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# String Mediated Phase Transitions [1]

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## Abstract

It is demonstrated from first principles how the existence of string like structures can cause a system to undergo a phase transition. In particular, we concentrate on the role of topologically stable cosmic string in the restoration of spontaneously broken symmetries. We discuss how the thermodynamic properties of strings alter when stiffness and nearest neighbour string-string interactions are included.

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# 1  Introduction

In this paper we present some recent calculations on string related phase transitions [1,2]. We begin by considering the statistical properties of the conformational phase transition that a single string undergoes and go on to discuss multiple-string driven phase transitions. The phase transition of particular interest to us is the restoration of a spontaneously broken gauge theory that permits topologically stable string-like structures. This is because the string-like structures in this case, cosmic strings, play a central role in one scenario for the formation of the large scale structures of the universe [3].

It has been suggested that a number of naturally occurring phase transitions can be understood in terms of string-like structures, the conformational phase transition for a single string, for example, can be used to understand the physics of some macro-molecules as well as the functioning of bipolymers ([4] and reference therein). Other phase transitions may be able to be understood in terms of a sudden proliferation in the number density of unbound string-like structures. Some better known examples are listed below:-

(1.)    The $\lambda$ transition in superfluid $^4He$[5,6]. (The strings are vortex rings.)

(2.)    The phase transition from superconducting to normal state in bulk superconductors ([7] and references therein).

(3.)    The melting of smetic-A liquid crystals [8]. (The string like structures in this case are lines of dislocations.)

(4.)    The deconfinement phase transition in QCD [10].

By deriving the statistical properties of these systems from our string model we can make verifiable predictions that can be used to check its validity.

The paper is divided into two parts. In the first we display the thermodynamic properties of strings that have stiffness and that interact by nearest neighbour string-string interactions only. Although incorporating only nearest neighbour interactions is perhaps naive, it has the advantage of being simple enough to enable the statistical mechanics of the system to be worked out in detail. We show from first principles the existence of a phase transition. In the second part we consider how the fact that cosmic strings and vortex lines are not fundamental strings, but rather collective excitations of an underlying theory, leads to a modification in the description of the phase transitions. In detail we consider Ñielsen-Olesen strings [11] and we derive Kibble's prediction of the string formation temperature [12] which we find to be independent of the properties of the strings we might have expected to be important, such as stiffness.

# 2   Thermodynamics of Classical Strings.

As already mentioned, there are many phase transitions that can be understood in terms of the condensation of string-like structures. We shall therefore, in order to keep the following discussions as general as possible, define a string to have the following properties:

(1.)    A constant intrinsic energy per unit length denoted by $\sigma$.

(2.)    A constant non-zero thickness 'a' that is much less than the string length 'L'. The simplest string-string force would be to let this thickness define an excluded volume that prevents two string segments having centre to centre separations of less than 'a'.

(3.)    A rigidity 'u', that measures the energy required to bend the string. For example, it has been suggested that for small radius of curvature bending $u > 0$ for bosonic strings [13] while for cosmic strings $u < 0 \sim O(\frac{\sigma a^2}{R})$ where R is the radius of curvature of the bent string [14].

(4.)    String-string forces. At the lowest level of approximation one might assume string-string forces were sufficient weak to be neglected. A more sophisticated approach might, for example, introduce a binding energy +b (-b) per unit length between two parallel strings (string-antistring) a distance 'a' apart.

Whether the strings are closed or open is dependent on the string type, for example, macromolecules and QCD flux tubes are typically open whilst cosmic strings are closed or infinite in length. This difference is not however of major interest, as only the fine details of the thermodynamics depend on whether the strings are open or closed.

The above criteria imply that the energy of a single string of length 'L' can be written as:

$$E(L, C) = \sigma L + \epsilon(C, L) \tag{1}$$

where $\epsilon(L, C)$ is the energy of the string configuration C due to its stiffness and self interactions. If the number density of configurations of energy E(L,C) is n(L,C) the partition function for a system of a **single** string in thermal equilibrium at a temperature $T = \beta^{-1}$ is

$$Z_1 = \sum_{L,C} n(L, C) exp(-\beta E(L, C)) \tag{2}$$

The sum is over all string configurations of differing energies. We can rewrite this expression as

$$Z_1 = \sum_L e^{-\beta \sigma L} \zeta(L) \tag{3}$$

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where

$$\zeta(L) = \sum_C n(L,C)e^{-\beta\epsilon(C)} \tag{4}$$

is the conformational partition function. Moreover, since the sum in (3) is really an integral over L, $\zeta(L)$ is the Laplace transform of $Z_1$.

To simplify the evaluation of the statistical properties of the strings from this partition function, we make two further approximations; we shall approximate the string to a polymer of hinged straight segments of length l=a and restrict the string segments to lie along the edges of a cubic lattice of lattice spacing 'a' [2]. We shall relax the latter condition when it becomes too restrictive. Thus, for example, it is possible to calculate $\zeta(L)$ for a string with no stiffness (u=0) and no binding energy (b=0). (i.e. $\epsilon(C) = 0$). The result is (for large (L/a))

$$\zeta(L) = (\frac{A}{2})(\frac{V}{a^3})(\frac{L}{a})^{-q-1}e^{\frac{\eta(L-a)}{a}} \tag{5}$$

where $V$ is the volume of the system, $\eta = ln(2d-1)$ (2d if back tracking is allowed) and d is the number of spatial dimensions[1]. q is a factor that depends both on 'd' and the nature of the strings. For example, for a non-interacting open string q=-1, for a non-interacting closed string q=d/2 and for a non-self-intersecting string at low string densities q=7/4 in three dimensions. The factor $(\frac{V}{a^3})$ accounts for the number of possible starting points for the string on the lattice, the factor $e^{\frac{\eta(L-a)}{a}}$ is the number of possible configurations of a non-interacting string with no restriction being placed on whether it is open or closed and the final factor is to restrict the number of configurations to those appropriate to the string type under discussion.

More generally, for non-vanishing $\epsilon(C), \zeta(L)$ would be expected to take the form

$$\zeta(L) = (\frac{A}{2})(\frac{V}{a^3})(\frac{L}{a})^{-q-1}e^{\frac{\eta(\beta,L)(L-a)}{a}} \tag{6}$$

where $\eta(\beta,L)$ contains the $\epsilon(C)$ dependency. All that is needed now to completely determine the equilibrium thermodynamics of the system is a knowledge of q and $\eta(\beta,L)$. Assuming that

$$Lim_{L\to\infty}\eta(\beta,L) = \eta(\beta) \tag{7}$$

we can substitute (6) into (3) to obtain:

$$Z_1 = \frac{A}{2}\frac{V}{a^3}\sum_{(\frac{L}{a})}(\frac{L}{a})^{-q-1}e^{-\beta L\sigma_{eff}(\beta,L)}e^{\frac{-\eta}{a}} \tag{8}$$

where

$$\sigma_{eff} = \sigma - \frac{\eta(\beta)}{\beta a} \tag{9}$$

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[2]modifying this to change the length of the segments to 'ka' where $k \sim O(1)$ instead of 'a' has little effect

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$\sigma_{eff}$ plays the role of an effective string tension for the string. It is the vanishing of $\sigma_{eff}$ that causes the phase transition to occur.

Let us consider a couple of examples where $\eta(\beta)$ can be determined exactly. For u=b=0, $\eta = ln(2d - 1)$ and therefore

$$\sigma_{eff} = \sigma\left(1 - \frac{T}{T_{st}}\right) \qquad (10)$$

where

$$T_{st} = \frac{\sigma a}{ln(2d - 1)} \qquad (11)$$

(Figure 1). The next simplest case to consider is that of a string which possesses a rigidity 'u' per right angle bend. $\eta(\beta)$ is then given by

$$\eta(\beta) = ln(1 + he^{-\beta u}) \qquad (12)$$

where h=2d-2 [2]. A plot of $\eta(\beta)$ versus T is shown in Figure 1. This can be generalized to the case where adjacent segments make arbitrary angles, so that 'u' is replaced by the distribution $u(\theta)$, with $u(\theta)d\theta$ the energy cost in bending two segments at an angle $\theta$, through an additional angle $d\theta$. Provided $u(\theta)$ does not change sign the qualitative behaviour of $\eta(\beta)$ is as given in Figure 1 (see [2] for full details).

The function $\eta(\beta)$ is important not only in determining the phase transition temperature but also in determining the mean number of folds per unit length ('n') of an open string of length L:

$$n = -\frac{T}{L}\frac{\partial}{\partial u}(\log(\zeta)) = \frac{1}{a}\frac{(e^\eta - 1)}{e^\eta} \qquad (13)$$

This is shown qualitatively in Figure 2. As would be expected for low temperatures and positive rigidity ($u > 0$) the string has very few folds per unit length, whereas for low temperature and anti-rigidity ($u < 0$) the string has a very large number of folds, the number being as close to the maximum as possible. The most important factor in determining the most probable string configuration at low temperatures is the energy associated with a configuration. At higher temperature the energy is no longer the dominant factor in determining the string's configuration, instead entropy is; the most probable configuration is the one that is most numerous.

The system undergoes a phase transition when $\sigma_{eff}$ vanishes and the whole of space becomes filled with string. The effect of stiffness (antistiffness) is to change the phase transition temperature by a fractional amount $\frac{\Delta T}{T_{st}} = \frac{|u|h}{(1+h)\sigma a}$ ($-\frac{|u|h}{(1+h)\sigma a}$) assuming $\frac{u}{T_{st}} << 1$ which is convenient for cosmic string calculations because, although $u, \sigma$ and $a$ are temperature dependent, $\frac{u}{\sigma a}$ is independent of temperature and easily evaluated [2].

The effect of string-string interactions (beyond excluded volume) are more difficult to gauge. At the moment we can only solve explicitly for particular unknotted

configurations like zips (eg. those that simulate the double-helix in D.N.A [15]), generalizations of zips like those of Figure 3, or folded configurations like those of the Lauritzen and Zwanzig model [16]. Preliminary calculations show that the effect of string interactions are qualitatively akin to those of rigidity. Attractive forces increase the temperature $T_{st}$ at which $\sigma_{eff}(T_{st}) = 0$ whereas, repulsive forces decrease $T_{st}$. Figures 1 and 4 reflect this, but more work needs to be done before any firm conclusions can be drawn.

So far we have just considered the conformational phase transition. The partition function for a system containing an arbitrary number of strings is

$$Z = exp(Z_1) \tag{14}$$

if interactions between strings can be neglected [1] (i.e. in the 'free gas approximation'). From (14) it can be shown that as $T_{st}$ is approached the number density of strings rapidly increases. $T_{st}$ defines a maximum temperature for the system (the Hagedorn Temperature) . As the temperature of the system approaches $T_{st}$ the increase in temperature as more and more energy is 'pumped' into the system becomes smaller and smaller. Instead of causing the temperature to increase the energy goes into creating more and more string length. We will see in the next section that this maximum temperature is peculiar to non-composite strings. A system containing strings which are collective excitations of some underlying theory **does not** posses a Hagedorn Temperature.

We observe that the mean number of loops is $Z_1$ [1] and that the mean number of loops of length L is:

$$R(L) \propto exp(-\beta L \sigma_{eff})L^{-q-1} \tag{15}$$

[1] For temperatures $T < T_{st}$ long strings are exponentially suppressed (Figure 5), while for temperatures close to $T_{st}$, $\sigma_{eff} \approx 0$ and

$$R(L) \propto L^{-q-1} \tag{16}$$

These results have been confirmed in 3D for the case of non-interacting loops $(q = \frac{3}{2})$ by Vilenkin and Smith [17].

From (14) we can evaluate the specific heat of our system. It is given by

$$C_v = \frac{1}{T^2}\frac{\partial^2}{\partial \beta^2}lnZ \tag{17}$$

Using the expansion

$$\sum_{n=1}^{\infty} n^{-a-1}e^{-n\varpi} = \Gamma(-a)\varpi^a + \sum_{m=0}^{\infty}\frac{(-1)^m}{m!}\zeta(a+1-m)\varpi^m \quad ( \text{ if } \varpi \geq 0) \tag{18}$$

(where $\zeta$ is Riemann's zeta function), it can be easily shown that, for a system of closed strings, the specific heat diverges as $T_{st}$ is approached like

$$C_v \sim (T - T_{st})^{q-2} \tag{19}$$

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if $q < 2$ and is otherwise finite [3]. For example, for non-interacting strings $q = \frac{d}{2}$ and so in this case $C_v$ is finite right up to $T_{st}$ in 5 or more spatial dimensions.

One might be worried about the use of the canonical ensemble to describe a system close to the phase transition temperature because there are large fluctuations in some thermodynamic quantities. Consider for example the mean energy density due to loops of strings. This is finite right up to the phase transition. The r.m.s. fluctuations in this quantity however diverges at the same rate as $C_v$. The mean energy density in loops is therefore not a sensible quantity to discuss at temperatures close to $T_{st}$. Note however, that even in four or or less dimensions, not all interesting quantities have such large r.m.s. fluctuations about their mean values. For example, the r.m.s. fluctuations in the mean number of loops of size nl, (R(nl)), is proportional to $R(nl)^{1/2}$ which allows us to sensibly discuss R(nl) even at temperatures very close to $T_{st}$. The point to note is that if you wish to use the canonical ensemble to describe a phase transition you should check that the mean quantities you discuss do not have large fluctuations about their mean values. The specific heat diverging does not in itself signal a break down of the canonical ensemble.

# 3    Thermodynamics of 'composite' strings

So far we have only considered the strings as fundamental objects. To understand the phase transition from superconducting to normal phase, the $\lambda$ transition and the restoration of a spontaneously broken symmetry we need to take into account the fact that the strings are not fundamental. Here we shall only consider scalar QED, the simplest theory permitting cosmic strings. Analogous methods can be used for other theories; those readers interested in the $\lambda$ transition are referred to Wiegel [5]. We will show how strings arise during the symmetry restoring phase transition and how the presence of the underlying theory alters our description of the phase transition and in particular alters our interpretation of the Hagedorn temperature.

Before proceeding, we should make it clear what assumptions we are making. In order to use the results of statistical mechanics we must assume that the strings are formed in thermal equilibrium. Although it is still open to debate, it would appear that cosmic strings could well be formed in such a state. This is because the effect of gravity at the scales under consideration is not strong, so the relaxation time of the string network is small in comparison with the expansion time of the universe. The second assumption we make is that the strings exist in flat space-time. (In a future publication we hope to relax these assumptions.) In the first instance we will also set u=0, b=0 thereby neglecting any interactions between, or rigidity of, the strings.

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[3]Note this result is independent of the temperature dependence of $\sigma$ provided that $\sigma_{eff}$ is a smooth function of T

Scalar QED is defined by the Lagrange density:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\mid(\partial_\mu + ieA_\mu)\phi\mid^2 + \frac{1}{2}m_0^2\mid\phi\mid^2 - \frac{\lambda}{4!}\mid\phi\mid^4 \tag{20}$$

where $\phi$ is a complex scalar field, $A_\mu$ is the gauge field with charge 'e' and $m_0^2 \geq 0$. The partition function for the theory can be written as

$$Z \propto \int \mathcal{D}\phi\mathcal{D}\phi^*\mathcal{D}A\, exp(-I_\beta[\phi,A]) \tag{21}$$

$$I_\beta[\phi,A] = -\int_0^\beta d\tau \int d^3x \mathcal{L}_E[\phi,A] \tag{22}$$

$\mathcal{L}_E$ is the Euclidean form of the Lagrangian (20) (We take the signature of our Euclidean space to be -4) and the sum over configurations of $\phi(\tau,x)$ and $A(\tau,x)$ is restricted to fields periodic in $\tau$ with period $\beta$. Since the fields $\phi(\tau,x)$ and $A(\tau,x)$ are periodic in $\tau$ they permit the Taylor expansion:

$$\phi(\tau,x) = \sum_n \phi_n(x)e^{\frac{i2\pi n\tau}{\beta}} \qquad A^\mu(\tau,x) = \sum_n A_n^\mu(x)e^{\frac{i2\pi n\tau}{\beta}} \tag{23}$$

in terms of denumerable sets of 3D fields. Substituting these expansions into $\mathcal{L}_E$ one can see that the mass of the $\phi_n(n \neq 0)$ and $A_n(n \neq 0)$ modes, (termed $\phi'$ and $A'$) are large and positive. We shall refer to $(\phi_0, A_0)$ and $(\phi', A')$ as the light and heavy modes respectively. The partition function Z can be rewritten as

$$Z \propto \int D\phi_0 DA_0\, exp(-\beta\bar{I}(\phi_0,A_0)) \tag{24}$$

where

$$exp(-\beta\bar{I}[\phi_0,A_0]) = \int D\phi' DA'\, exp(-\bar{I}_\beta[\phi,A]) \tag{25}$$

At high temperatures, assuming $\lambda >> e^2$ and working to order $\lambda$ one can integrate out the heavy modes to obtain a spatially local effective potential for the massless modes (for details see [1]) One obtains

$$Z = \int \mathcal{D}\phi_0\mathcal{D}A_{0\mu}\, exp(-\beta\bar{I}[\phi_0,A_{0\mu}]) \tag{26}$$

where in the covariant gauge

$$\bar{I}[\phi_0,A_{0\mu}] = \int d^3x[\frac{1}{4}F_{0ij}F_0^{ij} - \frac{1}{2}(\partial_i\phi_0)(\partial^i\phi_0^*)$$

$$-\frac{ie}{2}A_{0i}[\phi_0\partial^i\phi_0^* - \phi_0^*\partial^i\phi_0] - \frac{1}{2}m_0^2(1 - \frac{T^2}{T_{MF}^2})\mid\phi_0\mid^2$$

$$-\frac{1}{2}e^2\mid\phi_0\mid^2 A_{0i}A_0^i + \frac{\lambda}{4!}\mid\phi_0\mid^4 + \frac{1}{\xi}(\partial_i A_0^i)^2] \tag{27}$$

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and the term $\frac{1}{\xi}(\partial_i A_0^i)$ describes our gauge fixing[1]. $T_{MF}^2$ is a constant equal to $\frac{m_o^2}{\frac{\lambda}{18}+\frac{e^2}{4}}$ We have restricted ourselves to the regime $\lambda \gg e^2$ to avoid some technical difficulties in evaluating the functional integral over the heavy modes ( these difficulties are discussed in section 4). To evaluate (26) more fully we employ the saddle point method. The dominant contributions to the functional integral come from field configurations satisfying:

$$\frac{\delta I}{\delta \phi} \mid_{\phi=\phi_{saddle}, A^\mu=A^\mu_{saddle}} = 0$$

$$\frac{\delta I}{\delta A_\mu} \mid_{\phi=\phi_{saddle}, A^\mu=A^\mu_{saddle}} = 0$$

that is, from the field configurations that satisfy the equations of motion:

$$\partial^i F_{ji} = \frac{1}{2}ie(\phi^* \partial_j \phi - \phi \partial_j \phi^*) - e^2 A_j \mid \phi \mid^2 \tag{28}$$

$$(\partial_i + ieA_i)^2 \phi = -m_0^2(1 - \frac{T^2}{T_{MF}^2})\phi + \frac{\lambda}{3!} \mid \phi \mid^2 \phi \tag{29}$$

The contribution of any solution of these equations to the partition function can be found by substitution into (26). The solution $\phi = constant, A = 0$ is the minimum energy solution and therefore gives the maximum contribution. The approach in the early 70s to the evaluation of (26), [18], was to assume all other field configurations make a negligible contribution to the functional allowing the partition function to be written as

$$Z_{MF} = exp(\beta V(\frac{1}{2}m_s^2(T) < \phi >^2 - \frac{\lambda}{4!} < \phi >^4)) \tag{30}$$

where

$$m_s^2(T) = m_o^2(1 - \frac{T^2}{T_{MF}^2}) \text{ and } < \phi >^2 = \frac{6m_s^2}{\lambda} \tag{31}$$

to order $\lambda, e^2, \frac{Te^2}{m_s^2}$ and assuming $T \leq T_{MF}$. From this one can see that the system undergoes a second order phase transition at a temperature $T_{MF}$ (the order parameter $< \phi >$ vanishes smoothly).

This mean field approximation is clearly very crude. It assumes that the only important configuration near the critical point is one of uniform density, whereas we know that there are large fluctuations in $< \phi >$ and $< A >$ near the critical temperature. Ideally to improve on the approximation one would like to find all the maxima of the functional and sum their various contributions to Z but unfortunately in practice this is not really practical. Instead here we assume (ad hoc) that string-like field configurations make the dominant contribution to the partition function and that the contribution from other non-constant field configurations can be neglected. This assumption is not implausible when it is realised that the independent vortex

model of the $\lambda$ transition in superfluid helium includes the same assumptions (with no further justification) and yet gives good quantitative agreement with experiment.

The simplest string solution to the saddle point equations is an infinite straight string running for example along the z-axis. This is can be expressed as [11]:

$$\phi = \mid \phi(r) \mid e^{i\theta} \tag{32}$$

$$\underline{A} = \frac{r \wedge k}{r} \mid \underline{A}(r) \mid \tag{33}$$

where $\underline{k}$ is a unit vector in the z-direction, and $\theta$ is an arbitrary phase denoting the winding around the minimum of the potential.

The solutions to the equations of motion from (28,29) are shown schematically in Figure 6. At large distances from the string

$$Lim_{r\to\infty} \mid \phi(r) \mid \longrightarrow \mu(T)$$

where $\mu^2(T) = \frac{6m_s^2(T)}{\lambda}$. At the core $\mid \phi \mid$ vanishes. The thickness of the core is determined by $m_s^{-1}$, the Compton wavelength of the Higgs particle. The magnetic field is restricted to the core, the skin depth being determined by $m_v^{-1}$, the inverse of the vector mass $m_v$,

$$m_v(T) = e\mu(T) = \frac{e}{\sqrt{\lambda}}m_s(T)$$

The string tension, or mass per unit length is:

$$\sigma = \int d^2r[\frac{1}{2} \mid (\underline{\nabla} + ie\underline{A})\phi \mid^2 + \frac{\lambda}{4!} \mid \phi \mid^4 - \frac{1}{2}m_s^2 \mid \phi \mid^2 + \frac{1}{2}\underline{B}^2] \tag{34}$$

which is equal to

$$\sigma = f(\frac{\lambda}{e^2})2\pi\mu^2(T) \tag{35}$$

where 'f' is a slowly varying function well known from vortex studies [19] and f(1)=1. Thus we see as we approach $T_{MF}$, $\sigma(T)$ falls to zero, reducing the energy per unit length of the string, and increasing its width $\sim m_s^{-1}(T)$.

What is the contribution of the string solutions to the partition function (26)? Assuming we are in the regime where the strings are curved smoothly, so that any segment of length of order the width will also be straight, then, these strings are to a good approximation also solutions to (28,29), and have an energy per unit length approximately the same as the infinitely straight string. Z can be written as

$$Z = exp(Z_1) \tag{36}$$

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where

$$Z_1 = \sum_{L,C} n(L,C) exp(-\beta E(L,C)) \tag{37}$$

The analysis of section 2 goes through and the system undergoes a phase transition when

$$\sigma_{eff}(T_{st}) = \sigma(T_{st}) - \frac{\eta(\beta_{st})}{\beta_{st} a(T_{st})} = 0 \tag{38}$$

(assuming u=0,b=0) i.e. when

$$T_{st} = \frac{\sigma(T_{st})a(T_{st})}{\eta} \approx \gamma \mu^2(T_{st}) m^{-1}(T_{st}) \tag{39}$$

for some $\gamma \sim O(1)$ and $m = min(m_s, m_v)$ the inverse width of the string ( in our case, $\lambda >> e^2$, so $m = m_v$). Since the right hand side of (35) vanishes at $T = T_{MF}$, it follows that

$$T_{st} < T_{MF} \tag{40}$$

The difference between $T_{st}$ and $T_{MF}$ is small. Explicitly,

$$1 - \frac{T_{st}^2}{T_{MF}^2} \sim O(e^2) \qquad \text{(for } m = m_v) \tag{41}$$

We can also calculate the width of the strings at this temperature (this will be of the same order as their mean separation) By substitution

$$m_s(T_{st}) \sim O(em_s(T = 0)) \tag{42}$$

$$m_v(T_{st}) \sim O(em_v(T = 0)) \tag{43}$$

for $m = m_v$. That is, the network of strings at the phase transition has the separation of the centres of flux tubes scaled up by a factor $O(\frac{1}{e})$ compared to the closest packing of cold strings. What is the interpretation of $T_{st}$? It only makes sense to discuss string solutions when fluctuations that could remove them are still improbable. The free energy associated with such a fluctuation is, ignoring factors of unity

$$\Delta f(a)^3 \approx \frac{\mu^2(T)}{m_v} \tag{44}$$

where $a \sim$ mean string separation. This fluctuation will have a high probability so long as the free energy required is substantially less than the thermal energy T. The two are equal when

$$T \sim \mu^2(T) m_v^{-1} \tag{45}$$

i.e. $T \sim T_{st}$. Above $T_{st}$ it no longer makes sense to talk about strings. This suggests that we interpret $T_{st}$ as the string formation temperature. Note, unlike the case when

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strings are fundamental, $T_{st}$ is not a limiting temperature, above $T_{st}$ we are simply in a phase in which strings cannot exist.

As we mentioned earlier, realistic strings posses stiffness and interactions. Provided the stiffness 'u' is such that $\frac{|u|}{T_{st}} < 1$ ( as is the case for small bending) the analysis goes as before, leaving $T_{st}$ essentially unchanged. The case of interactions is however more complicated. The width of the string increases as we approach $T_{st}$, we expect the interactions of the strings to become increasingly important, because they will begin to overlap. This is a very difficult problem and we are currently trying to estimate the effect of incorporating nearest neighbour interactions, using Nambu strings, but with interaction energy 'b' per unit length [2]. In so far as $\frac{b}{\sigma}$ is independent of T interactions could still not effect the estimate that $(1 - \frac{T_{st}^2}{T_{MF}^2}) \sim O(e^2)$.

# 4 Discussion

Using the simple string model of Copeland et. al. [1] to describe phenomena such as, the superconducting to normal phase transition, the $\lambda$ transition, the melting of smetic-A liquid crystals and the restoration of spontaneously broken scalar QED, we would predict that they are all second order phase transition sharing the same critical exponents. However experimental results show this not to be the case, for example, melting is a first order phase transition whilst the $\lambda$ transition is observed to be second order. The incorrect prediction is due to the naivety of the model. By taking account of the varying natures of the string like structures it should be possible to produce a more accurate string model. Our first step has been to introduce to the model the possibility of strings with rigidity. We have shown that the variation in the rigidity of the strings like structures associated with the various phase transitions is not sufficient by itself to account for the experimental observations. Its most significant effect is to change the critical temperature by a fractional amount $\frac{\Delta T}{T_{st}(U=0)} \sim \frac{|u|}{\sigma a}$ ( if $\frac{|u|}{T_{st}} << 1$). In [2] we will show that regardless of $\frac{u}{T_{st}}$ the string phase transition is still second order and for scalar QED, $(1 - \frac{T_{st}^2}{T_{MF}^2})$ is still of order $e^2$ (if $\lambda >> e^2$). This estimate for $T_{st}$ was originally made by Kibble [12].

The next step is to evaluate the effect of the varying types of string string interactions. Up to now we have only estimated the effect of very simple string-string interactions, such as that described in figure 3. These types of interactions do not change the nature of the phase transition and their effects are qualitatively very similar to those of rigidity. We are currently working on incorporating more sophisticated string string interactions.

It is important to distinguish the meaning of $T_{st}$ between systems containing fundamental strings and those containing 'composite' strings. For the former it represents a maximum temperature for the system. For the latter it **does not**. Instead it describes the maximum temperature at which it is sensible to discuss string like

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field configurations. The equilibrium statistical properties of the field theory can be worked out at temperatures above $T_{st}$ but not in terms of strings. $T_{st}$ in this case represents the string formation temperature.

Of particular interest to us is the restoration of a spontaneously broken gauge theory that permits topologically stable string like structures. Before our work the mean field approach had been used as the basis for models of symmetry restoring phase transitions. In these models the system is described by constant fields of optimal strength. For example, for scalar QED this approach would approximate the partition function (26) to its absolute maxima only. Although at low temperatures this is a valid approximation, we have shown that as the temperature approaches $T_{st}$ this is no longer the case. To improve on this approximation we would ideally like to find *all* the maxima of the functional and sum their various contributions to Z. This however is not really practical and we have had to assume that string-like field configurations make the dominant contribution to the partition function and that contributions from other non-constant field configurations can be neglected.

The string model pictures the restoration of symmetry as being due to overlapping strings filling the whole of space, that it occurs at a temperature $T_{st} < T_{MF}$, and is second order. Because we have not yet fully assessed the effect of string string interactions in our model it is premature to rule out the possibility of the phase transition being first order. For scalar QED for example, it is possible to show that if $\frac{e^2}{\lambda} > 1$ ($\frac{e^2}{\lambda} < 1$) the effect of fluctuations about the mean field configurations near $T_{MF}$ is to make the phase transition weakly (strongly) first order [20]. In a future publication we hope to address the problem of whether or not this is true of our string model too.

The observational consequences of first order phase transitions in the early universe are interesting. If the cosmic string phase transition was strongly first order the initial string number density could very different from that estimated by Kibble [12]. This might have interesting consequences for the cosmic string and baryogenesis scenario of Brandenberger et al [22]. Perhaps a more interesting theory to investigate would be one that allows the formation of monopoles. If this phase transition was first order the initial monopole density might be small enough to resolve the 'monopole problem'. We are currently investigating this.

Several times during this meeting we have heard about the severe technical difficulties that exist in finite temperature gauge theories. These difficulties arise because the presence of the heat bath gives a preferential inertial frame. This inertial frame for example, leads to the temporal and spatial components of the gauge fields becoming decoupled, resulting in two possible independent masses for the gauge field. For a detailed discussion of the problems the reader is referred to the relevent articles in this volume. Of course in evaluating (26) these technical difficulties arose. It was to avoid these problems that we restricted ourselves to the regime $\lambda \gg e^2$ in which the gauge field contributions cannot be large. Terms of order $e^2 T^2$ are then constrained

by $e^2\mu^2$ and the vector mass is approximately unchanged, while at the same time the vector loop gives a small contribution to the effective scalar mass.

What are the astrophysical consequences of the distributions of string we have obtained? As $T_{st}$ is approached most of the string length goes into infinite strings with a scale invariant distribution of loops, both with approximately Brownian trajectories. If we had neglected string-string interactions completely we would have predicted that the string trajectories were exactly Brownian. This agrees with the results obtained by the rather different approach of Mitchell and Turok [21]. Shortly after the universe cooled through $T_{st}$ the string network would no longer be in thermal equilibrium. Our results indicate that the system would like to evolve to a state with an exponentially suppressed distribution of large loop sizes. This makes the string domination scenario of Kibble and Bennett [23,24] seem unlikely but only detailed simulations of string dynamics in an expanding universe could rule out this scenario altogether. Recently Hodges has numerically analysed the distribution of global strings as they are formed in a second order phase transition. He has the strings in an expanding universe, and finds that at equilibrium, for a horizon size ten times larger than the correlation length, at the Ginzburg temperature, a small fraction of the string length is in infinite string[25]. This appears to be in contradiction with the results of [17]. However, Hodges allows many time steps before looking at the string distribution, Vilenkin immediately freezes the Higgs fields and obtains the string distribution. The difference lies in the time scales and length scales that are used in the problem. Overall the behaviour of the smallest loops as dominating the partition function below the $T_{st}$ still appears to hold.

There remains a great deal to do in investigating phase transitions in the early universe. Can we quantify the effect of string-string interactions any more than we already have done? Will fluctuations around the known solutions cause the order of the phase transition to change? How will the distributions be affected by these interaction terms? What will the new string formation temperature be? For constant $\frac{u}{\sigma a}$ it appears to be little affected from the previous value (11), which didn't have any interactions included[1,2].

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# Figure Captions

(1)      $\eta$ as a function of T for strings with stiffness u and limited self-interaction $\epsilon$/unit length. The solid line corresponds to u=$\epsilon$=0, the dashed line to $u > 0$, $\epsilon = 0$, (*and* $\epsilon < 0, u = 0$), the dot-dashed line to $u < 0, \epsilon = 0$, (*and* $\epsilon > 0, u = 0$). We have $\eta \simeq \eta_0$ for $T \gg u; \epsilon a$.

(2)      The number of folds/unit length, n, as a function of temperature for a stiff string. The dashed, solid, and dot-dashed lines correspond to $u > 0, u = 0, u < 0$ respectively.

(3)      A class of (unknotted) string configurations in which string-string forces can be taken into account. The broken circles correspond to non- interacting loops, the double lines to interacting string segments.

(4)      The variation of $\sigma_{eff}$ with T. The solid line corresponds to $u = \epsilon = 0$, the dashed line to $u > 0, \epsilon = 0$, or $u = 0, \epsilon < 0$. The dot-dashed line corresponds to $u < 0, \epsilon = 0$ or $u = 0, \epsilon > 0$. Its intercept vanishes if $-u = O(\sigma a)$ or $\epsilon = O(\sigma)$.

(5)      The mean number of loops, R(L) as L varies for $T = T_0$ (solid line) and $T < T_0$ (dashed line). The solid line shows a scale invariant distribution, the dashed line exponential suppression of long loops.

(6)      An example of the field configurations for a vortex solution at temperature T.
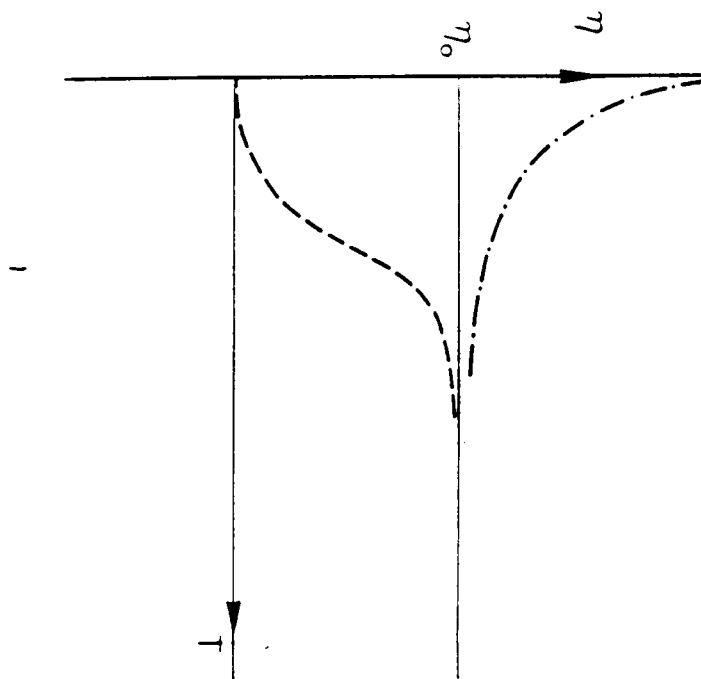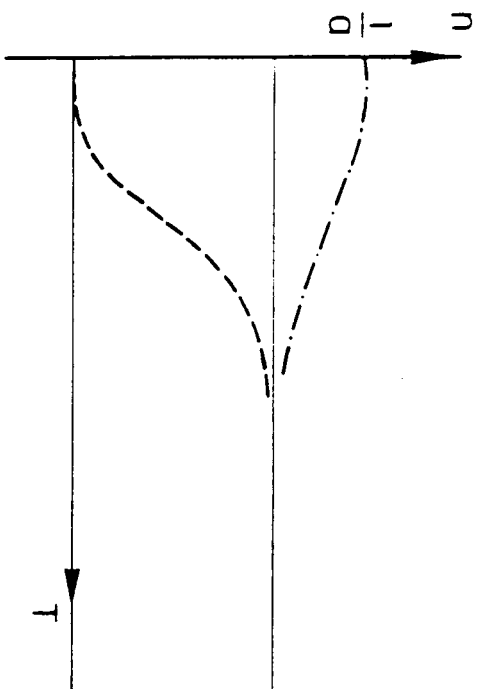
$$\zeta = O(m_s^{-1}) \qquad \lambda = O(m_v^{-1})$$

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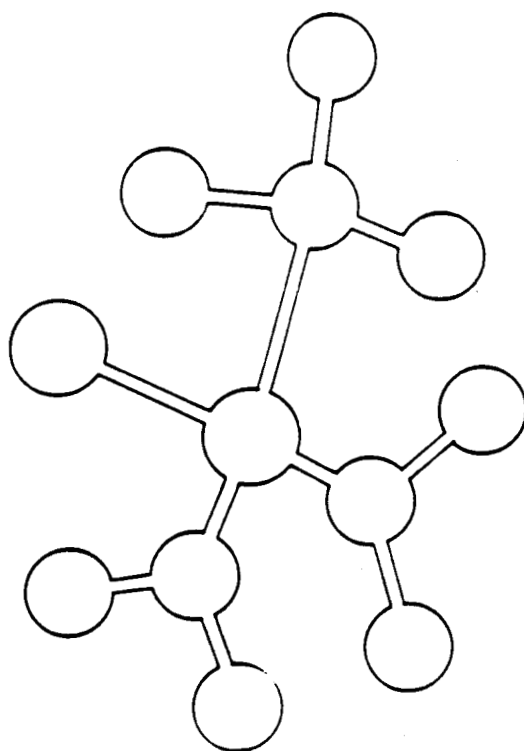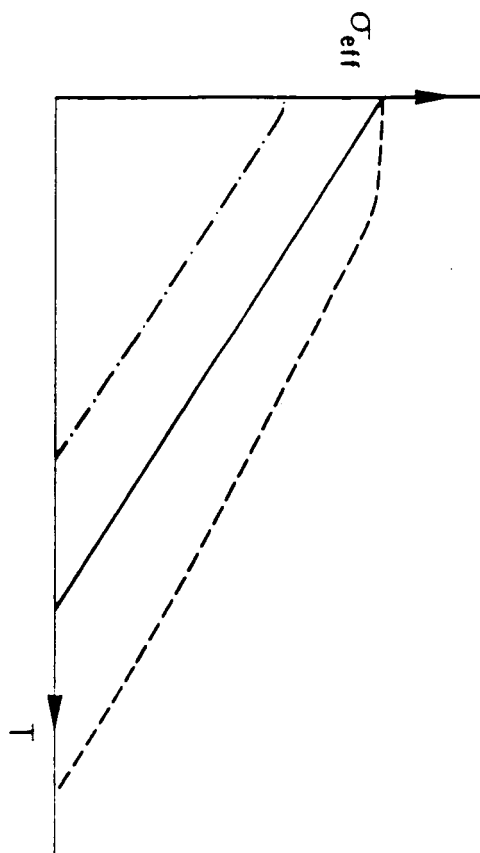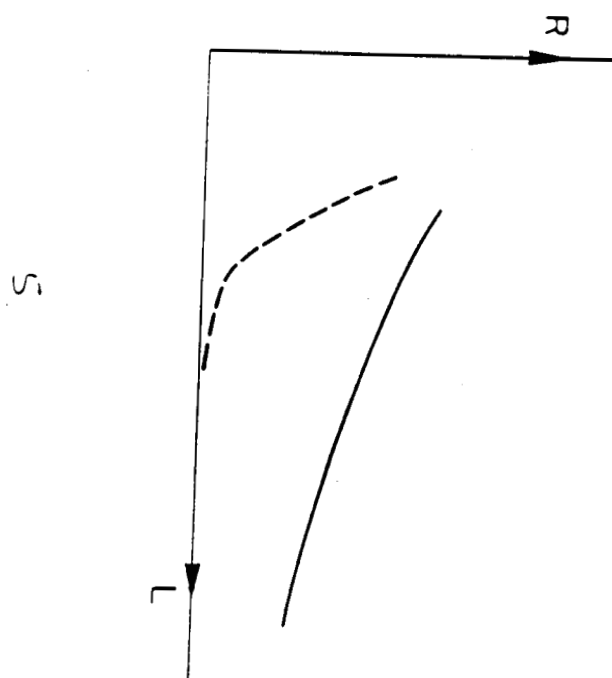
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