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Dynamic Reasoning in a Knowledge-based System

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Abstract

Any space based system, whether it is a robot arm assembling parts in the space or an onboard system monitoring the space station, has to react to *changes* which cannot be foreseen while on earth. As a result, apart from having domain-specific knowledge as in current expert systems, a space based AI system should also have *general principles of change*. This paper presents a modal logic which can not only represent change but also reason with it. Three primitive operations, *expansion*, *contraction* and *revision* are introduced and axioms which specify how the knowledge base should change when the external world changes are also specified. Accordingly the notion of *dynamic reasoning* is introduced, which unlike the existing forms of reasoning, provide general principles of change. Dynamic reasoning is based on two main principles, namely *minimize change* and *maximize coherence*. A possible-world semantics which incorporates the above two principles is also discussed. The paper concludes by discussing how the dynamic reasoning system can be used to specify actions and hence form an integral part of an autonomous reasoning and planning system.

1. Introduction

Due to the prohibitive costs of manned missions to outer space, unmanned explorations of distant objects is the only credible alternative. As man starts exploring deeper into space, the need for advanced *autonomous systems* becomes inevitable. Space based systems need to be autonomous with respect to two important aspects, namely, a) control and b) reasoning. As the communication delay between autonomous vehicles in outer space and earth is beyond acceptable limits, such vehicles should have independent control of their movement. For much the same reasons, they should also have independent decision making skills. Unless they are equipped with such capabilities they are unlikely to survive in an hostile environment. Also it is very difficult, if not impossible to preprogram such capabilities. This paper discusses only the reasoning capabilities of an autonomous system and not its control aspects.

Two main requirements for an autonomous reasoning system are

a. that the system be *dynamic*, i.e. any changes in the external environment should be immediately reflected in the systems view of the environment. In other words the system should be capable of *changing* its *knowledge-base* automatically based on the changes in the external environment.

b. that the system be *reactive* [8] i.e. it should be capable of changing its focus and pursuing an alternative goal if and when it is required. In other words the system should be capable of *changing* its *goals* automatically based on the changes in the external environment.

An example would help to illustrate the dynamic and reactive aspects of an autonomous, space based AI system. Consider a robot whose main goal is to explore the terrain of an hostile environment. During the exploration phase the sensors of the robot will constantly keep feeding information about the external environment. Some of this information might be in conflict with the robot's existing knowledge. The responsibility of the *dynamic reasoning* component of the robot is to accommodate this information from the external world with as little damage as possible to the current knowledge base. During the exploration phase if there is a sudden unexpected event, say a volcanic eruption, then the robot must be capable of *reacting* to this situation by abandoning its goal of exploration and instead acquire the goal of survival and try to achieve it. This process of changing the focus of attention or changing the goals is an integral part of the *reactive* component of the reasoning system. In [8] and [7] Georgeff et. al. describe the Procedural Reasoning System (PRS) and discuss its use in an autonomous mobile robot and in a diagnostic system for the space shuttle.

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It is quite clear that any space based system, whether it is a robot arm assembling parts in the space or an onboard system monitoring the space station, it has to react to *changes* which cannot be foreseen while on earth. As a result, apart from having domain-specific knowledge as in current expert systems, a space based AI system should also have *general principles of change*. Unfortunately, standard first-order logic and deductive reasoning have very little to say about dynamic reasoning. This is because most of the reasoning which is done using first-order logic has been restricted to computing the logical consequences of sentences, which can be categorized as *static reasoning*. This paper describes a modal logic and illustrates how dynamic reasoning can be carried out in this logic. The dynamic reasoning described in this paper incorporates two general, intuitive principles of change, namely, *minimize change* and *maximize coherence*. The axiomatization of this dynamic reasoning system is inspired by the postulates of theory change, first proposed by Gardenfors et. al. [6, 1]. Although, most of this paper will concentrate on the dynamic reasoning system, it also discusses how to integrate the dynamic reasoning system with a reactive planning system, to obtain a full-fledged embedded system capable of reasoning and planning autonomously in any hostile environment.

2. Statics of Belief Systems

Traditional knowledge-based systems, typically have two main modules:

a. Knowledge Base (KB) - which contains domain specific information in some formal language, which is normally first-order logic or a syntactic variant of first-order logic with procedural attachments.

b. Inference Engine (IE) - which performs *static* reasoning on the knowledge base.

The KB is taken to be a set of beliefs about a particular problem domain and the IE is capable of answering queries regarding the belief system. Such knowledge bases are essentially static, as they cannot represent or reason about how the KB changes.

This section will present a KB or belief system, which can model the evolving nature of knowledge bases. The static aspects of the belief system will be dealt in this section and the dynamic aspects of the belief system will be postponed to the next section.

The formal language $\mathcal{L}_1$ under consideration is a modal logic of beliefs. The objects of beliefs will be taken to be first-order formulas with equality [14]. The beliefs will be taken to be time-dependent. Thus the formula `BELIEF(t, ϕ)` represents the belief of the agent (or robot or the AI system) at time point `t`, that the formula ϕ is the case. For example, the formula `BELIEF(13:00, power_left(2))`, might be the belief of the robot that at 13:00 hrs, the number of hours of power left for it is 2 hours. In the language $\mathcal{L}_1$, quantifying temporal terms into the scope of the modal operator `BELIEF` is allowed, but quantifying individual terms is not allowed.

Usually the semantics of the `BELIEF` operator is given in terms of a possible-world accessibility relation, $\mathcal{B}$, which maps a possible world to a set of possible worlds. In the language $\mathcal{L}_1$ the relation $\mathcal{B}$, maps a possible world *at a given instant of time* to a set of possible worlds *at that given instant of time*. Also the satisfaction is with respect to a world *at a particular instant of time*. Thus if KI, is a Kripke interpretation of modal logic, and TA the term assignment, then the satisfaction of belief formulas with a particular variable assignment VA, is given as follows,

KI, w, TA(t) $\models$ `BELIEF(t, ϕ)`[VA] iff for all w′ such that $\mathcal{B}$(w, TA(t), w′), KI, w′, t $\models$ ϕ[VA].

The axiomatization for the above time-dependent belief system, called the B-modal system, are the axioms of first-order temporal logic and the standard KD45 axiomatization for beliefs [10, 17]. In a KD45-modal system the $\mathcal{B}$ relation has to be serial, transitive and euclidean. The class of models of the time-dependent belief system, whose $\mathcal{B}$ relation satisfies the above conditions are called $\mathcal{B}$-models. The soundness and completeness of the time-dependent belief system can be stated as follows,

Theorem 2.1: The B-modal system is sound and complete with respect to the class of $\mathcal{B}$-models.

Proof: The proof of the above theorem is straightforward and is proved in [17].

The `BELIEF` modal operator can also be treated as a self-belief operator as in autoepistemic (AE) logic [15]. In AE logic the agent reasons about his own beliefs and lack of beliefs. An agent believes a sentence if it is contained in his set of beliefs at that current instant of time. He does not believe it, if it is not contained in his set of beliefs. Thus AE logic is complete with respect to the beliefs and non-beliefs of the agent. A Kripke interpretation where the beliefs are treated as self-belief operators will be called an autoepistemic Kripke interpretation or AKI.

3. Semantics of Dynamic Belief Systems

In most of the commercially available knowledge-based systems or expert system shells, any changes to the KB have to be done manually. For a space-based AI system this solution is unacceptable for two main reasons,

1. the time delay for updates will be too costly and might endanger the mission and
2. maintaining the integrity of the KB will be difficult, especially if the input data is inconsistent with the existing KB.

Thus for any space-based AI system, automatic updates of the KB is a must. Two of the most popular systems for updating knowledge bases are the Truth Maintenance System (TMS) [4] and Assumption-based TMS (ATMS) [11]. TMS keeps track of how each and every formula was inferred, which are called *justifications* and incrementally modifies these justifications whenever there is an update. ATMS keeps track of the premises which are used in deriving a formula, called *assumptions* , and incrementally modifies them. However, in both these mechanisms the book-keeping required may not only be space consuming, but might also turn out to be time-consuming. Thus for a space-based AI system such mechanisms may be unsuitable.

In this section three basic *dynamic* operations, namely, *expansion*, *contraction* and *revision* are introduced. These operators are then used for reasoning about changing knowledge bases. The semantics of these operations are discussed in this section and the dynamic reasoning system is discussed in the next section.

Consider the situation where the agent (or robot) receives a first-order sentence ϕ from the external world. The relationships between this formula and the set of beliefs of the system at time `t`, can be enumerated as follows,

1. the agent believes in ϕ at `t`.
2. the agent believes in $\neg\phi$ at `t`.
3. the agent does not believe in ϕ nor $\neg\phi$ at `t` and hence the agent is *agnostic* about ϕ.
4. the agent does believes in ϕ and $\neg\phi$ at `t` and hence the agent has *inconsistent* beliefs about ϕ.

Inconsistent belief states are disallowed and hence, relation 4) above does not hold. The process by which an agent moves from one of the three regions(1, 2, or 3) to any of the other two is called the *dynamics* of the system. As there are three states and each transition involves two states there are totally 3^2 different transitions. Ignoring the trivial transitions of remaining in the same state six different transitions are left.

• **Expansion:** Agnostic state $\rightarrow$ Belief in ϕ.
• **Contraction**: Belief in $\phi \rightarrow$ Agnostic state.
• **Revision**: Belief in $\neg\phi \rightarrow$ Belief in ϕ.
• **N-Expansion**: Agnostic state $\rightarrow$ Belief in $\neg\phi$
• **N-Contraction**: Belief in $\neg\phi \rightarrow$ Agnostic state
• **N-Revision**: Belief in $\phi \rightarrow$ belief in $\neg\phi$.

The terminology and approach is an extension of the work done by Gardenfors and others [6, 1]. While their approach is non-modal and at the meta-level, we introduce modal operators and carry out the analysis at the object level.

The language of the time-dependent belief system is extended to the language $\mathcal{L}_2$, by introducing three *dynamic modal operators*- `EXPAND`, `CONTRACT` and `REVISE` to denote expansion, contraction and revision respectively. Thus `EXPAND(t,` ϕ`, u)` is read as 'the expansion by the agent at `t` with respect to ϕ is `u`', where ϕ is a first-order sentence and `t` and `u` are temporal terms (constants or variables). There is no need for any modal operators for N-expansion and N-contraction as they can be expressed by the modal operators `EXPAND` and `CONTRACT` respectively. Strictly speaking there is no need for the modal operator `REVISE` also as it can be defined using `EXPAND` and `CONTRACT`. Nesting of these operators are not allowed. As AE belief systems are uniquely determined by the first-order formulas, it is sufficient to consider the expansion, contraction and revision of first-order sentences alone. For example, expanding with respect to a belief sentence in an AE belief system is equivalent to expanding with respect to the object of the belief, and expanding with respect to a non-belief sentence is equivalent to contracting with respect to the object of the non-belief.

Given a set of beliefs at some particular instant of time t, and the nature of change, namely, expansion, contraction or revision the set of beliefs at the next time instant can be constructed and the semantics of the modal operators EXPAND, CONTRACT and REVISE are definable using this construction [17]. Alternatively, the semantics of the dynamic modal operators, can be given based on the autoepistemic Kripke interpretation, AKI. The latter approach is followed in this paper. The semantics of dynamic operators are based on *selection functions*, which select some possible worlds as being *closer* to the current world than the others [2, 12]. When the agent performs expansion or contraction, he is said to move into one of these closer worlds and designate these worlds as the worlds of the next time instant(s).

Consider an agent with a proposition p in some world w at time t1. Now the agent wants to expand with respect to the formula r. He can do so in many ways. He can move into a time point t2, where p, r are true or a time point t3 where only r is true or a time point t4 where p, q, r are true and so on. Amongst the different alternatives the agent should *choose* only some of them, based on certain criteria. The principles of minimal change and maximal coherence will be used in selecting the alternatives. According to these criteria, the selection function for expansion, or *expansion function*, denoted by $\mathcal{E}$, should choose the time point t2, where p, r are true. The other time points are also accessible from t1 but all of them have to pass through the time point t2. Thus the time point t3 can be obtained from t2 after a contraction with respect to p. Similarly, the time point t4 can be obtained from t2 after an expansion with respect to q. Also it should be noted that the expansion function $\mathcal{E}$ and the contraction function $\mathcal{C}$, depend on the current time point, the current world and the proposition with respect to which the expansion/contraction is performed. The result of the selection function is a set of time points.

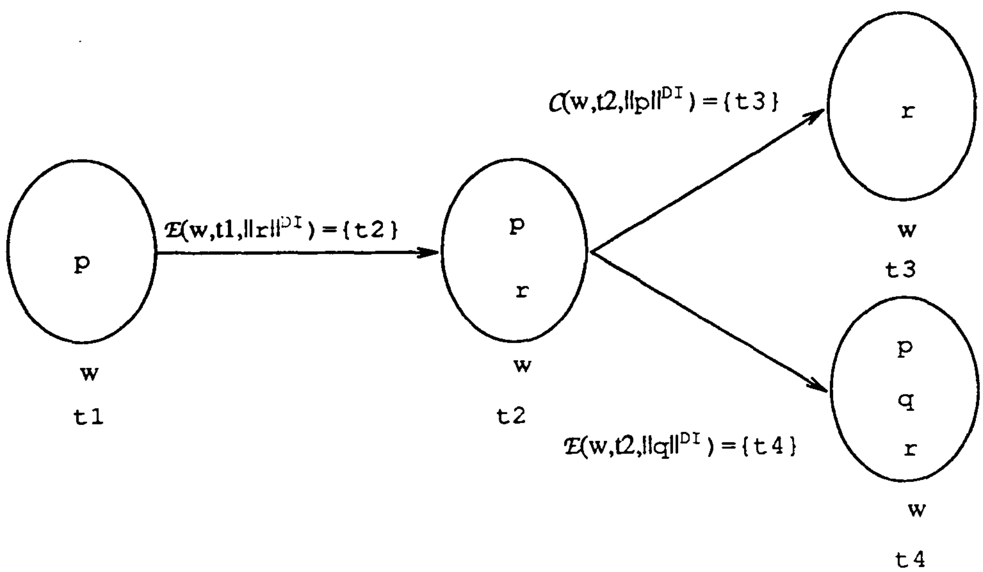

Figure 1. Expansion and Contraction Functions

Figure 1 shows the possible relationships between the different time points.

The selection functions discussed above can be generalized to handle first-order sentences, instead of just propositional formulas. The semantics of the dynamic belief system is given below.
Definition: The *dynamic interpretation* is a tuple, D = <$\mathcal{E}$, $\mathcal{C}$, AKI>, where

- $\mathcal{E}$ and $\mathcal{C}$ are functions which map, W (set of worlds), TU (set of time points), and $2^{2^{W}}$ (set of set of worlds) to a set of time points or 2^{TU}.
- AKI is an autoepistemic Kripke interpretation.

The satisfaction of belief formulas are as described before. Satisfiability of the dynamic modal operators is as follows,
Definition: A dynamic interpretation DI = <$\mathcal{E}$, $\mathcal{C}$, AKI>, satisfies a well formed formula ϕ, at world w and time t (written as DI, w, t $\models \phi$) given the following conditions,

1. DI, w, t $\models$ `EXPAND(t, ϕ, u)` iff $u \in \mathcal{E}(w, t, \|\phi\|^{DI})$, where ϕ is a first-order sentence.

2. DI, w, t $\models$ `CONTRACT(t, ϕ, u)` iff $u \in \mathcal{C}(w, t, \|\phi\|^{DI})$, where ϕ is a first-order sentence.

3. DI, w, t $\models$ `REVISE(t, ϕ, u)` iff DI, w, t $\models$ `CONTRACT(t, ¬ϕ, v)` and DI, w, v $\models$ `EXPAND(v, ϕ, u)`, where ϕ is a first-order sentence.

The notation $\|\phi\|^{DI}$ stands for all the worlds of the interpretation DI, which satisfies ϕ. More formally, $\|\phi\|^{DI} = \{w \mid DI, w, t \models \phi$ for any $t \in TU\}$. Condition 3 is an analog of the Levi identity, which is a theorem in Alchourron, Gardenfors and Makinson [1].

This completes the semantics of the dynamic operations. In the next section the axiomatization of the dynamic belief system is discussed. This axiomatization forms the basis of the dynamic reasoning system.

4. Dynamic Reasoning System

A reasoning system is specified by a set of axioms and inference rules. Given a set of formulas, the axioms and inference rules are used to infer more formulas which are called the logical consequences of the axiom system. In the case of static reasoning the formulas are first-order formulas which represent the world at the current time instant. Using the axioms and inference rules of first-order logic more formulas can be derived which also represent the world at the current instant of time. In the case of dynamic reasoning the formulas represent the world of the current time instant and the formulas which are added to the existing world or removed from the existing world. Based on the axioms and inference rules of the dynamic reasoning system more formulas are derived, which represent the state of the world at the *next* time instant. Thus the dynamic reasoning system will determine what will be true in the next time point, given the current time point and the nature of change. The next three subsections describe the axioms and inference rules of the dynamic reasoning system.

4.1 Expansion

By the very definition of expansion, the expansion of a FOE sentence ϕ, should result in a belief of ϕ at the next time instant. This is called the *Axiom of Inclusion* [6, 1].

Axiom of Inclusion

(AE1) `EXPAND(t, ϕ, u)` $\rightarrow$ `BELIEF(u, ϕ)`

Semantically the above axiom states that,

(CE1) If $u \in \mathcal{E}(w, t, \|\phi\|^{DI})$ then $w \in \|\mathtt{BELIEF(u,}\ \phi)\|^{DI}$.

Expansion is an operation which preserves the beliefs of an agent. If u is the world obtained after expanding t with respect to ϕ then all the belief formulas at t continue to hold at u although some of the non-belief formulas at t may be believed at u. In other words, expansion converts some of the agent's non-beliefs into beliefs, while at the same time preserving his existing beliefs. This property is called the *Axiom of Preservation of Beliefs.*

Axiom of Preservation of Beliefs

(AE2) `EXPAND(t, ϕ, u)` $\rightarrow$
`(BELIEF(t, α)` $\rightarrow$ `BELIEF(u, α))`

Semantically, the above axiom states that,

(CE2) If $u \in \mathcal{E}(w, t, \|\phi\|^{DI})$ and $w \in \|\mathtt{BELIEF(t,}\ \alpha)\|^{DI}$, then $w \in \|\mathtt{BELIEF(u,}\ \alpha)\|^{DI}$.

If the agent already believes in ϕ at t then expansion with respect to ϕ will yield no new beliefs. This is a case of trivial expansion where the agent's beliefs as well as non-beliefs are preserved. This is called the *Axiom of Trivial Expansion.*

Axiom of Trivial Expansion

(AE3) `BELIEF(t, ϕ)` $\wedge$ `EXPAND(t, ϕ, u)` $\rightarrow$ `(BELIEF(t, α)` $\equiv$ `BELIEF(u, α))`.

The semantic condition for the above axiom states is as follows,

(CE3) If $w \in \|\mathtt{BELIEF(t,}\ \phi)\|^{DI}$ and $u \in \mathcal{E}(w, t, \|\phi\|^{DI})$ then ($w \in \|\mathtt{BELIEF(t,}\ \alpha)\|^{DI}$, iff $w \in \|\mathtt{BELIEF(u,}\ \alpha)\|^{DI}$).

The operation of expansion is monotonic. Thus when belief in world t implies belief in world v then belief in the expanded world of t will imply belief in the expanded world of v. The following *Axiom of Monotonicity* mirrors this fact.

Axiom of Monotonicity

(AE4) $(\texttt{BELIEF}(t, \alpha) \rightarrow \texttt{BELIEF}(v, \alpha)) \wedge \texttt{EXPAND}(t, \phi, u) \wedge \texttt{EXPAND}(v, \phi, y) \rightarrow (\texttt{BELIEF}(u, \beta) \rightarrow \texttt{BELIEF}(y, \beta))$.

Semantically the above axiom translates into the following condition.

(CE4) (If $w \in \|\texttt{BELIEF}(t, \alpha)\|^{DI}$, then $w \in \|\texttt{BELIEF}(v, \alpha)\|^{DI}$) and $u \in \mathcal{E}(w, t, \|\phi\|^{DI})$ and $y \in \mathcal{E}(w, v, \|\phi\|^{DI})$ then (If $w \in \|\texttt{BELIEF}(u, \beta)\|^{DI}$, then $w \in \|\texttt{BELIEF}(y, \beta)\|^{DI}$).

The axioms AE1-AE3 state that if u is the expanded world of t with respect to ϕ then the beliefs at t are preserved at u and the agent acquires belief in ϕ at u. It does not rule out the possibility of the agent acquiring beliefs which are in no way related to ϕ nor the original world at time t. In other words the *only* beliefs at u are the beliefs at t and the belief in ϕ and all its consequences, where ϕ is a first-order sentence with respect to which the expansion is being carried out. This is equivalent to *minimizing the acquisition of beliefs* during expansion. Thus any belief formula at u which is not already in t has been obtained by an explicit expansion or is a consequence of an explicit expansion.

Axiom of Minimization of Beliefs

(AE5) $\texttt{BELIEF}(u, \phi) \rightarrow \texttt{BELIEF}(t, \phi) \vee (\texttt{EXPAND}(t, \alpha, u) \wedge \texttt{BELIEF}(u, \texttt{BELIEF}(u, \alpha) \supset \phi))$.

Semantically the above axiom translates to the following condition,

(CE5) If $w \in \|\texttt{BELIEF}(u, \phi)\|^{DI}$ then ($w \in \|\texttt{BELIEF}(t, \phi)\|^{DI}$ or ($u \in \mathcal{E}(w, t, \|\alpha\|^{DI})$ and $w \in \|\texttt{BELIEF}(u, \texttt{BELIEF}(u, \alpha) \supset \phi)\|^{DI}$)).

While axioms (AE2), (AE3) and (AE4) enforce the principle of *maximal coherence*, axioms (AE1) and (AE5) enforce the principle of *minimal change*.

The following inference rule which states that if ϕ_1 and ϕ_2 are equivalent then expanding with respect to either one of them will give the same result is also needed. Modal systems which are closed under this type of inference rule are called *classical* systems [3].

(RE1) From $\vdash \phi_1 \equiv \phi_2$ infer $\vdash \texttt{EXPAND}(t, \phi_1, u) \equiv \texttt{EXPAND}(t, \phi_2, u)$.

The axioms and inference rule (AE1)-(AE5) and (RE1) capture the proof-theoretic notion of *minimal change* and *maximal coherence* for expansion. The conditions (CE1)-(CE5), on the expansion function and the belief relation capture the semantic notion of the *closest* possible worlds which obey the principles of minimal change and maximal coherence. The semantic conditions (CE1)-(CE5) describe a *closest expanded world.* The class of models whose $\mathcal{E}$ selection function satisfies the conditions (CE1)-(CE5) are called the $\mathcal{E}$-models.

4.2 Contraction

The axioms of contraction are very similar to the axioms of expansion. The axiom corresponding to the axiom of inclusion for expansion, is the *Axiom of Exclusion* for contraction. Thus if the world t is contracted with respect to ϕ to give u then the agent must not believe ϕ at u.

Axiom of Exclusion

(AC1) $\texttt{CONTRACT}(t, \phi, u) \rightarrow \neg\texttt{BELIEF}(u, \phi)$

Semantically the above axiom states that,

(CC1) If $u \in \mathcal{C}(w, t, \|\phi\|^{DI})$ then $w \in W - \|\texttt{BELIEF}(u, \phi)\|^{DI}$.

Just as expansion preserves the beliefs of an agent one would expect contraction to preserve the non-beliefs of an agent. In fact, this is the case if one assumes that the world does not contain any AE belief formulas, i.e. formulas of the form $\neg\texttt{BELIEF}(t, \phi) \supset \psi$. For such cases, the following *Axiom of Preservation of Non-beliefs* which is analogous to the Axiom of Preservation of Beliefs holds.

Axiom of Preservation of Non-Beliefs

$\texttt{CONTRACT}(t, \phi, u) \rightarrow$
$\quad (\neg\texttt{BELIEF}(t, \alpha) \rightarrow \neg\texttt{BELIEF}(u, \alpha))$

However, in the more general case, where the world contains both AE and universal AE beliefs the above axiom does not hold. Consider the case where the agent contracts the formula ϕ to go into a time point u, where the AE belief $\neg\texttt{BELIEF}(u, \phi) \supset \psi$ is true. Now as the agent contracts ϕ, the formula $\neg\texttt{BELIEF}(u, \phi)$ will be true and this will cause ψ to be true and hence ψ to be believed. Thus in the presence of AE beliefs the agent can acquire new beliefs during contraction. The acquisition of these beliefs should be minimized. Hence the new beliefs acquired by the agent should be the logical consequences of $\neg\texttt{BELIEF}(u, \phi)$, where ϕ is the formula with respect to which the agent has performed contraction. The following axiom called the *Axiom of Minimization of Beliefs* is analogous to the corresponding axiom of expansion.

Axiom of Minimization of Beliefs

(AC2) BELIEF(u, ϕ) $\rightarrow$ BELIEF(t, ϕ) $\vee$ (CONTRACT(t, α, u) $\wedge$ BELIEF(u, $\neg$BELIEF(u, α) $\supset$ ϕ)).

Semantically the above axiom translates to the following condition,

(CC2) If w $\in$ $\|$BELIEF(u, ϕ)$\|^{DI}$ then (w $\in$ $\|$BELIEF(t, ϕ)$\|^{DI}$ or (u $\in$ $\mathcal{C}$(w, t, $\|\alpha\|^{DI}$) and w $\in$ $\|$BELIEF(u, $\neg$BELIEF(u, α) $\supset$ ϕ)$\|^{DI}$)).

If the agent does not believe ϕ at t then contraction with respect to ϕ will not increase the non-beliefs of the agent. In other words under the above conditions the contraction operation preserves the agent's beliefs as well as non-beliefs. This results in the following *Axiom of Trivial Contraction*,

Axiom of Trivial Contraction

(AC3) $\neg$BELIEF(t, ϕ) $\wedge$ CONTRACT(t, ϕ, u) $\rightarrow$ ($\neg$BELIEF(t, α) $\equiv$ $\neg$BELIEF(u, α)).

The semantic condition for the above axiom is as follows,

(CC3) If w $\in$ W - $\|$BELIEF(t, ϕ)$\|^{DI}$ and u $\in$ $\mathcal{C}$(w, t, $\|\phi\|^{DI}$) then (w $\in$ W - $\|$BELIEF(t, α)$\|^{DI}$, iff w $\in$ W - $\|$BELIEF(u, α)$\|^{DI}$).

The axiom of monotonicity is not satisfied by contraction. This is because contraction reduces the beliefs of the agent and is therefore non-monotonic in nature.

While axiom (AC2) stated what are the formulas which should be acquired in the contracted world, axiom (AC5) states what are the beliefs which should be given up during contraction. It states this indirectly by requiring whatever is *lost* during contraction should be *recovered* during expansion. As the agent gains as little as possible during expansion (due to the Axiom of Minimization of Beliefs), the agent has to loose as little as possible for the following axiom to hold. This implies that the following axiom performs the function of minimizing non-beliefs and is called the *Axiom of Minimization of Non-beliefs.*

Axiom of Minimization of Non-beliefs

(AC4) CONTRACT(t, ϕ, u) $\wedge$ EXPAND(u, ϕ, v) $\rightarrow$ (BELIEF(t, α) $\rightarrow$ BELIEF(v, α)).

Semantically, the above axiom is equivalent to the following condition,

(CC4) If u $\in$ $\mathcal{C}$(w, t, $\|\phi\|^{DI}$) and v $\in$ $\mathcal{E}$(w, u, $\|\phi\|^{DI}$) then (w $\in$ $\|$BELIEF(t, α)$\|^{DI}$, then w $\in$ $\|$BELIEF(v, α)$\|^{DI}$).

A strict equivalence of the consequent of (AC4) does not hold. Consider the case where ϕ is not present in t. Then according to axiom (AC3) the time points t and u will have identical beliefs. Now if u is expanded with respect to ϕ, then at v the agent will believe in ϕ but did not have this belief at t, which proves that the strict equivalence does not hold.

The following inference rule which states that if ϕ_1 and ϕ_2 are equivalent then contracting with respect to either one of them will give the same result is also needed.

(RC1) From $\vdash \phi_1 \equiv \phi_2$ infer $\vdash$ CONTRACT(t, ϕ_1, u) $\equiv$ CONTRACT(t, ϕ_2,u).

Axioms (AC1)-(AC4) together capture the proof-theoretic notion of *maximal coherence* and *minimal change* for contraction. The conditions (CC1)-(CC4), on the contraction function and the belief relation capture the semantic notion of the *closest* possible worlds which obey the principles of minimal change and maximal coherence. The semantic conditions (CC1)-(CC4) describe a *closest contracted world.* The class of models whose $\mathcal{C}$ selection function satisfies the conditions (CC1)-(CC4) are called the $\mathcal{C}$-models.

4.3 Revision

As revision can be expressed in terms of expansion and contraction only one axiom is needed for revision. This axiom states that revising with respect to ϕ is equivalent to contracting with respect to $\neg\phi$ followed by expansion with respect to ϕ. This axiom is a reformulation of Levi identity [6].

Axiom of Revision

(AR1) REVISE(t, ϕ, u) $\rightarrow$ CONTRACT(t, $\neg\phi$, v) $\wedge$ EXPAND(v, ϕ, u)

The axiom system for the language $\mathcal{L}_2$, called the *basic dynamic (BD)-model*, is the B-modal system together with the axioms and inference rules for expansion, contraction and revision. The *basic* $\mathcal{D}$-model is defined as a model of the dynamic interpretation DI, whose $\mathcal{B}$ relation is a $\mathcal{B}$-model, whose $\mathcal{E}$ function is a

$\mathcal{E}$-model and whose $\mathcal{C}$ function is a $\mathcal{C}$-model.

The soundness and completeness of the dynamic reasoning system is stated below. Once again the details of the proof can be found in [17].

Theorem 4.2: The BD-modal system is sound and complete with respect to the class of all basic $\mathcal{D}$-models.

4.4 Actions and Planning

Expansion, contraction and revision are the most primitive or fundamental dynamic operations. However, they are not the only dynamic operations. The dynamic operation which has received a great deal of attention in AI is the notion of *actions*. In the situation calculus [13] approach actions are treated as transformations from one situation to another. Situations are like possible worlds, introduced in the previous section. Actions can be defined in terms of the dynamic operations expansion, contraction and revision. The dynamic operations can also be used to define parallel actions. This helps in providing a unifying architecture for dynamic reasoning as well as planning. This section gives a brief description of how to define actions in terms of the dynamic operations and provides insights into a completely integrated autonomous reasoning and planning system.

Actions are normally defined in terms of *preconditions* and *postconditions*. If a certain precondition holds at the current instant of time and an action is carried out then the post-condition holds in the next time instant. This can be expressed as the modal formula `ACTION(t,` β`,` α`,` γ`, u)`, which states that at `t` if the agent believes β and the action α is carried out then the agent will believe in γ at the next time instant `u`. The axiom for action is defined as follows,

Axiom of Action

$\mathtt{ACTION(t,}\ \beta,\ \alpha,\ \gamma,\ \mathtt{u)} \equiv \mathtt{BELIEF(t,}\ \beta) \wedge \mathtt{REVISE(t,}\ \gamma,\ \mathtt{u)}.$

The same methodology as above can be used to define both *sequential* and *parallel* actions. Let α_1 ; α_2 denote two sequential actions and $\alpha_1 \parallel \alpha_2$ denote two parallel actions. The axioms for these actions can be defined as follows,

Axiom of Sequential Action

$\mathtt{ACTION(t,}\ \beta_1 \wedge \beta_2,\ \alpha_1 ; \alpha_2,\ \gamma_1 \wedge \gamma_2,\ \mathtt{u)} \equiv \mathtt{BELIEF(t,}\ \beta_1) \wedge \mathtt{REVISE(t,}\ \gamma_1,\ \mathtt{v)} \wedge \mathtt{BELIEF(v,}\ \beta_2) \wedge \mathtt{REVISE(v,}\ \gamma_2,\ \mathtt{u)}.$

Axiom of Parallel Action

$\mathtt{ACTION(t,}\ \beta_1 \wedge \beta_2,\ \alpha_1 \parallel \alpha_2,\ \gamma_1 \wedge \gamma_2,\ \mathtt{u)} \equiv \mathtt{BELIEF(t,}\ \beta_1 \wedge \beta_2) \wedge \mathtt{REVISE(t,}\ \gamma_1,\ \mathtt{u)} \wedge \mathtt{REVISE(t,}\ \gamma_2,\ \mathtt{u)}.$

The following example provides a simple situation, where the theory developed in this paper can be used.

Example:

Consider the blocks-world, at time `t1`, where there are two blocks A and B, such that the Red colored block, A, is at location L1 and the Blue colored block, B, is at location L2. Also both blocks A and B are clear. The laws in this domain can be stated as follows, a) No two blocks are on the same location, b) No block occupies more than one location, c) A clear location has nothing on top of it, and d) No block has more than one color. This information is stated as follows in the dynamic belief system.

1.1 $\forall \mathtt{t(BELIEF(t,\ on(x,l)} \wedge \mathtt{x} \neq \mathtt{y} \rightarrow \neg\mathtt{on(y,l)))}$

1.2 $\forall \mathtt{t(BELIEF(t,\ on(x,l)} \wedge \mathtt{l} \neq \mathtt{m} \rightarrow \neg\mathtt{on(x,m)))}$

1.3 $\forall \mathtt{t(BELIEF(t,\ clear(l)} \equiv \neg\mathtt{on(x,l)))}$

1.4 $\forall \mathtt{t(BELIEF(t,\ color(x,c)} \wedge \mathtt{c} \neq \mathtt{d} \rightarrow \neg\mathtt{color(x,d)))}$

The contingent information about the blocks world at time `t1` is stated as follows,

1.5 `BELIEF(t1, clear(A))`	1.6 `¬BELIEF(t1, ¬clear(A))`
1.7 `BELIEF(t1, clear(B))`	1.8 `¬BELIEF(t1, ¬clear(B))`
1.9 `BELIEF(t1, on(A,L1))`	1.10 `¬BELIEF(t1, ¬on(A,L1))`
1.11 `BELIEF(t1, on(B,L2))`	1.12 `¬BELIEF(t1, ¬on(B,L2))`
1.13 `BELIEF(t1, color(A,Red))`	1.14 `¬BELIEF(t1, ¬color(A,Red))`
1.15 `BELIEF(t1, color(B,Blue))`	1.16 `¬BELIEF(t1, ¬color(B,Blue))`

Using the laws 1.1 to 1.4 and the beliefs 1.5 to 1.16 the following additional beliefs can be derived,

1.17 `BELIEF(t1, ¬on(A,B))`	1.18 `¬BELIEF(t1, on(A,B))`
1.19 `BELIEF(t1, ¬on(B,A))`	1.20 `¬BELIEF(t1, on(B,A))`

1.21	BELIEF(t1, ¬on(A,L2))	1.22	¬BELIEF(t1, on(A,L2))
1.23	BELIEF(t1, ¬on(B,L1))	1.24	¬BELIEF(t1, on(B,L1))
1.25	BELIEF(t1, ¬color(A,Blue))	1.26	¬BELIEF(t1, color(A,Blue))
1.27	BELIEF(t1, ¬color(B,Red))	1.28	¬BELIEF(t1, color(B,Red))

Assume that there are two robots: the painting robot and the block moving robot. Let the painting robot paint the block B using Red paint and at the same time let the block-moving robot move the block B to location A. The action paint(B,Red) has no preconditions but it causes the color of B to be Red. The action move(B,A) requires that both the blocks B and A be clear. The post-condition of performing the action move(B,A) is that the block B is on A. As the preconditions of both the actions are believed at t1, both the actions can be carried out in parallel. From the axiom of parallel actions this is equivalent to revising t1 with respect to on(B, A) and with respect to color(B, Red), to give the time point t2. Using the axioms of expansion, contraction and revision the time point t2 can be *derived* from the time point t1. The state of the world at t2 is given below.

2.5	BELIEF(t2, ¬clear(A))	2.6	¬BELIEF(t2, clear(A))
2.7	BELIEF(t2, clear(B))	2.8	¬BELIEF(t2, ¬clear(B))
2.9	BELIEF(t2, on(A,L1))	2.10	¬BELIEF(t2, ¬on(A,L1))
2.11	BELIEF(t2, ¬on(B,L2))	2.12	¬BELIEF(t2, on(B,L2))
2.13	BELIEF(t2, color(A,Red))	2.14	¬BELIEF(t2, ¬color(A,Red))
2.15	BELIEF(t2, ¬color(B,Blue))	2.16	¬BELIEF(t2, color(B,Blue))
2.17	BELIEF(t2, ¬on(A,B))	2.18	¬BELIEF(t2, on(A,B))
2.19	BELIEF(t2, on(B,A))	2.20	¬BELIEF(t2, ¬on(B,A))
2.21	BELIEF(t2, ¬on(A,L2))	2.22	¬BELIEF(t2, on(A,L2))
2.23	BELIEF(t2, ¬on(B,L1))	2.24	¬BELIEF(t2, on(B,L1))
2.25	BELIEF(t2, ¬color(A,Blue))	2.26	¬BELIEF(t2, color(A,Blue))
2.27	BELIEF(t2, color(B,Red))	2.28	¬BELIEF(t2, ¬color(B,Red))

Note that not only the formula on(B,A) is believed, but all its consequences, namely ¬clear(A) and ¬on(B,L2) are also believed. Similarly the formula color(B,Red) and its consequence ¬color(B,Blue) are also believed. Thus the principles of minimal change and maximal coherence solves two of the important problems in planning; the frame problem [13] and ramification problem [9]. An alternative approach to planning using the principles of minimal change and maximal coherence at the meta-level is discussed in [5].

In order to obtain a completely autonomous reasoning system, capable of dynamic reasoning and reactive planning, the architecture mentioned in this paper should be extended to a belief-desire-intention architecture [16]. The principles of minimal change and maximal coherence can then be used not only for change in beliefs, but also for change in desires and intentions. Such a system would act like a dynamic reasoning system as described in this paper and as a reactive planning system as described in [8]. Such a system would be able to react rationally, within a reasonable amount of time, without any human intervention, to any unforeseen situation and will be ideal for space-based applications.

5. Conclusion

This paper presents a dynamic belief system capable of representing and reasoning about change. The dynamic reasoning system based on the dynamic belief system embodies two general principles of change, namely minimal change and maximal coherence. The dynamic reasoning system can be used as a module for a more general purpose autonomous reasoning system capable of reacting to any unforeseen circumstances. Such systems will prove to be crucial for future space based AI systems.

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