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ALGORITHM FOR IN-FLIGHT GYROSCOPE CALIBRATION

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ABSTRACT

An optimal algorithm for the in-flight calibration of spacecraft gyroscope systems is presented. Special consideration is given to the selection of the loss function weight matrix in situations in which the spacecraft attitude sensors provide significantly more accurate information in pitch and yaw than in roll, such as will be the case in the Hubble Space Telescope mission. The results of numerical tests that verify the accuracy of the algorithm are discussed.

I. INTRODUCTION

A spacecraft gyroscope system, sometimes referred to as the rate gyro assembly (RGA), is used to measure angular rotation rates of the spacecraft. This is required for proper control of the spacecraft, particularly for the proper positioning of spacecraft sensors with respect to desired targets. We present here an algorithm for RGA calibration that was used for the High Energy Astronomy Observatory (HEAO) missions and enhanced for use in the Hubble Space Telescope (HST) mission. Section II of this article presents the basic algorithm; Section III, the statistical weighting scheme; Section IV, the results of numerical tests of the algorithm; and Section V, our conclusions.

II. BASIC RGA CALIBRATION ALGORITHM

Any RGA must be composed of at least three gyroscopes whose axis directions taken together completely span the space of possible rotations (i.e., pitch, roll, and yaw). An RGA consisting of K gyros produces as output a response "vector" $\mathbf{R} = (r_1, r_2, \ldots, r_K)^T$, where r_k is the response of the kth gyro. The response vector is translated into a measured angular velocity, Ω_M, of the spacecraft (in the spacecraft frame) via the relation

$$\Omega_M = G_0 \mathbf{R} - \mathbf{D}_0 , \tag{1}$$

where G_0 is the RGA 3-by-K scale factor / alignment matrix, and $\mathbf{D}_0$ is the RGA drift rate bias. If G_0 and $\mathbf{D}_0$ deviate from their true values, because of either poor initial calibration or temporal changes of the RGA, then Ω_M will deviate from the true angular rate, Ω. The goal of the algorithm is to determine correction matrices M and $\mathbf{d}$ that may be applied to G_0 and $\mathbf{D}_0$ so that a modified equation (1) will yield the true angular rate:

$$G = M G_0 , \tag{2a}$$

$$\mathbf{D} = M \mathbf{D}_0 + \mathbf{d} , \tag{2b}$$

$$\Omega = G \mathbf{R} - \mathbf{D} = M \Omega_M - \mathbf{d} . \tag{2c}$$

The angular rate deviation, ω, between the measured and true rates is given by

$$\omega = \Omega_M - \Omega = -m \Omega_M + \mathbf{d} , \tag{3}$$

where $m = M - I$, and I is the identity matrix. The algorithm will solve for m and $\mathbf{d}$. The correction matrices m and $\mathbf{d}$ are dimensioned 3-by-3 and 3, respectively; we emphasize this because it ties directly to the fact that the algorithm being developed here provides correction information for the RGA as a whole in its capacity as a device for measuring three-dimensional angular motion. Unless the RGA under consideration consists of only three gyros, m and $\mathbf{d}$ will not contain sufficient information to allow separate calibration updates of scale, alignment, and drift for the individual gyros.

We wish now to relate the angular rate deviation as integrated over some calibration maneuver to the difference in attitude changes as determined for the maneuver by the RGA and by some independent attitude sensing devices (e.g., fixed-head star trackers). This derivation is conveniently done using quaternion notation; see Reference 1 for a discussion of the mathematics of quaternions. First, an expression for the time derivative of a quaternion is required. If $\mathbf{Q}(t)$ is the quaternion representing spacecraft attitude at time t, then the quaternion representing a change in attitude over a time interval Δt is given by $\mathbf{Q}^{-1}(t)\,\mathbf{Q}(t+\Delta t)$. If Δt is small, the attitude change may be expressed as

$$\mathbf{Q}^{-1}(t)\,\mathbf{Q}(t+\Delta t) = \mathbf{1} + \mathbf{q}(\Omega\Delta t/2)\,, \tag{4}$$

where $\mathbf{1}$ is the identity quaternion, and $\mathbf{q}(\Omega\Delta t/2)$ is a differential quaternion with vector component $\Omega\Delta t/2$ and scalar component zero. Defining $\Delta\mathbf{Q}(t)$ as $\mathbf{Q}(t+\Delta t) - \mathbf{Q}(t)$ and combining this definition with equation (4) yields

$$\Delta\mathbf{Q}(t) = \mathbf{Q}(t)\,\mathbf{q}(\Omega\Delta t/2)\,. \tag{5}$$

Dividing equation (5) by Δt produces the desired quaternion time derivative, $\mathbf{Q}'(t)$:

$$\mathbf{Q}'(t) = \Delta\mathbf{Q}(t)/\Delta t = \mathbf{Q}(t)\,\mathbf{q}(\Omega/2)\,. \tag{6}$$

Equation (6) applies as well for the quaternion time derivative corresponding to the attitudes as measured by the RGA, with subscript M placed appropriately.

Next, the time derivatives specified above are used to construct the time derivative of the attitude error quaternion and the definite integral of that quaternion over the time of the maneuver. The attitude error quaternion, $\delta\mathbf{Q}$, is defined as

$$\delta\mathbf{Q} = \mathbf{Q}_M\,(\mathbf{Q}_M^{-1}\,\mathbf{Q}\,)\,\mathbf{Q}_M^{-1} = \mathbf{Q}\,\mathbf{Q}_M^{-1}\,, \tag{7}$$

which is a quaternion expressing a rotation from the RGA-determined postmaneuver attitude to the true postmaneuver attitude, transformed to the premanuever reference frame. It follows by the chain rule of differentiation that

$$\delta\mathbf{Q}' = \mathbf{Q}'\,\mathbf{Q}_M^{-1} + \mathbf{Q}\,\mathbf{Q}'^{-1}_M \tag{8a}$$

$$= \mathbf{Q}\,\mathbf{q}(\Omega/2)\,\mathbf{Q}_M^{-1} + \mathbf{Q}\,\mathbf{q}^{-1}(\Omega_M/2)\,\mathbf{Q}_M^{-1}\,. \tag{8b}$$

Combining equation (8b) with the relations $\omega = \Omega_M - \Omega$ and $\mathbf{q}^{-1}(\Omega_M/2) = \mathbf{q}(-\Omega_M/2)$ produces

$$\delta\mathbf{Q}' = \mathbf{Q}\,\mathbf{q}(-\omega/2)\,\mathbf{Q}_M^{-1} \ . \tag{9}$$

Integrating both sides of equation (9) over the maneuver yields

$$\delta\mathbf{Q} - \mathbf{1} = \int \mathbf{Q}\,\mathbf{q}(-\omega/2)\,\mathbf{Q}_M^{-1}\ dt \ , \tag{10}$$

where the constant of integration (i.e., the identity quaternion) is removed from $\delta\mathbf{Q}$ because the integral in equation (10) is a definite integral. The attitude error quaternion can be expressed in terms of the rotation $\mathbf{Q}_{R1}^{-1}\,\mathbf{Q}_{R2}$ from the first reference attitude to the second (i.e., as determined using the attitude sensors against which the RGA is being calibrated) and the rotation $\mathbf{Q}_{G1}^{-1}\,\mathbf{Q}_{G2}$ between the first RGA-propagated attitude and the second. Equation (10) thereby becomes

$$(\mathbf{Q}_{R1}^{-1}\,\mathbf{Q}_{R2})\,(\mathbf{Q}_{G2}^{-1}\,\mathbf{Q}_{G1}) - \mathbf{1} = \int \mathbf{Q}\,\mathbf{q}(-\omega/2)\,\mathbf{Q}_M^{-1}\ dt \ . \tag{11}$$

No approximations have been made in the derivation to this point. We now make two approximations, each of which is accurate to first order in the error. First, $\mathbf{Q}_M$ is substituted for $\mathbf{Q}$ in the integrand in equation (11). This substitution yields

$$(\mathbf{Q}_{R1}^{-1}\,\mathbf{Q}_{R2})\,(\mathbf{Q}_{G2}^{-1}\,\mathbf{Q}_{G1}) - \mathbf{1} = \int \mathbf{Q}_M\,\mathbf{q}(-\omega/2)\,\mathbf{Q}_M^{-1}\ dt \ . \tag{12}$$

The integrand in equation (12) is simply the quaternion representation for a rotation of the vector $-\omega/2$ through a rotation defined by $\mathbf{Q}_M$, i.e., the rotation that transforms $-\omega/2$ from spacecraft coordinates at time t to spacecraft coordinates in the premaneuver reference frame. Equation (12) can therefore be written in matrix notation as

$$\mathbf{Z}_i = -1/2 \int \boldsymbol{T}_i\ \omega\ dt \tag{13a}$$

$$= 1/2 \int \boldsymbol{T}_i\,(\boldsymbol{m}\ \Omega_M - \mathbf{d})\,dt \ , \tag{13b}$$

where $\mathbf{Z}_i$ is the vector component of $\delta\mathbf{Q}$, $\boldsymbol{T}_i$ is the matrix for transforming vectors to premaneuver spacecraft coordinates, and i is a subscript designating maneuver number. The second approximation is made implicitly in the definition of $\mathbf{Z}_i$ as the vector component of $\delta\mathbf{Q}_i$; the fourth component of $\delta\mathbf{Q}_i$, which is actually equal to the cosine of the error rotation angle, is approximated as equal to 1. Because of the two approximations made in going from equation (11) to (13b), the calibration algorithm described here will be inherently iterative; the vector $\mathbf{Z}_i$ and matrix $\boldsymbol{T}_i$ must be reevaluated on each iteration.

Equation (13b) is linear in the unknowns $\boldsymbol{m}$ and $\mathbf{d}$ and thus lends itself naturally to standard least-squares techniques. First, the matrix equation that represents equation (13b) applied to N calibration maneuvers is written as

$$\mathbf{Z} = \boldsymbol{H}\,\mathbf{x}\,, \tag{14}$$

where $\mathbf{Z}$ and the state vector $\mathbf{x}$ are defined via

$$\mathbf{Z} = \{\, \mathbf{Z}_1^{\mathrm{T}}, \mathbf{Z}_2^{\mathrm{T}}, \ldots, \mathbf{Z}_N^{\mathrm{T}} \,\}^{\mathrm{T}}\,, \tag{15}$$

$$\mathbf{x} = 1/2\,\{\, m_{11}, m_{12}, m_{13}, m_{21}, m_{22}, m_{23}, m_{31}, m_{32}, m_{33}, d_1, d_2, d_3 \,\}^{\mathrm{T}}\,, \tag{16}$$

and $\boldsymbol{H}$ is a 3N-by-12 matrix of the form

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{U}_1 & -\boldsymbol{Y}_1 \\ \vdots & \vdots \\ \boldsymbol{U}_N & -\boldsymbol{Y}_N \end{bmatrix}. \tag{17}$$

Each $\boldsymbol{U}_i$ is a 3-by-9 matrix, the components of which are given by

$$(U_{j\,k+3(l-1)})_i = \int (T_{jk})_i\; (\Omega_M)_l\; dt\,, \tag{18}$$

and each Y_i is a 3-by-3 matrix with components

$$(Y_{jk})_i = \int (T_{jk})_i \, dt . \tag{19}$$

The least-squares solution for the state vector **x** in equation (14) minimizes the linear Bayesian weighted least-squares loss function, J, given by

$$J = 1/2 \, [E^T W E + (x - x_a)^T S_a (x - x_a)] , \tag{20}$$

where

$$E = Z - H x , \tag{21}$$

W and S_a are symmetric nonnegative definite weighting matrices, and x_a is an a priori estimate of **x**. The desired solution for **x** is given by

$$x = (H^T W H + S_a)^{-1} (H^T W Z + S_a x_a) . \tag{22}$$

Equations (20) and (22) include S_a and x_a for mathematical completeness. In what follows, we will assume $S_a = [0]$, i.e., no a priori knowledge of **x**. In applying equation (22) to determine **x**, it is clear that at least four calibration intervals are required and at least three of these must span the space of possible rotations. An acceptable minimum set of calibration intervals would be one maneuver each of pitch, roll, and yaw, together with a period of constant attitude to define the drift rate bias. In selecting calibration maneuvers to be used for the algorithm, a user should be aware that, at least in the equations specified above, a rotation of greater than 180 degrees is indistinguishable from a smaller rotation in the opposite direction. The use of such large rotations could lead to errors in the calibration and should therefore be avoided. The basic algorithm being discussed here lends itself easily to being broken into two separate algorithms, one to determine the scale factor / alignment portion of **x** and a separate one to determine the drift rate bias.

III. SPECIFICATION OF THE WEIGHT MATRIX

In principle, the specification of the weight matrix W in equation (22) depends on the scale size of random errors associated with the RGA itself, as well as the errors associated with the determination of reference attitudes. In practice, random errors associated with the RGA tend to be much smaller than those of the reference attitude sensors. This is true in particular for the sensors used for the HST mission. The HST uses two types of sensors for high-accuracy attitude determination: fixed-head star trackers (FHSTs) and fine guidance sensors (FGSs). The three FGS fields of view are clustered tightly (within 14 arc-minutes) about the principle axis of the spacecraft (hereafter called the V1-axis). When calibrated, the accuracy of the FGSs should be better than 0.010 arc-second. In most circumstances, however, their effective accuracy will be limited by the accuracy of the reference star catalog against which the FGS observations are compared during attitude computation; this will be about 0.3 arc-second. Because of the tight clustering of the FGSs about the V1-axis, this accuracy of 0.3 arc-second pertains only to the pitch and yaw components. The roll accuracy is determined by the accuracy of the FHSTs, whose fields of view are more widely distributed in direction about the spacecraft than those of the FGSs. The accuracy of the FHSTs used for the HST is about 10 arc-seconds. The fact that the reference attitudes for the HST are substantially better in pitch and yaw than in roll presented a special problem for the HST ground software. For spacecraft that have equal attitude accuracies about all axes, setting the weight matrix in equations (20) and (22) to the identity matrix, i.e., treating the accuracies of all components of all maneuvers as equal and uncorrelated, would be legitimate. This is not the case for the HST; consequently, significantly improved results for HST RGA calibration can be expected if a proper weight matrix is used. The need for a proper weight matrix is enhanced by the possible requirement to combine data sets for maneuvers in which some of the attitudes were determined using only FHSTs or only FGSs. This possibility arises because of potential sensor occultation by the Earth during parts of the spacecraft orbit.

The net effect of the considerations discussed above is that the simple product $\mathbf{E}^T \mathbf{E}$ ($\mathbf{E}$ defined in equation (21)) does not represent a squared sum of normalized, independent random variables as is required for an optimized least-squares loss function (e.g., see Reference 2). The determination of W depends upon the measurement uncertainties of the components of E, both in magnitude for the individual components and in any correlation of errors between the individual components. The 3N-vector $\mathbf{E}$ is composed of N 3-vectors, the relation being

$$E = (e_1^T, e_2^T, \ldots, e_N^T)^T, \tag{23}$$

where e_i is the error associated with the ith calibration maneuver. If the errors for the maneuvers are assumed to be independent, then W can be represented as a block diagonal matrix, with each block being a 3-by-3 matrix, w_i. The loss function becomes

$$J = \Sigma j_i, \tag{24a}$$

$$j_i = 1/2 \; e_i^T w_i e_i . \tag{24b}$$

The assumption is not strictly valid in cases where the same attitude data are used at both the end of one maneuver and the beginning of another. In such cases W would appropriately contain elements representing a cross-correlation of errors between maneuvers. Because this complication is both awkward to include computationally and of substantially smaller effect than that of the accuracy asymmetries in attitude produced by the use of FHSTs and FGSs together, we will neglect it. We will also make use of the fact that the random RGA errors are negligible when compared with the reference attitude errors. For notation simplification, hereafter we will suppress the subscript i (for maneuver number) unless it is explicitly required.

Using the approximations described above, the components of e may be written as

$$e_j = a_j + \Sigma t_{jk} b_k, \tag{25}$$

where a_j is the premaneuver attitude determination error about the jth spacecraft axis, b_k is the postmaneuver error about the kth axis, and is t_{jk} the (j,k) component of the postmaneuver-attitude-to-premaneuver-attitude transformation matrix. The numbers a_1, a_2, a_3, b_1, b_2, and b_3 may be assumed to be independent random variables, but the numbers e_1, e_2, and e_3 will in general be correlated because of the mixing of the postmaneuver errors via the maneuver transformation matrix. For cases where the a_j and b_k are all approximately equal (as would be true for attitudes determined using a number of well-separated sensors of equal accuracy), the correlation between the components of **e** would be small because of the combining in each of a number of random variables in different ways. Here, however, we are particularly interested in contexts where a_1 and b_1 (the roll errors) are large compared with a_2, a_3, b_2, and b_3. For such cases a maneuver with significant components about all axes will have errors given to first order by

$$e_1 = a_1 + t_{11}\, b_1 + O(\delta)\ , \qquad (26a)$$

$$e_2 = t_{21}\, b_1 + O(\delta)\ , \qquad (26b)$$

$$e_3 = t_{31}\, b_1 + O(\delta)\ , \qquad (26c)$$

where δ is a generic random variable with variance like that of a_2, a_3, b_2, or b_3. The correlation, particularly between e_2 and e_3, is clear. The goal now is to construct from the components of **e** three independent, normalized random variables that may be used in defining a least-squares loss function. The components of the weight matrix w can then be solved for by setting this new loss function equal to 1/2 ($\mathbf{e}^T w\, \mathbf{e}$). For this derivation we will consider two cases: (1) where the maneuver has a significant nonroll component (i.e., $t_{21}^2 \sigma_{b1}^2 > \sigma_\delta^2$ and/or $t_{31}^2 \sigma_{b1}^2 > \sigma_\delta^2$, where σ_{b1}^2 and σ_δ^2 represent the variances of b_1 and δ, respectively) and (2) where the maneuver is essentially pure roll (i.e., $t_{21}^2 \sigma_{b1}^2 < \sigma_\delta^2$ and $t_{31}^2 \sigma_{b1}^2 < \sigma_\delta^2$).

In case 1 (a maneuver with a significant nonroll component), the quantities e_1, e_2 and e_3 may be used to construct three essentially independent random variables $\langle a_1 \rangle$, $\langle b_1 \rangle$, and c, where $\langle a_1 \rangle$ and $\langle b_1 \rangle$ are estimators of a_1 and b_1, and c is a variable with variance like that of δ. We specifically construct c to eliminate the large roll errors:

$$c = t_{31}\, e_2 - t_{21}\, e_3\ . \qquad (27)$$

The variance of c over an ensemble of maneuvers is given by

$$\sigma_c^2 = \mathrm{var}\,\{ t_{31}\, e_2 - t_{21}\, e_3 \} \qquad (28a)$$

$$= (t_{31}^2\, C_2 + t_{21}^2\, C_3)\, \sigma_\delta^2\ , \qquad (28b)$$

where

$$C_2 = 1 + t_{22}^2 + t_{23}^2, \tag{29a}$$

$$C_3 = 1 + t_{32}^2 + t_{33}^2. \tag{29b}$$

We construct $<b_1>$ to be a good estimator of b_1:

$$<b_1> = [\, t_{21}^2 / (t_{21}^2 + t_{31}^2)\,]\,(e_2/ t_{21}) + [\, t_{31}^2 / (t_{21}^2 + t_{31}^2)\,]\,(e_3/ t_{31}). \tag{30}$$

The quantities e_2/t_{21} and e_3/t_{31} estimate b_1 to within δ/t_{21} and δ/t_{31}, respectively. The quantities in brackets are normalized weights inversely proportional to the square of the uncertainty of the corresponding estimate. For convenience we rewrite $<b_1>$ as

$$<b_1> = B_2\, e_2 + B_3\, e_3, \tag{31}$$

where

$$B_2 = t_{21} / (t_{21}^2 + t_{31}^2), \tag{32a}$$

$$B_3 = t_{31} / (t_{21}^2 + t_{31}^2). \tag{32b}$$

The variance of $<b_1>$ over an ensemble of maneuvers can be shown to be

$$\sigma_{<b1>}^2 = \sigma_{b1}^2 + (B_2^2\, C_2 + B_3^2\, C_3)\, \sigma_\delta^2. \tag{33}$$

The variance of $<b_1>$ increases as the maneuver approaches being pure roll; if $t_{21}^2\sigma_{b1}^2 = t_{31}^2\sigma_{b1}^2 = \sigma_\delta^2$, then $\sigma_{<b1>}^2 = (1 + C_2/4 + C_3/4)\, \sigma_{b1}^2 = 2\sigma_{b1}^2$. Finally, we construct $<a_1>$ to be a good estimator of a_1:

$$<a_1> = e_1 - t_{11} <b_1> = e_1 - A_2\, e_2 - A_3\, e_3, \tag{34}$$

where

$$A_2 = t_{11}\, B_2, \tag{35a}$$

$$A_3 = t_{11}\, B_3. \tag{35b}$$

The variance of $<a_1>$ over an ensemble of maneuvers can be shown to be

$$\sigma_{<a1>}{}^2 = \sigma_{a1}{}^2 + (t_{12}{}^2 + t_{13}{}^2 + A_{22} C_2 + A_{32} C_3) \sigma_\delta{}^2 . \tag{36}$$

The variance of $<a_1>$ also increases as the maneuver approaches being pure roll; if $t_{21}{}^2\sigma_{b1}{}^2 = t_{31}{}^2\sigma_{b1}{}^2 = \sigma_\delta{}^2$, then $\sigma_{<a1>}{}^2 = \sigma_{a1}{}^2 + (C_2/4 + C_3/4) \sigma_{b1}{}^2 = \sigma_{a1}{}^2 + \sigma_{b1}{}^2$. Although $< a_1 >$ is defined using $<b_1>$, it is specifically tailored to remove the correlation with b_1 from e_1. To lowest order $<a_1> = a_1$, $<b_1> = b_1$, and $c = f(a_2, a_3, b_2, b_3)$, from which it is clear that $<a_1>$, $<b_1>$, and c are essentially independent. To find expressions for the weight matrices w, we construct a loss function from the squared sum of $<a_1>$, $<b_1>$, and c after normalization and set it equal to the original loss function, i.e.,

$$2j = \mathbf{e}^T w\, \mathbf{e} = c / \sigma_c{}^2 + <b_1>^2 / \sigma_{<b1>}{}^2 + <a_1>^2 / \sigma_{<a1>}{}^2 . \tag{37}$$

The corresponding elements of w are

$$w_{11} = 1 / \sigma_{<a1>}{}^2 , \tag{38a}$$

$$w_{22} = A_2{}^2 / \sigma_{<a1>}{}^2 + B_2{}^2 / \sigma_{<b1>}{}^2 + t_{31}{}^2 / \sigma_c{}^2 , \tag{38b}$$

$$w_{33} = A_3{}^2 / \sigma_{<a1>}{}^2 + B_3{}^2 / \sigma_{<b1>}{}^2 + t_{21}{}^2 / \sigma_c{}^2 , \tag{38c}$$

$$w_{12} = w_{21} = - A_2{}^2 / \sigma_{<a1>}{}^2 , \tag{38d}$$

$$w_{13} = w_{31} = - A_3{}^2 / \sigma_{<a1>}{}^2 , \tag{38e}$$

$$w_{23} = w_{32} = A_2 A_3 / \sigma_{<a1>}{}^2 + B_2 B_3 / \sigma_{<b1>}{}^2 + t_{21} t_{31} / \sigma_c{}^2 . \tag{38f}$$

In case 2 (an essentially pure roll maneuver), the components of $\mathbf{e}$ can be expressed as

$$e_1 = a_1 + b_1 , \tag{39a}$$

$$e_2 = a_2 + t_{21} b_1 + (\cos\theta) b_2 + (\sin\theta) b_3 , \tag{39b}$$

$$e_3 = a_3 + t_{31} b_1 - (\sin\theta) b_2 + (\cos\theta) b_3 , \tag{39c}$$

where θ is the roll angle. Because b_2 and b_3 are assumed to have equal variance, the variables r_+ and r_-, defined as

$$r_+ = (\cos\theta)\, b_2 + (\sin\theta)\, b_3 , \tag{40a}$$

$$r_- = -(\sin\theta)\, b_2 + (\cos\theta)\, b_3 , \tag{40b}$$

are independent random variables with the same variance σ_δ^2. The components of **e** are therefore of the form

$$e_1 = a_1 + b_1 , \tag{41a}$$

$$e_2 = r_2 + t_{21}\, b_1 , \tag{41b}$$

$$e_3 = r_3 + t_{31}\, b_1 , \tag{41c}$$

where r_2 and r_3 are independent random variables with variance $2\sigma_\delta^2$. The components of **e** are mildly correlated via b_1; b_1 contributes half of the variance of e_1 and at most one-third of the variances of e_2 and e_3 (for $t_{21}^2 = t_{31}^2 = \sigma_\delta^2 / \sigma_{b1}^2$). We neglect this mild correlation for maneuvers that are essentially pure roll by treating them as exactly pure roll, i.e., by setting $t_{21} = t_{31} = 0$. The weight matrix elements that follow from this assumption are

$$w_{11} = 1 / (\sigma_{a1}^2 + \sigma_{b1}^2) , \tag{42a}$$

$$w_{22} = w_{33} = 1 / 2\sigma_\delta^2 , \tag{42b}$$

$$w_{12} = w_{21} = w_{23} = w_{32} = w_{13} = w_{31} = 0 . \tag{42c}$$

The equations specified above provide the functional relationship between the elements of *w* and the uncertainties in attitude determination with respect to the spacecraft axes. These uncertainties can be derived from the attitude covariance matrix (e.g., see Reference 3), given by

$$P = \sigma_t^2 [I - \sum_{k=1,n} (\sigma_t^2 / \sigma_k^2)\, V_k\, V_k^T]^{-1} \tag{43}$$

where

n = number of measurements,

V_k = kth star vector used for attitude determination, as expressed in the spacecraft frame,

σ_k = uncertainty of kth measurement,

$\sigma_t^2 = [\ \Sigma\ (\ \sigma_k^2\ )^{-1}\ ]^{-1}$.

For spacecraft such as the HST, whose sensor orientations allow attitude determinations with uncorrelated estimates of pitch, roll, and yaw, the diagonal elements of the matrix ***P*** may be used as the attitude error variance required in the expressions for the elements of ***w***.

IV. NUMERICAL TESTING

The scheme specified above for applying statistical weights to RGA maneuver data has been implemented in the Payload Operations Control Center (POCC) Applications Software Support (PASS) system to be used in support of the HST mission. As indicated at the end of Section II, the algorithm was implemented in a way that allows independent calibration of the RGA scale factor / alignment and the RGA drift rate bias. RGA, FGS, and FHST data appropriate for a number of different calibration intervals were generated using the PASS attitude simulator (this is a simulator that produces data like that expected from the HST). The FGS and FHST errors were of order 0.5 and 10 arc-seconds, respectively. The data consisted of nine independent 90-degree maneuvers (three each of pitch, roll, and yaw) and one extended period of constant attitude data. The 90-degree maneuvers would require approximately 16 minutes of spacecraft time, whereas the constant attitude data represented approximately 45 minutes of spacecraft time. These data were in turn processed through the PASS attitude determination and RGA calibration software. Final residuals were calculated for each maneuver component by comparing the RGA-measured maneuver with the "true" maneuver as supplied to the simulator. The results were good. The final residuals for the components of **E** were in magnitude appropriate for the sensor type governing those residuals (e.g., e_2 for a pure pitch maneuver was in size like the simulated FGS errors, whereas e_1 was like the simulated FHST errors). Furthermore, the scheme described above allows for the combining of data sets in which different attitudes were determined with various different sensor combinations (i.e., FHST and FGS, FHST only, and FGS only). To test this aspect, we processed our simulated data through two RGA calibration scenarios: (1) with both FHST and FGS data used for all attitude determinations and (2) with each maneuver processed twice, once with FHSTs only and once with FGSs only. The final accuracy of the RGA calibration was essentially the same for both scenarios. For the nine-maneuver simulation, the largest deviation (when comparing the measured maneuver magnitude after calibration with the true maneuver magnitude) was found to be about 30 parts per million. This is both appropriate for the magnitude of the sensor errors and sufficiently accurate to support the needs of the HST mission.

V. CONCLUSIONS

We have presented a general algorithm for the calibration of a spacecraft rate gyro assembly, as well as a data weighting scheme that produces a statistically optimal solution. The weighting scheme, although explicitly tailored for use during the Hubble Space Telescope mission, is applicable to any three-axis stabilized spacecraft. Numerical simulations demonstrate that the algorithm works as expected in theory and is capable of supporting the needs of the HST mission.

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