Coupled Rotor/Fuselage Dynamic Analysis of the AH-1G Helicopter and Correlation with Flight Vibrations Data

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Bell Helicopter Textron Inc. (BHTI) has been conducting a study of finite element modeling of helicopter airframes to predict vibration. This work is being performed under U.S. Government Contract NAS1-17496. The contract is monitored by the NASA Langley Research Center, Structures Directorate.

This report summarizes the procedure used at BHTI for predicting coupled rotor/fuselage vibrations with an application to the AH-1G two-bladed rotorcraft including comparisons with flight test vibrations. Key NASA and BHTI personnel are listed below:

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1. INTRODUCTION
INTRODUCTION

The NASA Langley Research Center is sponsoring a rotorcraft structural dynamics program with the overall objective to establish in the United States a superior capability to utilize finite element analysis models for calculations to support industrial design of helicopter airframe structures. Viewed as a whole, the program is planned to include efforts by NASA, universities, and the U.S. helicopter industry. In the initial phase of the program, teams from the major U.S. manufacturers of helicopter airframes will apply extant finite element analysis methods to calculate static internal loads and vibrations of helicopter airframes of both metal and composite construction, conduct laboratory measurements of the structural behavior of these airframes, and perform correlations between analysis and measurements to build up a basis upon which to evaluate the results of the applications. To maintain the necessary scientific observation and control, emphasis throughout these activities will be on advance planning, documentation of methods and procedures, and thorough discussion of results and experiences, all with industry-wide critique to allow maximum technology transfer between companies. The finite element models formed in this phase will then serve as the basis for the development, application, and evaluation of both improved modeling techniques and advanced analytical and computational techniques, all aimed at strengthening and enhancing the technology base which supports industrial design of helicopter airframe structures. Here again, procedures for mutual critique have been established, and these procedures call for a thorough discussion among the program participants of each method prior to the applications and of the results and experiences after the applications. The aforementioned rotorcraft structural dynamics program has been given the acronym DAMVIBS (Design Analysis Methods for VIBrationS).

Under the DAMVIBS program, the four industry participants (BHTI, Boeing Helicopters, McDonnell-Douglas Helicopter, and Sikorsky Aircraft) are to apply existing company methods for coupled rotor-fuselage analysis to calculate vibrations of the AH-1G helicopter and to correlate with data available from an Operational Load Survey (OLS) flight test program (References 1 and 2). In support of this common activity, BHTI, the manufacturer of the subject aircraft, was tasked to prepare and provide to the other participants the data needed to independently make these analyses and correlations. Specifically, BHTI was tasked to:

1. Present a detailed description of the modeling rationale and techniques used to develop the AH-1G NASTRAN fuselage finite element vibration model under a previous contract (Reference 3). A NASTRAN data deck of this model was provided to the other participating manufacturers.

2. Present a detailed description of all previous correlation work used to verify the finite element model (two versions - stick and built-up tailboom), including the following:
a. Ground vibration tests (GVT), static deflection tests and in-flight excitation simulation (References 4 and 5).

b. Application of the built-up tailboom model predictions to the static and ground vibration tests of Reference 4.

c. Correlation of both models with other prior AH-1G results contained in References 6 and 7.

3. Describe the OLS flight test program on the AH-1G and assemble the vibration data to be used in the correlations.

4. Present the AH-1G rotor system mechanical and aerodynamic coefficient data to all participants.

References 1-7 were used to develop the necessary background on the FEM and flight loads data for the current rotor/fuselage coupling analysis task as summarized in References 8 and 9.

This report describes work conducted by BHTI to evaluate the adequacy of current theoretical methods for predicting coupled rotor/fuselage vibration. The analysis methods described herein represent BHTI's advanced analysis rotorcraft flight simulation computer program C81 (Reference 10) and the industry's primary structural analysis computer program NASRAN. These analytical methods form the basis of an approach to helicopter dynamic analysis that has been used successfully at BHTI for many years. This report describes the analytical formulation of rotor dynamic equations, fuselage dynamic equations, coupling between the rotor and fuselage, and solutions to the total system. The coupled analysis is applied to an AH-1G two-bladed rotor system and results compared with measured OLS flight test vibrations.
2. DESCRIPTION OF ROTOR/FUSELAGE COUPLING METHOD
GENERAL PROCEDURE

The general procedure used in this study for calculation of AH-1G vibration characteristics is depicted on the following page. A Myklestad-type of analysis is used to calculate rotating elastic blade modal properties. A NASTRAN finite element method is used to form isolated fuselage modal characteristics. C81 is then used to couple the rotor and airframe math models, and then calculate rotor hub loads. Finally, the calculated hub loads are used as a forcing function on the NASTRAN finite element model to calculate the vibration responses.

The Myklestad family of programs has been used at BHTI to calculate helicopter rotor blade natural frequencies and mode shapes for many years. The capabilities of the program have been modified to include a complete elastic and inertial representation of the blade, pitch control systems, and pylon impedances. Recent modifications have been made to enhance user convenience, to give more accurate results, and to model more advanced rotor configurations.

C81 is a comprehensive rotorcraft flight simulation program used to calculate the aeroelastic response of the coupled rotor/fuselage system. The structural analysis is based on a modal technique while rotor aerodynamics are modeled using strip theory and bivariant tables to represent stall and compressibility effects over the rotor disk.
GENERAL PROCEDURE

MYKLESTAD
ISOLATED ROTOR BLADE MODAL ANALYSIS

NASTRAN
ISOLATED FUSELAGE MODAL ANALYSIS

C81
COUPLE ROTOR AND FUSELAGE, COMPUTE HUB LOADS

NASTRAN
COMPUTE FUSELAGE VIBRATION USING C81 HUB LOADS AS FORCING FUNCTION
C81 HISTORICAL DEVELOPMENT

Initially, C81 rotor performance and handling qualities were based on an actuator disk representation with a simple 6 degree-of-freedom rigid fuselage. In 1968, rotor aerodynamics graduated to strip theory representations with analytic functions dependent on blade radius, location, and time representing air loads. In addition, the fuselage and rotor were dynamically coupled. In 1971, \( C_L \) and \( C_D \) coefficients were added to the strip theory as well as an aeroelastic rotor model and rigid pylon representation. 1973 saw the first acceptance of a C81 deck in an analytic competition for rotorcraft design. In 1977, Floquet theory was added to address the stability of systems whose motion was governed by differential equations with periodic coefficients (e.g. ground resonance effects). Optimization capabilities were added in 1983 to improve the usefulness of C81 in design and in 1985 the capability to handle 10 elastic fuselage modes (in addition to six rigid modes) was included in the coupled rotor-fuselage program.
C81 HISTORICAL DEVELOPMENT

1965  ROTOR AERODYNAMIC MODEL
1968  COUPLED ROTOR / FUSELAGE (RIGID)
1971  AEROELASTIC ROTOR
1973  REQUIRED IN AAH / UTTAS COMPETITION
1976  DISSIMILAR BLADES
1977  FLOQUET THEORY ANALYSIS
1983  OPTIMIZATION
1985  ELASTIC FUSELAGE EFFECTS
C81 ANALYSIS CAPABILITY

C81 has the capability to model a wide variety of aircraft and hub configurations as shown on the following page. BHTI's comprehensive rotorcraft dynamics analysis code C81 is capable of modeling the following components of a rotorcraft: A fuselage; two rotors, each with a modal pylon, aeroelastic blades, and a nacelle; a wing; four stabilizing surfaces, none of which must be purely vertical or horizontal; four external stores or aerodynamic brakes; a nonlinear, coupled control system including a collective bobweight, stability and control augmentation system, and maneuver autopilot simulator; two jets; and a weapon.
C81 ANALYSIS CAPABILITY

AIRCRAFT CONFIGURATIONS

- CONVENTIONAL MR/TR
- TANDEM
- SIDE BY SIDE
- AIRPLANE
- WINDMILL
- TILT ROTOR
- COAXIAL
- COMPOUND
- DIRIGIBLE
- WIND TUNNEL

HUB CONFIGURATIONS

- GIMBALLED
- TEETERING
- ARTICULATED
- HINGELESS
A C81 analysis employs five coordinate systems that describe the behavior of various rotorcraft components. Each coordinate system is denoted by the following subscript notation.

**Subscript**

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>1. <strong>Ground Reference</strong> - A non-rotating coordinate system taken to be the inertial reference system.</td>
</tr>
<tr>
<td>f</td>
<td>2. <strong>Fuselage Reference</strong> - A non-rotating coordinate system centered at the fuselage center of gravity.</td>
</tr>
<tr>
<td>m</td>
<td>3. <strong>Mast Reference</strong> - A non-rotating coordinate system centered at the top of the mast.</td>
</tr>
<tr>
<td>h</td>
<td>4. <strong>Hub Reference</strong> - A rotating coordinate system that shares the same origin as the mast reference system.</td>
</tr>
<tr>
<td>b</td>
<td>5. <strong>Blade Reference</strong> - The origin of the blade rotating coordinate system is at the inboard end of the feathering bearings.</td>
</tr>
</tbody>
</table>
C81 COORDINATE SYSTEMS

NONROTATING

ROTATING
Elastic rotor blade dynamic characteristics are calculated for use in C81 by a Myklestad-based analysis implemented in a computer code called DNAM06. Blade modal natural frequencies, generalized inertias, and mode shapes are calculated by DNAM06 in rotating coordinates. All linear mass and spring terms are included in the DNAM06 analysis. Terms that were deleted from the DNAM06 analysis in order to simplify the Myklestad analysis, such as the Coriolis acceleration terms, are included in the forcing function of the C81 rotor analysis. Nonlinear terms such as flapping springs and flap-lag-torsional moments are also handled by the C81 analysis, as are the coupling effects of the rotor hub's motion.

The following figure lists the assumptions which underlie the linear blade equations of motion in the Myklestad analysis.
BHTI ELASTIC ROTOR BLADE DYNAMIC ANALYSIS

ASSUMPTIONS:

- BLADE CROSS SECTIONS ARE NOT SYMMETRIC.
- ELASTIC AXIS IS A PIECEWISE PARALLEL STRAIGHT LINE.
- STRUCTURAL PRINCIPAL AXES AND MASS PRINCIPAL AXES ARE PARALLEL, BUT NOT COINCIDENT.
- EFFECTS OF LINEAR AND ANGULAR DISPLACEMENTS AND ACCELERATIONS OF THE ROTOR HUB ARE INCLUDED.
- EFFECTS OF STEADY BLADE FEATHERING MOTION ARE INCLUDED.
- SHEAR DEFORMATION IS NEGLECTED.
- PRECONE AND UNDERSLING ARE NOT INCLUDED.
The DNAM06 analysis uses a finite element transfer matrix approach to represent blade properties. The rotor blade is represented by a series of lumped, rigid 3-D dumbell inertias connected by untwisted, massless, elastic beams. Built-in twist of the blade is introduced incrementally at the blade stations where the inertias are located. Rotating fully-coupled inplane, out-of-plane, and torsional deflections of the blade are also considered. Radial extension of the blade is neglected, however.

A representative blade element is shown in the following figure.
BHTI ELASTIC ROTOR BLADE REPRESENTATION

3-D INERTIA DUMBBELL

ELASTIC AXIS

C.G.

STA. \( i + 1 \)

STA. \( i \)

PITCH CHANGE AXIS (REF AXIS SYSTEM)

\( L_i \)

\( u_dh \)
DNAM06 uses the state vector and transfer matrix type of formulation developed by Myklestad. The state vectors consist of two linear displacements, three angular rotations, and the shears and moments corresponding to these displacements and rotations.

The state vector contains the following quantities:

\[ u_i \] - inplane displacement
\[ w_i \] - out-of-plane displacement
\[ u'_i \] - in-plane slope
\[ w'_i \] - out-of-plane slope
\[ V_{xi} \] - in-plane shear
\[ V_{zi} \] - out-of-plane shear
\[ M_{zi} \] - out-of-plane moment
\[ M_{xi} \] - in-plane moment
\[ \phi_i \] - torsional deflection about y axis
\[ M_{yi} \] - torsional moment about y axis

The general form of the transfer matrix equation is given on the following page.
MYKLESTAD STATE VECTOR SOLUTION

\{S_i\} = [M_i][f_i]\{S_{i+1}\}

WHERE:

\{S_i\} - STATE VECTOR AT STATION I

\{M_i\} - TRANSFER MATRIX ACROSS INERTIA DUMBELL

\[f_i\] - TRANSFER MATRIX ACROSS ELASTIC ELEMENT OF LENGTH \(L_i\)
The time variant aeroelastic rotor representation in C81 is based on a modal analysis approach. Some of the assumptions contained in this analysis are presented on the following page.
ASSUMPTIONS:

- The rotor blade is divided into the same radial segments for both aerodynamic and dynamic calculations.

- Each segment face has three degrees of freedom that are used in the generalized force calculation. (w - out of plane, u - in plane, Φ - twist about y axis).

- DNAM06 (elastic blade analysis) supplies the normal modes that describe u, w, and Φ displacements for each segment face of every blade mode.

- Linear interpolation is used to define mode shapes between two adjacent faces.

- Effects of steady blade feathering motion are included.

- 20 segments per blade, maximum.

- 11 input blade modes (from DNAM06) per rotor, maximum.

- 2 rotors, maximum.

- 7 blades per rotor, maximum.
The C81 rotor analysis is designed to handle the fully-coupled blade mode shapes calculated by DNAM06. These mode shapes have several attributes that are important in the modal technique. Each mode shape is a solution to the coupled differential equations for the free vibration of the total rotor system. The solution, or mode shape, is obtained by deletion of all velocity dependent and nonlinear terms. The collection of mode shapes forms an orthogonal set. Each mode shape has a natural frequency, and it satisfies the appropriate set of boundary conditions imposed on the blade.
ATTRIBUTES OF BLADE MODE SHAPES:

• EACH IS A SOLUTION TO THE LINEAR PORTION OF THE COUPLED DIFFERENTIAL EQUATIONS OF FREE VIBRATION

• EACH HAS A NATURAL FREQUENCY

• THE COLLECTION FORMS AN ORTHOGONAL SET

• EACH SATISFIES IMPOSED BOUNDARY CONDITIONS
ROTOR BLADE ROOT BOUNDARY CONDITIONS

The behavior of the rotor hub can be related to the blade mode shapes that are used to describe the entire rotor system by the boundary conditions presented on the next page. A hub impedance model is used to include the effects of an isotropic support system in the blade modes. For collective modes, out-of-plane motion is restrained by one input spring constant and one input mass. In-plane slope changes are opposed by a torsional spring. For cyclic modes, the in-plane motion is restrained by one input spring and one input mass.
## Rotor Blade Root Boundary Conditions

<table>
<thead>
<tr>
<th>Mode Type</th>
<th>Motion</th>
<th>Two Bladed Rotor Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collective</td>
<td>Out-of-Plane</td>
<td>Cantilever</td>
</tr>
<tr>
<td></td>
<td>In Plane</td>
<td>Pinned</td>
</tr>
<tr>
<td></td>
<td>Torsion</td>
<td>Cantilever</td>
</tr>
<tr>
<td>Cyclic</td>
<td>Pinned</td>
<td>Cantilever</td>
</tr>
<tr>
<td></td>
<td>Cantilever</td>
<td>Cantilever</td>
</tr>
</tbody>
</table>
A separation of variables approach is employed in the solution of the coupled equations as depicted in the figure. The independent variables are blade radial location $y$, and time $t$. The deflections $u_n$, $w_n$, and $\phi_n$ of the $n^{th}$ elastic blade mode shape are only a function of blade radial position $y$. On the left hand side of the equation, $u$, $w$, and $\phi$ are the total elastic deformation of the blade which are dependent upon both blade radial position and time. The modal participation factor, $\xi_n$, for the $n^{th}$ mode is only a function of time.
SOLUTION FORM OF ROTOR DYNAMIC SYSTEM

SEPARATION OF VARIABLES

\[
\begin{bmatrix}
    u(y, t) \\
    w(y, t) \\
    \phi(y, t)
\end{bmatrix}
= \sum_{n=1}^{NM} \begin{bmatrix}
    u_n(y) \\
    w_n(y) \\
    \phi_n(y)
\end{bmatrix}\delta_n(t)
\]

\(u_n, w_n, \phi_n\) – COMPONENTS OF NTH BLADE MODE SHAPE.

\(\delta_n\) – PARTICIPATION FACTOR FOR NTH BLADE MODE.

\(u, w, \phi\) – TOTAL BLADE ELASTIC DEFORMATION.

NM – NUMBER OF BLADE MODES.
A brief summary of C81 rotor aerodynamic capabilities is listed below:

1. Maximum of 20 radial aerodynamic segments.

2. The airfoil sectional $C_l$, $C_d$, and $C_m$ are tabular functions of Mach number and angle of attack.

3. Momentum theory induced velocity is calculated based either on equations internal to the program or user input tables.

4. Unsteady aerodynamic options include Theodorsen and Carta theories.

5. The effects of blade elasticity are included.

The rotor blade aerodynamic reference system is shown below.
ROTOR BLADE AERODYNAMIC REPRESENTATION
MODAL FORM OF ROTOR AEROELASTIC EQUATIONS

Application of the separation of variables technique to the solution of the rotor blade equations of motion results in the modal blade equations presented in the figure. Here, \( F_u \) is the applied inplane force, \( F_w \) is the applied out-of-plane force, \( M_\phi \) is the applied twisting moment and \( I_n \) is the generalized inertia of the \( n^{th} \) blade mode. \( F_n \) is the forcing function due to the aerodynamic forces and inertial forces, including those forces which were deleted from the Myklestad analysis. It should be noted that the left hand side of the rotor equations is uncoupled due to the orthogonality of the mode shapes obtained from DNAM06.
 inertial loads

\[ \phi_I, \phi_I^n \]

aerodynamic loads

\[ \phi_A, \phi_A^n, \phi_A^n \]

where:

\[ \phi_I + \phi_A = \phi_W \]

\[ \phi_I^n + \phi_A^n = \phi_A^n \]

\[ \phi_I + \phi_A^n = \phi_A^n \]

\[ \frac{u_I}{u_D} = \frac{u_I}{\Delta p(\mu \phi_W + u_m \phi_A^n + u_n \phi_A^n)} = u_0 \frac{u_0}{\phi_W + u_0} \]

modal form of rotor aerelastic equations
C81 ROTOR AERODYNAMIC MODULE

The forcing function terms $A_u$, $A_w$, $A_\psi$ on the right hand side of the rotor blade equations of motion are calculated in the C81 rotor aerodynamic module. Representative inputs and outputs of this module are shown in the diagram below.
C81 ROTOR AERODYNAMIC MODULE

BLADE DISPLACEMENTS
BLADE VELOCITIES
FLIGHT CONDITION
RPM

COLLECTIVE
CYCLIC
FUSELAGE ATTITUDE

TWIST
CHORD
AIRFOIL DATA

ROTOR
AERODYNAMIC
MODULE

THRUSt
H-FORCe
Y-FORCe

POWER REQUIRED
F/A FLAPPING
LATERAL FLAPPING
INDUCED VELOCITY

AERODYNAMIC TERMS
$a_u, a_w, a_\phi$

RHS OF ROTOR EQUATION
The set of previously presented modal equations is sufficient to describe the time variant aeroelastic response of the rotor blade because of two reasons:

1. The linear effects of blade mass, blade elasticity, and blade geometry are inherently included in the calculated blade mode shapes and natural frequencies.

2. Aerodynamic and aeroelastic effects are included in the forcing terms on the right hand side of the blade modal equation. Dynamic terms which were deleted from the blade natural frequency and mode shape calculation are also included on the right hand side of this equation. These dynamic terms arise from angular velocities and accelerations of the rotating coordinate system, from linear rotor hub accelerations, and from inertial terms such as Coriolis accelerations.

The following figure presents the general form of the blade equation of motion along with an indication of the analysis in which each of the terms is calculated.
GENERAL FORM OF BLADE EQUATIONS

\[ m \frac{\partial^2 \mathbf{r}}{\partial t^2} + m \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}) + \mathbf{F}_{\text{spring}} = \mathbf{F}_{\text{AERO}} - m \frac{\partial^2 \mathbf{R}}{\partial t^2} - 2m \mathbf{\Omega} \times \frac{\partial \mathbf{r}}{\partial t} - m \left( \frac{\partial \mathbf{\Omega}}{\partial t} \right) \times \mathbf{r} \]

CALCULATED IN MYKLESTAD BLADE MODE ANALYSIS

CALCULATED IN C81

NON-ROTATING SYSTEM

ROTATING SYSTEM
The aeroelastic behavior of a rotor is highly dependent upon the dynamic characteristics of the fuselage to which it is attached. The fuselage (or nonrotating components) is represented in C81 by the previously discussed modal technique. This modal form of the fuselage equations was selected in order to provide general motions at the rotor hub with a minimum number of additional equations (thus reducing computational requirements). This technique allows the use of fuselage modes based on extremely complex and detailed fuselage/support system models which can be obtained from structural analysis finite element codes such as NASTRAN. Each fuselage mode shape can have three linear displacements and three angular rotations at the rotor hub as part of the modal information. The modal information also contains fuselage natural frequencies, generalized inertias, and damping where applicable. The C81 analysis is currently designed to handle up to ten elastic fuselage modal equations. For convenience, C81 has an option to input fuselage modes that were calculated with or without full rotor mass (included as a point mass), for use in coupled rotor/fuselage dynamic analysis. A simplifying assumption used by the C81 fuselage dynamic analysis is that the forcing function comes from the rotor, and that there are no other aerodynamic or inertial loads applied to the fuselage model.
FUSELAGE DYNAMIC ANALYSIS

- NO OSCILLATORY AERODYNAMIC FORCES ARE APPLIED TO AERODYNAMIC SURFACES

- HUB DEGREES OF FREEDOM AND HUB LOADS (FROM THE ROTOR) ARE USED IN THE GENERALIZED FORCE CALCULATIONS.

  (THREE LINEAR DISPLACEMENTS: \( X_j, \ Y_j, \ Z_j \) AND THREE ANGULAR ROTATIONS: \( \theta_{xj}, \ \theta_{yj}, \ \theta_{zj} \) OF THE HUB)

- NASTRAN (FINITE ELEMENT ELASTIC FUSELAGE ANALYSIS) SUPPLIES THE NORMAL MODE DATA

- 10 INPUT ELASTIC FUSELAGE MODES (FROM NASTRAN), MAXIMUM
The modal form of the fuselage equations of motion is presented on the following page, where $p_j$ is the fuselage modal participation factor, $\omega_j$ is the natural frequency of the $j$th mode, $\zeta_j$ is the damping ratio specified for the $j$th mode, $G_{ij}$ is the generalized inertia of the $j$th mode, and $F_j$ is the $j$th modal forcing function.
FUSELAGE MODAL EQUATIONS. OF MOTION

\[ \ddot{p}_j + 2 \zeta_j \bar{\omega}_j \dot{p}_j + \bar{\omega}_j^2 p_j = \frac{F_j}{G I_j} \]
The modal forcing function $F_j$ on the right hand side of the fuselage modal equation of motion is given below. The quantities $V_{mx}$, $V_{my}$, $V_{mz}$, $M_{mx}$, $M_{my}$, and $M_{mz}$ are the shear and moment components at the top of the rotor mast. These shears and moments are calculated by a modal displacement technique, which combines the hub shear and moment coefficients obtained from the Myklestad analysis and the solution of the rotor's modal equations of motion. $F_{mx}$, $F_{my}$, and $F_{mz}$ are force components due to any translation of the rotor mass not included in either the rotor or fuselage analysis. Since there is no radial degree of freedom in the rotor dynamics model, $F_{mx}$ and $F_{my}$ also include the corrections needed to account for the inertial forces associated with radial foreshortening.
FUSELAGE MODAL FORCING FUNCTION

\[ F_j = X_{mj} V_{mx} + Y_{mj} V_{my} + Z_{mj} V_{mz} \]
\[ + \theta_{mxj} M_{mx} + \theta_{myj} M_{my} + \theta_{mzj} M_{mz} \]
\[ + X_{mj} F_{mx} + Y_{mj} F_{my} + Z_{mj} F_{mz} \]

OR,

\[ F_j = F_{oj} + X_{mj} F_{mx} + Y_{mj} F_{my} + Z_{mj} F_{mz} \]
TRANSFORMATION OF FUSELAGE COORDINATES

The $j^{th}$ fuselage mode shape is defined in terms of the following displacements at the top of the mast expressed with respect to the fuselage reference system:

- $x_j$ - in $x_f$ direction
- $y_j$ - in $y_f$ direction
- $z_j$ - in $z_f$ direction
- $\theta_{xj}$ - about $x_f$ axis
- $\theta_{yj}$ - about $y_f$ axis
- $\theta_{zj}$ - about $z_f$ axis

In order to perform the coupled rotor/fuselage analysis, the fuselage mode shapes must be transformed into the mast reference system. The transformation equation is presented below, where the subscript "m" refers to the mast reference system and no subscript refers to the fuselage reference system. This transformation references the fuselage equations to the same coordinate system as the rotor equations.
TRANSFORMATION OF FUSELAGE COORDINATES

\[(X_{mj}, Y_{mj}, Z_{mj}) = [T_{mf}] (X_{i}, Y_{i}, Z_{i})\]

\[(\theta_{xmj}, \theta_{ymj}, \theta_{zmj}) = [T_{mf}] (\theta_{xij}, \theta_{yij}, \theta_{zij})\]

WHERE: \([T_{mf}] = FUSELAGE-TO-MAST COORDINATE TRANSFORMATION MATRIX.\]
The total linear acceleration at the rotor hub is the sum of fuselage elastic contributions and fuselage rigid body contributions. The terms $X_m$, $Y_m$, and $Z_m$ are the rigid body fuselage acceleration components written with respect to the mast reference axis system. The terms $a_x$, $a_y$, and $a_z$ represent the hub accelerations which come from the term $\frac{d^2 R}{dt^2}$ on pg 35. NP is the number of fuselage elastic modes used in the analysis.
ROTOR HUB ACCELERATION COMPONENTS

\[(a_{xm}, a_{ym}, a_{zm}) = \sum_{j=1}^{NP} \ddot{p}_j (x_{mj}, y_{mj}, z_{mj}) + (\ddot{x}_m, \ddot{y}_m, \ddot{z}_m)\]

- NP = NUMBER OF FUSELAGE ELASTIC MODES USED IN ANALYSIS
- MAST REFERENCED (NON-ROTATING) COORDINATES
- \(\ddot{x}_m, \ddot{y}_m, \ddot{z}_m\) - RIGID FUSELAGE ACCELERATION COMPONENTS
IN-PLANE INERTIAL FORCES DUE TO ROTOR HUB MOTION

The inplane rotor hub accelerations, $a_{xm}$ and $a_{ym}$, produce in-plane forces, $F_I$ and $F_R$ perpendicular and parallel to the blade, respectively. The inertial loading is separated in this fashion because for an elastic rotor the $F_I$ force is absorbed into the rotor modal equations as part of the forcing function for the rotor inplane degree of freedom, while the $F_R$ term has to be handled separately since there is no radial degree of freedom in the blade modes.
IN-PLANE INERTIAL FORCES DUE TO ROTOR HUB MOTION

\[ dF_i = m (a_{xm} \sin \psi_i + a_{ym} \cos \psi_i) \]

\[ dF_R = m (a_{xm} \cos \psi_i - a_{ym} \sin \psi_i) \]
After integrating over each blade and summing the contribution of all blades, the inplane rotor hub forces $F_{xm}$ and $F_{ym}$ are given by the following equations. For an elastic rotor analysis the $Fi$ terms in the equation must be omitted because their effect will be already accounted for in the modal analysis.
INERTIAL HUB FORCES DUE TO ROTOR HUB MOTION

\[ F_{xm} = \sum_{i=1}^{NB} \left( f \int_{0}^{R} \left( -dF_i \sin \Psi_i - dF_R \cos \Psi_i \right) dy \right) \]

\[ F_{ym} = \sum_{i=1}^{NB} \left( f \int_{0}^{R} \left( -dF_i \cos \Psi_i + dF_R \sin \Psi_i \right) dy \right) \]

\( F_1 \) omitted for elastic rotor analysis
IN-PLANE INERTIA FORCES FOR AN ELASTIC ROTOR

For an elastic rotor, the inplane force perpendicular to the rotor blade, \( F_I \), is already included in the forcing function that is applied to the elastic rotor blade modal equations, and therefore the hub loads due to these terms are computed by summation of the load over the number of blade modes. The inplane force in the radial direction, \( F_R \), has to be integrated separately and included as a hub force in the fuselage equations. \( F_R \) has to be handled separately because, as stated earlier, there is no radial degree of freedom in the elastic blade model in Myklestad. The hub shears due to \( F_R \) are presented on the following page.

It should be noticed that the out-of-plane inertial force included arising from \( a_m \) must be included. For an elastic rotor, a distributed inertia force is included as part of the rotor forcing function, and the vertical shear that is applied at the rotor hub is computed from the modal response of the rotor.

Also, the inertia forces on the rotor caused by angular velocities and angular accelerations of the fuselage are also included in the rotor forcing function. These effects are transmitted to the fuselage at the rotor hub by means of the net beamwise bending moments computed from the modal response of the rotor.
IN-PLANE INERTIA FORCES FOR AN ELASTIC ROTOR

\[ F_{xm} = -M_B \sum_{i=1}^{NB} \sum_{j=1}^{NP} \ddot{p}_j (x_{mj} \cos^2 \psi_i - y_{mj} \cos \psi_i \sin \psi_i) \]

\[ -M_B \sum_{i=1}^{NB} (x_m \cos^2 \psi_i - y_m \cos \psi_i \sin \psi_i) \]

\[ F_{ym} = M_B \sum_{i=1}^{NB} \sum_{j=1}^{NP} \ddot{p}_j (x_{mj} \cos \psi_i \sin \psi_i - y_{mj} \sin^2 \psi_i) \]

\[ +M_B \sum_{i=1}^{NB} (x_m \cos \psi_i \sin \psi_i - y_m \sin^2 \psi_i) \]

WHERE: \( M_B = \) MASS OF ONE BLADE.

\( NB = \) NUMBER OF BLADES
The fuselage modal equations of motion, which include rotor mass, are given on the following page for a two-bladed elastic rotor. The terms with a single underline are needed to account for the rotor mass that was included in the NASTRAN analysis. The double underlined terms are the result of the rotor's radial inertial terms which are not handled by the rotor modal analysis since there is no radial degree of freedom. The terms on the right hand side of the equation are due to the rigid body motions ($\ddot{x}_m, \ddot{y}_m, \ddot{z}_m$) of the airframe.
MODAL FORM OF FUSELAGE EQUATIONS OF MOTION

- TWO BLADED ROTOR
- ROTOR MASS INCLUDED

\[
\begin{align*}
\dot{\mathbf{p}}_j + \frac{2M_B}{Gl_j} \sum_{i=1}^{NP} \dot{\mathbf{p}}_i & \left[ x_{mi} x_{mj} (\cos^2 \Psi - 1) + y_{mi} y_{mj} (\sin^2 \Psi - 1) \right] \\
- \frac{z_{mi} z_{mj}}{\cos \Psi \sin \Psi} & \left( y_{mi} x_{mj} + x_{mi} y_{mj} \right) + 2 \zeta_j \omega_j \mathbf{p}_j \\
+ \ddot{\mathbf{p}}_j & = \frac{F_{ij}}{Gl_j} \left[ \dot{x}_{mi} x_{mj} (\cos^2 \Psi - 1) + \dot{y}_{mi} y_{mj} (\sin^2 \Psi - 1) \right] \\
- \frac{z_{mi} z_{mj}}{\cos \Psi \sin \Psi} & \left( \dot{y}_{mi} x_{mj} + \dot{x}_{mi} y_{mj} \right)
\end{align*}
\]

WHERE: \( M_B = \text{MASS OF ONE BLADE} \)
SOLUTION SCHEME FOR DYNAMICALLY COUPLED EQUATIONS OF MOTION

The following sequence of calculations is used to solve the coupled rotor/fuselage dynamic system. This sequence was developed to provide a consistent set of equations without the need to solve the full set of equations simultaneously.

1. Find aerodynamic loading for rotors, empennage, and fuselage. These forces depend only upon displacements and velocities.

2. Compute that portion of the rotor modal forcing functions not dependent on fuselage accelerations (such terms as nonlinear flapping springs and dampers and unsteady aerodynamic effects).

3. Solve for rigid body fuselage accelerations due to rotor excitation.

4. Solve for the accelerations of the fuselage generalized coordinates due to rotor excitation.

5. Add the inertia loads caused by fuselage motion to the rotor modal forcing function.

6. Solve for rotor accelerations.

Hammings Predictor-Corrector Method of numerical integration is used to integrate the equations of motion.
SOLUTION SCHEME FOR DYNAMICALLY COUPLED EQUATIONS OF MOTION

It = t + Δt

STOP

t = tf

YES

COMPUTE HUB SHEARS AND MOMENTS BY MODAL APPROACH

COMPUTE AIRFRAME AERODYNAMIC FORCES FOR STEADY TRIM ONLY

COMPUTE ROTOR AERODYNAMIC FORCES

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INTEGRATE ACCELERATIONS FOR HUB & BLADE MOTION
3. APPLICATION TO THE AH-1G OPERATIONAL LOAD SURVEY (OLS) FLIGHT TEST PROGRAM
Summary of Required C81 Inputs

The inputs to C81 that were necessary to calculate AH-1G OLS hub loads are presented in the figure on the opposite page. These inputs consist of rotor blade modal data, fuselage modal data, and basic C81 input data. The rotor blade modal data is calculated by the Myklestad computer program which used OLS inputs from Reference 2 to describe the physical rotor blade properties. Blade modal data calculated by Myklestad and used as inputs to C81 consists of natural frequencies, generalized inertias, and mode shapes. Isolated fuselage modal data is calculated by NASTRAN (based on OLS data) and these data (in the form of natural frequencies, generalized inertias, and mode shapes) are used in C81 to describe fuselage dynamic behavior. All other basic C81 inputs were taken from Reference 8 and these inputs, combined with the rotor and fuselage modal input data, comprise the total AH-1G OLS C81 input deck. Each of these three sets of data are discussed on the following pages.
SUMMARY OF REQUIRED C81 INPUTS

AH-1G OLS
ISOLATED ROTOR BLADE
MODAL DATA
MYKLESTAD (DNAM06)

AH-1G OLS
ISOLATED FUSELAGE
MODAL DATA
NASTRAN

AH-1G OLS
C81
BASIC DATA

AH-1G OLS
C81
INPUT DECK
The rotor blade modal data required by C81 is calculated by the BHTI Myklestad (DNAM06) computer program. OLS data is used as input to this program to describe the physical rotor blade properties. This program then calculates the blade natural frequencies, generalized inertias, and mode shapes for input to C81. For the present analysis, nine modes were chosen to represent the flexible blade characteristics. Five of these modes are of the cyclic variety which have pinned out-of-plane, cantilever inplane, and cantilever torsional blade boundary conditions. The other four modes are collective in nature and have cantilevered out-of-plane, pinned inplane, and cantilevered torsional blade boundary conditions.
**ROTOR BLADE C81 INPUT DATA**

- CALCULATED BY MYKLESTAD (DNAM06)

<table>
<thead>
<tr>
<th>MODE</th>
<th>FREQUENCY (p)</th>
<th>TYPE</th>
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<td>CYCLIC</td>
</tr>
<tr>
<td>1ST IN-PLANE BENDING</td>
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</tr>
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<td>1ST TORSION</td>
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<td>CYCLIC</td>
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<tr>
<td>2ND OUT-OF-PLANE BENDING</td>
<td>2.7489</td>
<td>CYCLIC</td>
</tr>
<tr>
<td>3RD OUT-OF-PLANE BENDING</td>
<td>4.5301</td>
<td>CYCLIC</td>
</tr>
<tr>
<td>1ST OUT-OF-PLANE BENDING</td>
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<td>COLLECTIVE</td>
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<tr>
<td>1ST TORSION</td>
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<td>COLLECTIVE</td>
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</tr>
<tr>
<td>3RD OUT-OF-PLANE BENDING</td>
<td>4.7384</td>
<td>COLLECTIVE</td>
</tr>
</tbody>
</table>

- PLUS RESPECTIVE GENERALIZED INERTIAS AND MODE SHAPE DATA
NASTRAN was used to calculate the fuselage modal data for use in C81. The full rotor weight, which includes the rotor, hub, and R-MUX (OLS rotating multiplex instrumentation box) box, was included as a point mass at the top of the rotor mast in this analysis. The calculated NASTRAN data is then expressed in a format consistent with C81 input requirements. The calculated mode shapes, for example, must be transformed into the C81 coordinate system. C81 is capable of handling ten elastic fuselage modes and thus, in addition to six rigid body modes of the fuselage calculated by NASTRAN, ten elastic modes were used to represent the fuselage in the C81 analysis. These modes are listed below and contain the modes that are most important to a rotor dynamics analysis, namely the pylon pitch and roll modes, and the fuselage first bending and torsion modes.
FUSELAGE C81 INPUT DATA

- CALCULATED BY NASTRAN

<table>
<thead>
<tr>
<th>MODE</th>
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<td>PYLON PITCH</td>
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<td>PYLON ROLL</td>
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<td>1ST FUSELAGE LATERAL BENDING</td>
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<tr>
<td>1ST FUSELAGE VERTICAL BENDING</td>
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<tr>
<td>1ST FUSELAGE TORSION</td>
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<tr>
<td>2ND FUSELAGE VERTICAL BENDING</td>
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<td>2ND FUSELAGE LATERAL BENDING</td>
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</tr>
<tr>
<td>FUSELAGE ROLL/ENGINE LATERAL</td>
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</tr>
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<td>MAIN ROTOR MAST LATERAL BENDING</td>
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</tr>
<tr>
<td>MAIN ROTOR MAST FORE-AFT BENDING</td>
<td>27.10</td>
</tr>
</tbody>
</table>

- PLUS RESPECTIVE GENERALIZED INERTIAS AND MODE SHAPES
Two helicopter C81 trim options were employed in this study. The first technique is called "trim to cyclic - fall through trim". This is basically a "wind tunnel" type of trim and therefore rigid body effects are not included. These rigid body dynamics are important, however, for accurate loads calculations. To account for rigid body effects, the helicopter is allowed to fall through trim for a length of time corresponding to ten rotor revolutions. Ten rotor revolutions were chosen so that no significant changes between measured and calculated flight conditions would develop. The key point here is that measured OLS control position data is used as input. The second trim procedure is called "full aircraft trim" and is representative of the type of analysis that is used during design phases of helicopter development. Here, only measured OLS flight conditions are used as inputs to C81 and C81 calculates all control positions required to trim the helicopter. The C81 input deck was then submitted for execution and calculation of rotor hub loads.
AH-1G OLS C81 TRIM OPTIONS

TRIM TO CYCLIC FALL THROUGH TRIM

FULL AIRCRAFT TRIM

CALCULATED ROTOR BLADE LOADS
ROTOR HUB LOADS
Basic C81 Input Data

The remaining portion of the basic AH-1G OLS C81 input deck contains mainly physically descriptive data for items such as the main rotor, fuselage, tail rotor, rotor aerodynamics, fuselage aerodynamics, wings, stabilizing surfaces, controls, and flight conditions obtained from Reference 2. Also included are groups that control the basic execution of the program, i.e., error limits, iteration limits, and trim options which are found in Reference 8.
BASIC C81 INPUT DATA

- DATA OBTAINED FROM REFERENCE 2 (OLS DATA) THAT IS USED FOR "TRIM TO CYCLIC-FALL THROUGH TRIM" OPTION

1) ROTOR  
2) FUSELAGE  
3) ROTOR AERODYNAMICS  
4) FLIGHT CONDITIONS

- ADDITIONAL DATA OBTAINED FROM REFERENCE 8 THAT IS USED FOR "FULL AIRCRAFT TRIM" OPTION

5) TAIL ROTOR  
6) FUSELAGE AERODYNAMICS  
7) WINGS  
8) STABILIZING SURFACES  
9) CONTROLS
Comparisons between the measured blade feathering and flapping angles and the C81 calculated feathering and flapping angles are presented in the following two figures, while a comparison of the measured fuselage pitch and the C81 computed pitch angles is given in the third figure. The test data are represented by the open symbols while the C81 results are represented by the solid and dashed lines. The solid line is the result of the "trim to cyclic-fall through trim" option. The dashed line is the result of using the "full aircraft trim" option.
MIR HUB FEATHERING ANGLE, DEG

AIRSPEED, KTS

MIR HUB FEATHERING ANGLE, DEG

AIRSPEED, KTS

FULL AIRCRAFT TRIM
TRIM TO CYCLIC FALL THROUGH TRIM
TEST DATA

AH-1G OLS TRIM CORRELATION
AH-1G OLS Trim Correlation (Concluded)
Harmonic Analysis of Calculated Time History Data

C81 computes all loads in time history format and thus requires a harmonic analysis to separate out the magnitudes of the various harmonic loads found in the system. This analysis is "self-contained" in C81 and represented by the equation on the next page. C81 determines the coefficients $a_0$, $a_k$, $b_k$ where $k$ in this case is 2, 4, or 6. The term $a_0$ represents the steady load component and the $a_k$ and $b_k$ are the oscillatory cosine and sine components of the time history data. Measured OLS data are harmonically analyzed in a similar manner.
HARMONIC ANALYSIS OF CALCULATED TIME HISTORY DATA

- CALCULATE 2, 4, 6 PER REV HUB LOADS FROM CALCULATED TIME HISTORY DATA.

\[ f(t) = a_0 + \sum_{k=1}^{N} \left[ a_k \cos(2\pi k \omega t) + b_k \sin(2\pi k \omega t) \right] \]

- COMPARE COMPONENTS TO MEASURED OLS TEST DATA
Comparisons between measured OLS test data and C81 calculated rotor blade loads are presented in the figures on the next three pages. Some measure of the capability of C81 to predict blade loads (and thus hub loads) can be gleaned from this comparison. It is these blade loads that sum together to form the overall hub loads that will be used in the fuselage vibration prediction. Results are presented for three airspeeds: 67, 114, and 142 KTAS. This covers the low, medium, and high speed flight regimes of the AH-1G OLS data. The rotor blade beam bending moments, chord bending moments, and torsional moments are presented as a function of blade radial station at each airspeed considered. The test data are represented by the solid symbols and the "trim to cyclic-fall through trim" and "full aircraft trim" analyses are represented by the solid and dashed lines, respectively.
AH-1G OLS ROTOR BLADE LOADS CORRELATION

\[ \text{TEST DATA} \]

- TRIM TO CYCLIC-FALL THROUGH TRIM
- FULL AIRCRAFT TRIM

\( V = 67 \text{ kts} \)
\( V = 114 \text{ kts} \)
\( V = 142 \text{ kts} \)

BEAM BENDING MOMENT (IN-lb x 10^-3)

RADIAL POSITION (%)
AH-1G OLS ROTOR BLADE LOADS CORRELATION (CONT’D)

△ TEST DATA
TRIM TO CYCLIC-FALL THROUGH TRIM
--- FULL AIRCRAFT TRIM

V = 67 kts

V = 114 kts

V = 142 kts

CHORD BENDING MOMENT (IN-LB x 10^-3)

RADIAL POSITION (%)

76
TORSIONAL MOMENT (IN-LB x 10^-3)

- Full Aircraft Trim
- Trim to Cyclic-Fall Through Trim
- Test Data ▽

AH-1G OLS ROTOR BLADE LOADS CORRELATION (CONCLUDED)
The table shows the hub shears that were predicted by C81 using the "trim to cyclic-fall through trim" procedure outlined earlier. Here, the 2, 4, and 6-per-rev sine and cosine components are tabulated in the x, y, and z directions for each of the six airspeeds considered. These shears are expressed with respect to the C81 mast coordinate system and thus a transformation must be performed before they can be used as a forcing function on a NASTRAN model.
C81 AH-1G OLS HUB SHEAR PREDICTIONS
(TRIM TO CYCLIC - FALL THROUGH TRIM)

<table>
<thead>
<tr>
<th>Airspeed and Direction</th>
<th>2p (lb) SINE</th>
<th>2p (lb) COSINE</th>
<th>4p (lb) SINE</th>
<th>4p (lb) COSINE</th>
<th>6p (lb) SINE</th>
<th>6p (lb) COSINE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>67 kn</strong></td>
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<td></td>
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<td></td>
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<tr>
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<td>-211.473</td>
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<tr>
<td><strong>85 kn</strong></td>
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<td></td>
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<tr>
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<td><strong>101 kn</strong></td>
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<tr>
<td>x shear</td>
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<td><strong>114 kn</strong></td>
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</table>
The table shows the hub shears that were predicted by C81 using the "full aircraft trim" procedure outlined earlier. Here, the 2, 4, and 6 per rev sine and cosine components are tabulated in the x, y, and z directions for each of the six airspeeds considered. These shears are expressed with respect to the C81 mast coordinate system and thus a transformation must be performed before they can be used as a forcing function on a NASTRAN model.
## C81 AH-1G OLS HUB SHEAR PREDICTIONS
(FULL AIRCRAFT TRIM)

<table>
<thead>
<tr>
<th>Airspeed and Direction</th>
<th>2p (lb)</th>
<th>4p (lb)</th>
<th>6p (lb)</th>
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<tbody>
<tr>
<td></td>
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<td>SINE</td>
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<td></td>
</tr>
<tr>
<td>x shear</td>
<td>-665.538</td>
<td>-1001.118</td>
<td>150.310</td>
</tr>
<tr>
<td>y shear</td>
<td>860.691</td>
<td>-422.950</td>
<td>-422.327</td>
</tr>
<tr>
<td>z shear</td>
<td>527.788</td>
<td>1377.519</td>
<td>-77.821</td>
</tr>
<tr>
<td>142 kn</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x shear</td>
<td>-860.880</td>
<td>-1019.420</td>
<td>244.469</td>
</tr>
<tr>
<td>y shear</td>
<td>865.340</td>
<td>-500.791</td>
<td>-413.128</td>
</tr>
<tr>
<td>z shear</td>
<td>522.532</td>
<td>1834.928</td>
<td>3.573</td>
</tr>
</tbody>
</table>
The sine and cosine components of the harmonics of hub shears calculated by C81 must be converted to the proper sign for NASTRAN. The coordinate systems for C81 and NASTRAN are shown in the figure.

The sign change is accounted for in the phase angle representation of NASTRAN dynamic loading. Care must be used by the analyst when determining phase input because the phase angle equation has limitations. A ± 180° change may be required to the predicted phase angle because of these limitations. The limitations stem from the inability of the arctangent function to distinguish between quadrants I and III and quadrants II and IV for phase angle predictions (i.e., a positive cosine, positive sine input and a - cosine, -sine input yield identical phase angle results when in reality they are 180° out of phase).
CONVERSION OF C81 HUB LOADS FOR NASTRAN INPUT

C-81 HARMONIC SERIES LOAD OUTPUT

\[ f(t) = a_o + \sum_{k=1}^{N} \left[ a_k \cos(2nk \omega t) + b_k \sin(2nk \omega t) \right] \]

where

\[ f(t) = \text{TIME HISTORY DYNAMIC LOAD VECTOR} \]

\[ a_o, a_k, b_k = \text{INTEGRATION CONSTANTS} \]

\[ \omega = \text{HARMONIC FREQUENCY OF INTEREST}(2p, 4p, 6p) \]

\[ t = \text{TIME} \]

NASTRAN DYNAMIC LOAD INPUT

\[ X = a_k^2 + b_k^2 \]

\[ \phi = \tan^{-1} \left( \frac{b_k}{a_k} \right) \]
Trim to Cyclic Hub Loads (NASTRAN Format)

The hub load predictions for the 2, 4, and 6-per-rev main rotor harmonics, when converted to NASTRAN amplitude and phase dynamic load format for the trim-to-cyclic ("wind tunnel") case, are shown below.
# TRIM TO CYCLIC HUB LOADS (NASTRAN FORMAT)

<table>
<thead>
<tr>
<th>Airspeed and Direction</th>
<th>Amplitude</th>
<th>Phase</th>
<th>Amplitude</th>
<th>Phase</th>
<th>Amplitude</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>67 kn</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x shear</td>
<td>-908.25</td>
<td>184.82</td>
<td>-402.28</td>
<td>-18.07</td>
<td>-94.29</td>
<td>143.25</td>
</tr>
<tr>
<td>y shear</td>
<td>887.45</td>
<td>103.75</td>
<td>370.88</td>
<td>250.22</td>
<td>83.57</td>
<td>53.85</td>
</tr>
<tr>
<td>z shear</td>
<td>550.40</td>
<td>128.72</td>
<td>-259.77</td>
<td>144.50</td>
<td>-96.52</td>
<td>109.19</td>
</tr>
<tr>
<td>85 kn</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x shear</td>
<td>-1015.63</td>
<td>196.91</td>
<td>-484.72</td>
<td>-2.79</td>
<td>-156.96</td>
<td>161.97</td>
</tr>
<tr>
<td>y shear</td>
<td>945.31</td>
<td>109.22</td>
<td>400.88</td>
<td>-89.27</td>
<td>130.62</td>
<td>72.31</td>
</tr>
<tr>
<td>z shear</td>
<td>-650.88</td>
<td>85.07</td>
<td>-27.18</td>
<td>116.70</td>
<td>-36.64</td>
<td>80.61</td>
</tr>
<tr>
<td>101 kn</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x shear</td>
<td>-1145.42</td>
<td>192.70</td>
<td>-563.01</td>
<td>-11.07</td>
<td>-203.01</td>
<td>153.69</td>
</tr>
<tr>
<td>y shear</td>
<td>1024.43</td>
<td>105.67</td>
<td>448.74</td>
<td>265.19</td>
<td>167.44</td>
<td>63.90</td>
</tr>
<tr>
<td>z shear</td>
<td>-873.07</td>
<td>57.31</td>
<td>-119.46</td>
<td>-60.73</td>
<td>-39.68</td>
<td>8.89</td>
</tr>
<tr>
<td>114 kn</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x shear</td>
<td>-1347.38</td>
<td>192.17</td>
<td>-668.98</td>
<td>-10.34</td>
<td>-233.12</td>
<td>150.89</td>
</tr>
<tr>
<td>y shear</td>
<td>1144.03</td>
<td>103.48</td>
<td>555.92</td>
<td>264.07</td>
<td>190.04</td>
<td>62.04</td>
</tr>
<tr>
<td>z shear</td>
<td>-1104.63</td>
<td>46.23</td>
<td>-140.45</td>
<td>-86.22</td>
<td>-67.09</td>
<td>-3.64</td>
</tr>
<tr>
<td>128 kn</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x shear</td>
<td>-1496.91</td>
<td>184.26</td>
<td>-763.66</td>
<td>-17.84</td>
<td>-257.49</td>
<td>140.87</td>
</tr>
<tr>
<td>y shear</td>
<td>1200.43</td>
<td>94.90</td>
<td>654.90</td>
<td>254.54</td>
<td>211.01</td>
<td>51.69</td>
</tr>
<tr>
<td>z shear</td>
<td>-1356.54</td>
<td>37.57</td>
<td>-219.21</td>
<td>232.85</td>
<td>-19.65</td>
<td>-31.98</td>
</tr>
<tr>
<td>142 kn</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x shear</td>
<td>-1897.28</td>
<td>223.20</td>
<td>-922.69</td>
<td>27.02</td>
<td>-242.80</td>
<td>178.69</td>
</tr>
<tr>
<td>y shear</td>
<td>1360.04</td>
<td>128.46</td>
<td>788.93</td>
<td>-63.50</td>
<td>210.04</td>
<td>95.39</td>
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<tr>
<td>z shear</td>
<td>-1672.52</td>
<td>27.22</td>
<td>-206.26</td>
<td>208.15</td>
<td>-31.73</td>
<td>231.19</td>
</tr>
</tbody>
</table>
Full Aircraft Trim Hub Loads (NASTRAN Format)

The hub load predictions for the 2, 4, and 6-per-rev main rotor harmonics, when converted to NASTRAN amplitude and phase dynamic load format for the full aircraft trim case, are shown below.
### FULL AIRCRAFT TRIM HUB LOADS (NASTRAN FORMAT)

<table>
<thead>
<tr>
<th>Airspeed and Direction</th>
<th>2p Amplitude</th>
<th>2p Phase</th>
<th>4p Amplitude</th>
<th>4p Phase</th>
<th>6p Amplitude</th>
<th>6p Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>67 kn</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x shear</td>
<td>-731.22</td>
<td>217.95</td>
<td>-260.38</td>
<td>41.56</td>
<td>-63.50</td>
<td>213.44</td>
</tr>
<tr>
<td>y shear</td>
<td>817.47</td>
<td>116.15</td>
<td>203.12</td>
<td>-50.76</td>
<td>50.74</td>
<td>128.73</td>
</tr>
<tr>
<td>z shear</td>
<td>-708.46</td>
<td>123.92</td>
<td>-338.93</td>
<td>130.16</td>
<td>-123.50</td>
<td>61.03</td>
</tr>
<tr>
<td><strong>85 kn</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x shear</td>
<td>-848.41</td>
<td>213.43</td>
<td>-309.22</td>
<td>23.73</td>
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<td>170.04</td>
</tr>
<tr>
<td>y shear</td>
<td>873.23</td>
<td>113.64</td>
<td>281.70</td>
<td>-56.82</td>
<td>74.47</td>
<td>85.02</td>
</tr>
<tr>
<td>z shear</td>
<td>-638.40</td>
<td>72.40</td>
<td>-48.25</td>
<td>172.67</td>
<td>-57.28</td>
<td>15.00</td>
</tr>
<tr>
<td><strong>101 kn</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x shear</td>
<td>-968.56</td>
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<td>-144.64</td>
<td>160.81</td>
</tr>
<tr>
<td>y shear</td>
<td>937.31</td>
<td>113.47</td>
<td>338.52</td>
<td>-61.57</td>
<td>107.88</td>
<td>74.46</td>
</tr>
<tr>
<td>z shear</td>
<td>-897.93</td>
<td>43.62</td>
<td>-133.62</td>
<td>-83.98</td>
<td>-77.03</td>
<td>-20.80</td>
</tr>
<tr>
<td><strong>114 kn</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x shear</td>
<td>-1083.36</td>
<td>211.08</td>
<td>-444.54</td>
<td>13.87</td>
<td>-147.75</td>
<td>159.43</td>
</tr>
<tr>
<td>y shear</td>
<td>949.93</td>
<td>114.23</td>
<td>403.64</td>
<td>-67.81</td>
<td>114.06</td>
<td>75.30</td>
</tr>
<tr>
<td>z shear</td>
<td>-1149.20</td>
<td>31.18</td>
<td>-133.48</td>
<td>248.08</td>
<td>-91.56</td>
<td>-24.32</td>
</tr>
<tr>
<td><strong>128 kn</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x shear</td>
<td>-1202.16</td>
<td>213.62</td>
<td>-499.08</td>
<td>17.53</td>
<td>-132.39</td>
<td>157.18</td>
</tr>
<tr>
<td>y shear</td>
<td>959.00</td>
<td>116.17</td>
<td>449.79</td>
<td>-69.88</td>
<td>108.58</td>
<td>76.45</td>
</tr>
<tr>
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<td>20.96</td>
<td>-237.79</td>
<td>199.10</td>
<td>-47.65</td>
<td>-40.27</td>
</tr>
<tr>
<td><strong>142 kn</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x shear</td>
<td>-1334.29</td>
<td>220.18</td>
<td>-526.81</td>
<td>27.65</td>
<td>-102.43</td>
<td>159.47</td>
</tr>
<tr>
<td>y shear</td>
<td>999.80</td>
<td>120.06</td>
<td>470.53</td>
<td>-61.40</td>
<td>84.97</td>
<td>82.75</td>
</tr>
<tr>
<td>z shear</td>
<td>-1907.88</td>
<td>15.90</td>
<td>-341.38</td>
<td>179.40</td>
<td>-5.65</td>
<td>151.06</td>
</tr>
</tbody>
</table>
Regardless of the sophistication of the FEM used for the airframe vibration predictions, the accuracy of the predicted response depends largely on the accuracy of the predicted rotor-induced loads transmitted to the fuselage. This loading environment is very complex. Two different harmonic load cases from C81 analyses will be presented: (1) trim to cyclic-fall through trim and (2) full aircraft trim. In addition to the rotor-induced harmonic loads acting directly on the fuselage, loads transmitted to the airframe through the control actuators and aerodynamic forces acting directly on the airframe also affect the dynamic response. These load conditions are depicted in the figure.
NASTRAN LOAD CONDITIONS

2, 4, AND 6/REV HUB LOADS (C81)*

2/REV CONTROL LOADS (OLS DATA)

2/REV FIN LOAD AT 142 KT (KAMAN DATA)

*C81 HUB SHEAR PREDICTIONS

1. TRIM TO CYCLIC ("WIND TUNNEL" CONDITION)
2. FULL AIRCRAFT TRIM
Measured 2p boost cylinder axial loads were obtained from the OLS report (Reference 1) for application to the NASTRAN fuselage model. The effect of these loads on the vibratory response calculations will be assessed.
### 2p CONTROL LOADS FROM OLS DATA

<table>
<thead>
<tr>
<th>AIRSPEED (kt)</th>
<th>F/A CYCLIC</th>
<th>LATERAL CYCLIC</th>
<th>COLLECTIVE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AMP</td>
<td>PHASE</td>
<td>AMP</td>
</tr>
<tr>
<td>67</td>
<td>336.8</td>
<td>139.9</td>
<td>502.2</td>
</tr>
<tr>
<td>85</td>
<td>347.4</td>
<td>137.2</td>
<td>570.1</td>
</tr>
<tr>
<td>101</td>
<td>416.8</td>
<td>122.1</td>
<td>600.1</td>
</tr>
<tr>
<td>114</td>
<td>455.6</td>
<td>120.5</td>
<td>640.4</td>
</tr>
<tr>
<td>128</td>
<td>560.4</td>
<td>116.6</td>
<td>728.3</td>
</tr>
<tr>
<td>142</td>
<td>721.5</td>
<td>126.3</td>
<td>815.8</td>
</tr>
</tbody>
</table>
KAC Test Data for Force Determination

2p hub shears and tail fin lateral loading at 142 kt-level flight from the Kaman AH-1G force determination tests (Reference 7, pp. 127 and 134) are shown below. Only the lateral fin load was applied to the 2p lateral response case to assess the effects on vibratory response calculations.
KAC TEST DATA FOR FORCE DETERMINATION

<table>
<thead>
<tr>
<th>FORCE DIRECTION</th>
<th>FORCE DETERMINATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MAG (lb)</td>
</tr>
<tr>
<td>VERTICAL AT HUB*</td>
<td>1342.</td>
</tr>
<tr>
<td>LONGITUDINAL AT HUB*</td>
<td>309.</td>
</tr>
<tr>
<td>LATERAL AT HUB*</td>
<td>205.</td>
</tr>
<tr>
<td>LATERAL AT TAIL ROTOR GEARBOX</td>
<td>146.</td>
</tr>
</tbody>
</table>

*C81 calculated hub loads were used rather than force determination hub loads
2, 4, and 6-per-Rev Flight Vibration Comparisons

Vibration response comparisons of the coupled rotor/fuselage analyses and OLS test measurements are presented in this section. The comparisons are presented in the following sequence:

1. 2, 4, and 6-per-rev vertical and lateral response comparisons (hub shears only)
   a. C81 Hub Shears - Trim-to-cyclic
   b. C81 Hub Shears - Full Trim

2. 2-per-rev vertical and lateral response comparisons (combined loads)
   a. C81 Hub Shears (Trim-to-cyclic) + Control Loads
   b. C81 Hub Shears (Trim-to-cyclic) + Control Loads + Fin Loads
2, 4, AND 6-PER-REV FLIGHT VIBRATION COMPARISONS

2, 4, AND 6/REV VERTICAL AND LATERAL RESPONSE COMPARISONS -

(HUB SHEARS ONLY):

C81 HUB SHEARS - TRIM-TO-CYCLIC

C81 HUB SHEARS - FULL TRIM

2/REV VERTICAL AND LATERAL RESPONSE COMPARISONS

COMBINED LOADS

- C81 HUB SHEARS (TRIM-TO-CYCLIC)
- CONTROL LOADS
- LATERAL FIN LOAD AT 142 KT
TWO-PER-REV VERTICAL RESPONSE - HUB SHEARS ONLY

Response calculations for six airspeeds from 67-142 kn with hub shears applied for the trim-to-cyclic and full aircraft trim dynamic analyses are compared for the following response points:

1. Nose  
2. Gunner  
3. Pilot  
4. Engine Deck  

<table>
<thead>
<tr>
<th></th>
<th>FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Nose</td>
<td>46</td>
</tr>
<tr>
<td>2. Gunner</td>
<td>93</td>
</tr>
<tr>
<td>3. Pilot</td>
<td>148</td>
</tr>
<tr>
<td>4. Engine Deck</td>
<td>250</td>
</tr>
</tbody>
</table>

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.

There is generally good agreement between analysis and test.
TWO-PER-REV VERTICAL RESPONSE - HUB SHEARS ONLY

- **NOSE**
- **GUNNER**
- **PILOT**
- **ENGINE**

- **TEST**
- **TRIM TO CYCLIC**
- **FULL AIRCRAFT TRIM**
TWO-PER-REV VERTICAL RESPONSE - HUB SHEARS ONLY

Response calculations for six airspeeds from 67-142 kn with hub shears applied for the trim-to-cyclic and full aircraft trim dynamic analyses are compared for the following response points:

<table>
<thead>
<tr>
<th></th>
<th>Response Points</th>
<th>FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Tailboom Junction</td>
<td>300</td>
</tr>
<tr>
<td>2.</td>
<td>Elevator</td>
<td>400</td>
</tr>
<tr>
<td>3.</td>
<td>Tailboom, Aft</td>
<td>485</td>
</tr>
<tr>
<td>4.</td>
<td>Tail Rotor Gearbox</td>
<td>518</td>
</tr>
</tbody>
</table>

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.
T/R GEARBOX

AFT TAIL

FULL AIRCRAFT TRIM

TRIM TO CYCLIC

TEST

ELEVATOR

T/B JUNCTION

TWO-PER-REV VERTICAL RESPONSE - HUB SHEARS ONLY
TWO-PER-REV LATERAL RESPONSE - HUB SHEARS ONLY

Response calculations for six airspeeds from 67-142 kn with hub shears applied for the trim-to-cyclic and full aircraft trim dynamic analyses are compared for the following response points:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Nose</td>
<td>FS</td>
</tr>
<tr>
<td>2</td>
<td>Gunner</td>
<td>46</td>
</tr>
<tr>
<td>3</td>
<td>Pilot</td>
<td>93</td>
</tr>
<tr>
<td>4</td>
<td>Engine Deck</td>
<td>148</td>
</tr>
<tr>
<td></td>
<td></td>
<td>250</td>
</tr>
</tbody>
</table>

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.

The correlation between test and both analysis cases is poor. The calculations are much lower than test. Neither case predicts any significant vibration levels in the lateral direction. This is suspected to be due to not accounting for lateral 2/rev fin loading due to main rotor downwash which is evaluated in this section of the report.
TWO-PER-REV LATERAL RESPONSE - HUB SHEARS ONLY

NOSE

GUNNER

PILOT

ENGINE

* TEST
--- TRIM TO CYCLIC
----- FULL AIRCRAFT TRIM
TWO-PER-REV LATERAL RESPONSE - HUB SHEARS ONLY

Response calculations for six airspeeds from 67-142 kn with hub shears applied for the trim-to-cyclic and full aircraft trim dynamic analyses are compared for the following response points:

1. Tailboom Junction 300
2. Elevator 400
3. Tail Fin 521

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.

This is again thought to be due to not accounting for the 2/rev lateral tail fin loading.
TWO-PER-REV LATERAL RESPONSE - HUB SHEARS ONLY

T/B JUNCTION

ELEVATOR

TAIL FIN

- TEST
- TRIM TO CYCLIC
- FULL AIRCRAFT TRIM
Response calculations for six airspeeds from 67-142 kn are compared at the main rotor hub. Only hub shears are applied to the NASTRAN model for the trim-to-cyclic and full aircraft trim dynamic analyses. Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.
Two-per rev main rotor hub response - hub shears only
FOUR-PER-REV VERTICAL RESPONSE - HUB SHEARS ONLY

Response calculations for six airspeeds from 67-142 kn with hub shears applied for the trim-to-cyclic and full aircraft trim dynamic analyses are compared for the following response points:

1. Nose: F_s = 46
2. Gunner: F_s = 93
3. Pilot: F_s = 148
4. Engine Deck: F_s = 250

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.
FOUR-PER-REV VERTICAL RESPONSE - HUB SHEARS ONLY

- Test
- Trim to Cyclic
- Full Aircraft Trim
Response calculations for six airspeeds from 67-142 kn with hub shears applied for the trim-to-cyclic and full aircraft trim dynamic analyses are compared for the following response points:

<table>
<thead>
<tr>
<th>Response Point</th>
<th>FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Tailboom Junction</td>
<td>300</td>
</tr>
<tr>
<td>2. Elevator</td>
<td>400</td>
</tr>
<tr>
<td>3. Tailboom, Aft</td>
<td>485</td>
</tr>
<tr>
<td>4. Tail Rotor Gearbox</td>
<td>518</td>
</tr>
</tbody>
</table>

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.
FOUR-PER-REV VERTICAL RESPONSE - HUB SHEARS ONLY
FOUR-PER-REV LATERAL RESPONSE - HUB SHEARS ONLY

Response calculations for six airspeeds from 67-142 kn with hub shears applied for the trim-to-cyclic and full aircraft trim dynamic analyses are compared for the following response points:

1. Nose
2. Gunner
3. Pilot
4. Engine Deck

<table>
<thead>
<tr>
<th></th>
<th>FS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>46</td>
<td>93</td>
<td>148</td>
<td>250</td>
<td></td>
</tr>
</tbody>
</table>

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.
FOUR-PER-REV LATERAL RESPONSE - HUB SHEARS ONLY

- **Nose**
  - Test
  - Trim to cyclic
  - Full aircraft trim

- **Gunner**
  - Test
  - Trim to cyclic
  - Full aircraft trim

- **Pilot**
  - Test
  - Trim to cyclic
  - Full aircraft trim

- **Engine**
  - Test
  - Trim to cyclic
  - Full aircraft trim

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Response calculations for six airspeeds from 67-142 kn with hub shears applied for the trim-to-cyclic and full aircraft trim dynamic analyses are compared for the following response points:

1. Tailboom Junction
2. Elevator
3. Tail Fin

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.
FOUR-PER-REV LATERAL RESPONSE - HUB SHEARS ONLY

T/B JUNCTION

ELEVATOR

TAIL FIN

- TEST
- TRIM TO CYCLIC
- FULL AIRCRAFT TRIM
Response calculations for six airspeeds from 67-142 kn are compared at the hub. Only hub shears are applied to the NASTRAN model for the trim-to-cyclic and full aircraft trim dynamic analyses. Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations. Note the differences in scale between the fore-aft/lateral response plots and the vertical response plot for proper comparison.
FOUR-PER-REV MAIN ROTOR HUB RESPONSE - HUB SHEARS ONLY

FORE/AFT

LATERAL

VERTICAL

TEST

TRIM TO CYCLIC

FULL AIRCRAFT TRIM
Response calculations for six airspeeds from 67-142 kn with hub shears applied for the trim-to-cyclic and full aircraft trim dynamic analyses are compared for the following response points:

1. Nose
2. Gunner
3. Pilot
4. Engine Deck

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.
SIX-PER-REV VERTICAL RESPONSE - HUB SHEARS ONLY

- NOSE
- GUNNER
- PILOT
- ENGINE

TEST
TRIM TO CYCLIC
FULL AIRCRAFT TRIM
Response calculations for six airspeeds from 67-142 kn with hub shears applied for the trim-to-cyclic and full aircraft trim dynamic analyses are compared for the following response points:

1. Tailboom Junction 300
2. Elevator 400
3. Tailboom, Aft 485
4. Tail Rotor Gearbox 518

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.
SIX-PER-REV VERTICAL RESPONSE - HUB SHEARS ONLY

**T/B JUNCTION**

- **TEST**
- **TRIM TO CYCLIC**
- **FULL AIRCRAFT TRIM**

**ELEVATOR**

**AFT TAIL**

**T/R GEARBOX**

Airspeed: 65, 85, 105, 125, 145 knots

Acceleration: 0.0, 0.1, 0.2, 0.3, 0.4 g
SIX-PER-REV LATERAL RESPONSE - HUB SHEARS ONLY

Response calculations for six airspeeds from 67-142 kn with hub shears applied for the trim-to-cyclic and full aircraft trim dynamic analyses are compared for the following response points:

1. Nose 46  
2. Gunner 93  
3. Pilot 148  
4. Engine Deck 250

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.
SIX-PER-REV LATERAL RESPONSE - HUB SHEARS ONLY

NOSE

GUNNER

PILOT

ENGINE

- TEST
- TRIM TO CYCLIC
- FULL AIRCRAFT TRIM
Response calculations for six airspeeds from 67-142 kn with hub shears applied for the trim-to-cyclic and full aircraft trim dynamic analyses are compared for the following response points:

1. Tailboom Junction
2. Elevator
3. Vertical Tail Fin

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.
SIX-PER-REV LATERAL RESPONSE - HUB SHEARS ONLY

**T/B JUNCTION**

**ELEVATOR**

**TAIL FIN**

- **TEST**
- **TRIM TO CYCLIC**
- **FULL AIRCRAFT TRIM**
Response calculations for six airspeeds from 67-142 kn are compared at the main rotor hub. Only hub shears are applied to the NASTRAN model for the trim-to-cyclic and full aircraft trim dynamic analyses. Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations. Note the differences in scale between the fore-aft/lateral response plots and the vertical response plot for proper comparison.
SIX-PER-REV MAIN ROTOR HUB RESPONSE - HUB SHEARS ONLY

**FORE/AFT**

![Graph of Fore/Aft acceleration vs. Airspeed](image)

- **TEST**
- **TRIM TO CYCLIC**
- **FULL AIRCRAFT TRIM**

**LATERAL**

![Graph of Lateral acceleration vs. Airspeed](image)

**VERTICAL**

![Graph of Vertical acceleration vs. Airspeed](image)
Response calculations for six airspeeds from 67-142 kn with hub shears and boost cylinder loads applied simultaneously for the trim-to-cyclic and full aircraft trim dynamic analyses are compared for the following response points:

1. Nose
2. Gunner
3. Pilot
4. Engine Deck

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.
TWO-PER-REV VERTICAL RESPONSE - HUB AND CONTROL LOADS

NOSE

GUNNER

PILOT

ENGINE

TEST

TRIM TO CYCLIC

FULL AIRCRAFT TRIM
Response calculations for six airspeeds from 67-142 kn with hub shears and boost cylinder loads applied simultaneously for the trim-to-cyclic and full aircraft trim dynamic analyses are compared for the following response points:

1. Tailboom Junction: 300
2. Elevator: 400
3. Tailboom, Aft: 485
4. Tail Rotor Gearbox: 518

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.
T/R GEARBOX

AFT TAIL

FULL AIRCRAFT TRIM
TRIM TO CYCLIC
TEST

ELEVATOR

T/R JUNCTION

TWO-PER-REV VERTICAL RESPONSE - HUB AND CONTROL LOADS
In addition to the hub shears and boost cylinder loads being applied simultaneously, a tail fin lateral load was added at 142 kn (see Page 92) for the trim-to-cyclic and full aircraft trim dynamic analyses. Response calculations for six airspeeds from 67-142 kn are compared for the following response points:

1. Nose 46
2. Gunner 93
3. Pilot 148
4. Engine Deck 250

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.
In addition to the hub shears and boost cylinder loads being applied simultaneously, a tail fin lateral load was added at 142 kn (see Page 92) for the trim-to-cyclic and full aircraft trim dynamic analyses. Response calculations for six airspeeds from 67-142 kn are compared for the following response points:

1. Tailboom Junction  
   FS  
   300  
2. Elevator  
   400  
3. Vertical Tail Fin  
   521  

Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.
TWO-PER-REV LATERAL RESPONSE - HUB, CONTROL, AND FIN LOAD

T/B JUNCTION

LEGEND

- TEST
- TRIM TO CYCLIC
- FULL AIRCRAFT TRIM
- FIN LOAD ADDED

ELEVATOR

TAIL FIN

AIRSPEED (knots)
ACC. (g)

1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
65  85  105  125  145

AIRSPEED (knots)

1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
65  85  105  125  145

AIRSPEED (knots)

1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
65  85  105  125  145

135
Response calculations for six airspeeds from 67-142 kn are compared for the hub response point with hub shears and boost cylinder loads applied simultaneously for the trim-to-cyclic and full aircraft trim dynamic analyses. Test data from the Operational Loads Survey Report (Reference 1) are used for comparison with the calculated vibrations.
TWO-PER-REV MAIN ROTOR HUB RESPONSE - HUB AND
CONTROL LOADS

**FORE/AFT**

![Graph showing acceleration (g) vs. airspeed (kts) for FORE/AFT loads.]

**LATERAL**

![Graph showing acceleration (g) vs. airspeed (kts) for LATERAL loads.]

**VERTICAL**

![Graph showing acceleration (g) vs. airspeed (kts) for VERTICAL loads.]

**LEGEND**

- • TEST
- • TRIM TO CYCLIC
- •• FULL AIRCRAFT TRIM

The page is of poor quality.
5. CONCLUSIONS
CONCLUSIONS

Test data from an AH-1G Operational Load Survey (OLS) were used for correlation of a coupled rotor/fuselage analysis. Analytic predictions of hub shears (C81), OLS measured control loads, and a tail rotor gearbox lateral force were used to excite the NASTRAN model. The control loads and gearbox lateral force only apply to 2p main rotor harmonic excitation while the hub shears were determined for 2p, 4p, and 6p harmonics.

The correlation was based on comparing vibration amplitudes in the lateral and vertical directions at selected fuselage locations for six airspeeds from 67-142 kn.

Conclusions from the rotor/fuselage coupling analysis and flight vibration correlation study are as follows:

1. An existing analysis method for coupling rotor and fuselage dynamic analyses using the BHTI Rotorcraft Flight Simulation Program C81 and NASTRAN demonstrated the capability to produce reasonable response predictions for rotorcraft.

2. There is good agreement between calculated and measured vertical two-per-rev vibration. This is the predominant excitation frequency for a two-bladed teetering rotor. The inclusion of 2p control loads significantly affects the response level of certain response locations.

3. Lateral two-per-rev vibration levels predicted by the coupled rotor/fuselage analyses were much lower than the measured vibrations at most of the correlation points. Main rotor downwash on the fin is suspected to be a factor since the correlation was noted to improve significantly when the two-per-rev fin load was included in the calculations for the 142 kn condition.

4. Calculated and measured four- and six-per-rev vibration responses agree fairly well. It is not surmised that the accuracy of the analysis at these frequencies can be judged, however, since the vibration response at these frequencies was not strongly influenced by modes in close proximity to the forcing frequency. From the results of vibration test correlations, the airframe vibration prediction was quantitatively accurate through four-per-rev, but deviated from measured results significantly at six-per-rev. In addition, when exciting at
CONCLUSIONS (Continued)

the main rotor hub through the pylon, the correlation obtained was poorer than that obtained when exciting directly on the airframe. Considering these factors, the agreement between test and analyses for the four- and six-per-rev responses may have been coincidental. More information is needed in order to judge the prediction of airframe vibration at these frequencies.

Recommendations for further investigations are as follows:

1. Vibration prediction in the four-per-rev frequency range and above needs further investigation.

2. Investigate the effect of pylon dynamics on airframe vibration by a combined analytical and test correlation program.

3. Investigate the main rotor two-per-rev downwash environment on the AH-1G fin.

4. A convenient method for measuring hub shears should be developed for direct correlation with the analysis.
CONCLUSIONS

- SUCCESSFUL DEMONSTRATION OF ROTOR/AIRFRAME COUPLING ANALYSIS
  - ROTOR ANALYSIS - C81
  - FUSELAGE ANALYSIS - NASTRAN

- TWO-, FOUR-, AND SIX-PER-REV CORRELATION
  - TWO-PER-REV VERTICAL GOOD
  - TWO-PER-REV LATERAL POOR (FIN DOWNWASH LOADING SUSPECTED)
  - FOUR- AND SIX-PER-REV FAIR

- FURTHER WORK NEEDED
  - INVESTIGATE HIGHER FREQUENCY (> FOUR-PER-REV) VIBRATION PREDICTIONS
  - INVESTIGATE PYLON DYNAMICS
  - MEASURE MAIN ROTOR DOWNWASH ON FIN
  - DEVELOP HUB SHEAR MEASUREMENT TECHNOLOGY
6. REFERENCES
REFERENCES


**Abstract**

Under a research program designated DAMVIBS (Design Analysis Methods for VibrationS), four U. S. helicopter industry participants (BHTI, Boeing Helicopter, McDonnell Douglas Helicopter, and Sikorsky Aircraft) are to apply existing analytical methods for calculating coupled rotor-fuselage vibrations of the AH-1G helicopter for correlation with flight test data from an AH-1G Operational Load Survey (OLS) test program. The analytical representation of the fuselage structure is based on a NASTRAN finite element model (FEM) developed by BHTI, which has been extensively documented and correlated with ground vibration test.

This report summarizes the procedure used at BHTI for predicting coupled rotor-fuselage vibrations using the advanced Rotorcraft Flight Simulation Program C81 and NASTRAN. The summary includes detailed descriptions of the analytical formulation of rotor dynamics equations, fuselage dynamic equations, coupling between the rotor and fuselage, and solutions to the total system of equations in C81.

Analytical predictions of hub shears for main rotor harmonics 2\(p\), 4\(p\), and 6\(p\) generated by C81 are used in conjunction with 2\(p\) OLS measured control loads and a 2\(p\) lateral tail rotor gearbox force, representing downwash impingement on the vertical fin, to excite the NASTRAN model. NASTRAN is then used to correlate with measured OLS flight test vibrations. Blade load comparisons predicted by C81 showed good agreement. In general, the fuselage vibration correlations show good agreement between analysis and test in vibration response through 15 to 20 Hz. Recommendations for further work are included.

**Key Words**

- Rotor-Fuselage Coupling
- Airframe Vibrations
- AH-1G Helicopter
- Structural Analysis
- Finite Element
- Helicopter
- Flight Test
- NASTRAN
- Rotorcraft
- Dynamic

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