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(PRELIMINARY) MODELING, SIMULATION AND CONTROL FOR A CRYOGENIC FLUID MANAGEMENT FACILITY

How relevant to the complex space station applications are academic formulation and solution of control problems, based on recently published textbook methodology complemented with limited laboratory scale experiments? The textbook abstractions are often stripped of consideration of constraints of prime concern to the field application: process capacities, user demands, economics, hazards analysis and fault tolerance. However, the approach of the classroom -- simplistic of necessity due to man-hour and funding constraints -- serves as a starting point for formulating a "top-down modular" definition of the problem and development of an overall perspective for the research professor or student. The individual is thus conditioned to readily adapt to a position in team efforts with major funding.

As one of an ongoing series of term projects in Process Monitoring and Control at UH-CL, the class in PROC 5232: Process Modeling, Simulation and Control, has studied the synthesis of a control system for a cryogenic fluid management facility. The severe demands for reliability as well as instrumentation and control unique to the space station environment are prime considerations.

Realizing that the effective control system depends heavily on quantitative description of the facility dynamics, a methodology for process identification and parameter estimation is postulated. A block diagram of the associated control system is also postulated. Finally, an on-line adaptive control strategy is developed utilizing optimization of the velocity form control parameters -- proportional gains, integration and derivative time constants -- in appropriate difference equations for direct digital control.

Of special concern are the communications, software and hardware supporting interaction between the ground and orbital systems. It is visualized that specialists in the OSI/ISO utilizing the Ada programming language will influence further development, testing and validation of the simplistic models here presented for adaptation to the actual flight environment.

MODELING, SIMULATION AND CONTROL
FOR
CRYOGENIC FLUID MANAGEMENT FACILITY

Abstract Outline

1. PROCESS DESCRIPTION

1.1 Baseline Configuration: CFMFE Flight System

The initial concept is diagrammed in Figure 1.1. Assembled as a module for mounting in the shuttle, it consists of three submodules identified with successive operational stages:

a) chilldown of the Ground Fill Line on the pad;

b) chilldown and filling of the Supply Tank on the pad;

c) chilldown of the Transfer Line combined with chilldown and filling of the Receiver Tank in orbit.

The submodules for operational stages a) and b) are detailed in Figure 1.1.1. The submodules for operational stages c) and d) are detailed in Figure 1.1.2.

N O T I C E

At the deadline for submitting manuscripts this paper was incomplete.

Copies of the completed version will be made available at the presentation to those who desire one.

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CFMF SIMPLIFIED SCHEMATIC

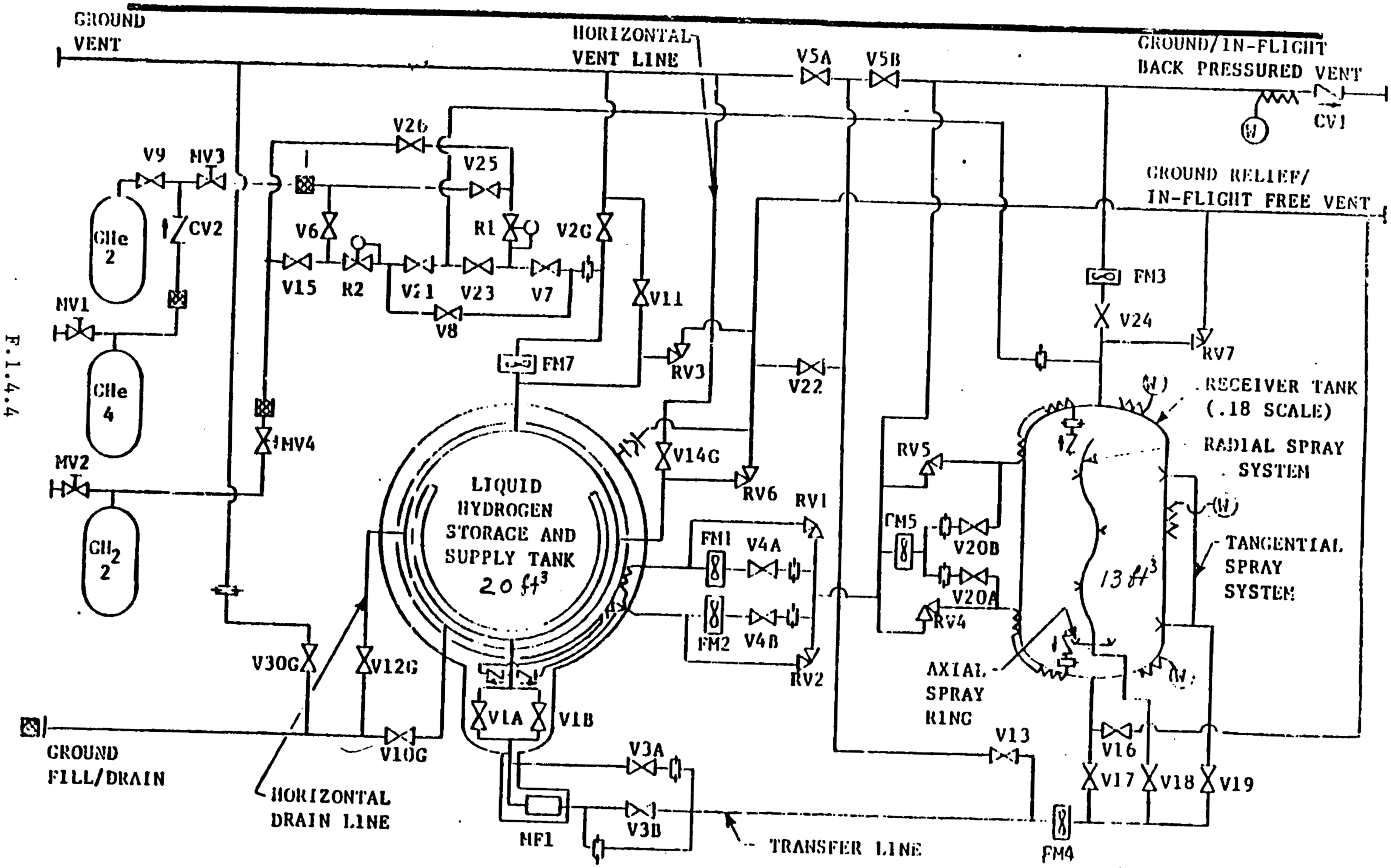

FIGURE 1.1 INITIAL CONCEPT

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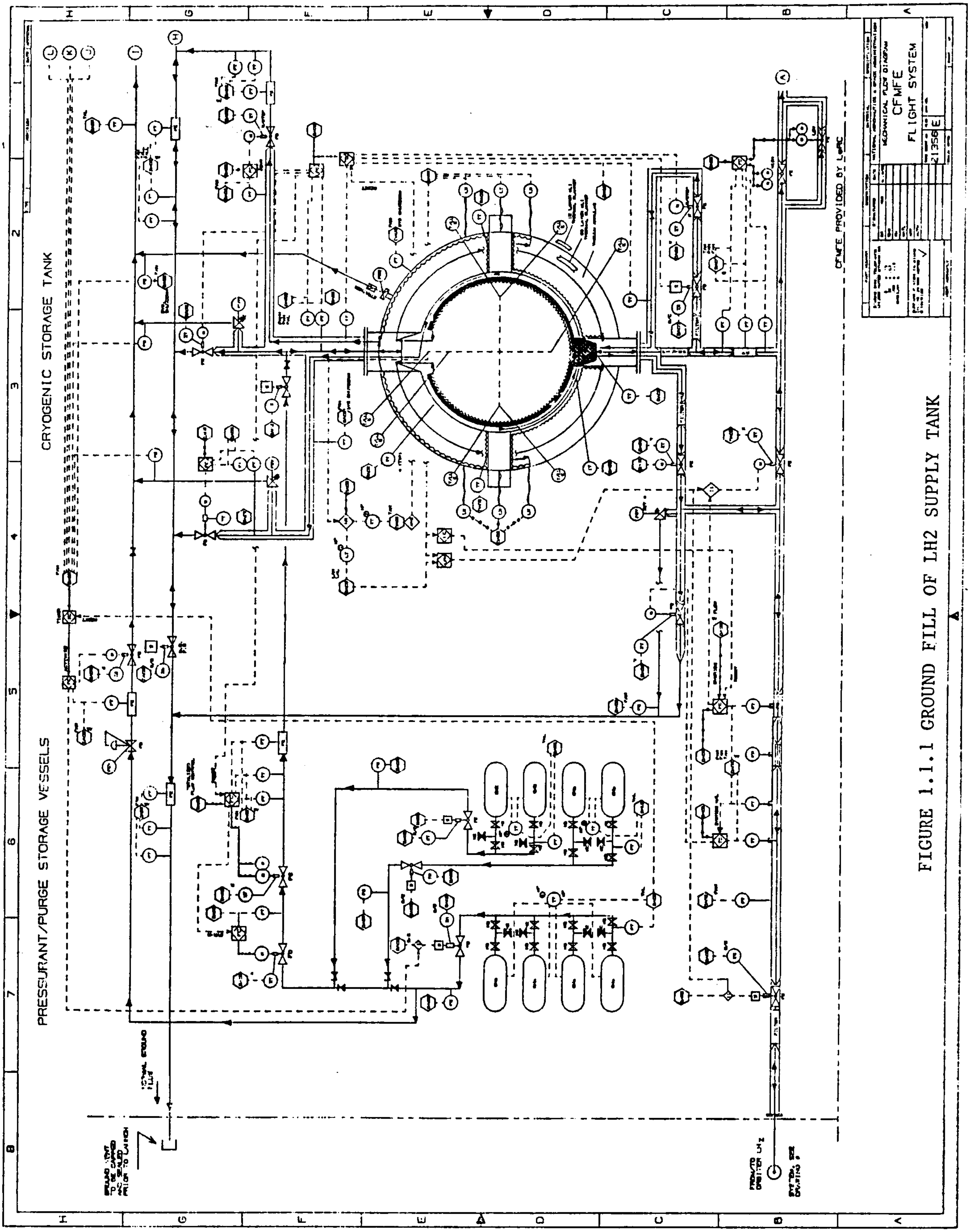


FIGURE 1.1.1 GROUND FILL OF LH2 SUPPLY TANK

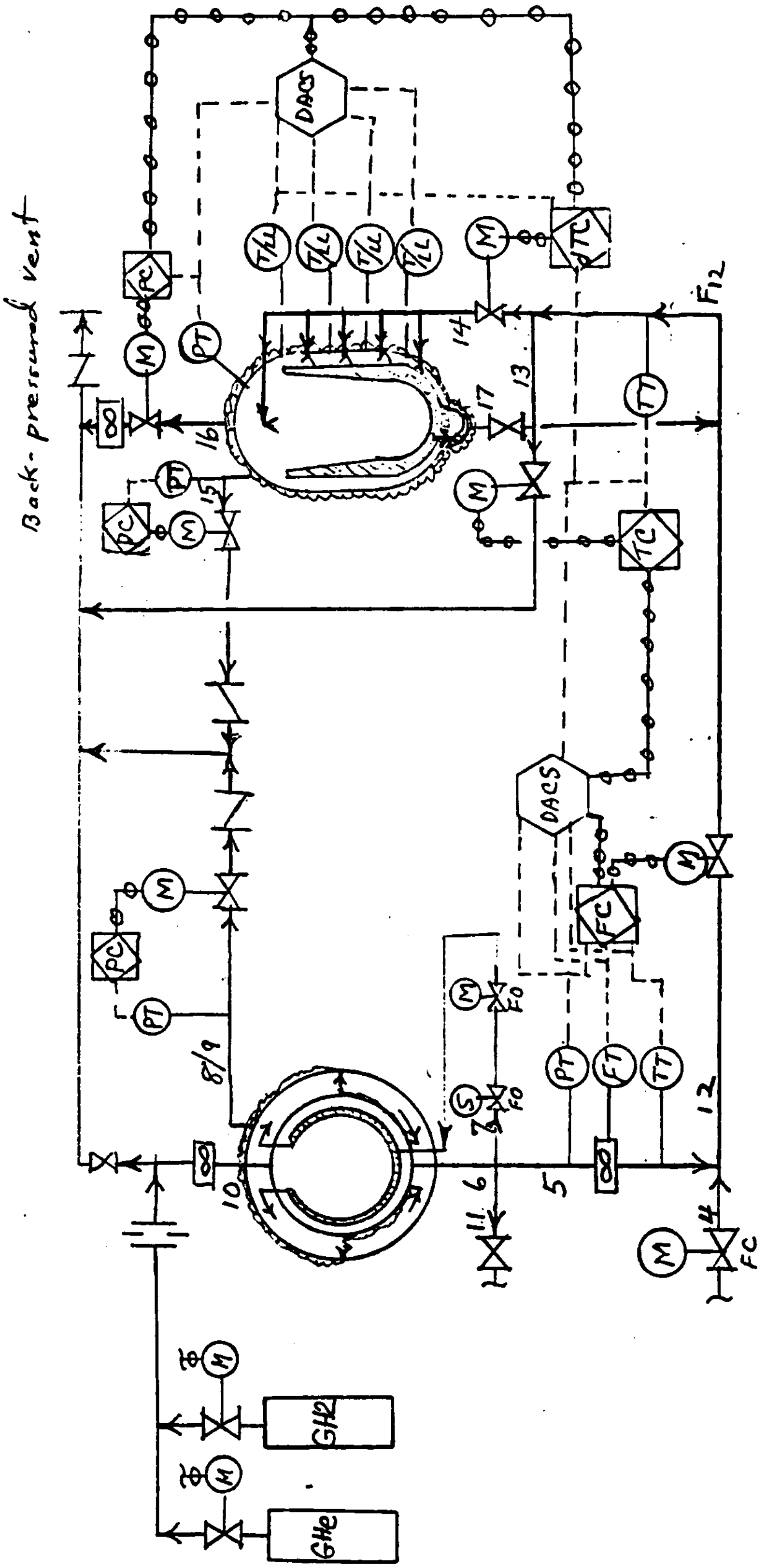


FIGURE 1.1.1 LH2 TRANSFER TO RECEIVER TANK

CFMFE

1. PROCESS DESCRIPTION

1.2 Functional Requirements and Constraints

Time:	Chilldown time,	$\theta_c = 15$ min.
	Fill time,	$\theta_F = 60$ min.
Pressure:	Minimum	$P_{min} = 1$ atm
	Maximum excursion	$P_{max} = 85$ psig (PSV spec)
Temperature:	Minimum	$T_{min} = 36.7^{o}R$ ($20.4^{o}K$)
	Ambient	$T_a = 530^{o}R$
	Maximum ΔT_{87}	ΔT_{87max} = (TBD)
Conservation of H2:	(TBD)	
Hazards:	Explosion and fire (TBD)	
	Destructive vibration (TBD) and shock	
	Stress fractures (TBD)	
	Loss of power (TBD)	
Zero-gravity:	Liquid pressurization (TBD)	
	Chilldown of receiver tank system (TBD)	
	Filling receiver tank (TBD)	
	Contingency respondent and fault tolerant (TBD)	

CFMFE

1. PROCESS DESCRIPTION

1.3 Problem Identification

1.3.1 Thermal balances and minimized system chilldown and fill times

On the pad:

1. The Ground Fill line
2. The CH2 Storage and Supply Tank

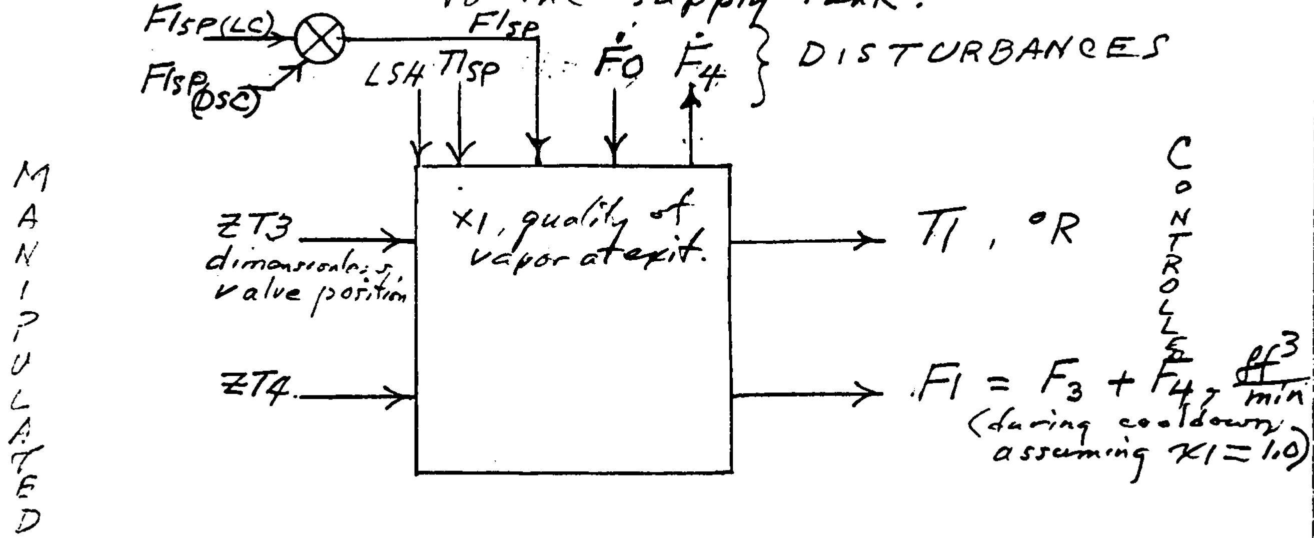
In orbit:

3. Chilldown time for the transfer line from the Supply Tank to the Receiver Tank
4. Chilldown and fill time for the Receiver Tank

Special problems:

5. Overpressures and destructive stresses
6. Delayed GH2 boiloff due to heat transfer

The I/O model for the transfer line from the LH2 source to the supply tank:



disturbances:

$T1_{SP(t)} = A_T + B_T t_D$, °R (TBD) (dictated by computer)

where $t_D = t/\theta$
θ = cooldown time, i.e., the time until $T_1 - T_{LH2} < D_{1max} \cong 5°K$ (?)
LC = average measured LH2 level in supply tank.

$F1_{SP(t)} = F1(LC, TS)$, ft³/min (TBD)
(during cooldown $x1 = 1.0$;
when filling the supply tank,
$x1$ = small fraction or negligible)
(dictated by computer)

$\dot{F}0 = \dot{F}0$ (input LH2 flowrate defined by the fuel cell facility) (TBD)

$\dot{F}_4 = \dot{F}_4$ (demanded by the follow-on supply tank module for cooldown and filling period) (TBD)

LSH

controlled variables:

$T1$ = line 1 temperature subject to constraints $|dT_1/dt| < D_{max}$, $P1_{Low} < P1 < P1_{HIGH}$ (TBD)

$F1$ = line 1 volumetric flowrate controlled mainly to limit maximum possible pressure surges in the line (TBD)

manipulated variables

$ZT3 = PI(T1)$

$ZT4 = PI(F1)$

$ZS1 = DDC \cdot \overline{LSH}$ (logical)

Problems: (omit)

1.2 Draw the appropriate block diagram analogous to Fig. 29.10 in Stephanopoulos.

1.3 Write the closed-loop transfer functions.

on the pad

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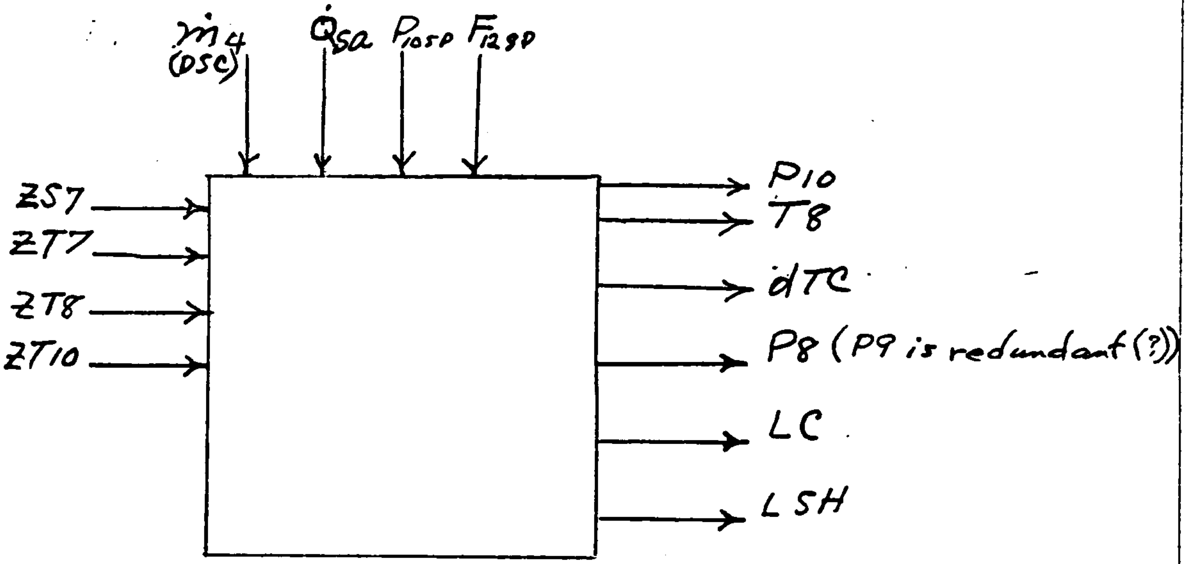

<u>disturbance</u>: $\dot{m}_4 = \dot{m}_4(F1, ZT3, ZT4, T1, P1, \dot{Q}_1)$ (TBD)
$\dot{Q}_a$ = nil for loading but contributes to boiloff afterwards (TBD)

<u>controlled</u>:
P10 ≈ Supply tank pressure, $P10 < P_{10max}$ (TBD)
T8 = Cooling coil vent gas temperature
dTC ≡ $T8 - T7 < D_{max}$ (TBD)
P8 = Cooling coil vent gas pressure $< P8_{max}$ "
LC = average of 4 supply tank measurement
LSH = supply tank level high causing interrupt closing of ZT4

<u>manipulated</u>
ZS7 = ($dTC < dTC_{max}$) (TBD)
ZT7 = PID($dTC = T_8 - T_7$) "
ZT8 = PI (P8) "
ZT10 = PI (P10) "

in orbit

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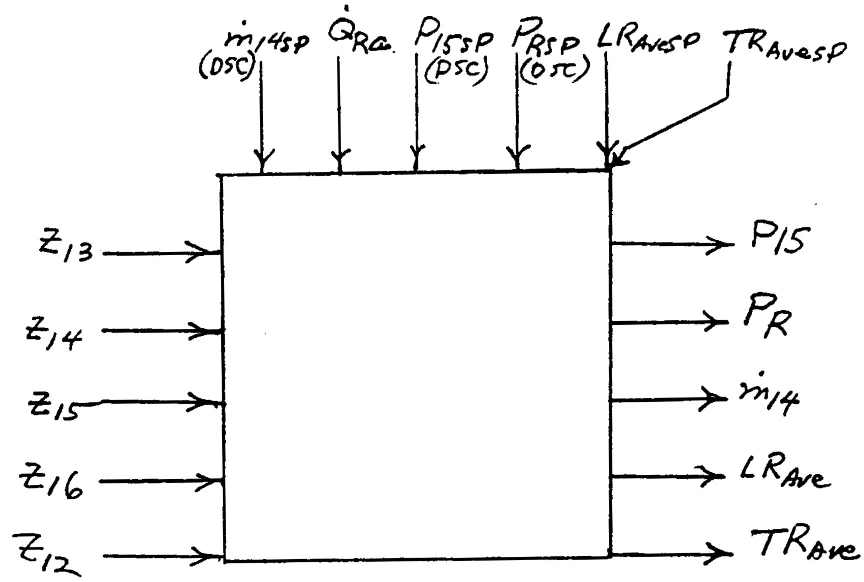


disturbances: $\dot{Q}_{Ra} = \dot{Q}_{Ra}$ (energy stored in concentrated masses, heat leakage from environment)
$\dot{m}_{14SP} = \dot{m}_{14SP}$ (supervisory control by DACS)
$P_{15SP}, P_{RSP}, L_{AveSP}, TR_{AveSP}$ = fcns(time, supervisory control by the DACS)

controlled: P_{15} = cooling coil vent gas pressure, $P_{15} < P_{15max}$ (TBD)
P_R = receiver tank pressure, $P_R < P_{Rmax}$
$\dot{m}_{14}$ = H2 mass flow rate, a fcn of time responding to the setpoint scheduled by the DACS.
LR_{Ave} = average of liquid level measurements in tank, a fcn of time reponding to the set point scheduled by the DACS
TR_{Ave} = average of tank temperature measurements, a fcn of time responding to the set point scheduled by the DACS.

manipulated: $Z_{12} = PI(F5)$
$Z_{13} = PI(T12)$
$Z_{14} = PID(\delta TC_{TR,13})$
$Z_{15} = P(P_{15})$
$Z_{16} = P(P_R)$

2. MATHEMATICAL MODELING

2.2 Analysis, Degrees of Freedom and Control Loops

2.2.1 The Ground Fill Line

Physical model for an energy balance:
Assumptions:

1. The aluminum tube is perfectly insulated
2. LH2 enters with quality $x = 0$
3. Until childown is essentially complete, the exit GH2 has a quality of $x = 1$
4. Maximum childown rates are limited by the venting capacity of the line
5. Significant thermal energy sources which limit the minimum cooldown time are the concentrated masses associated with stainless steel control valves and sensors.
6. The enthalpy of LH2 at near atmospheric pressure is given by:

$$h = 278.4 + 441.8x + 10.13\,(T-21)$$

$$= 507.47 + 10.13T,\ \text{kJ/kg} \quad \text{using the unit } {}^{o}K$$

$$= 218.63 + 2.425T,\ \text{Btu/lb}_m \quad \text{using the unit } {}^{o}R$$

Reference: Perry and Green, Ch.E. Handbook, McGraw-Hill 1984, pp 3-1958

$$C_{pf} = C_{pg} = 10.13\ \text{kJ/kg}^{o}K \text{ at } 21^{o}K$$

7. The heat capacity of Al is:

$$C_{vAl} = -\,0.1362 + 0.007528T - 0.00001356T^2 \quad \text{kJ/kg}^{o}K$$

with T in ^{o}K

$$C_{vAl} = -\,0.03254 + 0.000999T - 9.99 \times 10^{-7}T^2 \quad \text{Btu/lb}_m{}^{o}R$$

with T in ^{o}R

Reference: Perry and Green, 1984, pp 3-135

8. The heat capacity of stainless steel is:

$$C_v = -\,0.0586 + 0.003219T - 5.078 \times 10^{-6}T^2 \quad \text{kJ/kg}^{o}K$$

with T in ^{o}K

$$C_v = -\,0.0140 + 0.000428T - 3.75 \times 10^{-7}T^2 \quad \text{Btu/lb}_m{}^{o}R$$

with T in ^{o}R using $1\ \text{Btu/lb}_m{}^{o}R = 4.178\ \text{kJ/kg}^{o}K$

2.2.1 The Ground Fill Line

Assumptions (continued)

9. Radiation heat transfer rates across the annulus of concentric tubes or spheres is nil compared to convective heat transfer rates from Al to LH2:

$$q/A \overset{0}{=} 300 \text{ Btu/hrft}^2 \text{ from Al to LH2 at } 36.7^oR$$

$$q/A = F_{12}\sigma(T^4 - T^4_{LH2}) \overset{0}{=} 12.7 \text{ Btu/ft}^2\text{hr from StSt to Al}$$

$$\text{at } 530^oR \text{ where } F_{12} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \overset{0}{=} \frac{1}{\frac{1}{.1} + \frac{1}{.2} - 1} \overset{0}{\simeq} 0.094$$

$$\sigma = .1713 \times 10^{-8} \text{ Btu/(ft}^2\text{hr}^oR^4)$$

Reference: Perry and Green, 1984

10. Thermal diffusivities, $k/\rho C_v$:

	Al	St.St.
20^oK	0.5	0.040
100^oK	0.00023	$\sim 9 \times 10^{-5}$
300^oK	0.00011	3.3×10^{-5}

Reference: Perry and Green, 1984, pp 3-263

11. $\rho_{GH2} = 0.104 \text{ kg/m}^3 \text{ at } 20.4^oK$

$\rho_{LH2} = 70.57 \text{ kg/m}^3 \text{ at } 20.4^oK \text{ or } 4.72 \text{ lb}_m/\text{ft}^3$

11. Densities: $\rho_{Al} = 2723 \text{ kg/m}^3 \text{ or } 170 \text{ lb}_m/\text{ft}^3$

$\rho_{StSt} = 7900 \text{ kg/m}^3 \text{ or } 492 \text{ lb}_m/\text{ft}^3$

Reference: Perry and Green, 1984, pp 3-96

12. Thermal conductivities:

Reference: Perry and Green, 1984, pp 3-261

13. Convective heat transfer coefficients:

References:
1) Perry and Green, 1984, pp 10-23
2) H.H. Walters, AiResearch Manufacturing Compant "Single-Tube Heat Transfer Tests with Liquid Hydrogen", (see WADC Technical Report 59-423)
3) Drake et al., Arthur D. Little, Inc. "Pressurized Cool-Down of a Cryogenic Liquid Transfer system Containing Vertical Sections", (tests with LO2)

2.2.1 The Ground Fill Line

Assumptions (continued)

Walters -- LH2 tests:

film boiling: $h = 460$ to 540 Btu/hrft^{2o}R
for inlet (?) = 1.6 to 1.7 atm

$Re \overset{0}{=} 3 \times 10^5$

nucleate boiling: h = 10 x value for film boiling

Drake et al. -- LO2 tests:

film boiling: $h = 300$ Btu/ft^{2}hr^{o}R
for inlet pressure = 20 psig
outlet pressure = 10 psig

$h = 200$ Btu/ft^{2}hr^{o}F
for inlet pressure = 10 psig
outlet pressure = 5.5 psig

LH2 -- assume: $h_{max} = 500$ Btu/hrft^{2o}R (uncontrolled)

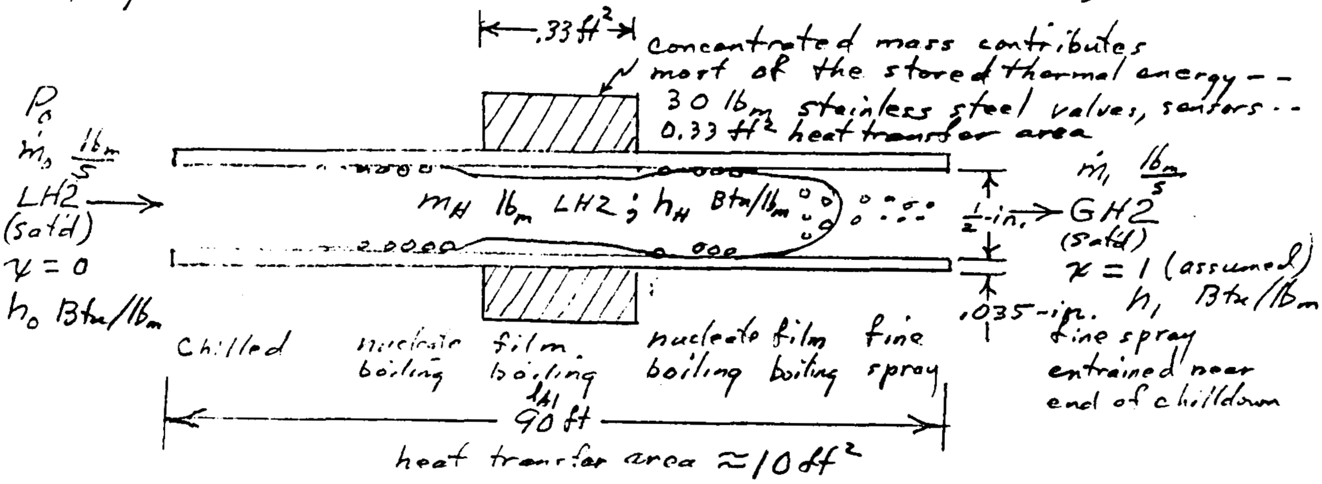
$h_{ave} = 300$ Btu/hrft^{2o}R (controlled)

$Re \overset{0}{=} 3 \times 10^5$

14. Critical constants of H2: $P_c = 12.8$ atm

$T_c = 33.3^oK = 60.0^oR$

physical description (nominal dimensions and masses)



thermal balances and heat transfer problems

Initially, the Ground Fill Line is at ambient temperature. As LH2 is introduced the liquid phase moves through the line in such a way that the internal Al tubing is chilled within a short distance behind the advancing LH2 front. Prolonged boiling occurs over contact surfaces associated with the concentrated masses.

If the LH2 is introduced at an adequate rate, the maximum cooldown rate at near atmospheric outlet pressure (P_1) is limited by the venting capacity of the tube at the given inlet pressure (P_o)

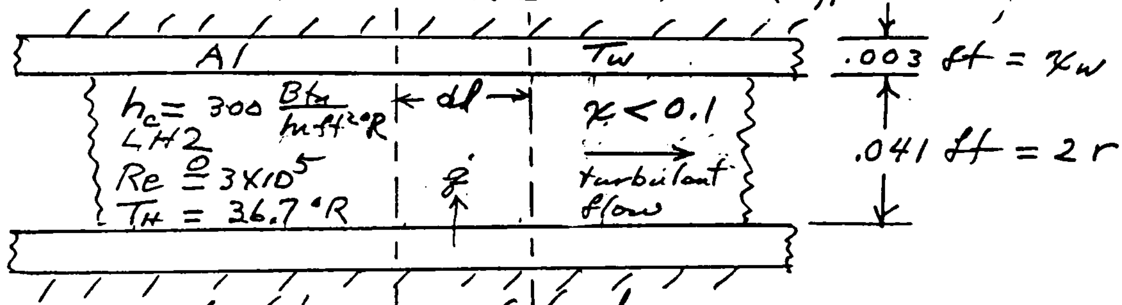
If the LH2 is introduced at an excessive rate, explosive overpressures and thermal contraction can result in destructive stresses.

Four problems are addressed:

Prob. 1. Cooldown time for a perfectly insulated infinitely long Al tube at ambient temperature suddenly filled with LH2 at atmospheric pressure;
Prob. 3. cooldown time for a finite concentrated mass joined to the Al tubing over a finite area;
Prob. 2, potential overpressures during the cooldown of the infinitely long Al tube suddenly filled with LH2; and
Prob. 4. minimum cooldown time dictated by the venting capacity of a finite tube to which concentrated masses are joined.

(A hypothetical case not physically possible.)

Problem 1: Cooldown time for a perfectly insulated infinitely long Al tube at ambient temperature suddenly filled with LH2 at constant pressure of 1 atmosphere, and with $Re \cong 3 \times 10^5$.



perfectly insulated

$$\dot{q} = 2\pi r\, dl\, h_c (T_W - T_H)$$

$$= -\rho_W 2\pi r\, x_W\, dl\, c_W \frac{dT_W}{dt}$$

$$= \pi r^2 dl\, \rho_H\, h_{fg} \frac{dx}{dt} \quad ; \quad Btu/hr$$

where heat of vaporization, $h_{fg} = 441.8$ kJ/kg $= 190$ Btu/lbm

$dT_H/dt = 0$ (assumed)

density, $\rho = 170$ lbm/ft³

$$c_W \approx -0.014 + 0.000428\,T - 3.75 \times 10^{-7}\,T^2 \quad Btu/lbm\text{-}°R$$

$= 0.0509$ Btu/lbm°R at 180°R

$= 0.0915$ " at 360°R

dividing by $2\pi r\, dl\, h$

$$T_W - T_H = -(\rho_W x_W c_W / h) \frac{dT_W}{dt} = -\tau_W \frac{dT_W}{dt}$$

$$= \frac{1}{2} r\, \rho_H \frac{h_{fg}}{h_c} \frac{dx}{dt}$$

assuming T_H is constant and heat transfer to LH2 merely converts the liquid to vapor

$$\tau_W \frac{dT_W}{dt} + T_W = T_H$$

define $D_W = T_W - T_H$, $\frac{dT_W}{dt} = \frac{dD_W}{dt}$

$$\tau_W \frac{dD_W}{dt} + D_W = 0$$

$$D_W(t) = D_{W(0)}\, e^{-t/\tau_W}$$

(later: integrate for $c_W = c_W(t)$)

First approximation

$$T_{W(t)} = (T_{W(0)} - 36.7) \exp\{-t/[(170 \times .003 \times .09/300) \cdot 3600]\} + 36.7\ °R$$

$$T_{W(t)} = (T_{W(0)} - 36.7)\, e^{-2.977t} + 36.7\ ,\ °R$$

with time in seconds

F.1.4.16

problem 1 (cont'd):

For $T_{w(0)} = 536.7\,^\circ R$ (298.2 °K)

$$T_{w(t)} = 500 \exp(-2.977 t) + 36.7, \quad ^\circ R$$

t, sec	$T_{W(t)}$, °R	
0	536.7	
1	62.2	
2	38.0 (1.3 °R above b. pt.)	} nucleate boiling reduces these values for $T_W - T_H < 5\,^\circ R$
3	36.77 (0.07 °R above b. pt.)	

problem 2: the potential overpressure during cooldown of the infinitely long Al tube of problem 1 suddenly filled with LH2

if assume $x_{(0)} = 0$, i.e., the line is completely filled with LH2; and, further, if assume absolutely no strain in the tube wall as well as zero compressibility of the LH2, then all heat transfer yields an increase in enthalpy (h_{LH2}) of the LH2:

$$2\pi r x_W \, d\ell \, \rho_{AL} \int_{T_a}^{T_{LH2},P_{atm}} c_{AL}\, dT \;\stackrel{=}{=}\; \pi r^2 d\ell \, \rho_{LH2} \int^{T_{LH2},P_{atm}} c_{pf}\, dT$$

$$2 \times .003 \times 170 \int_{T_a}^{T_{LH2},P_{atm}} [-0.03254 + 0.000999T - 9.99 \times 10^{-7} T^2] dT \quad T_{LH2}, 1atm$$

$$= .041 \times 4.72 \int_{T_{LH2},1atm}^{T_{LH2},P_{atm}} 2.425\, dT$$

$$-0.0325(T_{LH2,P_{atm}} - 530) + \frac{0.000999}{2}(T^2_{LH2,P_{atm}} - 530^2) - 3.33 \times 10^{-7}(T^3_{LH2,P_{atm}} - 530^3) \;\stackrel{=}{=}\; \frac{.041 \times 4.72 \times 2.425}{2 \times .003 \times 170}(T_{LH2,P_{atm}} - 36.7)$$

$$T_{LH2,P_{atm}} \approx 185\,^\circ R \quad \text{(supercritical)}$$

$$P_{LH2} \stackrel{\sim}{=} \frac{\rho RT}{144} = 4.72 \times 766 \times 185/144 = 4650 \text{ lbf/in}^2$$

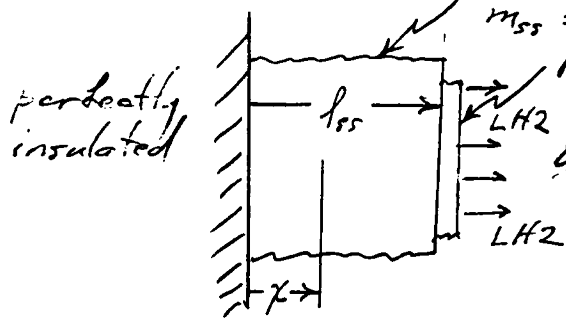
potential overpressure $\stackrel{\sim}{=}$ 300 atm.
within 3 seconds (?)

problem 3: Cooldown time for a concentrated mass:

Reference: J.C. Burke et al, Arthur D. Little, Inc.
"Pressurized Cooldown of Cryogenic ~~Transfer~~ Lines" F-5, (?)
-- based on tests with LN2.

Stainless Steel slab with mass $m_{ss} \cong 30\ lb_m$

Al tube wall



$$q = hA_c(T_{(l_{ss},t)} - T_{LH2})$$

$$\overset{\text{O}}{=} 300 \times .33(T_{(l_{ss},t)} - 36.7)\ \frac{Btu}{hr}$$

$$l_{ss} \approx \frac{30}{492} \times \frac{1}{.33} \overset{\text{O}}{=} 0.185\ ft. \quad (2.2\text{-in})$$

assume: $\frac{dT}{dx} = 0$ in the Al tube wall
$P = 1\ atm$

By Fourier field equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad \frac{°R}{hr} \quad \text{within the steel slab} \quad (a)$$

where

$$\alpha = \frac{k}{\rho c_v} \approx 8 \times 10^{-5} \frac{m^2}{s}$$

initial & boundary conditions

$t = 0 \quad T_{(x,0)} = T_a = 530°R$

$x = 0 \quad \frac{\partial T(0,t)}{\partial x} = 0$

$x = l_{ss} \quad q = hA_c(T_{(l_{ss},t)} - T_{LH2}) = kA_c \frac{\partial T_{(l_{ss},t)}}{\partial x}$

in dimensionless form

$\Theta \equiv$ cooldown time, i.e., the time required to cool the slab to negligible $\Delta T = T_{ave} - T_{LH2} \leq E_{\Delta T}$ (TBD) (?)

$t_D = t/\Theta \quad 0 < t_D < 1.$

$\alpha_D = \frac{\alpha_{ss}\Theta}{l_{ss}^2}$

$k_D = \frac{k}{l_{ss}} \longrightarrow$

$x_D = x/l_{ss} \quad 0 < x_D < 1$

$$\boxed{\begin{gathered} \frac{\partial T}{\partial t_D} = \alpha_D \frac{\partial^2 T}{\partial x_D^2} \qquad (b) \\ t = 0, \quad T_{(x_D,0)} = T_a \\ x_D = 0, \quad \frac{dT_{(0,t_D)}}{dx_D} = 0 \\ x_D = 1, \quad \frac{\partial T_{(1,t_D)}}{\partial x_D} = \frac{h}{k_D}(T_{(1,t_D)} - T_{LH2}) \end{gathered}}$$

F

note: $\frac{h}{k_D} \overset{\circ}{\approx} \frac{500}{.0566/.185} \overset{\circ}{\approx} 1634$

where $k_{SiSt} = 0.0566 \frac{Btu}{ft\,hr\,°R}$ at 200 R

$\therefore$ assume

$$\frac{h}{k_D} = \infty$$

$$\frac{\partial T(1, t_D)}{\partial x_D} = \infty$$

Laplace domain

$$s\,T_{(x_D,s)} - T_{(x_D,0)} = \alpha_D \frac{\partial^2}{\partial x^2} T_{(x_D,s)}$$

$$\frac{\partial^2}{\partial x^2} T_{(x_D,s)} - \frac{s}{\alpha_D} T_{(x_D,s)} = -\frac{1}{\alpha_D} T_a$$

which yields a general solution in the Laplace domain as a function of x_D

$$T_{(x_D,s)} = A_{(s)}\, e^{\sqrt{\frac{s}{\alpha_D}} x_D} + B_{(s)}\, e^{-\sqrt{\frac{s}{\alpha_D}} x_D} - \frac{T_a}{\alpha_D}$$

$$T_{(0,s)} = A_{(s)} + B_{(s)} - T_a/\alpha_D$$

$$B_{(s)} = T_{(0,s)} + T_a/\alpha_D - A_{(s)}$$

$$T_{(1,s)} = A(s)\, e^{\sqrt{\frac{s}{\alpha_D}}} + \left(T_{(0,s)} + \frac{T_a}{\alpha_D} - A_{(s)}\right) e^{-\sqrt{\frac{s}{\alpha_D}}} - \frac{T_a}{\alpha_D} \approx \frac{T_{LN2}}{s} \qquad (C)$$

since by the boundary condition at $x_D = 1$:

$$\frac{\partial}{\partial x_D} T_{(1,s)} = \frac{h}{k_D}\left(T_{(1,s)} - \frac{T_{LN2}}{s}\right)$$

$$T_{(1,s)} = \frac{T_{LN2}}{s} + C e^{-\frac{h}{k_D}} \quad \text{where } C e^{-1634} \approx C e^{-\infty}$$

$$= \frac{T_{LN2}}{s}$$

$A_{(s)}$ = (explicit relationship from eq (C)) which can be expressed as a series relationship in s.

(TB Completed Later)

Solution of eq.(6) by separation of variables

$$T = \Theta(t)\,\psi(x)$$

$$\frac{\partial T}{\partial t_D} = \psi(x)\frac{\partial \Theta}{\partial t_D}, \quad \frac{\partial^2 T}{\partial x_D^2} = \Theta(t)\frac{\partial^2 \psi(x)}{\partial x_D^2}$$

$$\psi\frac{\partial \Theta}{\partial t_D} = \alpha_D\,\Theta\frac{\partial^2 \psi}{\partial x_D^2}$$

$$\frac{1}{\Theta}\frac{\partial \Theta}{\partial t_D} = \alpha_D\frac{1}{\psi}\frac{\partial^2 \psi}{\partial x_D^2} = -H$$

(a positive H yields a solution that increases without limit as $t \to \infty$)

$$\Theta = C_0 e^{-Ht_D}$$

$$\psi = C_1 e^{j\sqrt{\frac{H}{\alpha_D}}x_D} + C_2 e^{-j\sqrt{\frac{H}{\alpha_D}}x_D}$$

$$T = e^{-Ht_D}\left(C_1 e^{j\sqrt{\frac{H}{\alpha_D}}x_D} + C_2 e^{-j\sqrt{\frac{H}{\alpha_D}}x_D}\right)$$

$$\frac{\partial T}{\partial x_D} = e^{-Ht_D}\,j\sqrt{\frac{H}{\alpha_D}}\left(C_1 e^{j\sqrt{\frac{H}{\alpha_D}}x_D} - C_2 e^{-j\sqrt{\frac{H}{\alpha_D}}x_D}\right)$$

$$\left.\frac{\partial T}{\partial x_D}\right|_{x_D=0} = e^{-Ht}\,j\sqrt{\frac{H}{\alpha_D}}\left(C_1 - C_2\right) = 0$$

$$C_2 = +\,C_1$$

$$T = e^{-Ht_D}\,2C_1\cos\sqrt{\frac{H}{\alpha_D}}x_D$$, a particular soln which leads to a general solution

$$T = \sum_{n=0}^{\infty} e^{-H_n t_D} K_n \cos\left(\sqrt{\frac{H}{\alpha_D}}x_D + 2n\pi\right) \quad (7)$$

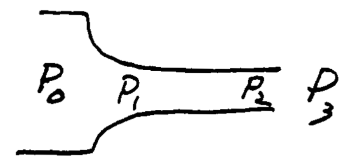
(To be completed later)

Problem 4. minimum cooldown time dictated by the venting capacity of a finite tube to which concentrated masses are attached

at the tube exit assume;

sonic velocity for saturated GH2 at 1 atmosphere

for $k = 1.6$
the limiting ratio



$$\frac{P_2}{P_0} = \frac{P_1}{P_0} = 0.497$$

and the maximum mass velocity for isentropic flow is

$$G^\circ = P_0 \sqrt{\frac{g_c M k}{R T_0}\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}}, \ kg/s \cdot m^2$$

$$= 1.01\times10^{6}\sqrt{\frac{1\times2\times1.6}{8314\times36.7}\left(\frac{2}{1.6+1}\right)^{(1.6+1)/(1.6-1)}}$$

$$= 583 \ kg/s\,m^2$$

$$= 119 \ lb_m/s \cdot ft^2$$

$$G^\circ A_{tube} = 119 \times \frac{\pi}{4} \times \left(\frac{.5}{12}\right)^2$$

$$= 0.163 \ lb_m \ GH2/s$$

lb_m LH2 required to remove sensible heat from Ground Fill Line

$$\left[30 \ lb_m SS \times .09 \frac{Btu}{lb_m SS\,^\circ R} + 7 \ lb_m Al \times .19 \frac{Btu}{lb_m Al\,^\circ R}\right] \times (536.7 - 36.7)\,^\circ R$$

$$= \qquad Btu$$

$$\text{cooldown time} \geq \frac{2017 \ Btu}{0.163 \ \frac{lb_m GH_2}{s} \times (441.8/1.056 \times 2.2) \frac{Btu}{lb_m GH_2}}$$

$$\geq 65 \ \text{seconds}$$

twophase flow rate through 90 ft. of tubing will entail significant frictional losses!
(T B Completed later)

Background: The previous presentation for the Transfer Line
Perry & Green, Ch.E. Handbook, p10-59:
eq.(10-195) where $A_1\mathcal{F}_{12} \equiv A_2\mathcal{F}_{21}$

$$\sigma = (0.1713)(10^{-8})\frac{Btu}{ft^2\text{-}h\text{-}{}^{\circ}R^4} \qquad = \frac{1}{\frac{1}{A_1\epsilon_1} + \frac{1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}$$

$\epsilon_1 \approx 0.10$ for Al

$\epsilon_2 \approx 0.60$ for St-Steel

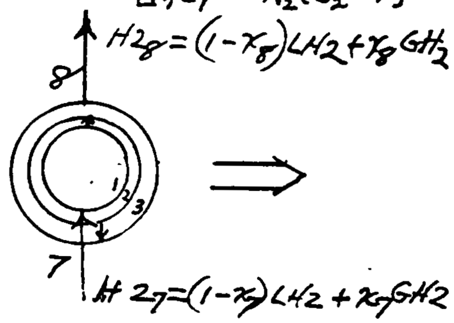
$$\dot{Q}_{i\to j} = A_i\mathcal{F}_{ij}\sigma T_i^4 - A_j\mathcal{F}_{ji}\sigma T_j^4$$

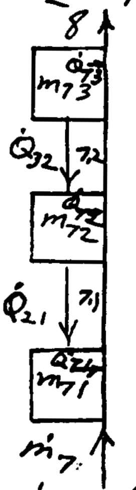
$$= A_1\mathcal{F}_{12}\sigma(T_1^4 - T_2^4)$$

$$= A_1\mathcal{F}_{12}\sigma(T_1^3 + T_1^2T_2 + T_1T_2^2 + T_2^3)(T_1 - T_2)$$

FIGURE 2.1

$$A_1\mathcal{F}_{12} = \frac{1}{\left[\frac{1}{A_1\epsilon_1} + \frac{1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)\right]}$$





$h_7 = (1-x)h_{LH2} + xh_{GH2}$
at 20.4+ °K

Problems:
1.5 Write the energy balances over submasses accounting for transfer of heat between submasses as well as to the H2 coolant.

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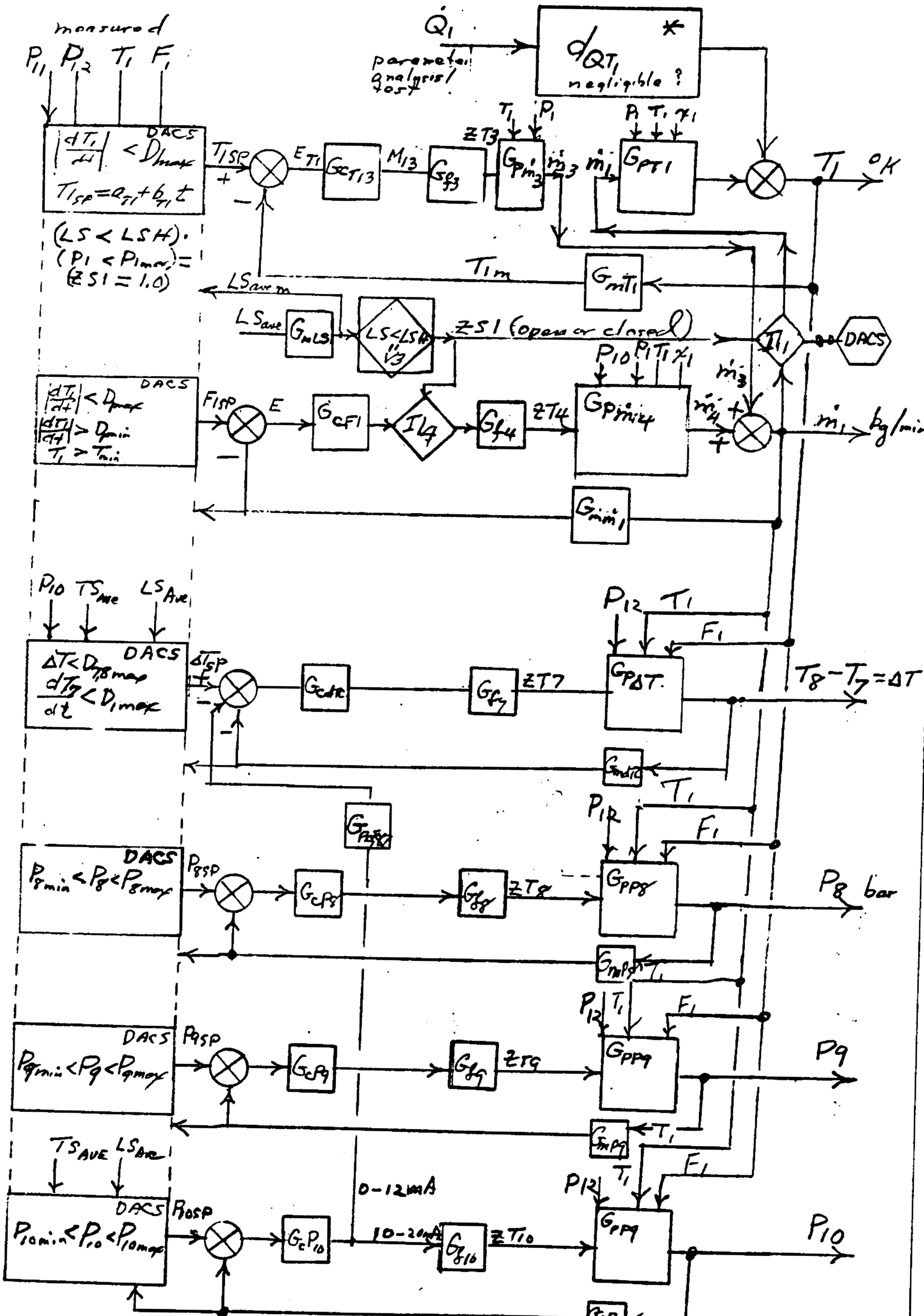

FIGURE 2.3.1 Loading the Supply Tank

* Note ; With good thermal design, heat leakage into the system is negligible during the cooldown and loading process, It becomes the primary concern when steady state boiloff occurs after loading is complete. There remains heat exchange between components 8, 9 & 10.

1 bar ≡ 100 kN/m² (kPa)
0.987 atm.

F.1.4.23