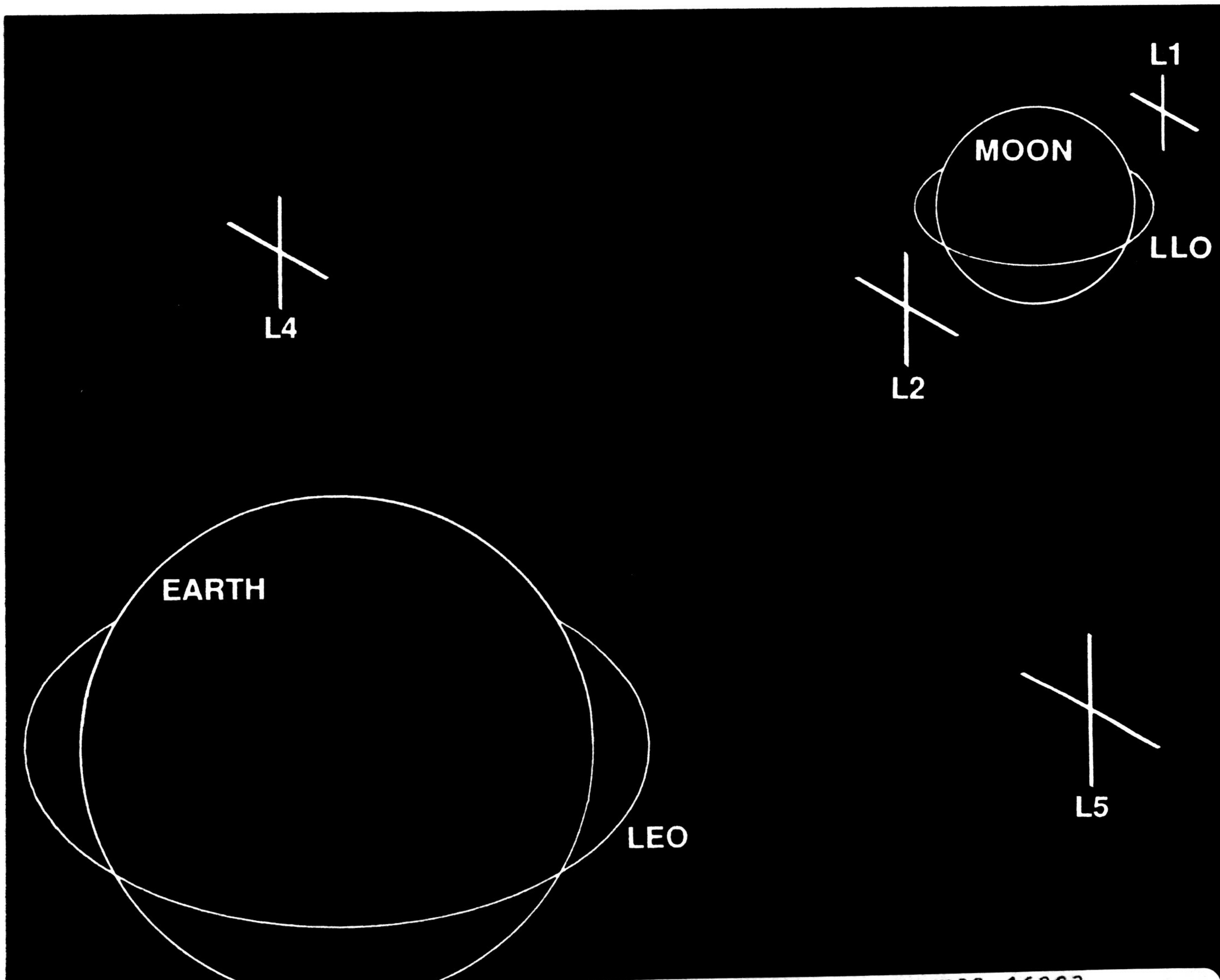
NASA CR 172087

LBSS

Velocity Deltas for LEO to L2, L3, L4, & L5, and LLO to L1 & L2



(NASA-CR-172087) USER AND TECHNICAL DOCUMENTATION (Eagle Engineering) 56 p CSCL 22C

N89-16893

Unclas

G3/13 0179858

NASA Contract Number NAS9-17878
EEI Report Number 88-208
September 30, 1988

EAGLE

LIBRATE

User and Technical Documentation

National Aeronautics and Space Administration
Lyndon B. Johnson Space Center

Advanced Projects Office

Eagle Engineering, Inc.
Houston, Texas
September, 1988

NASA Contract NAS9-17878
Eagle Engineering Report No. 88-208

Foreword

This program is an important tool in the study of alternative routes between the Earth and the Moon. Dr. John Alred was the NASA Technical Monitor for the contract under which this program was produced. Mr. Andy Petro was the NASA Task Manager for this particular task. Mr. Bill Stump was the Eagle Project Manager for the contract under which this program was produced. The program was written by Jack Funk, originally in Quick Basic, and translated into Fortran by Mr. Bill Engblom. Mr. Engblom also prepared the documentation.

Table of Contents

1.0 Introduction

The program LIBRATE calculates velocities for trajectories from low earth orbit (LEO) to four of the five libration points (L2, L3, L4, L5), and from low lunar orbit (LLO) to libration points L1 and L2. Librations points (LP) are defined as locations in space that orbit the Earth such that they are always stationary with respect to the Earth-Moon line. Libration point #2 (L2) is located between the Earth and Moon where the gravitational attraction from both bodies are equal. L1 and L3 are located behind the Moon and Earth, respectively, such that the pull of the Earth and Moon together just cancel the centrifugal acceleration associated with the libration point's orbit. L4 and L5 are located half-way between the Earth and the Moon and 60° off the Earth-Moon line to the left and right, respectively. Hence, the Earth, Moon, and all libration points, lie in the same plane.

The flight to be analyzed departs from a circular orbit of any altitude and inclination about the Earth or Moon and finishes in a circular orbit about the Earth at the desired libration point within a specified flight time. First, the departure orbit is made into a more eccentric orbit (ellipse or hyperbola) with an initial ΔV in order to reach the libration point while meeting the flight time constraint. The less the desired flight time, the more eccentric the orbit, and the larger the initial ΔV required. The least eccentric elliptic orbit would require the minimum ΔV and the maximum flight time. A second ΔV is then needed once the elliptic or hyperbolic flight path has reached the libration point in order to change the velocity vector of the eccentric trajectory to that of the libration point's orbit (circularize). So, the more eccentric the orbit, the larger the velocity change. This second burn must also account for the inclination of the eccentric trajectory with respect to the Earth-Moon-LP plane.

This program produces a matrix of the ΔV's needed to complete the desired flight. The user specifies the departure orbit (location and altitude), and the maximum flight time. A matrix is then developed with 10 inclinations (with respect to the Earth-Moon-LP plane), ranging from 0° to 90°, forming the columns, and 19 possible flight times, ranging from the flight time (input) to 36 hours less than the input value, in decrements of 2 hours, forming the rows. This matrix is presented in three different reports including the total ΔV's, and both of the ΔV components discussed above.

Section 2.0 of this document describes the input required from the user to define the flight. Section 3.0 describes the contents of the three reports that are produced as outputs. Section 4.0 includes the instructions needed to execute the program.

A more detailed description of the process used in LIBRATE has been included as Appendix D (main program), Appendix E (in-program subroutine), and Appendices F, G, H, and I (external subroutines). LIBRATE was derived in part from the PLANECHG program (also produced under this contract), discussed in a different, more detailed documentation report. Therefore, for a more in-depth look at many of the equations, variables, and conventions used in LIBRATE, please consult the PLANECHG documentation.

2.0 Program Inputs

The following paragraphs discuss the inputs provided by the user. The prompt is the message displayed by the program onto the screen. The input variable is the variable assigned to the user's response. The description provides information about how to respond to the prompt.

1. Prompt: INPUT EARTH OR MOON

   Input variable: BODY

   Description: Enter either MOON or EARTH as a departure orbit location. EARTH is the default value.

2. Prompt: INPUT PERIGEE ALTITUDE (NMI)

   Input variable: HPE

   Description: Enter departure orbit altitude above the surface of the departure location (Earth or Moon), in nautical miles. A circular orbit is assumed.

3. Prompt: INPUT LIBRATION POINT NUMBER

   Input variable: NLP

   Description: Enter the number (1, 2, 3, 4 or 5) of the desired libration point. Remember that L1 (behind the Moon), L2 (between Earth and Moon), and L3 (behind Earth) are all on the Earth-Moon line. L4 and L5 are located midway between the Earth and Moon and 60° off the Earth-Moon line to the left and right, respectively. Recall that LIBRATE <u>cannot</u> calculate trajectories from the Earth to L1 or the Moon to L3, L4, or L5. Other programs produced under this contract do these calculations, however.

3. Prompt: INPUT FLIGHT TIME

Input variable: FLTIM

Description: Enter the flight time allowed for the transfer, in hours. If this value is larger than the calculated maximum flight time for such a trajectory then a message will appear indicating the maximum flight time allowed followed by another flight time input prompt. This may occur several times because the program approximates the maximum flight time based on the flight time input. Simply continue to enter flight times less than those time constraints issued by the program until a value is accepted (no error message).

3.0 Program Outputs

This section describes the contents of each of the three reports generated by the program. These reports may be found in the output file, LIBRATE.OUT. Samples of report #1, #2, and #3 have been included herein, starting on the following page.

Report #1: Total Delta Velocity for Earth (or Moon) Transfer Trajectory to Libration Point

This report is a matrix of total delta velocities in ft/sec that are needed to complete the transfer from the departure location (Earth or Moon) to the desired libration point. Each cell corresponds to a particular flight time and inclination (of departure orbit with respect to Earth-Moon-LP plane).

Report #2: Delta Velocity at Libration Point for Earth (or Moon) Transfer Trajectory to Libration Point

This report is a matrix of delta velocities in ft/sec that are needed at the libration points to correct the velocity vectors of the eccentric trajectories to that of the libration point's orbit. Each cell corresponds to a particular flight time and inclination (of departure orbit with respect to Earth-Moon-LP plane).

Report #3: Delta Velocity at Earth (or Moon) Orbit for Transfer Trajectory to Libration Point

This report is a matrix of delta velocities in ft/sec that are needed to make the circular departure orbit into a trajectory (ellipse or hyperbola) that will reach the libration point while meeting the flight time constraint. Each cell corresponds to a particular flight time and inclination (of departure orbit with respect to Earth-Moon-LP plane).

MAP OF TOTAL VELOCITY NEEDED FOR EARTH TRANSFER TRAJECTORY TO LIBRATION POINT #2

RUN DATE 23-SEP-88 RUN TIME 13:47:50

CIRCULAR ORBIT ALTITUDE 250.0

INCL> FLIGHT TIME (HR)	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
80.0	12167.	12180.	12226.	12299.	12394.	12507.	12631.	12762.	12895.	13026.
78.0	12190.	12206.	12251.	12323.	12417.	12528.	12652.	12781.	12913.	13044.
76.0	12223.	12238.	12282.	12353.	12446.	12556.	12678.	12806.	12937.	13066.
74.0	12264.	12279.	12323.	12392.	12484.	12592.	12712.	12839.	12968.	13096.
72.0	12316.	12330.	12373.	12441.	12531.	12637.	12755.	12880.	13007.	13133.
70.0	12378.	12392.	12434.	12501.	12588.	12692.	12808.	12930.	13055.	13179.
68.0	12453.	12466.	12507.	12571.	12657.	12758.	12871.	12991.	13113.	13235.
66.0	12540.	12553.	12592.	12655.	12738.	12836.	12946.	13063.	13183.	13302.
64.0	12642.	12654.	12692.	12752.	12832.	12927.	13034.	13148.	13265.	13381.
62.0	12758.	12771.	12807.	12865.	12941.	13033.	13136.	13247.	13360.	13473.
60.0	12891.	12903.	12938.	12993.	13067.	13155.	13254.	13361.	13471.	13580.
58.0	13042.	13053.	13086.	13139.	13210.	13294.	13390.	13492.	13598.	13704.
56.0	13212.	13223.	13254.	13304.	13371.	13452.	13544.	13642.	13744.	13846.
54.0	13403.	13413.	13443.	13491.	13554.	13631.	13718.	13812.	13910.	14008.
52.0	13617.	13627.	13655.	13700.	13760.	13833.	13916.	14005.	14098.	14192.
50.0	13857.	13865.	13892.	13934.	13991.	14060.	14138.	14223.	14312.	14401.
48.0	14124.	14132.	14157.	14196.	14250.	14315.	14389.	14469.	14554.	14639.
46.0	14422.	14430.	14453.	14490.	14540.	14601.	14671.	14747.	14827.	14907.
44.0	14755.	14762.	14784.	14819.	14866.	14923.	14989.	15060.	15135.	15211.

MAP OF DELVEL AT LIBRATION POINT FOR EARTH TRANSFER TRAJECTORY TO LIBRATION POINT #2

RUN DATE 23-SEP-88 RUN TIME 13:47:50

CIRCULAR ORBIT ALTITUDE 250.0

INCL> FLIGHT TIME (HR)	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
80.0	2174.	2188.	2234.	2307.	2402.	2515.	2639.	2770.	2903.	3034.
78.0	2196.	2212.	2257.	2329.	2423.	2534.	2657.	2787.	2919.	3050.
76.0	2226.	2241.	2286.	2357.	2450.	2560.	2681.	2810.	2941.	3070.
74.0	2265.	2280.	2323.	2393.	2484.	2592.	2712.	2839.	2968.	3096.
72.0	2312.	2327.	2370.	2438.	2527.	2634.	2752.	2877.	3004.	3130.
70.0	2371.	2385.	2426.	2493.	2580.	2684.	2800.	2922.	3048.	3172.
68.0	2440.	2454.	2494.	2559.	2644.	2745.	2858.	2978.	3101.	3222.
66.0	2521.	2534.	2573.	2636.	2718.	2817.	2927.	3044.	3164.	3283.
64.0	2615.	2628.	2665.	2726.	2805.	2901.	3007.	3121.	3238.	3354.
62.0	2723.	2735.	2771.	2829.	2906.	2998.	3101.	3211.	3325.	3438.
60.0	2846.	2857.	2892.	2947.	3021.	3109.	3209.	3315.	3425.	3535.
58.0	2984.	2996.	3028.	3081.	3152.	3236.	3332.	3434.	3540.	3646.
56.0	3140.	3151.	3182.	3232.	3299.	3380.	3471.	3570.	3672.	3774.
54.0	3315.	3325.	3354.	3402.	3466.	3542.	3629.	3723.	3821.	3919.
52.0	3509.	3519.	3547.	3592.	3652.	3725.	3807.	3897.	3990.	4084.
50.0	3726.	3734.	3761.	3803.	3860.	3929.	4007.	4092.	4181.	4270.
48.0	3966.	3974.	3999.	4039.	4092.	4157.	4231.	4312.	4396.	4481.
46.0	4232.	4240.	4263.	4301.	4351.	4412.	4482.	4558.	4637.	4718.
44.0	4528.	4536.	4557.	4592.	4639.	4696.	4762.	4833.	4908.	4985.

MAP OF DELVEL AT EARTH ORBIT FOR EARTH TRANSFER TRAJECTORY TO LIBRATION POINT #2

RUN DATE 23-SEP-88 RUN TIME 13:47:50

CIRCULAR ORBIT ALTITUDE 250.0

INCL> FLIGHT TIME (HR)	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
80.0	9992.	9992.	9992.	9992.	9992.	9992.	9992.	9992.	9992.	9992.
78.0	9994.	9994.	9994.	9994.	9994.	9994.	9994.	9994.	9994.	9994.
76.0	9997.	9997.	9997.	9997.	9997.	9997.	9997.	9997.	9997.	9997.
74.0	10000.	10000.	10000.	10000.	10000.	10000.	10000.	10000.	10000.	10000.
72.0	10003.	10003.	10003.	10003.	10003.	10003.	10003.	10003.	10003.	10003.
70.0	10008.	10008.	10008.	10008.	10008.	10008.	10008.	10008.	10008.	10008.
68.0	10013.	10013.	10013.	10013.	10013.	10013.	10013.	10013.	10013.	10013.
66.0	10019.	10019.	10019.	10019.	10019.	10019.	10019.	10019.	10019.	10019.
64.0	10027.	10027.	10027.	10027.	10027.	10027.	10027.	10027.	10027.	10027.
62.0	10035.	10035.	10035.	10035.	10035.	10035.	10035.	10035.	10035.	10035.
60.0	10046.	10046.	10046.	10046.	10046.	10046.	10046.	10046.	10046.	10046.
58.0	10058.	10058.	10058.	10058.	10058.	10058.	10058.	10058.	10058.	10058.
56.0	10072.	10072.	10072.	10072.	10072.	10072.	10072.	10072.	10072.	10072.
54.0	10089.	10089.	10089.	10089.	10089.	10089.	10089.	10089.	10089.	10089.
52.0	10108.	10108.	10108.	10108.	10108.	10108.	10108.	10108.	10108.	10108.
50.0	10131.	10131.	10131.	10131.	10131.	10131.	10131.	10131.	10131.	10131.
48.0	10158.	10158.	10158.	10158.	10158.	10158.	10158.	10158.	10158.	10158.
46.0	10189.	10189.	10189.	10189.	10189.	10189.	10189.	10189.	10189.	10189.
44.0	10227.	10227.	10227.	10227.	10227.	10227.	10227.	10227.	10227.	10227.

4.0 Program Execution Instructions

The following instructions describe the steps to be taken by the user to execute this program:

A. Obtain access to the DEC VAX minicomputer and sign on with user identification.

B. At the $ prompt, type <u>RUN LIBRATE</u>.

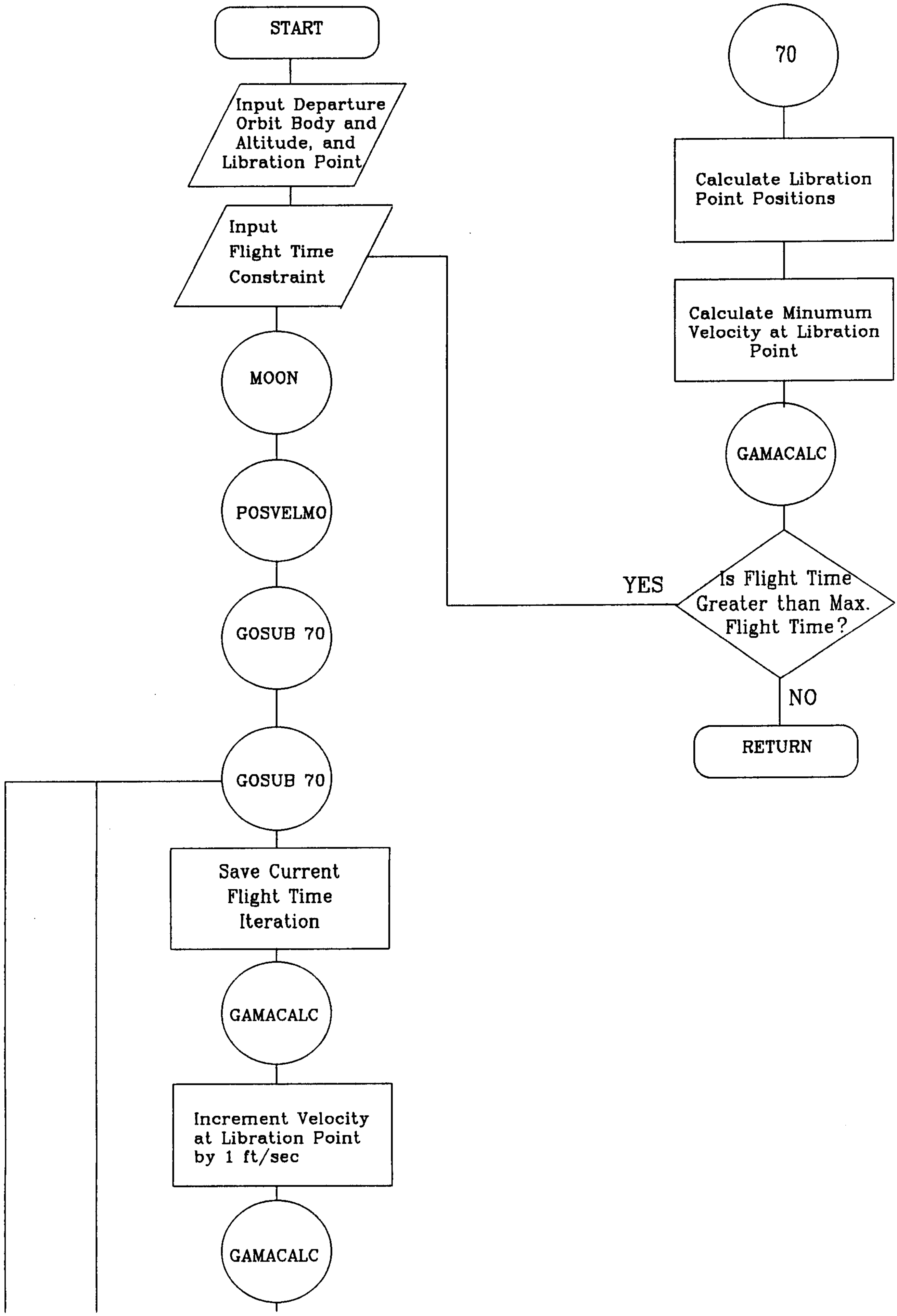
C. When prompted by the program, enter the program inputs. See section 2.0 for a discussion of the inputs.

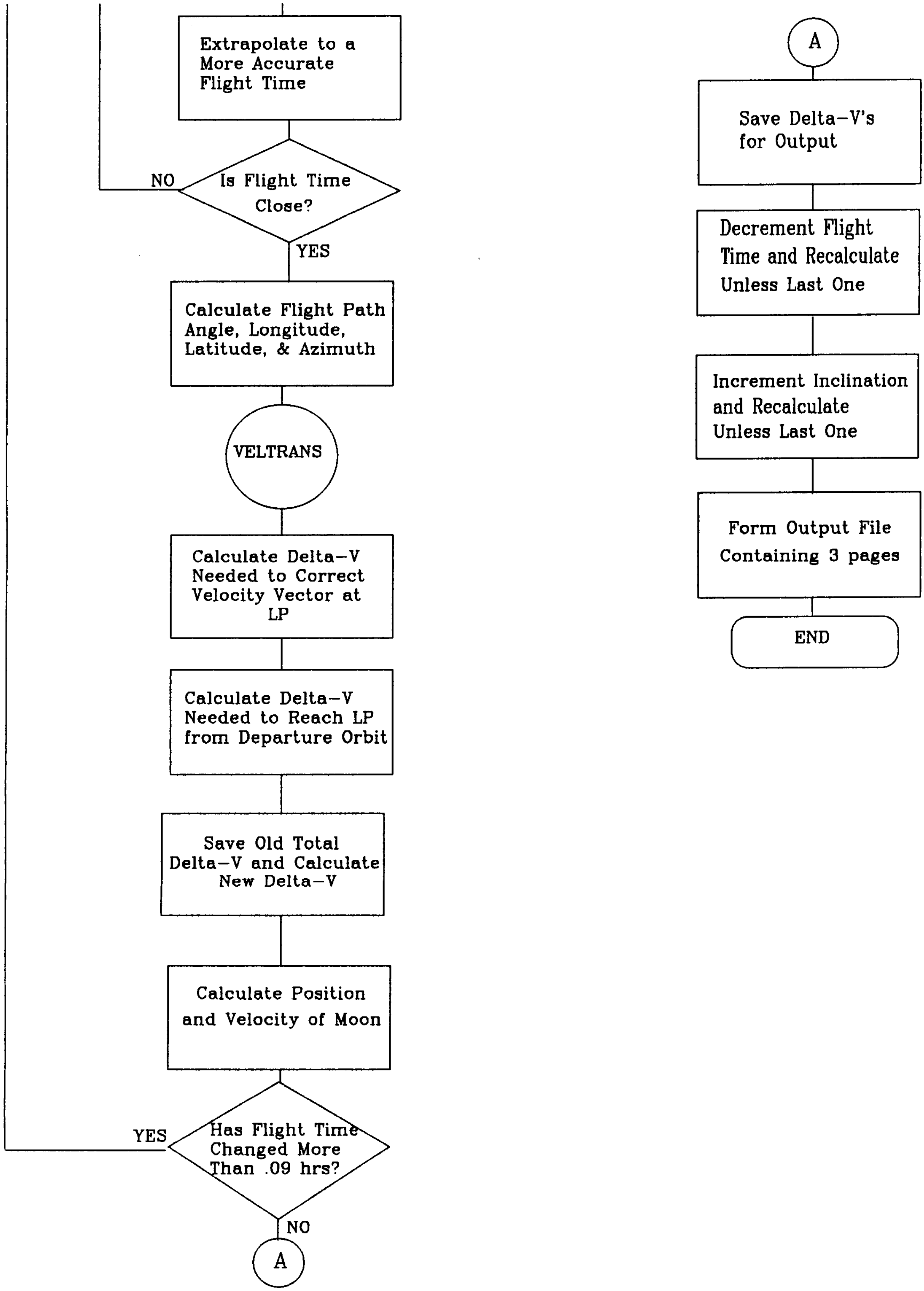
D. After the last input has been entered, the program will execute for approximately 1 minute. Upon completion, the message FORTRAN STOP will appear, followed by the $ prompt.

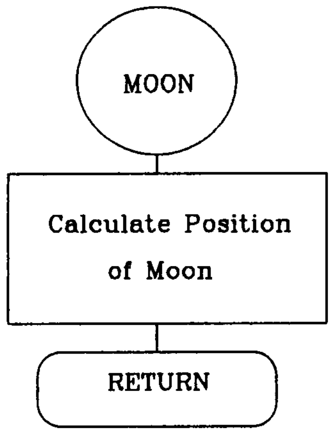
E. The program outputs will be placed in a file named LIBRATE.OUT;### where ### is a system generated number of the report. To print the most recently generated report, type the following at the $ prompt: <u>TYPE LIBRATE.OUT</u>

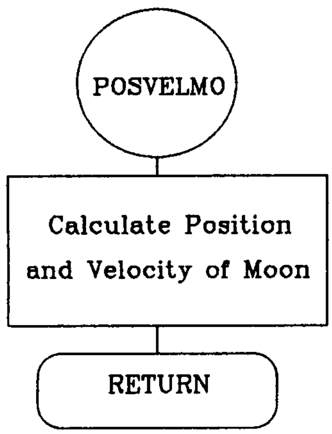
F. To re-execute the program with new parameters, begin again at step (B) above.

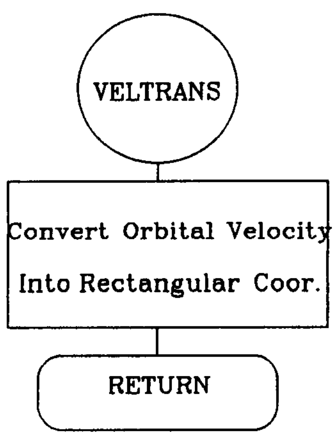
Appendix A. Program Flow Chart

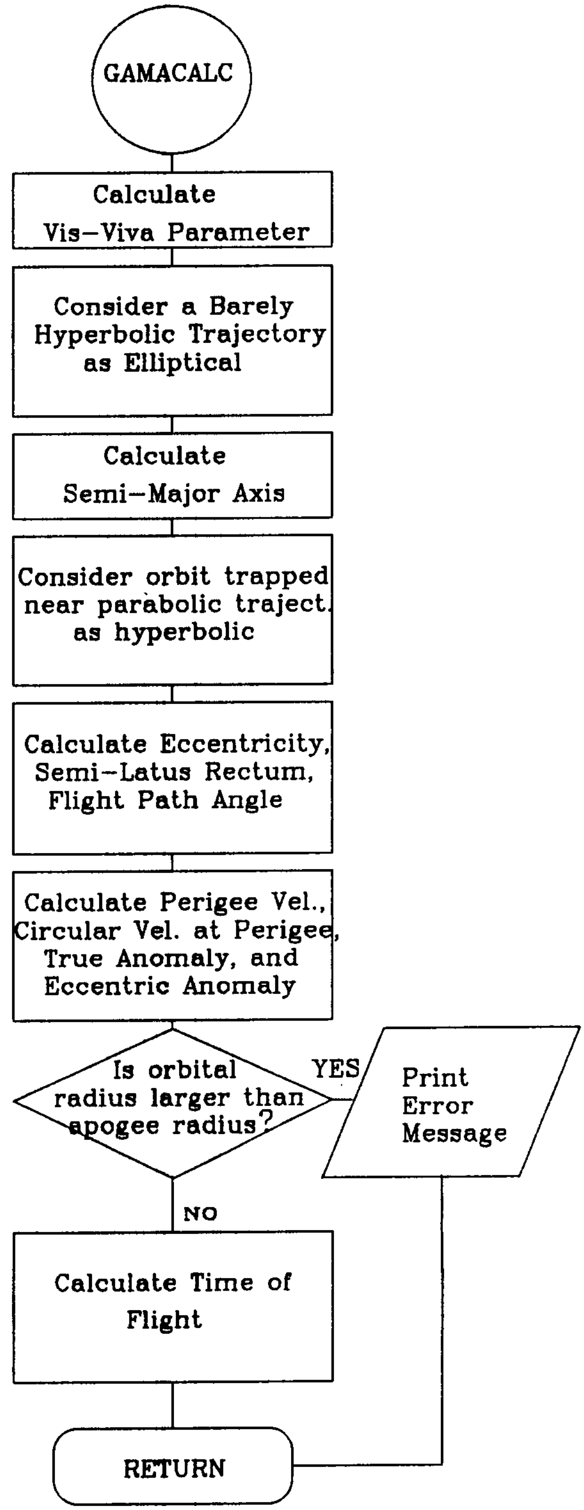

START
Input Departure Orbit Body and Altitude, and Libration Point
Input Flight Time Constraint
MOON
POSVELMO
GOSUB 70
GOSUB 70
Save Current Flight Time Iteration
GAMACALC
Increment Velocity at Libration Point by 1 ft/sec
GAMACALC
70
Calculate Libration Point Positions
Calculate Minumum Velocity at Libration Point
GAMACALC
Is Flight Time Greater than Max. Flight Time?
YES
NO
RETURN


Extrapolate to a More Accurate Flight Time
NO
Is Flight Time Close?
YES
Calculate Flight Path Angle, Longitude, Latitude, & Azimuth
VELTRANS
Calculate Delta-V Needed to Correct Velocity Vector at LP
Calculate Delta-V Needed to Reach LP from Departure Orbit
Save Old Total Delta-V and Calculate New Delta-V
Calculate Position and Velocity of Moon
YES
Has Flight Time Changed More Than .09 hrs?
NO
A
A
Save Delta-V's for Output
Decrement Flight Time and Recalculate Unless Last One
Increment Inclination and Recalculate Unless Last One
Form Output File Containing 3 pages
END


MOON
Calculate Position of Moon
RETURN


POSVELMO
Calculate Position and Velocity of Moon
RETURN


VELTRANS
Convert Orbital Velocity Into Rectangular Coor.
RETURN


GAMACALC
Calculate Vis-Viva Parameter
Consider a Barely Hyperbolic Trajectory as Elliptical
Calculate Semi-Major Axis
Consider orbit trapped near parabolic traject. as hyperbolic
Calculate Eccentricity, Semi-Latus Rectum, Flight Path Angle
Calculate Perigee Vel., Circular Vel. at Perigee, True Anomaly, and Eccentric Anomaly
Is orbital radius larger than apogee radius?
YES
Print Error Message
NO
Calculate Time of Flight
RETURN

Appendix B. Code Listing

```
C  ************************************************************
C  *** Libration Point (LP) Program ***                       *
C  ************************************************************
C  *   from Earth to Points 2, 3, 4, and 5 and                *
C  *   from Moon to Points 1 and 2                             *
C  ************************************************************
C  *   Written in Quick Basic by: Jack Funk                   *
C  *   Translated into Fortran and documented by: Bill Engblom *
C  ************************************************************
C
C     MAIN
C
      IMPLICIT REAL*16 (A-H,O-Z)
      CHARACTER*5 TRAJE, BODY
      CHARACTER*32 HEAD1, HEAD2, HEAD3
      CHARACTER*10 TIMP, DATP
C
C
      DIMENSION XXL(5), YYL(5), AINCEP(10), FLTIMP(19)
      DIMENSION PAGE1(15,19), PAGE2(15,19), PAGE3(15,19)
C
C     OPEN OUTPUT FILE
C
      OPEN (UNIT = 1,FILE = 'LIBRATE.OUT' ,STATUS = 'NEW')
C     OUTPUT TO FILE; IP:  OUTPUT TO SCREEN; IS
      IP    = 1
      IS    = 5
      II    = 1
      JJ    = 1
      NSTOP = 1
      DPR   = 57.29578
      PI    = 3.141593
      CMUE  = 1.407647E+16
      CMUM  = 1.731400E+14
      FTNM  = 6076.115
      REE   = 20925741.
      REMO  = 5.7039E+6
      FTM   = .3048
C     OUTBOUND TRAJECTORIES (MODE = 1)
C     MODE HAS BEEN CHANGED TO MD
      MD = 1
      CALL DATE(DATP)
      CALL TIME(TIMP)
      PRINT *,'INPUT EARTH OR MOON '
      READ (6,5) BODY
    5 FORMAT (A5)
      IF (BODY .NE. 'MOON') BODY = 'EARTH'
      PRINT *, 'INPUT PERIGEE ALTITUDE (NMI) '
      READ  *, HPE
```

```
      RPE = HPE * FTNM + REE
      PRINT *, 'INPUT LIBRATION POINT NUMBER '
      READ  *, NLP
      AINCEO = 0.
      AINCE = AINCEO / DPR
  20  CONTINUE
      PRINT *,'INPUT FLIGHT TIME (HR) '
      READ *, FLTIM
      IF (FLTIM .EQ. 0) GOTO 20
      FTIM = FLTIM
  25  CONTINUE
C     CALCULATE EARTH-MOON DISTANCE
      CALL MOON(T, RAM, DECM, RM)
      CALL POSVELMO(TIEM, RREMER, XDLO, YDLO)
      RREM = RM * REE
C      WRITE (IS,30)
C  30  FORMAT (' FTIM  DVTOT   DVCIR DELV    VELX    VXX     VYX     VZX
C     *    VXLP    VYLP    RREM')
C     CALCULATE LIBRATION POINT LOCATIONS, MAX FLIGHT TIME
      ICALL = 1
      GOTO 70
 1000 CONTINUE
C
  35  CONTINUE
C
C     UPDATE LIBRATION POINT LOCATIONS
      ICALL = 2
      GOTO 70
 2000 CONTINUE
  40  CONTINUE
C     TIMEE HAS BEEN CHANGED TO TIEM
      TIEMS = TIEM
      CALL GAMACALC(RPE, VELE, RRL, CMUE, CSGAME, VPE, VCIRE,
     * TIEM1, TRAJE, THETAE)
      VELE2 = VELE + 1.
      CALL GAMACALC(RPE, VELE2, RRL, CMUE, CSGAME, VPE, VCIRE,
     * TIEM2, TRAJE, THETAE)
      DELVELE = 1./(TIEM2 - TIEM1) * (FTIM - TIEM1 - TIEMM)
      IF (DELVELE .LT. 500.) THEN
        VELE = VELE + DELVELE
      ELSE
        VELE = VELE + DELVELE/QABS(DELVELE) * 500.
      ENDIF
      IF (VELE .LT. VPMIN) VELE = VPMIN
      CALL GAMACALC(RPE, VELE, RRL, CMUE, CSGAME, VPE, VCIRE,
     * TIEM, TRAJE, THETAE)
C
      IF (TIEM .EQ. 0.) GOTO 60
      TIEMT = TIEM
```

```
      IF (QABS(FTIM - TIEMT) .GT. 1.) GOTO 35
C
C     MODE HAS BEEN CHANGED TO MD
      GAMAE = FLOAT(MD) * QATAN(QSQRT(1. - CSGAME**2.)/CSGAME)
      ALONE = 180./DPR + ALONX
      ALATE = QATAN(ZZLP/QSQRT(XXLP**2 + YYLP**2))
   50 CONTINUE
C     CALCULATION OF EARTH ORBIT AZIMUTH AT SPHERE OF INFLUENCE
      IF(AINCE .NE. 0.) THEN
        PHIE = PI-QASIN(ZZ/(RRL*QSIN(AINCE)))
        SAZM = QCOS(AINCE) / QCOS(ALATE)
        CAZM = QSIN(AINCE) * QCOS(PHIE) / QCOS(ALATE)
        AZME = QATAN2(SAZM, CAZM)
      ELSE
        AZME = PI/2.
      END IF
C
      CALL VELTRANS(VELE,GAMAE,AZME,ALATE,ALONE,VXE,VYE,VZE,VELE)
      DELVEL = QSQRT((VXE+OMEGM*YYLP)**2. + (VYE-OMEGM*(XXLP
     * - XXBC))**2. + VZE**2.)
      DVCIRE = VPE - VCIRE
      DVTSAV  = DVTOTAL
      DVTOTAL = DELVEL + DVCIRE
C
C     CALCULATE POSITION AND VELOCITY OF MOON
C     ITTERATE FOR FLIGHT TIME TO SPHERE OF INFLUENCE
C
      TIM = (TIMJ-2451545.)/36525.+TIEM/876600.
      CALL POSVELMO(TIM,RREMER,XDLO,YDLO)
      RREM = RREMER*REE
      IF (QABS(TIEM-TIEMS) .GT. .09) GOTO 35
   60 CONTINUE
C      WRITE(IS,65) TIEM,DVTOTAL,DVCIRE,DELVEL,VELE,VXE,VYE,VZE,
C     * -OMEGM*YYLP,OMEGM*(XXLP-XXBC),RREMER
C  65       FORMAT(F5.1,2X,F6.0,1X,F6.0,1X,F5.0,2X,F5.0,1X,3(F6.0,1X)-
,1X,F6.0,
C     * 1X,F6.0,2X,F6.3)
      AINCEP(II) = AINCE * DPR
      FLTIMP(JJ) = FTIM
      PAGE1(II,JJ) = DVTOTAL
      PAGE2(II,JJ) = DELVEL
      PAGE3(II,JJ) = DVCIRE
      FTIM = FTIM - 2.
      JJ = JJ + 1
      NSTOP = NSTOP + 1
      IF (NSTOP .LT. 20) GOTO 40
      FTIM = FLTIM
      JJ = 1
      NSTOP = 1
```

```
      AINCE = AINCE + 10./DPR
      II = II + 1
      IF (AINCE .LT. 91./DPR) GOTO 40
      HEAD1 = 'MAP OF TOTAL VELOCITY NEEDED'
      CALL LPPAGE (PAGE1, BODY, HEAD1, NLP, DATP, TIMP, HPE, AINCEP,
     * FLTIMP)
      HEAD2 = 'MAP OF DELVEL AT LIBRATION POINT'
      CALL LPPAGE (PAGE2, BODY, HEAD2, NLP, DATP, TIMP, HPE, AINCEP,
     * FLTIMP)
      IF (BODY .EQ. 'EARTH') THEN
        HEAD3 = 'MAP OF DELVEL AT EARTH ORBIT'
      ELSE
        HEAD3 = 'MAP OF DELVEL AT LUNAR ORBIT'
      ENDIF
      CALL LPPAGE (PAGE3, BODY, HEAD3, NLP, DATP, TIMP, HPE, AINCEP,
     * FLTIMP)
      GOTO 3000
   70 CONTINUE
C     CALCULATION OF LIBRATION POINT POSITIONS
      RREM = RREMER * REE
      XXBC = .01215052 * RREM
      OMEGM = YDLO/(RREM - XXBC)
      XXL(1) = 1.15567 * RREM
      YYL(1) = 0.
      XXL(2) = 0.83691 * RREM
      YYL(2) = 0.
      XXL(3) = -1.005062 * RREM
      YYL(3) = 0.
      XXL(4) = 0.5 * RREM
      YYL(4) = -RREM * QSIN(60./DPR)
      XXL(5) = 0.5 * RREM
      YYL(5) = RREM * QSIN(60./DPR)
      ZZLP = 0.
C     CHANGE DATA FROM EARTH TO MOON
      IF (BODY .EQ. 'MOON') THEN
        RPE = HPE*FTNM + REMO
        XXLP = XXL(NLP) - RREM
        CMUE = CMUM
      ELSE
        XXLP = XXL(NLP)
      ENDIF
C      CALCULATE Y POSITION, RADIUS, AND LONGITUDE OF LIBRATION POINT
NLP
      YYLP = YYL(NLP)
      RRL = QSQRT(XXLP**2. + YYLP**2. + ZZLP**2.)
      ALONX = QATAN2(YYLP, XXLP)
C     CALCULATION OF MIN VEL AT LIB POINT FOR ORBIT CONTAINING
C     RRL AND RPE
      AAE = (RRL + RPE)/2.
```

```
      VPMIN = QSQRT(CMUE/AAE*RPE/RRL) * 1.001
      IF(VELE .LT. VPMIN) VELE = VPMIN
      CALL GAMACALC(RPE,VPMIN,RRL,CMUE,CSGAME,VPE,VCIRE,AMAXFT,
     * TRAJE,THETAE)
      IF (AMAXFT .LT. FTIM) THEN
        PRINT *,'FLIGHT TIME IS GREATER THAN MAX FLIGHT TIME ',
     *  AMAXFT
        GOTO 20
      ENDIF
      IF (ICALL. EQ. 1) GOTO 1000
      IF (ICALL .EQ. 2) GOTO 2000
 3000 STOP
      END



      SUBROUTINE LPPAGE(PAGE,BODYP,HEADP,NLP,DATP,TIMP,HPE,AINCEP,
     * FLTIMP)
      IMPLICIT REAL*16 (A-H,O-Z)
      CHARACTER*5 BODYP
      CHARACTER*10 TIMP, DATP
      CHARACTER*32 HEADP
      DIMENSION PAGE(15,19), AINCEP(10), FLTIMP(19)
      IP = 1
      IS = 5
      WRITE (IP,5) CHAR(12)
    5 FORMAT (' ',A1)
      WRITE (IP,10) HEADP, BODYP, NLP
   10 FORMAT (T2,A32,' FOR ',A5,' TRANSFER TRAJECTORY
     * TO LIBRATION POINT #',I1)
      WRITE (IP,20) DATP, TIMP
   20 FORMAT (T25,'RUN DATE ',A9,'   RUN TIME ',A8)
      WRITE (IP,25) HPE
   25 FORMAT (T25,'CIRCULAR ORBIT ALTITUDE  ',F6.1)
      WRITE (IP,30) AINCEP(1), AINCEP(2), AINCEP(3), AINCEP(4),
     *AINCEP(5),AINCEP(6),AINCEP(7),AINCEP(8),AINCEP(9),AINCEP(10)
   30 FORMAT (/,' INCL>',10(2X,F5.1),/,' FLIGHT',/,' TIME (HR)')
      DO 40 NPI = 1,19
        WRITE (IP,35) FLTIMP(NPI),PAGE(1,NPI),PAGE(2,NPI),
     *  PAGE(3,NPI),PAGE(4,NPI),PAGE(5,NPI),PAGE(6,NPI),PAGE(7,NPI),
     *  PAGE(8,NPI),PAGE(9,NPI),PAGE(10,NPI)
   35   FORMAT (1X,F6.1,10(1X,F6.0))
   40 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE POSVELMO(TIM,RRM,XDLO,YDLO)
      IMPLICIT REAL * 16 (A-H,O-Z)
      DELT = 0.5/36525./24./3600.
      T1 = TIM-DELT
      CALL MOON(T1,RAM,DECM,RM1)
      T2 = TIM+DELT
      CALL MOON(T2,RAM,DECM,RM2)
      XDLO = (RM2-RM1)*20925741.
      RRM = (RM2+RM1)/2.
      RRM = RRM
      RRMB = RRM -7.412789E-01
      YDLO = 200570.2/RRMB
      RETURN
      END



      SUBROUTINE MOON (T, RAM, DECM, RM)
C     FINDS LOCATION OF MOON IN EQUATORIAL COORDS. AT ANY TIM
C     REF:      '87 ASTRONOMICAL ALMANAC
C                T IS JULIAN CENTURIES SINCE YEAR 2000
C                LAM IS MOON'S ECLIPTIC LONGITUDE
C                BETA IS MOON'S ECLIPTIC LATITUDE
C                PIE IS HORIZONTAL PARALLAX
C                RM IS DIST. TO MOON IN EARTH RADII
C                RAM IS RT. ASCENSION OF MOON
C                DECM IS MOON'S DECLINATION
C                SD IS SEMIDIAMETER OF MOON'S ORBIT

      IMPLICIT REAL * 16 (A-Z)
C      PRINT *, ' MOON'

      P = 3.1415926535
      C = P / 180.

      LAM = C*218.32+C*481267.883*T+C* 6.29 * QSIN(C * 134.9 + C *
     *477198.85 * T) - C * 1.27 * QSIN(C * 259.2 - C * 413335.38 *
     *T) + c * .66 * QSIN(C * 235.7 + c * 890534.23*T)

      LAM = LAM + C * .21 * QSIN(C * 269.9 + C * 954397.7*T) - C *
     *.19 *  QSIN(C * 357.5 + C * 35999.05 * T) - C * .11 *
     *SIN(C * 186.6 + C * 966404.05*T)

      beta = c*5.13*QSIN(c*93.3 + c * 483202.03 * T) + c *
     * .28 * QSIN(C * 228.2 +  C * 960400.87 * T) -c*.28*QSIN
     *(c*318.3+c* 60003.18*T)-c*.17*QSIN(c*217.6-c*407332.2 * T)

      pie = c*.9508+c*.0518*COS(c*134.9 + c * 477198.85 * T) +
```

```
     *c*.0095*QCOS(c*259.2-c*413335.38*T)+c* .0078*COS(c * 23
     *5.7+c*890534.23*T)+c*.0028*QCOS(c*269.9+c*954397.7* T)

      SD = .2725 * pie
      RM = 1. / QSIN(pie)

      l      = QCOS(beta)  * QCOS(LAM)
      M      = .9175 * QCOS(beta)  * QSIN(LAM)  - .3978 * QSIN(beta)
      n      = .3978 * QCOS(beta)  * QSIN(LAM)  + .9175 * QSIN(beta)
      RAM    = QATAN2(M, l)
      DECM   = QASIN(n)
      RETURN
      END



      SUBROUTINE GAMACALC(RPX,VV,RRX,CMUX,COSGAMX,VPX,VCIRX,
     *TIMX,TRAJ,THETAX,DPR,FTNM)
      IMPLICIT REAL * 16 (A-H, O-Z)
      CHARACTER*5 TRAJ
      IHYPER = 1
      TRAJ = 'ELIPT'
      QQX = RRX*VV**2./CMUX
      IF(QQX-2 .LT. 1.0E-06) QQX = QQX - 1.0E-06
      IF (QQX .GT. 2.) THEN
        IHYPER = -1
        TRAJ = 'HYPER'
C       PRINT *,' ***** TRAJECTORY IS HYPERBOLIC '
      ENDIF
      AAX = RRX/(2.-QQX)
      IF (AAX. GT . 1.0E12 .OR. AAX. LT. -1.0E12) AAX = -1.0E12
      EEX          = 1.-RPX/AAX
      PPX          = AAX*(1. -EEX ** 2.)
      COSGAMX      = QSQRT(RPX/RRX*(1.+EEX)/QQX)
      GAMAX        = QACOS(COSGAMX)
      VPX          = QSQRT(CMUX*(1.+EEX)/RPX)
      VCIRX        = QSQRT(CMUX/RPX)
      COSTHETAX =  (PPX/RRX-1.)/EEX
      THETAX       = QACOS(COSTHETAX)
      COSAEX       =  (EEX+COSTHETAX)/(1.+EEX*COSTHETAX)
      ERRRP        = RRX-AAX*(1+EEX)
      IF(ERRRP .GT. 0. .AND. AAX .GT. 0.) THEN
        WRITE (IS,407) ERRRP/6076.1155, ALAT*DPR, ALON*DPR
 407    FORMAT (' RADIUS > APOGEE BY NMI ',F7.5, F8.0, F7.5)
        WRITE (IS,417) QQX, AAX/FNTM, EEX, GAMAX*DPR, VPX
 417    FORMAT (' QQX = ',F7.5,' AAX = ',F8.0,' EEX = ',F7.5,
     *  ' GAMAX = ', F5.1,' VPX = ',F7.1)
      ENDIF
```

```
      IF(QQX .GT. 2.) GOTO 101
C       CALC FLIGHT TIME FOR ELIPTICAL ORBITS
        IF (ERRRP .GT. 0.) THEN
          TIMX = 0.
          GOTO 199
        ENDIF
        AEX    = QACOS(COSAEX)
        SINAEX = QSQRT(1-COSAEX**2.)
        TIMX   = QSQRT(AAX ** 3. / CMUX)*(AEX-EEX*SINAEX)/3600.
        GOTO 199
 101  CONTINUE
C     CALC FLIGHT TIME FOR HYPERBOLIC ORBITS
      COSHF = COSAEX
      SINHF = QSQRT(COSHF**2.-1.)
      FFX   = QLOG(COSHF+QSQRT(COSHF**2.-1.))
      TIMX = QSQRT(-1. *AAX*AAX*AAX/CMUX)*(EEX*SINHF-FFX)/3600.
 199  CONTINUE
      RETURN
      END


      SUBROUTINE VELTRANS(VEL,GAMA,AZM,ALAT,ALON,VXX,VYX,VZX,VMAG)
      IMPLICIT REAL * 16 (A-Z)
      RRD   = VEL * QSIN (GAMA )
      RPHID = VEL * QCOS (GAMA )
      VLON  = RPHID * QSIN (AZM )
      VLAT  = RPHID * QCOS (AZM )
      VXR   = -RRD * QCOS (ALAT ) * QCOS (ALON )
      VYR   = -RRD * QCOS (ALAT ) * QSIN (ALON )
      VZR   =  RRD * QSIN (ALAT )
      VXLA  = VLAT * QSIN (ALAT ) * QCOS (ALON )
      VYLA  = VLAT * QSIN (ALAT ) * QSIN (ALON )
      VZLA  = VLAT * QCOS (ALAT )
      VXLO  = VLON * QSIN (ALON )
      VYLO = -VLON * QCOS (ALON )
      VZLO = 0.0
      VXX  = VXR + VXLA + VXLO
      VYX  = VYR + VYLA + VYLO
      VZX  = VZR + VZLA + VZLO
      VMAG = QSQRT (VXX **2. + VYX ** 2.+ VZX ** 2. )
      RETURN
      END
```

Appendix C. Program Variables

INPUT VARIABLE	DESCRIPTION
AINCEO	Inclination of departure orbit (LEO or LLO) with respect to the Earth-Moon-LP plane (degrees)
BODY	Body of departure ('EARTH' or 'MOON')
FLTIM	Flight time constraint for trajectory (hours)
HPE	Holding perigee altitude of departure orbit (nautical miles)

CONSTANT	VALUE	DESCRIPTION
C	Π/180	Degrees per radian (deg./rad.)
CMUE	1.407647E+16	Gravitational parameter of the Earth (ft^3/sec^2)
CMUM	1.731432E+14	Gravitational parameter of the Moon (ft^3/sec^2)
DPR	57.29578	Degrees per radian (deg./rad.)
FTM	0.3048	Meters per foot (m/ft)
FTNM	6,076.115	Feet per nautical mile (ft/nm)
MD	'OUTBOUND'	Leg of trip on which to perform calculations (outbound)
PI	3.141593	Π (dimensionless)
REE	20,925,741	Radius of the Earth (ft)
REMO	5,703,900	Radius of the Moon (ft)

VARIABLE	DESCRIPTION
AAE	Semi-major axis of least eccentric Earth (or Moon)-to-LP trajectory. Perigee at departure orbit, apogee at libration point
AAX	Semi-major axis of one of the transfer orbits (Gamacalc Subroutine)
AEX	Eccentric anomaly of one of the transfer orbits (Gamacalc Subroutine)
AINCE	Departure orbit inclination (rad.) with respect to Earth-Moon-LP plane
AINCEP	Array of departure orbit inclinations (Lppage Subroutine)
ALAT	Latitude of LP point, measured from Earth-Moon plane, always 0° (Gamacalc Subroutine)
ALATE	Latitude of LP, measured from Earth-Moon plane, always 0°
ALATMIN	Latitude of LP associated with the minimum total ΔV
ALATSIM	Latitude of LP associated with the minimum heading correction ΔV
ALON	Longitude of LP, measured from Earth-Moon line zero longitude at departure body, starts from -X direction (Veltrans Subroutine)
ALONE	Longitude of LP, measured from zero longitude at Earth, starts from -X direction
ALONMIN	Longitude of LP associated with the minimum total ΔV
ALONSIM	Longitude of LP associated with the minimum heading correction ΔV
ALONX	Longitude of the LP, measured from the Earth-Moon line at the departure body, starts from -X direction
AMAXFT	Maximum flight time to LP from perigee altitude (least eccentric trajectory)
AZM	Azimuth of the transfer orbit upon reaching the LP, measured clockwise from +Y direction of departure body (Veltrans Subroutine)
AZME	Azimuth of the transfer orbit upon reaching the LP, measured clockwise from +Y direction of departure body
BETA	Moon's ecliptic latitude
CAZM	Cosine of the azimuth
CMUX	Earth or Moon gravitational parameter (Gamacalc Subroutine)

COSAEX	Cosine of the eccentric anomaly of one of the transfer orbits
CSGAME	Cosine of the flight path angle at LP of the Earth (or Moon)-to-LP trajectory
COSGAMX	Cosine of the flight path angle at LP of one of the transfer orbits (Gamacalc Subroutine)
COSHF	Hyperbolic cosine of the eccentric anomaly of one of the transfer orbits
COSTHETAX	Cosine of the true anomaly of one of the transfer orbits
DATP	Today's date
DECM	Declination of the Moon (not used)
DELT	A fraction of TIM that represents a half-second
DELVEL	ΔV needed at the LP point to correct the velocity vector
DELVELE	Extrapolated ΔV increment which is added to existing guess for the velocity at the LP of the Earth-to-LP trajectory (VELE)
DVCIRE	ΔV between departure circular orbit and Earth (or Moon)-to-LP trajectory
DVMIN	Hold variable for minimum total ΔV
DVSIM	Hold variable for minimum heading correction ΔV
DVTOTAL	Total ΔV for flight
DVTSAV	Temporary storage for total ΔV
ERRRP	Difference between orbital range at LP and apogee range. Orbital radius must be less than apogee radius or error message results
EEX	Eccentricity of one of the transfer orbits
FFX	Eccentric anomaly of one of the transfer orbits
FLTIMP	Array of flight times (Lppage Subroutine)
GAMA	Flight path angle at LP of Earth (or Moon)-to LP trajectory (Veltrans Subroutine)
GAMAE	Flight path angle at LP of Earth (or Moon)-to LP trajectory
GAMAX	Flight path angle of one of the transfer orbits (Gamacalc Subrou
HEADP	Heading for report (Lppage Subroutine)

HEAD1	Heading for report #1
HEAD2	Heading for report #2
HEAD3	Heading for report #3
IHYPER	Indicator that describes whether an orbit is hyperbolic
ICALL	Flag signifying where the program will resume after the internal subroutine (GOTO 70) has been completed
II	Counter for rows of PAGE arrays
IP	Unit number for formatted writes to output file
IS	Unit number for formatted writes to the screen
JJ	Counter for columns of PAGE arrays
L	A geocentric direction cosine
LAM	Moon's ecliptic longitude
M	A geocentric direction cosine
N	A geocentric direction cosine
NSTOP	Counter used to determine when to get a new inclination and reset the flight time
OMEGM	Angular velocity of the Moon
OMEGE	Argument of perigee of departure orbit
PAGE	Array of output matrix data (Lppage Subroutine)
PAGE1	Array holding matrix of total ΔV data
PAGE2	Array holding matrix of heading correction ΔV data
PAGE3	Array holding matrix of departure ΔV data
PHIE	Angle between the Earth-to-Moon plane and the departure body-to-LP line (always 180°). Used to determine azimuth angle
PIE	Horizontal parallax
PPX	Semi-latus rectum of one of the transfer orbits

QQX	Vis-viva parameter for an orbit
RAM	Right ascension of the Moon (not used)
RM	Distance from the Earth to the Moon in Earth radii at time T
RM1	Distance from the Earth to the Moon in Earth radii at time T1
RM2	Distance from the Earth to the Moon in Earth radii at time T2
RPE	Distance from departure body center to perigee altitude
RPHID	Orbital path component of the velocity vector
RPX	Distance from Earth or Moon center to perigee altitude (Gamacalc Subroutine)
RRD	Radial component of the velocity vector
RREM	Distance from the Earth to the Moon in feet
RREMER	Moon's distance from the Earth in Earth radii
RRL	Distance from the departure body center to the libration point
RRM	Average distance from the Earth to the Moon, using RM1 and RM2
RRMB	Distance from the Earth-Moon baricenter to the Moon
RRX	Distance from departure body (Earth or Moon) to LP
SAZM	Sine of the Azimuth
SD	Semi-diameter of the Moon's orbit
SINAEX	Sine of the eccentric anomaly of one of the transfer orbits
SINHF	Hyperbolic sine of the eccentric anomaly of one of the transfer orbits
T	Number of Julian centuries since the year 2000 AD
T1	TIM minus half a second
T2	TIM plus half a second
THETAE	True anomaly of Earth or Moon orbit
THETAX	True anomaly of one of the transfer orbits (Gamacalc Subroutine)

TIEM	Iterated Earth (or Moon)-to-LP time of flight (seconds)
TIEM1	Earth or Moon-to-LP time of flight guess (seconds), using VELE1
TIEM2	Earth or Moon-to-LP time of flight guess (seconds), using VELE2
TIEMS	Temporary storage for Earth-to-LP time of flight
TIEMT	Total time of flight
TIM	Time of arrival at LP from Earth (or Moon), in centuries since the year 2000 AD
TIMP	Time now
TIMX	Time of flight from Earth or Moon perigee to LP (Gamacalc Subroutine)
TIMJ	Earth departure Julian date (where January 1, 2000 is day 2,451,545. Refer to Section C of The Astronomical Almanac of the Year 1988).
TRAJ	Text that describes trajectory as hyperbolic or elliptical (Gamacalc Subroutine)
TRAJE	Text that describes trajectory as hyperbolic or elliptical
T1	One-half second before TIM
T2	One-half second after TIM
VCIRE	Velocity of Earth circular orbit
VCIRX	Velocity of the Earth or Lunar circular orbit (Gamacalc Subroutine)
VEL	Velocity at LP of one of the transfer orbits (Veltrans Subroutine)
VELE	Velocity at LP of the Earth (or Moon)-to-LP trajectory
VELE2	One foot per second more than VELE
VLAT	Latitude component of the velocity vector
VLON	Longitude component of the velocity vector
VMAG	Velocity vector magnitude
VPE	Perigee velocity of the departure body-to-LP trajectory
VPX	Perigee velocity for one of the transfer orbits (Gamacalc Subroutine)

VV	Velocity at LP of one of the transfer orbits (Gamacalc Subroutine)
VPMIN	Minimum velocity at LP for Earth-to-LP trajectory with maximum flight time
VXE	X-coordinate of velocity vector at LP for Earth (or Moon)-to-LP trajectory
VXLA	X-component of the latitude component of the velocity vector
VXLO	X-component of the longitude component of the velocity vector
VXR	X-component of the radial component of the velocity vector
VXX	Total X-component of the velocity vector
VYE	Y-coordinate of velocity vector at LP for Earth (or Moon)-to-LP trajectory
VYLA	Y-component of the latitude component of the velocity vector
VYLO	Y-component of the longitude component of the velocity vector
VYR	Y-component of the radial component of the velocity vector
VYX	Total Y-component of the velocity vector
VZE	Z-coordinate of velocity vector at LP for Earth (or Moon)-to-LP trajectory
VZLA	Z-component of the latitude component of the velocity vector
VZLO	Z-component of the longitude component of the velocity vector
VZR	Z-component of the radial component of the velocity vector
VZX	Total Z-component of the velocity vector
XDLO	X-coordinate of the velocity of the Moon
XXBC	Distance from Earth geometric center to baricenter
XXL	Array of distances in the X-direction from the Earth to each LP
XXLP	Distance in the X-direction from the departure body (Earth or Moon) to the LP's X-coordinate
YDLO	Y-coordinate of the velocity of the Moon
YYL	Array of distances in the Y-direction from the Earth to each LP
YYLP	Distance in the Y-direction from the departure body to the LP's Y-coordinate

ZZ	Distance in the Z-direction from the Earth-Moon plane to the LP (always 0)
ZZLP	Distance in the Z-direction from the Earth-Moon plane to the LP (always 0)

Appendix D. Detailed Program Description

Librate Main Program

1. Declare matrices (XXL, YYL, PAGE1, PAGE2, PAGE3, AINCEP, FLTIMP).
2. Open the output file (LIBRATE.OUT).
3. Define the program constants.
4. Record the current date and time (DATP and TIMP).
5. Read the program inputs:
    a. BODY (EARTH or MOON)
    b. HPE (perigee altitude of Earth orbit)
    c. NLP (libration point number)
    d. FLTIM (flight time constraint)
6. Calculate RPE, the distance from the Earth's center to Earth perigee orbit.
7. Calculate RM, Earth-Moon distance (Call Moon Subroutine).
8. Calculate YDLO, the orbital velocity of the Moon (Call Posvelmo Subroutine).
9. Calculate RREM, Earth-Moon distance in feet.
10. Calculate libration point locations, maximum flight time (AMAXFT), minimum velocity at libration point (VPMIN) -- In-program subroutine. If flight time input (FLTIM) is greater than the maximum flight time (AMAXFT) then re-input flight time (step 5).
11. Update libration point locations, maximum flight time (AMAXFT), minimum velocity at libration point (VPMIN) -- In-program subroutine.
12. Save current time of flight estimate (TIEMS)

13. Calculate time of flight from perigee to LP (TIEM1), cosine of the flight path angle (COSGAMX) at LP, velocity at perigee needed to reach LP (VPX), velocity for circular orbit at perigee (VCIRX) from current values of LP distance (RRL), and velocity at LP (VELE) -- Call Gamacalc Subroutine.

14. Increment velocity at LP (VELE) by 1 ft/sec to get VELE2.

15. Calculate another time of flight (TIEM2), cosine of the flight path angle (COSGAMX), velocity at perigee needed to reach LP (VPX), velocity for circular orbit at perigee (VCIRX) from current values of LP distance (RRL), and velocity at LP (VELE2) -- Call Gamacalc Subroutine.

16. Estimate a delta-v needed to be added to VELE satisfy the time of flight constraint using a linear extrapolation of the two values determined in steps 13 and 15 (DELVELE).

17. If magnitude of DELVELE increment is larger than 500 ft/sec than limit magnitude of DELVELE to 500 ft/sec. Add the velocity increment to VELE.

18. If the new velocity at the LP (VELE) is smaller than the previously calculated minimum velocity (VPMIN) then set VELE = VPMIN.

19. Calculate new time of flight (TIEM), cosine of the flight path angle (COSGAMX), velocity at perigee needed to reach LP (VPX), velocity for circular orbit at perigee (VCIRX) from current values of LP distance (RRL), and velocity at LP (VELE) -- Call Gamacalc Subroutine.

20. If the time of flight is zero then go to output section of program.

21. Total time of flight (TIEMT) equals time of flight from Earth or Moon (TIEM).

22. If calculated flight time (TIMEE) is not within 1 hr. of the desired time then iterate the flight times again starting at step 11.

23. Determine the flight path angle from the cosine of the angle (COSGAMX).

24. Calculate longitude of libration point with respect to -X axis.

25. Calculate latitude of libration point with repect to Earth-Moon line.

26. Calculate azimuth angle of trajectory at LP.

27. Convert velocity at LP to rectangular coordinates (Call Veltrans)

28. Calculate the velocity increment needed to adjust the velocity heading at the LP such that the resulting orbit is stationary with respect to the Earth-Moon line (DELVEL) -- angular velocity at LP, with respect to the Earth, is equal to that of the Moon (OMEGM).

29. Determine velocity increment needed at perigee of initial departure orbit to attain an orbit containing the LP (DVCIRE).

31. Save previous delta-v total as DVTSAV.

32. Calculate total delta-v (DVTOTAL) from DELVEL + DVCIRE.

33. Calculate position and velocity of the Moon: Determine exact Julian date and Call Posvelmo.

34. Update distance from Earth to Moon in feet (RREM).

35. If time of flight (TIEM) is not within 0.09 hours of the time of flight before the last iteration (TIEMS) then go back to step 11 for more iterations.

36. Save current inclination, flight time, total delta-v, heading correction ΔV, and departure ΔV in the arrays AINCEP, FLTIMP, PAGE1, PAGE2, and PAGE3, respectively.

37. Decrement flight time (FTIM) by 2 ft/sec.

38. Go back to step 12 to calculate the trajectory for the new flight times unless the entire column has been completed (19 flight times for each inclination).

39. Reset flight time back to input value (FLTIM).

40. Increment inclination (AINCE) by 10°.

41. Go back to step 12 to calculate the trajectory for the next column until the inclination has reached 90°.

42. Produce three pages of output in the file LIBRATE.OUT using the saved arrays in step 36 via Subroutine LPPAGE.

Appendix E. Detailed In-Program Subroutine Description

Librate In-Program Subroutine

1. Update values for Earth-Moon position in feet (RREM), Earth-Moon baricenter (XXBC), and the angular velocity of the Moon (OMEGM).
2. Update all libration point positions with respect to Earth-Moon line (XXL, YYL, ZZLP)
3. Update libration point position in use (XXLP, YYLP), and correct values if departing from the Moon.
4. Update distance from departure body to libration point (RRL), and longitude of LP with respect to departure body.
5. Calculate minimum velocity (VPMIN) at libration point for orbit with apogee RRL and perigee RPE.
6. If current iterated velocity at the LP (VELE) is less than minimum velocity then reset VELE to VPMIN.
7. Call Gamacalc Subroutine to determine the maximum flight time (AMAXFT) for an orbit containing RRL and RPE, and having an apogee velocity of VPMIN.
8. If input value for flight time (FLTIM) is greater than the maximum flight time then print the maximum flight time allowed (AMAXFT) and go back to step 5 to re-input the flight time constraint.
9. Return

Appendix F. Subroutine GAMACALC

The subroutine GAMACALC receives the parameters orbital perigee radius (RPX), orbital velocity at LP (VV), orbital radius at LP (RRX), and the gravitational parameter of the body being orbited (CMUX). It calculates and returns time of flight (TIMX), the cosine of the flight path angle (COSGAMX), velocity at periapses (VPX), circular velocity at periapses (VCIRX), true anomaly (THETAX), and an indicator describing whether the orbit is elliptical or hyperbolic (TRAJ$).

1. Initialize indicators to presume an elliptical orbit. (IHYPER, TRAJ$).

2. Calculate the vis-viva parameter
   QQX = (RRX * VELX^2) / CMUX.

3. If the orbit is just barely hyperbolic (QQX is within one-millionth of 2), force the calculation to consider it elliptical (reduce QQX to just under 2).

4. If the orbit is stil hyperbolic, reset the indicators to show this. Print a message on the screen announcing a hyperbolic orbit.

5. Calculate the semi-major axis of the orbit
   AAX = RRX / (2-QQX).

6. If the semi-major axis is very large (greater than 10^12) or very small (less than -10^12), the orbit is trapped near a parabolic trajectory. Make it hyperbolic:
   AAX = -10^12.

7. Calculate the orbit eccentricity
   EEX = 1 - (RPX/AAX).

8. Calculate the semi-latus rectum
   PPX = AAX * (1 - EEX).

9. Calculate the flight path angle

   a. The angular momentum of the orbit at perigee is
      HP = RPX * VELPERIGEE * cos(GAMAX).
      But at perigee, GAMAX is zero, so
      HP = RPX * VELPERIGEE.

   b. At LP, angular momentum is
      HX = RRX * VELX * cos(GAMAX).

   c. Angular momentum is constant along a given orbit, so
      HP = HX
      RPX * VELPERIGEE = RRX * VELX * cos(GAMAX)
      GAMAX = arccos((RPX * VELPERIGEE) / (RRX * VELX))

d. The velocity at perigee is

VELPERIGEE = sqr((CMUX * RAPOGEE) / (AAX * RPX)).

Since RAPOGEE / AAX = (1 + EEX), then

VELPERIGEE = sqr (CMUX * (1 + EEX) / RPX) or

RPX * VELPERIGEE = sqr(RPX * (1 + EEX) * CMUX).

e. Substituting (d) into (c) above yields

GAMAX = arccos((sqr(RPX * (1 + EEX) * CMUX) / (RRX * VELX))

f. Substituting from (2) above yields

GAMAX = arccos(sqr((RPX * (1 + EEX)) / (RRX * QQX))).

10. Calculate the perigee velocity

VPX = sqr(CMUX * (1 - EEX) / RPX).

11. Calculate the circular velocity at perigee

VCIRX = sqr(CMUX / RPX).

12. Calculate the true anomaly

THETAX = arccos(((PPX / RRX) - 1) / EEX)

13. Calculate the eccentric anomaly

AEX = arccos((EEX + cos(THETAX)) / (1 + EEX * cos(THETAX))).

14. Compare the orbital radius at LP (RRX) to the apogee radius (AAX * (1 + EEX)). If the orbital radius at LP is greater than the apogee radius, print a message on the screen indicating the difference in nautical miles. Also display the following:

a. LP latitude (ALAT)
b. LP longitude (ALON)
c. vis-viva parameter (QQX)
d. semi-major axis (AAX)
e. eccentricity (EEX)
f. flight path angle (GAMAX)
g. perigee velocity (VPX).

15. Calculate the time of flight.

a. If the orbit is elliptical:

TIMX = sqr(AAX^3/CMUX) * (AEX - EEX * sin(AEX)).

b. If the orbit is hyperbolic:

TIMX = sqr(-AAX^3/CMUX)*(EEX*sinh(EEX) - log(cosh(EEX) + sinh(EEX))).

Appendix G. Subroutine POSVELMO

The subroutine POSVELMO receives the parameter TIM (number of Julian centuries from the year 2000) and returns the moon's position and velocity at that time. Specifically, it returns the moon's distance from the Earth's center, in Earth radii (RRM); velocity in the x-direction (along the Earth-Moon line), in feet per second (XDLO); and velocity in the y-direction (direction of the moon's orbit), in feet per second (YDLO).

1. Calculate a fraction of time that represents half a second.
   DELT = 0.5 seconds / (36525 days/century * 24 hrs/day * 3600 seconds/hr)
   = 1.58440E-10 centuries.

2. Call the subroutine MOON with the parameter (TIM minus DELTA) to determine the moon's distance in Earth radii at half a second before TIM. This distance is RM1.

3. Call the subroutine MOON with the parameter (TIM plus DELTA) to determine the moon's distance in Earth radii at half a second after TIM. This distance is RM2.

4. Calculate the velocity of the moon in the -x (radial) direction.
   XDLO = [(RM2 Earth radii - RM1 Earth radii) / (1 sec)] * 20,925,741 ft/radii.

5. Calculate the average radius of the Lunar orbit during the one second centered on TIM.
   RRM = (RM1 + RM2) / 2.

6. Determine the radius of the Lunar orbit from the Earth-Moon barycenter.
   RRMB = RRM - 0.7412789 Earth radii.

7. Calculate the moon's velocity in the direction of its orbit.

   a. Moon's apogee (Apo) = 62.83308 Earth radii.

   b. Moon's perigee (Per) = 55.68264 Earth radii.

   c. Eccentricity (e) = (Apo - Per) / (Apo + Per) = 0.06033.

   d. Semi-latus rectum (p) = Apo(1 -e^2) = 62.60439 Earth radii
      = 1,310,038,967 feet.

   e. Earth's gravitational parameter (mu) = 1.407646822E+16 ft^3/sec^2.

   f. Angular momentum (h) = sqr(mu * p)
      = 4.29427E+12 ft^2/sec * (1 earth radii / 20,925,672.57 ft)
      = 205,215.4 ft*Earth radii/second.

   g. Y-velocity (YDLO) = h / RRMB = 205,215.4/RRMB.

Appendix H. Subroutine MOON

The subroutine MOON receives the parameter T (number of Julian centuries from the year 2000) and returns the approximate location of the moon in geocentric coordinates at that time. Specifically, it returns the right ascension of the moon (RAM), declination of the moon (DECM), and distance to the moon in Earth radii (RM). The formulae are from The Astronomical Almanac of the Year 1984, page D46. All degrees are converted to radians with the conversion factor $C = \pi/180$.

1. Calculate the ecliptic coordinates of the moon.

a. Moon's ecliptic longitude

$$\begin{aligned} LAM = {} & 218^\circ.32 + 481267^\circ.833T \\ & + 6^\circ.29 * \sin(134^\circ.9 + 477198^\circ.85T) \\ & - 1^\circ.27 * \sin(259^\circ.2 - 413335^\circ.38T) \\ & + 0^\circ.66 * \sin(235^\circ.7 + 890534^\circ.23T) \\ & + 0^\circ.21 * \sin(269^\circ.9 + 954397^\circ.70T) \\ & - 0^\circ.19 * \sin(357^\circ.5 + 35999^\circ.05T) \\ & - 0^\circ.11 * \sin(186^\circ.6 + 966404^\circ.05T) \end{aligned}$$

b. Moon's ecliptic latitude

$$\begin{aligned} BETA = {} & 5^\circ.13 * \sin(93^\circ.3 + 483202^\circ.03T) \\ & + 0^\circ.28 * \sin(228^\circ.2 + 960400^\circ.87T) \\ & - 0^\circ.28 * \sin(318^\circ.3 + 6003^\circ.18T) \\ & - 0^\circ.17 * \sin(217^\circ.6 - 407332^\circ.20T) \end{aligned}$$

c. Horizontal parallax

$$\begin{aligned} PIE = {} & 0^\circ.9508 \\ & + 0^\circ.0518 * \cos(134^\circ.9 + 477198^\circ.85T) \\ & + 0^\circ.0095 * \cos(259^\circ.2 - 413335^\circ.38T) \\ & + 0^\circ.0078 * \cos(235^\circ.7 + 890534^\circ.23T) \\ & + 0^\circ.0028 * \cos(269^\circ.9 + 954397^\circ.70T) \end{aligned}$$

d. Semi-diameter of moon's orbit

$$SD = 0.2725 * PIE$$

e. Distance to the moon in Earth radii

$$RM = 1 / \sin(PIE)$$

2. Form the geocentric direction cosines to rotate into geocentric coordinates.

a. $l = \cos(BETA)\cos(LAM)$
b. $m = 0.9175*\cos(BETA)\sin(LAM) - 0.3978*\sin(BETA)$
c. $n = 0.3978*\cos(BETA)\sin(LAM) + 0.9175*\sin(BETA)$
where $l = \cos(DECM)\cos(RAM)$, $m = \cos(DECM)\sin(RAM)$, $n = SIN(DECM)$.

3. Then:

   a. RAM = arctan(m/l) [right ascension]
   b. DECM = arcsin(n) [declination]

The errors will rarely exceed 0.2 Earth radii in distance (RM), 0.3° in right ascension (RAM), and 0.2° in declination.

Appendix I. Subroutine VELTRANS

The subroutine VELTRANS converts an orbital velocity vector into rectangular coordinates (see figure J-1). Parameters received by the subroutine are velocity (VEL), flight path angle (GAMA), azimuth (AZM), latitude above the Earth-Moon plane (ALAT), and longitude from the Earth-Moon line (ALON). A set of intermediate calculations is performed to express the velocity vector in terms of a radial component, a latitudinal component, and a longitudinal component. Each of these three components is further resolved into x-, y-, and z-components. Finally, all three x-components, all three y-components, and all three z-components are summed to provide the total x-, y-, and z-components of velocity.

1. Conversion of velocity vector into spherical coordinates.

From the geometry, the radial component of velocity, $\dot{R}$, is calculated to be
VEL * sin(GAMA). (See figure J-2).
The component along the orbital path, $R\dot{\Phi}$, is
VEL * cos(GAMA).
This orbital path component of velocity is further resolved into a latitude component, $\dot{LAT}$, and a longitude component, $\dot{LON}$ (see figure J-3). Again, from the geometry,
$\dot{LON} = R\dot{\Phi}$ * sin(AZM) and
$\dot{LAT} = R\dot{\Phi}$ * cos(AZM).

2. Conversion of spherical coordinates into rectangular coordinates.

a. Conversion of radial component into rectangular coordinates.

Refer to figure J-4. The projection of R onto the x-y plane is
$\dot{R}$ * cos(LAT).
The negative x-component of this is
$\dot{R}$ * cos(LAT) * cos(LON)
so the x-component, $\dot{X}$, is
-$\dot{R}$ * cos(LAT) * cos(LON).
The negative y-component of R is
$\dot{R}$ * cos(LAT) * sin(LON)
so the y-component, $\dot{Y}$, is
-$\dot{R}$ * cos(LAT) * sin(LON).
The z-component, $\dot{Z}$, is
$\dot{R}$ * sin(LAT).

b. Conversion of latitude component into rectangular coordinates.

Refer to figures J-5 and J-6. $\dot{LAT}$ is perpendicular to the radial vector, R. A line in the z-direction that meets the tip of $\dot{LAT}$ and intersects the radius vector produces the angles a and b, where
b = 90 - LAT and
a + b + 90 = 180. Therefore,
a = LAT.
From the geometry, the z-component of $\dot{LAT}$, $Z\dot{LAT}$, is
$\dot{LAT}$ * cos(LAT).

The projection of $L\dot{A}T$ onto the x-y plane is

$L\dot{A}T * \sin(LAT)$. (see figures J-7).

The x-component of this, $XL\dot{A}T$, is

$L\dot{A}T * \sin(LAT) * \cos(LON)$.

The y-component of this, $YL\dot{A}T$, is

$L\dot{A}T * \sin(LAT) * \sin(LON)$.

c. Conversion of longitude component into rectangular coordinates.

Refer to figures J-8 and J-9. $L\dot{O}N$ is always parallel to the x-y plane, so the z-component of $L\dot{O}N$, $ZL\dot{O}N$, is always zero. Using the same proof as in (b) above, it can be seen that the angle between $L\dot{O}N$ and the y-component of $L\dot{O}N$ is equal to LON. From the geometry, the x-component of $L\dot{O}N$, $XL\dot{O}N$, is

$L\dot{O}N * \sin(LON)$.

The negative y-component of $L\dot{O}N$ is

$L\dot{O}N * \cos(LON)$,

so the y-component, $YL\dot{O}N$, is

$-L\dot{O}N * \cos(LON)$.

3. Sum of the rectangular coordinates.

All of the x-, y-, and z-components are summed to provide the complete rectangular coordinates of the velocity vector.

$VXX = \dot{X} + XL\dot{A}T + XL\dot{O}N$

$VYX = \dot{Y} + YL\dot{A}T + YL\dot{O}N$

$VZX = \dot{Z} + ZL\dot{A}T + ZL\dot{O}N$.

Appendix J. Figures

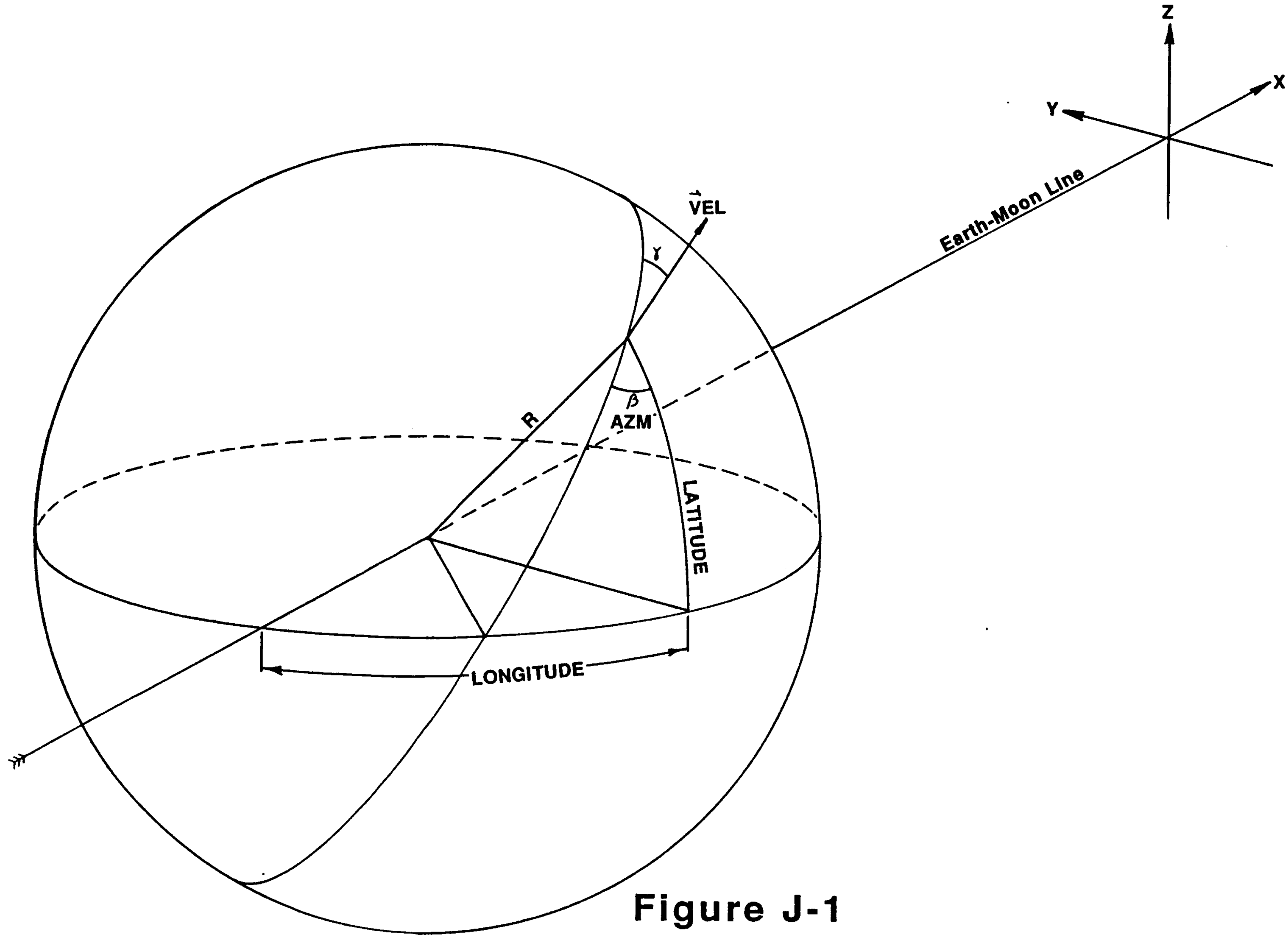


Figure J-1

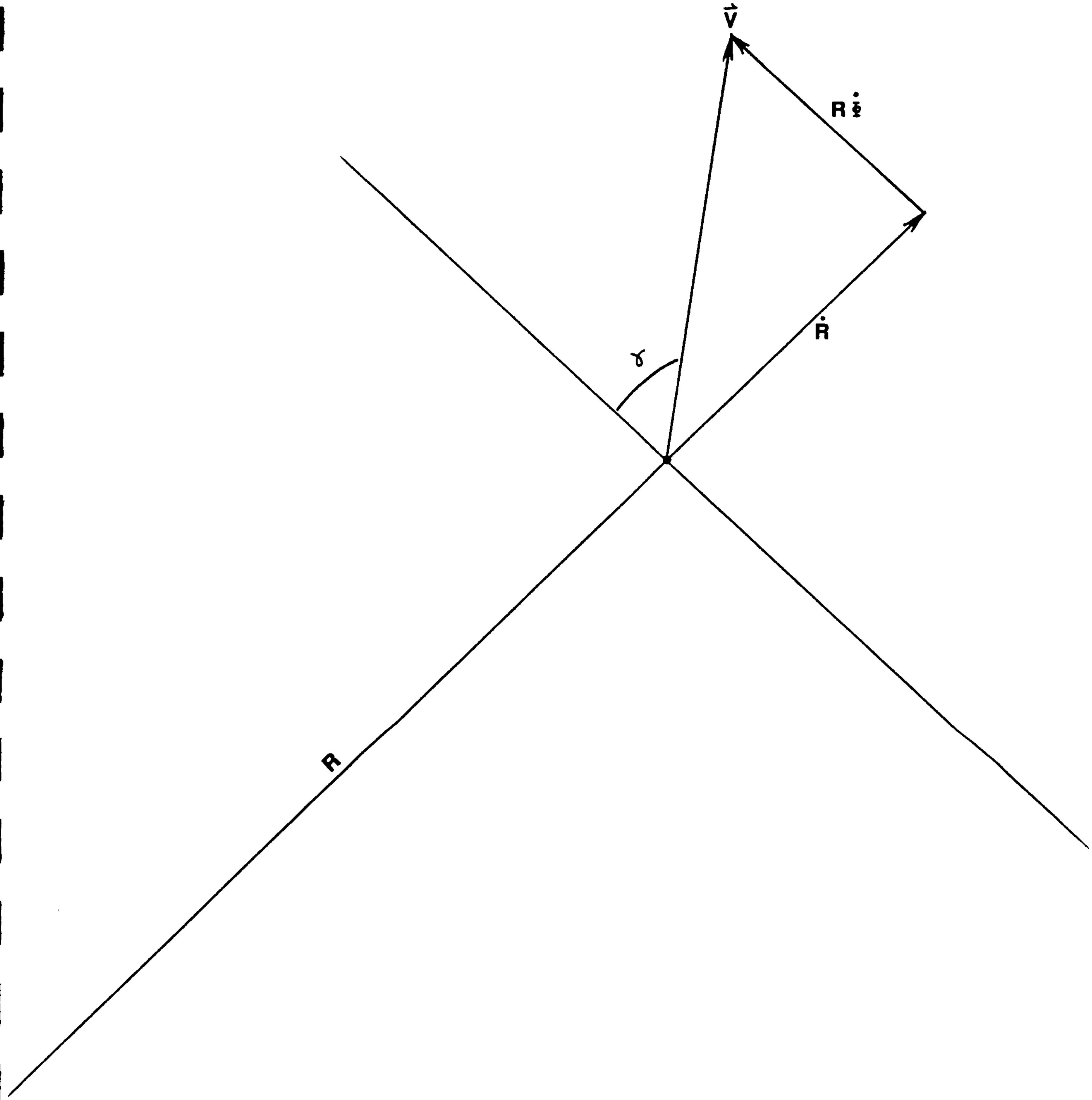


Figure J-2

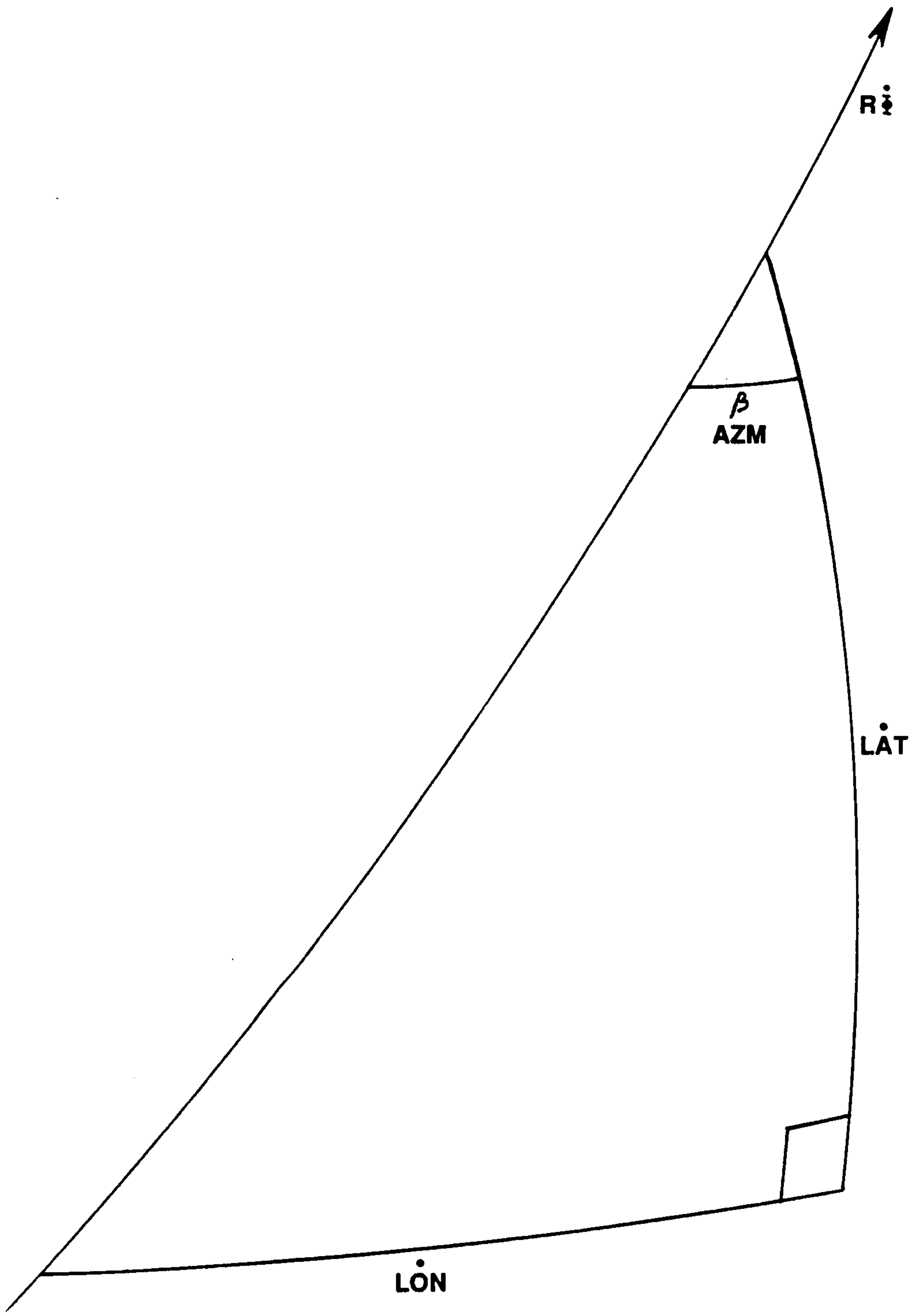


Figure J-3

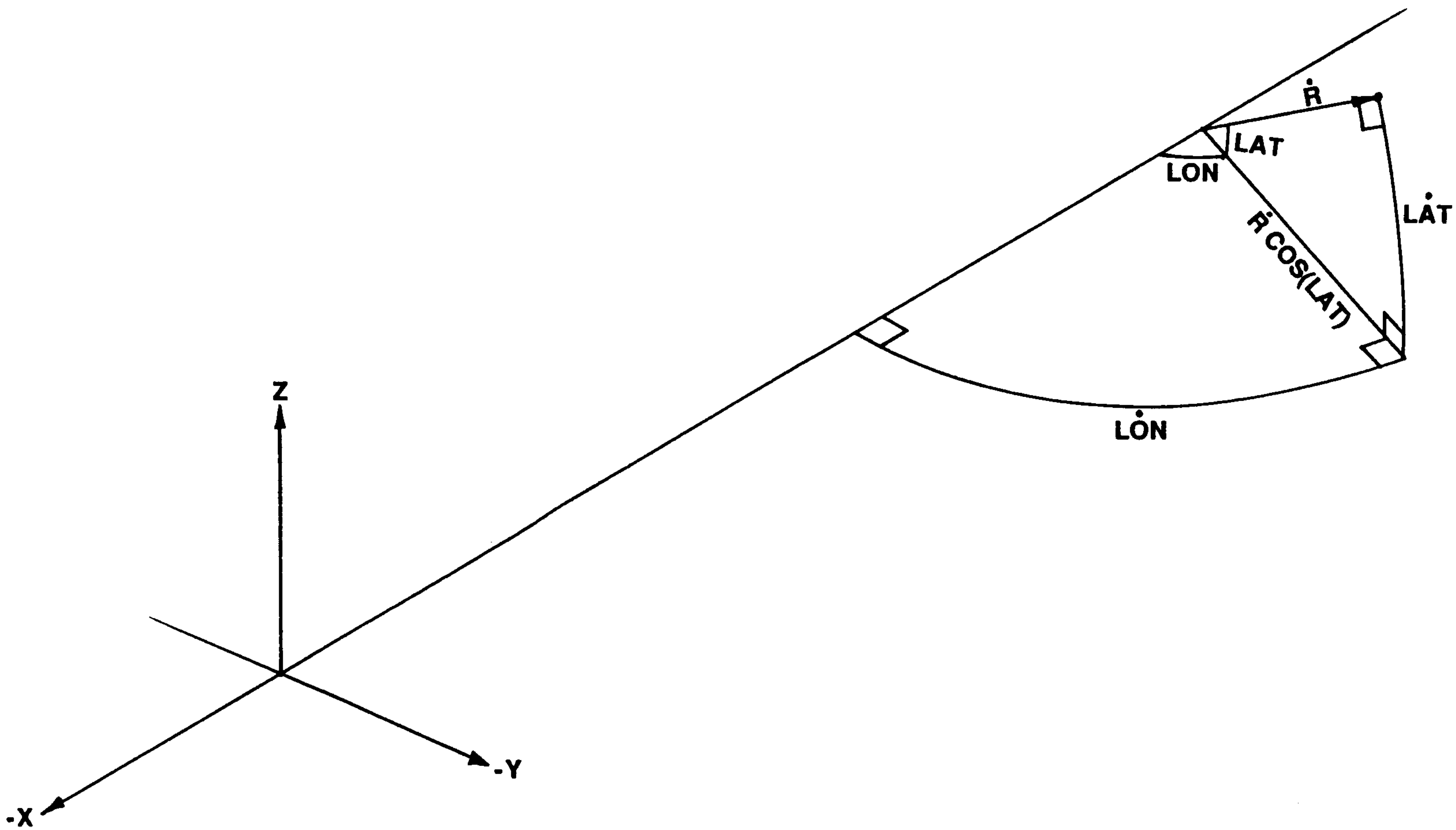


Figure J-4

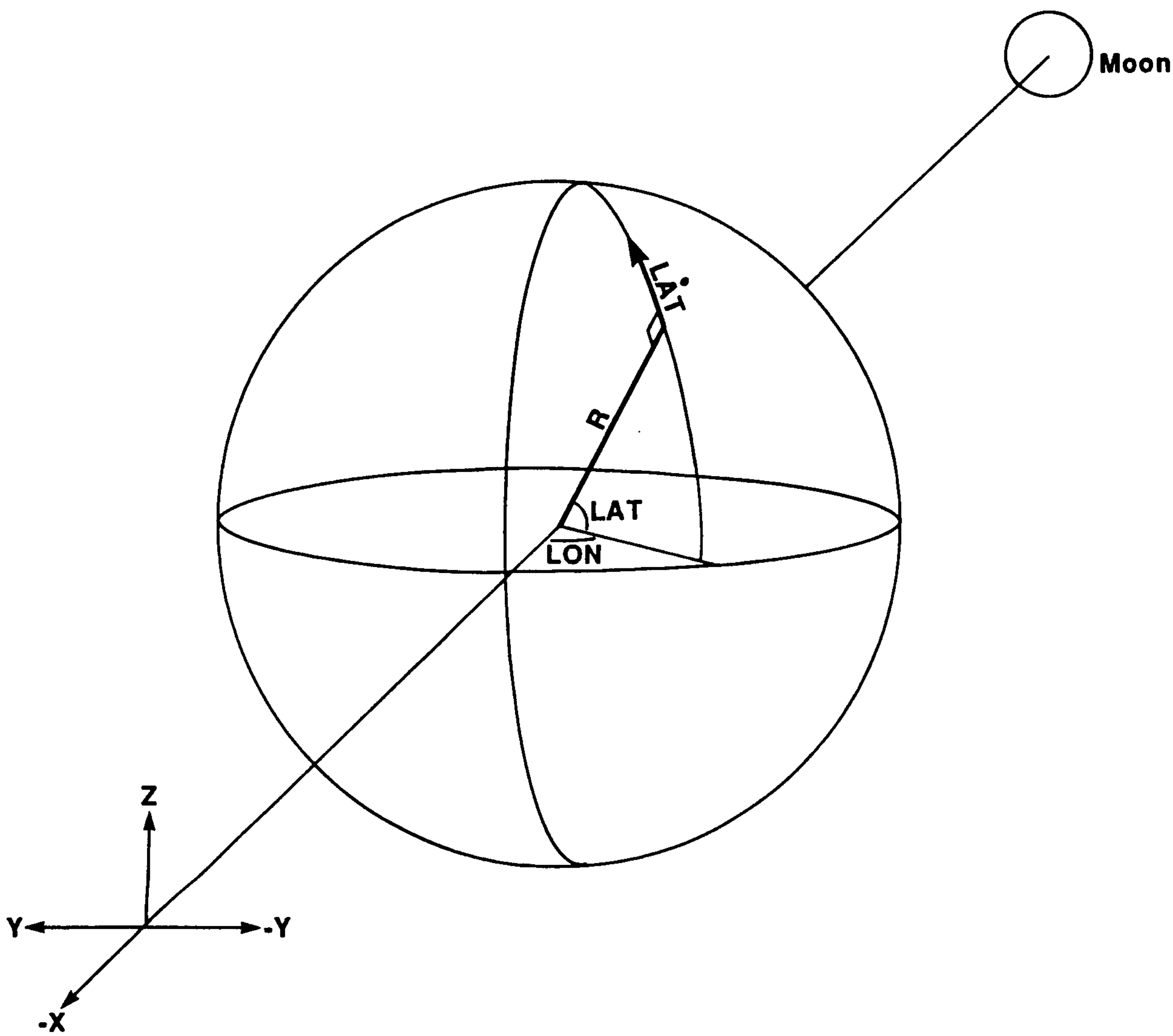


Figure J-5

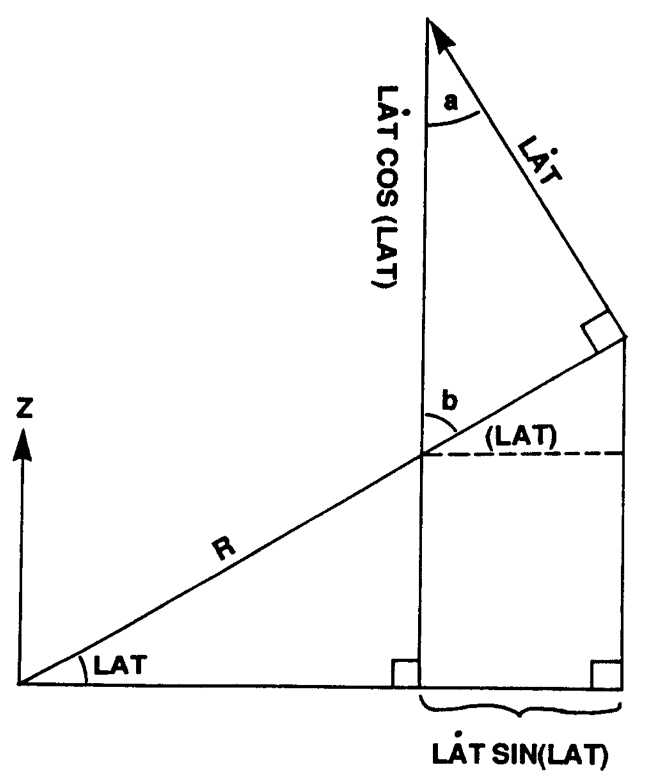


Figure J-6

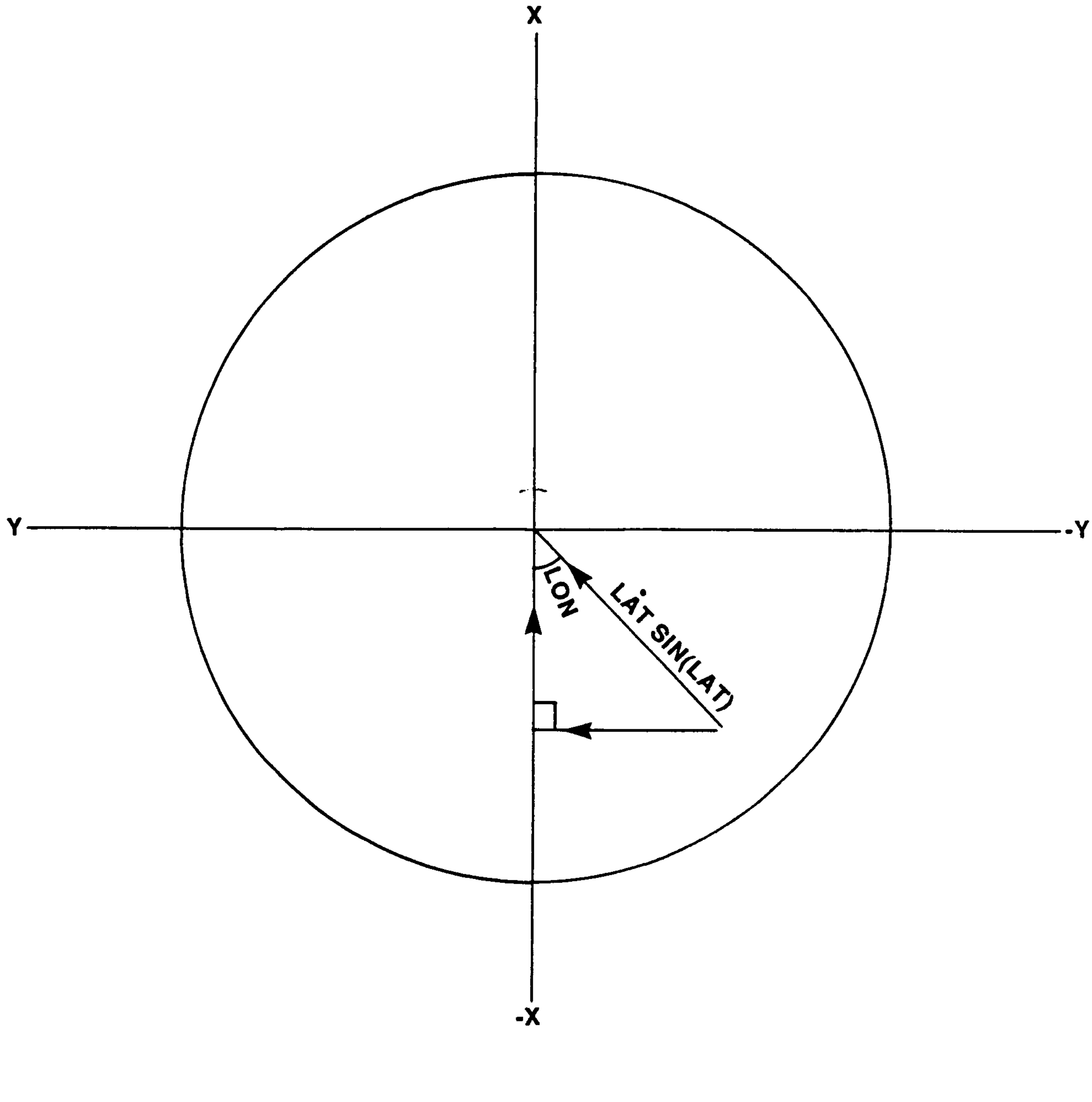


Figure J-7

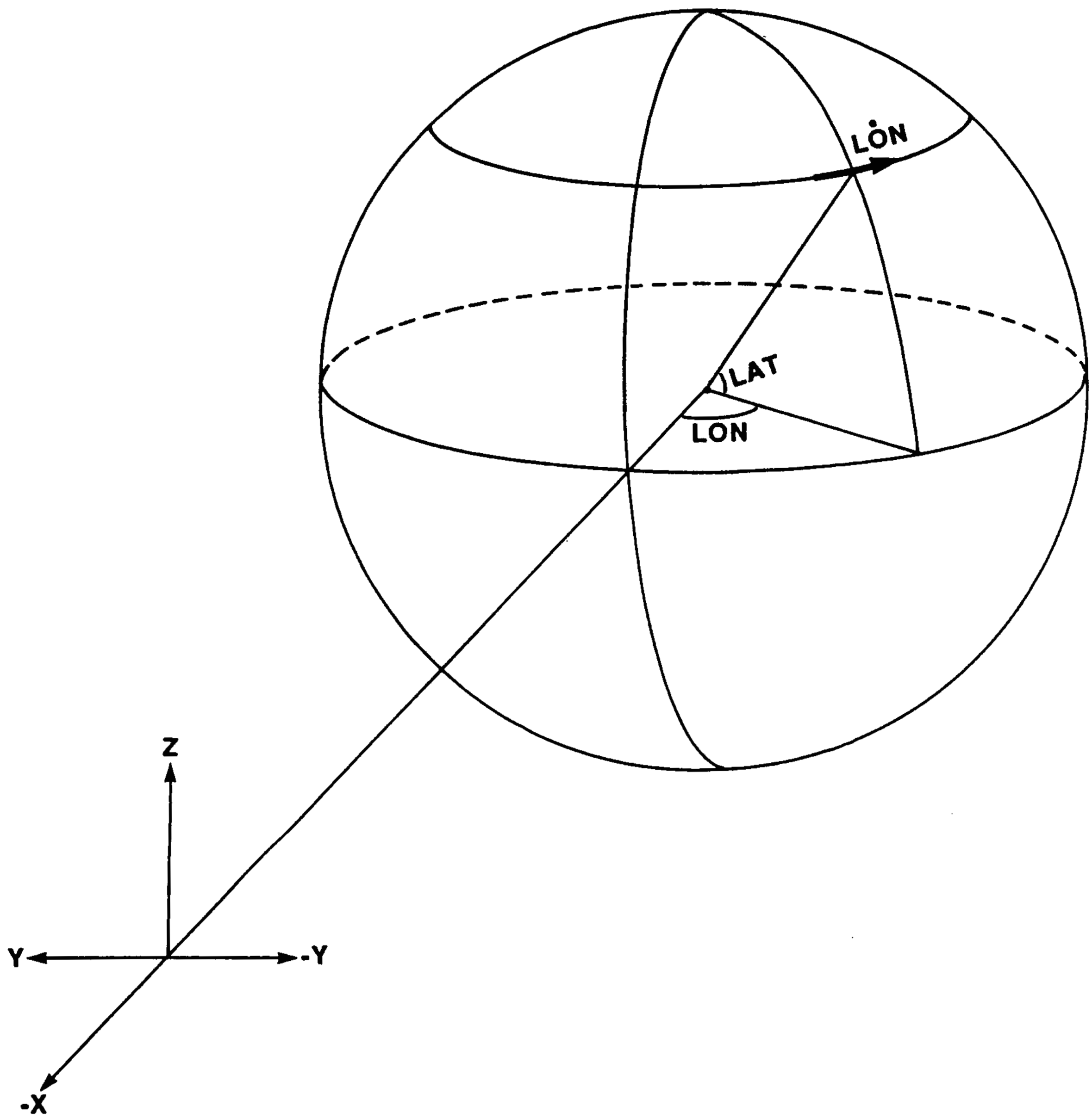


Figure J-8

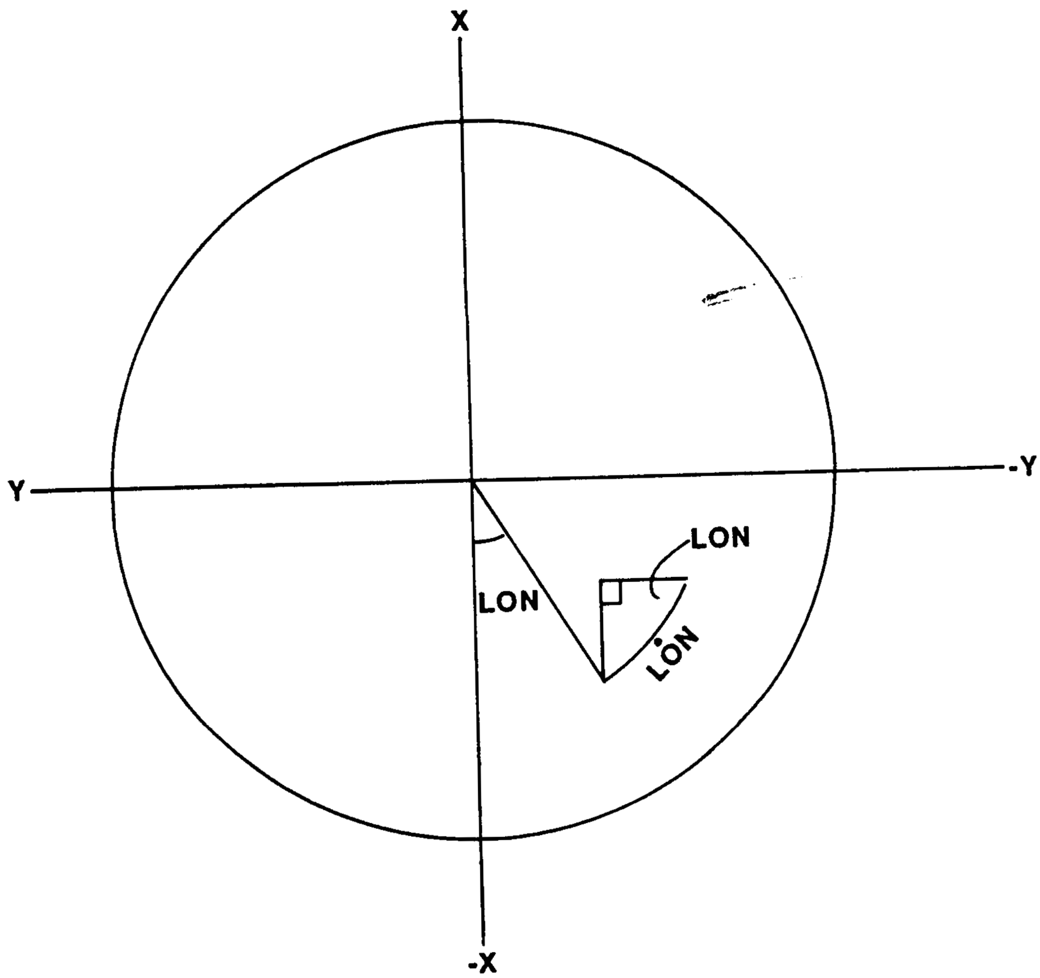


Figure J-9