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A New Algorithm for Modeling Friction in Dynamic Mechanical Systems

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A new method of modeling friction forces that impede the motion of parts of dynamic mechanical systems is described. Conventional methods in which the friction effect is assumed a constant force, or torque, in a direction opposite to the relative motion, are applicable only to those cases where applied forces are large in comparison to the friction, and where there is little interest in system behavior close to the times of transitions through zero velocity. This article describes a new algorithm that provides accurate determination of friction forces over a wide range of applied force and velocity conditions. The method avoids the simulation errors resulting from a finite integration interval used in connection with a conventional friction model, as is the case in many digital computer-based simulations. The new algorithm incorporates a predictive calculation based on initial conditions of motion, externally applied forces, inertia, and integration step size. The predictive calculation in connection with an external integration process provides an accurate determination of both static and Coulomb friction forces and resulting motions in dynamic simulations. Accuracy of the results is improved over that obtained with conventional methods and a relatively large integration step size is permitted. A function block for incorporation in a specific simulation program is described. The general form of the algorithm facilitates implementation with various programming languages such as Fortran or C, as well as with other simulation programs.

I. Introduction

Recent interest in certain limit cycle oscillatory modes of operation of the 70-m antenna at DSS 14 has intensified the need for dynamic analysis and simulation of the axis servos. Limit cycle oscillations of phyiscal positioning systems, such as the antenna axis servos, result from nonlinearities associated with the position sensors and the control actuation devices. To support limit cycle investigations it is necessary to model and simulate all the identified nonlinearities in the system. Because of the large magnitude of axis friction, which equals roughly 20 percent of the maximum available control effort, accurate modeling of friction effects is of critical importance.

The basic physical laws of friction are discussed in numerous textbooks on mechanics and are briefly summarized here. First, the friction force between two bodies lies in the tangent plane of the contact point between the bodies. In the absence of relative motion, its magnitude is less than or equal to the product of the normal force between the bodies and a constant coefficient of static friction. Relative tangent plane motion between the bodies cannot commence until an externally applied force exceeds the maximum magnitude of the friction force. Second, the Coulomb (as opposed to viscous) friction in the presence of relative motion between the two bodies, the friction force equals the product of the normal force and a coefficient of friction which may be different from the static coefficient and is in a direction opposite to the relative motion.

In those applications where the normal force is constant and static friction forces are of no great concern, the friction force can be modeled by a constant force directed opposite the relative motion. This conventional model combined with an inertia is illustrated in transfer function form in Fig. 1 where motion is restricted to a single coordinate. The nonlinear function block in Fig. 1 has three possible outputs: a unit amplitude with algebraic sign the same as that of the velocity; $\dot{x}$, (when $\dot{x}$ is nonzero); and zero output for zero $\dot{x}$. The constant of multiplication, F_c, in the constant block is the product of the normal force and the friction coefficient. The net input force to the integration block is thus equal to the difference between the applied force and the friction force.

By inference, the behavior of the model of Fig. 1 can be predicted for a number of simple cases. First, when the applied force is zero and the initial rate of motion is nonzero, the motion will decay to zero under the influence of friction. Next, when the applied force exceeds the magnitude of the friction, the motion will accelerate in direct proportion to the difference between the applied and friction forces. For these two cases the model is seen to provide a reasonable representation of motion in the presence of friction. Examining next the case where the applied force is less in magnitude than the friction and the initial rate is zero, the model is seen to deviate from the physical law because the modeled friction force is zero for the zero rate condition and an erroneous acceleration of the inertia results.

Assuming the conventional model is evaluated using fixed step size numerical integration, the zero rate case above produces a friction force which, because it exceeds the applied force, causes a rate reversal and leads to a sustained oscillatory process. While the amplitude of the rate excursions can be reduced through a reduction of integration step size, it will be seen that regardless of step size, the rate oscillates about a nonzero mean due to the nonzero input applied force. A special computation is thus necessary to determine a step size sufficiently small to control both the mean and amplitude of rate error. In the 70-m antenna axis servos the ratio of friction to inertia is 102 millidegrees/sec^2 (1.77 milliradians/sec^2) for the azimuth axis and roughly 1.5 times that ratio for elevation. It can be shown that controlling the above rate errors to less than 0.1 mdeg/sec requires a step size of roughly 1.0 msec, which is unreasonably small and leads to excessive computation time and data storage for small computer-based simulations.

II. Derivation of Equations for Modeling Friction

From the foregoing discussion it is evident that accurate computer modeling of motion involving friction is based on knowledge of both the applied force and the velocity of the body influenced by the friction. The velocity determination is thus an essential adjunct of any friction model. When that velocity is determined by a finite step size integration process, the effects of friction reversals resulting from mid-integration-step zero crossings of velocity must be considered. The modeling problem thus becomes the determination of net effective impulse such that the velocity change resulting from the finite step integration is reasonably accurate.

The derivation of the equations for modeling friction is equally applicable to translational or rotational systems. For rotation, the derivation assumes a slowly varying externally applied torque to a constant inertia body in the presence of both static and invariant Coulomb friction and a known, fixed integration interval. The following conditions are considered separately.

(1) The applied torque is greater than the static friction torque and the inertia is initially at rest.

(2) The applied torque and initial rate are such that the rate of motion will not reach zero within the next integration interval.

(3) The applied torque is less than the static friction and the initial rate of motion is such that the rate will reach zero within the integration interval.

(4) The applied torque is greater than the static friction and the initial rate of motion is such that the rate will reach zero within the integration interval.

For Condition (1) above, the net torque acting on the inertia is simply the applied torque diminished in magnitude by the Coulomb friction torque. The effective friction torque, T_f, is a constant with direction opposite to the applied torque

$$T_f = -F_c \cdot \operatorname{sign}(T_{ap}) \tag{1}$$

for $\dot{\theta} = 0$ and $|T_{ap}| > F_c$ where F_c is the Coulomb friction torque, the sign function is unit amplitude with the algebraic sign of its argument, T_{ap} is the applied torque, and $\dot{\theta}$ is the rate of motion. The use of the Coulomb rather than the static value in this case is based on the assumption of an instantaneous transition from the static to the sliding friction case. It will be seen that this assumption results in a minimum net torque equal to the difference between the static and Coulomb values. The resulting rate impulse can be adjusted to better comply with known physical behavior by selection of the integration step size.

The necessary condition for (2) above is determined from the equation of motion in the presence of friction

$$\dot{\theta}(t) = \dot{\theta}(0) + \left(\frac{1}{J}\right)(T_{ap} + T_f)\, t \qquad (2)$$

where for rotational motion, J is the inertia moment, $\dot{\theta}(t)$ is the rate at time t, T_{ap} is the applied torque and T_f, the friction torque. Substituting $-F_c \cdot \mathrm{sign}(\dot{\theta})$ for the friction, T_f, solving for $\dot{\theta}(t_{oc}) = 0$, and dividing by the integration step size, t_i, yields

$$\frac{t_{oc}}{t_i} = \frac{-J\,\dot{\theta}(0)}{T_{ap} - F_c \cdot \mathrm{sign}\,[\dot{\theta}(0)]} \qquad (3)$$

Negative values of t_{oc}/t_i imply a level of applied torque in excess of the friction and in the same direction as the rate. Positive values imply an applied torque either in a direction opposite the rate, or having a magnitude less than the friction, or both. A negative or unity or greater than unity value of t_{oc}/t_i is a necessary and sufficient condition for Condition (2) above. The net torque in this case is the algebraic difference between the applied and friction torques where the friction is opposite in direction to the rate.

$$T_f = -F_c \cdot \mathrm{sign}\,(\dot{\theta}) \qquad (4)$$

for $t_{oc}/t_i < 0$ or $t_{oc}/t_i \geqslant 1$.

In Condition (3) above, the rate will reach zero at some time within the integration interval and the applied torque will be insufficient to produce a rate in the reversed direction. Because the net torque acts on the inertia for the full interval, it must then decelerate the inertia to precisely zero rate at the end of the interval. The required net torque and necessary conditions are thus

$$T_{\mathrm{net}} = \frac{-J\,\dot{\theta}}{t_i} \qquad (5)$$

for $|T_{ap}| < F_s$ and $0 < t_{oc}/t_i < 1$.

In Condition (4) above, the applied torque is sufficient to overcome the static friction level and reverse the rate within the integration interval. The actual friction torque in the physical system will thus reverse coincidental with the rate reversal. The effective friction torque is obtained by averaging the instantaneous friction torque over the integration interval. Thus

$$T_f = -F_c \cdot \mathrm{sign}(\dot{\theta}) \left(\frac{[t_{oc} - (t_i - t_{oc})]}{t_i}\right)$$

$$T_f = F_c \cdot \mathrm{sign}(\dot{\theta}) \left[1 - 2\left(\frac{t_{oc}}{t_i}\right)\right] \qquad (6)$$

for $|T_{ap}| > F_s$ and $0 < t_{oc}/t_i < 1$.

If the initial rate is zero and Condition (1) above is not satisfied, the friction equals the applied torque and the net torque becomes zero. Further, since Conditions (1) through (4) encompass all possible torque and rate conditions of interest, Eqs. (1) through (6) together with their conditions of applicability form the basis for defining effective friction torque and the net torque.

III. Application to Practice

A function block incorporating the logic and equalities of Eqs. (1) through (6) was developed for incorporation into a dynamic simulation model of the 70-m azimuth axis servo using MATRIXx, a copyrighted software program from Integrated Systems, Inc. for simulation of dynamic systems. The friction model utilizes four general equation building blocks and one standard function block from the MATRIXx utilities. Because the simulation program does not facilitate conditional branching in function blocks, it was necessary to structure the algorithm to employ eight logical variables whose one/zero values define Conditions (1) through (4) discussed in relation to Eqs. (1) through (6). The logical variables are then used in one equation for the net torque.

The algorithm inputs are $U1$, applied torque, $U2$, output rate from an adjoint integration process, and $U3$, a unit variable with the algebraic sign of $U2$. The output is $Y22$, the net torque to the integrator. Parameters are the static and Coulomb friction levels, F_s and F_c, the inertia moment, J, and the integration interval t_i.

The computations are grouped in function blocks as shown in Fig. 2 to avoid intermixing relational and arithmetic operators. The logical assignments use the convention where the lefthand variable is true (one) if the righthand condition is satisified, and false (zero) otherwise. A listing of variables and equations is provided below.

Friction logical variables

$$YY1 = U1 > F_c$$

$$YY2 = U1 < -F_c$$

$$YY3 = U2 > 0$$

$$YY4 = U2 < 0$$

$$YY5 = U1 > F_s \text{ and not } (YY3 \text{ or } YY4)$$

$$YY6 = U1 < -F_s \text{ and not } (YY3 \text{ or } YY4)$$

$$YY7 = YY1 \text{ or } YY2$$

Numeric variables, $YN1$ = critical torque, $YN2 = t_{oc}/t_i$

$$YN1 = U2 \frac{J}{t_i}$$

$$YN2 = \frac{YN1}{U3F_c - U1}$$

Logical variable

$$YY10 = YN2 > 0 \text{ and } YN2 < 1$$

Algebraic friction equation

$$Y21 = (2 \cdot YN2 - 1) F_c$$

$$\begin{aligned} Y22 = & [YY3 (U1 - Y21) + YY4 (U1 + Y21)] \; YY7 \cdot YY10 \\ & + [YY3 (U1 - F_c) + YY4 (U1 + F_c)] \; (1 - YY10) \\ & + YY5 (U1 - F_s) + YY6 (U1 + F_s) \\ & - (1 - YY7) YY10 \cdot YN1 \end{aligned}$$

IV. Simulation Test Results

Performance of the conventional friction model of Fig. 1 and the new model of Fig. 2 was compared in a dynamic simulation of the 70-m azimuth axis position servo. To simplify the simulation results, the flexible dynamics of the antenna structure were replaced with equivalent rigid-body parameters, thereby reducing the dynamic system to eighth order. The simulations are otherwise representative of actual system performance. The system excitation was a small (1.0 millidegree) position step transient. The control torque, measured in units of psi of hydraulic differential pressure, and axis rate in millidegrees/sec were recorded for comparison. The simulation was run for a total time of 5.0 sec with a 10 msec integration step size for both friction models. Results for the conventional model are shown in Fig. 3 and for the new model in Fig. 4.

The position loop dynamics simulated are such that the control torque changes slowly in response to the small transient applied here. The oscillatory behavior of the conventional friction model is evident during the intervals when the applied torque is less than the friction. The nonzero mean rate during these intervals is erroneous as the rate should be zero until the applied torque exceeds the 400 psi friction threshold. The irregularities on the rising and descending portions of the torque graph appear to be the spurious result of the oscillations coupling back through the rate loop.

The new friction model produces smooth torque transitions and zero rate in the intervals between the static friction levels (425 psi) in conformance with expectations based on the physical laws of friction. The ripple in the rate result is most likely the 7.0-Hz mode of the gear actuator stiffness included in the model.

V. Summary and Conclusions

An improved method for modeling dynamic motion in the presence of friction has been described. Simulation test results demonstrated that the anomalies of more conventional methods are corrected without increasing computer processing time. While the new algorithm is based on an external Euler integration, it should be capable of extension to incorporate a trapezoid- or possibly a polynomial-based integration method. The increased complexity of the predictive calculation with a polynomial integration may, however, negate any advantage to be gained with the more efficient integration methods.

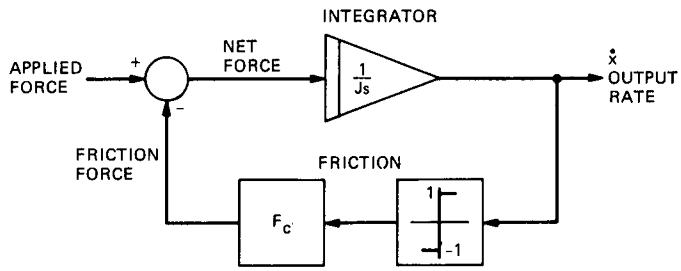

Fig. 1. Conventional friction model for single coordinate motion.

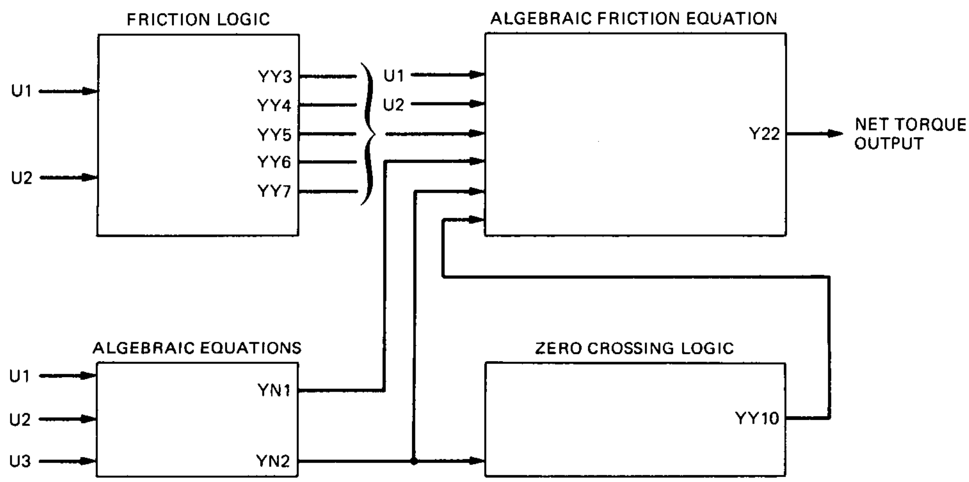

Fig. 2. Simulation function block implementation of the new algorithm.

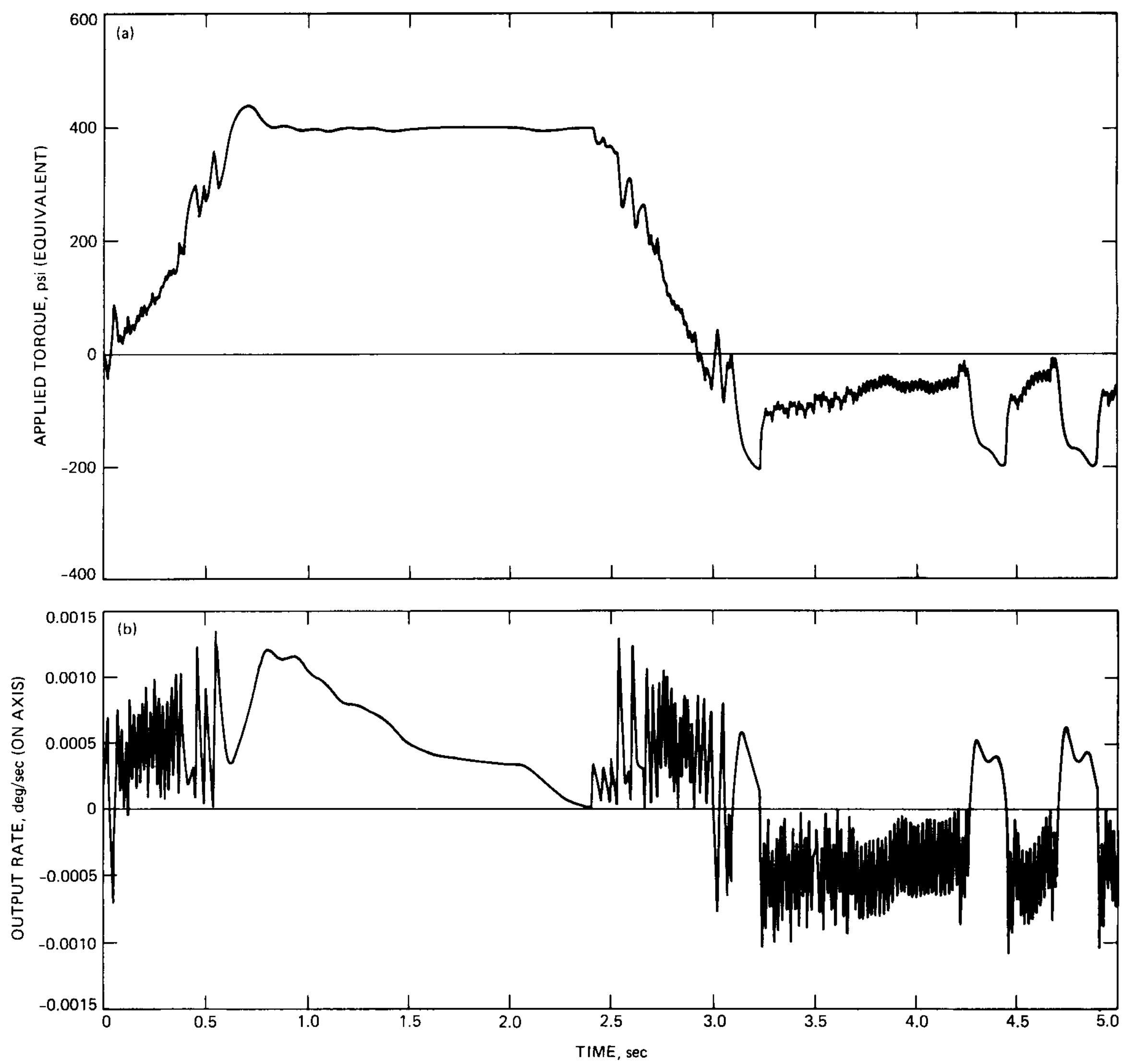

Fig. 3. Conventional friction simulation algorithm: (a) applied torque and (b) output rate.

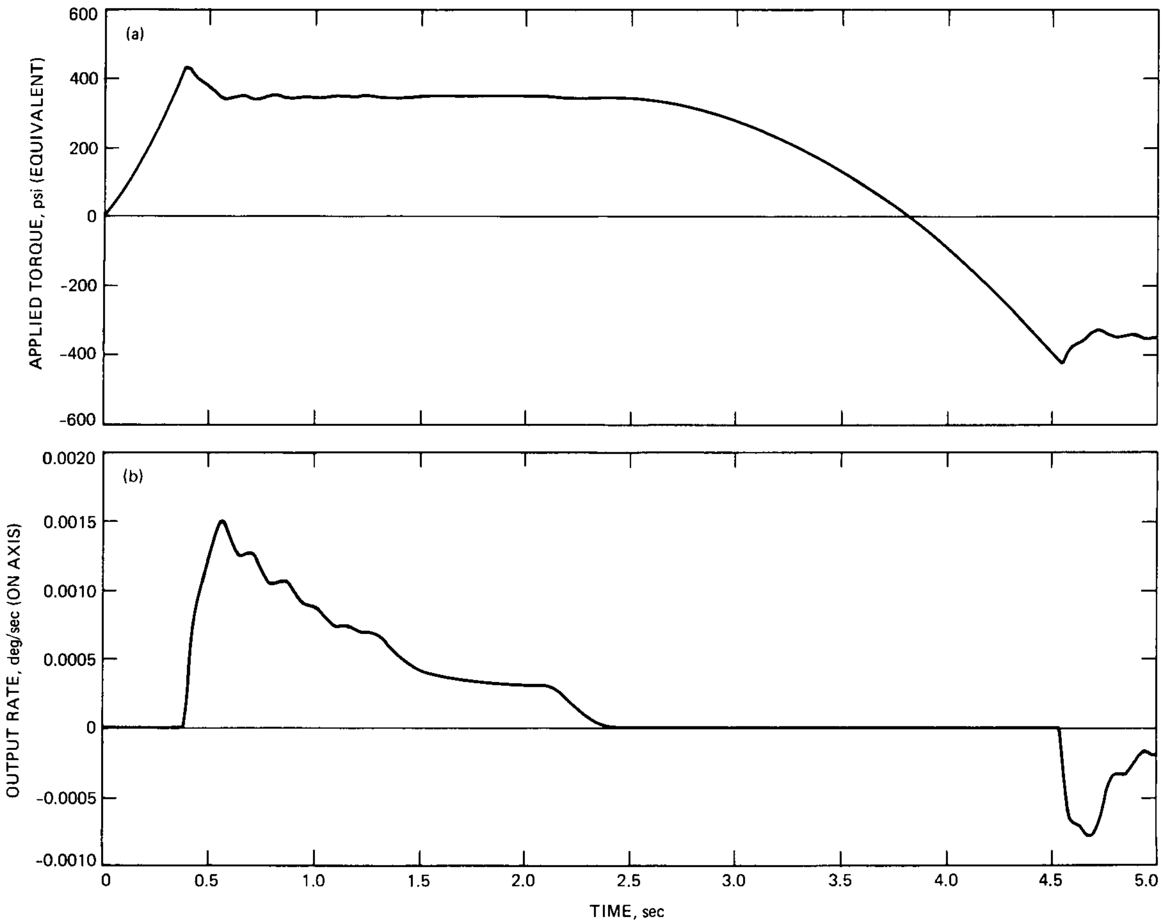

Fig. 4. New friction simulation algorithm: (a) applied torque and (b) output rate.