584	4008.09973414	2670.40559244	1332.71145074	-4.98269096
1.00	446.83474448	6857.50214542	5171.81141375	3486.12068207	1800.42995039	114.73921872	-1570.95151296