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THREE-DIMENSIONAL SINGULAR POINTS IN AERODYNAMICS

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Abstract

When three-dimensional separation occurs on a body immersed in a flow governed by the incompressible Navier-Stokes equations, the geometrical surfaces formed by the three vector fields (velocity, vorticity, and skin friction) and a scalar field (pressure) become interrelated through topological maps containing their respective singular points and extremal points.

A mathematically consistent description of these singular points becomes essential when we want to study the geometry of the separation. A separated stream surface requires, for example, the existence of a saddle-type singular point on the skin-friction surface. This singular point is actually, in the proper language of mathematics, a saddle of index two. The index is a measure of the dimension of the outset (set leaving the singular point). Hence, when we specify a saddle of index two, a two-dimensional surface that becomes separate from the osculating plane of the saddle is implied.

In this short paper, we will show how we can interpret the three-dimensional singular points mathematically and discuss the most common aerodynamical singular points via this perspective.

Introduction

When three-dimensional separation occurs on a body immersed in a flow governed by the incompressible Navier-Stokes equations, the geometrical

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surfaces formed by the three vector fields (velocity, vorticity, and skin friction) and a scalar field (pressure) become interrelated through topological maps containing their respective singular points and extremal points. For example, a nodal singular point on the skin-friction surface (a surface everywhere tangent to the skin-friction vector) is found to correspond to an extremal point in the surface pressure.[1]

Some special curves present themselves in the geometry of the flow. Among these curves, we shall distinguish and define two: the line of separation and the vortex core. The line of separation represents a special skin-friction line. It is the one skin-friction line that emanates from a saddle-type singular point and ends at a focus-type singular point on the skin-friction surface.

The vortex core represents a special streamline. It is the one streamline that emanates from (or moves through) a particular singular point, namely, a focus located either in the skin-friction surface or in the flow itself. The vortex core constitutes a limiting line in the flow field around which stream surfaces (surfaces tangent everywhere to the velocity vector field) wind.

Our premise is that if a curve can be distinguished from its neighbors by its having a topological property that they do not share, then that distinction must be reflected in other discoverable singular properties as well. In the search for these singular properties, we make use of differential geometric methods that originated with Andronov's study[2] of the singular points of phase portraits. Such methods become relevant based on the postulate that the vector fields in question (velocity, vorticity, skin friction) remain continuous and analytic throughout. In this search, when we try to refer to the three-dimensional singular points, a certain ambiguity presents itself as follows: a saddle on the skin-friction surface is actually a half-node on the separation stream surface. The purpose of this short note is to introduce the nomenclature used by Abraham and Shaw[5] to describe the three-dimensional singular points of aerodynamics and show that in separation-related studies such nomenclature proves to be helpful in avoiding ambiguities of referencing to different three-dimensional singular points.

Furthermore, the questions related to whether it is a real separation or not can be resolved once and for all with the usage of the correct nomenclature. For example, the ongoing controversy about the open separation (local separation) can be resolved in the proper language of the three-dimensional singular points: Since there will be no saddle of index two in the case of a local separation, there will be no outset of dimension two leaving the skin-friction surface (i.e., no separated stream surface; hence, no separation).

Three-Dimensional Singular Points in Aerodynamics

Singular points are points where the field under consideration vanishes. We shall adopt the geometrical point of view of Abraham and Shaw[5] and treat the singular points as limiting sets. Three-dimensional singular points will be constructed by combining the limit sets of one- and two-dimensional state spaces.

If we expand the space variables of the vector field in a Taylor series around a singular point, the lowest-order terms normally will be linear in the variables. An analysis of the linear form provides the eigenvalues which in turn yield the characteristic exponents. For a spiral-type singular point in a plane, for example, we have two complex-conjugate numbers as characteristic exponents, whereas for a nodal-type singular point in a plane, we have two real numbers. If we plot these characteristic exponents (or eigenvalues) in a complex plane, we obtain the spectrum for the corresponding two-dimensional singular point. By linearity, the spectrum of a three-dimensional singular point will be a superposition of the spectra of its lower-dimensional components. For example, for a three-dimensional saddle point we can combine a one-dimensional repellor (a limiting set from which all the neighboring trajectories are escaping) with a nodal attractor in a plane (a limiting set to which all the trajectories are converging). The incoming flow, in this case the nodal attractor, constitutes the inset; and the outgoing flow, in this case the one-dimensional repellor, constitutes the outset. The index is defined as the dimension of the outset. In this case of a three-dimensional, saddle-type singular point, the index is unity. The trajectories (vector lines) which are neither in the inset nor in the outset pass by hyperbolically (fig. 1).

Typical Three-Dimensional Singular Points in Aerodynamics

We recognize the following three-dimensional singular points as occurring typically in aerodynamic applications:

1. A focus of attachment. When a streamline attaches itself to a body immersed in the flow at a focus (spiral) which is a planar repellor (the plane being the osculating plane to the body at the point of attachment), the streamline is a linear attractor. A focus of attachment is a saddle of index 2 (Fig. 2). Neither streamlines in the linear attractor nor in the spiral repellor spiral past the singular point.

2. A focus of separation. If we reverse the flow direction of the previous singular point we obtain a focus of separation, which is a saddle of index 1 (Fig. 2).

3. A node of attachment. This type of singular point consists of a linear attractor and a planar nodal repellor and is a saddle of index 2

(Fig. 3). The trajectories which are neither in the inset nor in the outset pass by hyperbolically.

4. A node of separation. This type of singular point is obtained from the previous one by a reversal of the directions of flow. A node of separation is a saddle of index 1 (Fig. 3).

Repellors of index 3 and attractors of index 0 are impossible in aerodynamics in the absence of sources and sinks. In this paper we shall consider elementary singular points (i.e., singular points with distinct eigenvalues) that are hyperbolic. Among the most important nonhyperbolic singular points, we cite centers, which are degenerate foci. The worst case of nonhyperbolicity is depicted in Fig. 4.

As a result of the previous discussion we are able to state the following theorem.

Theorem 1. Three-dimensional hyperbolic singular points of aerodynamics are either saddles of index one or saddles of index two. However, in the latter case, one anticipates the existence of a separation stream surface. In the former case, we anticipate the existence of a special limiting curve.

We believe that the following vocabulary, which arises out of a rigorous mathematical setting, is a more concise geometric description of the singular points than the various ad hoc descriptions that exist in the aerodynamics literature. For example, the index indicates immediately whether it is a separating streamline or a stream surface. Index 1 indicates that only one streamline is leaving the singular point and index 2 indicates that a surface is leaving the singular point.

Singular Points on the Surface or in the Flow

We can distinguish between the singular points on the skin-friction surface and those in the flow by defining the former ones as half-saddle points of index 2. This notion of half-singular points is not new.[3,4] The singular points on the skin-friction surface are definitely quite different from the ones in the flow.

A saddle point in the skin-friction surface is actually a half-saddle point of index 2 on the line of intersection of two surfaces, namely, skin-friction surface and the stream surface.

A focus on the skin-friction surface is actually a half-saddle point of index 1 on the intersection of a surface (skin-friction surface) and a line (vortex core).

A node on the skin-friction surface is actually a half saddle of index 2 on the intersection of a surface (skin-friction surface) and a line (a streamline) for the node of attachment and a half saddle of index 1 for the node of separation (detachment) on the intersection of a surface (skin-friction surface) and a line (a streamline).

In the case of singular points in a flow, the halves disappear from the singular points. To emphasize this characteristic we will refer to a full saddle of index 2 and a full saddle of index 1 (Fig. 5): a saddle-focus--saddle of index 1; a node--saddle of index 2.

Application of the Vocabulary--An Example

To show that the preceding mathematically consistent vocabulary does indeed help us not to have any ambiguity, let us consider the following typical geometry of the separated stream surfaces: A stream surface separates along the line of separation from the skin-friction surface. The line of separation emanates from a saddle point of the skin-friction surface. This particular singular point which is a saddle-type singular point on the skin-friction surface is actually a half-node on the separated stream surface (Fig. 6). If we use the vocabulary to refer to the three-dimensional singular point in terms independent of the special surfaces on which they are projected, we would call it a saddle of index 2. The term index 2 indicates that there is a surface (a sheet) formed by the streamlines emanating from this singular point (Fig. 6).

Again, in this same example, it would be very difficult to refer to the nature of the singular point in the flow; in the rolled-up surface, it is a focus but in the developed stream surface (i.e., the stretched-out stream surface), it is a full-saddle point. To eliminate the dependency of the description on the surface type which surface 1 must project the singular point, we can simply say, again using the preceding vocabulary, a saddle of index 1. Index 1 indicates that there is a line leaving the singular point. The dimension of the outset is one or equivalently, there can be no surface leaving the singular point.

Concluding Remarks

We think the representation of the three-dimensional singular points in the vocabulary that has been proposed can be useful for work in, or the study of, the geometry of separated and rolled stream surfaces. The precise use of the names of the singular points might help prevent ambiguities in description.

Another important application of the preceding analysis is anticipated for computer-graphical representations of flow problems. A coordinate-independent approach (i.e., a tensorial approach) would be necessary for efficient computations on the computer. The preceding description of the three-dimensional singular points is very useful for the language of tensorial stream surface computations.

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[2]Andronov, A. A., Leontovich, E. A., Gordon, I. I., and Maier, A. G., "Qualitative Theory of Second-Order Dynamic Systems," Wiley Publishing Co., New York, 1973.

[3]Tobak, M. and Peake, D. J., "Topology of Three-Dimensional Separated Flows," Annual Review of Fluid Mechanics, Vol. 14, 1982, pp. 61-85.

[4]Chapman, G. T., "Topological Classification of Flow Separation on Three-Dimensional Bodies," AIAA Paper 86-0485, 1986.

[5]Abraham, G. and Shaw, D., "Dynamics: The Geometry of Behaviour," Aerial Press Inc., 1984.

Figure Captions

Fig. 1 Construction of a three-dimensional saddle point and its spectrum.

Fig. 2 A focus of attachment and a focus of separation and their spectra.

Fig. 3 A node of attachment and a node of separation and their spectra.

Fig. 4 The worst case nonhyperbolic three-dimensional singular point and its spectrum.

Fig. 5 Construction of a three-dimensional half saddle and its spectrum.

Fig. 6 Separated stream surfaces and their developments.

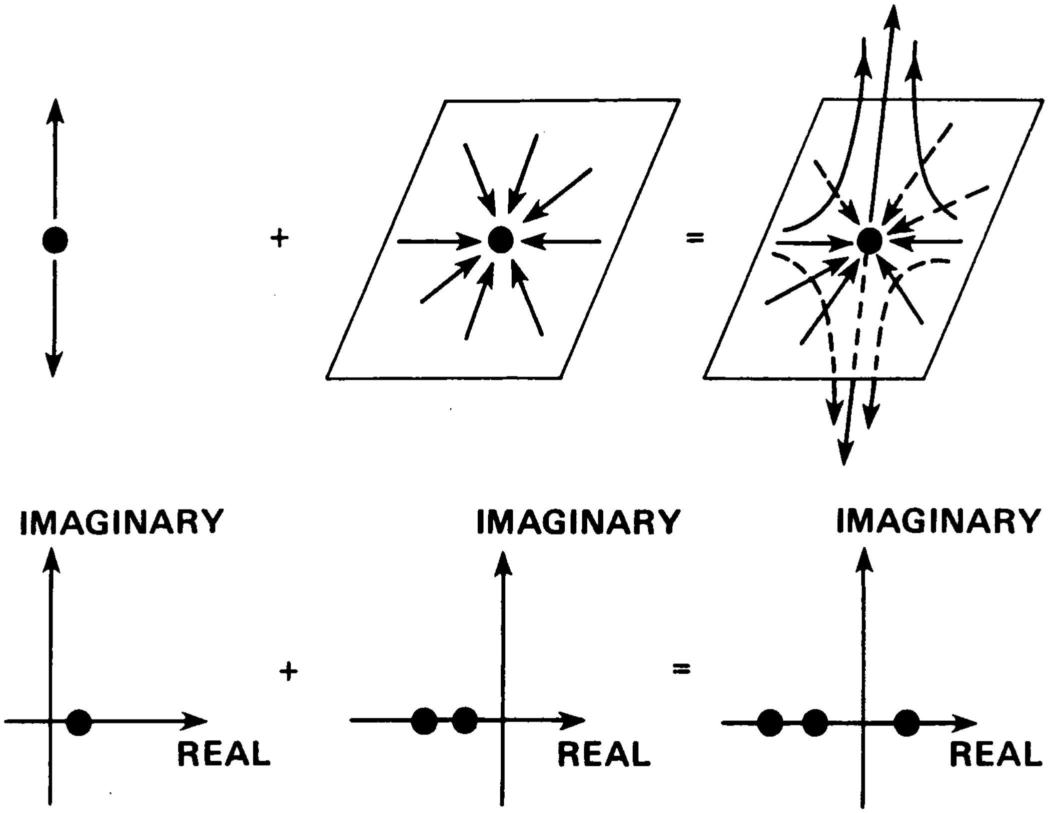
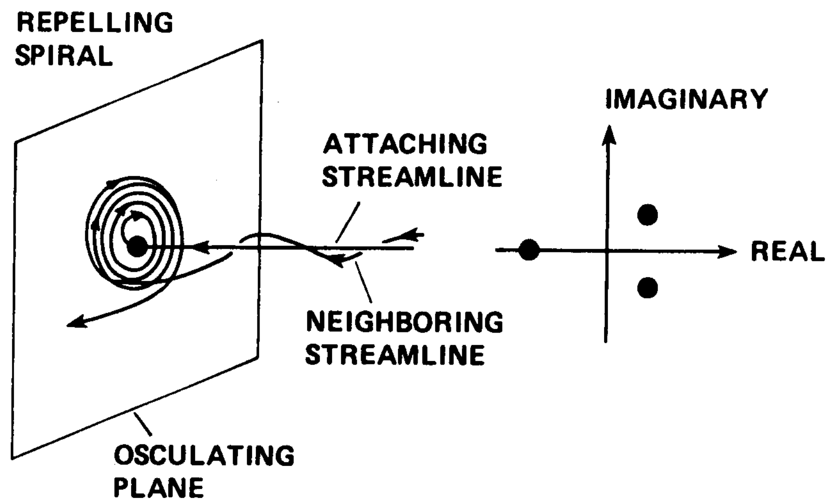
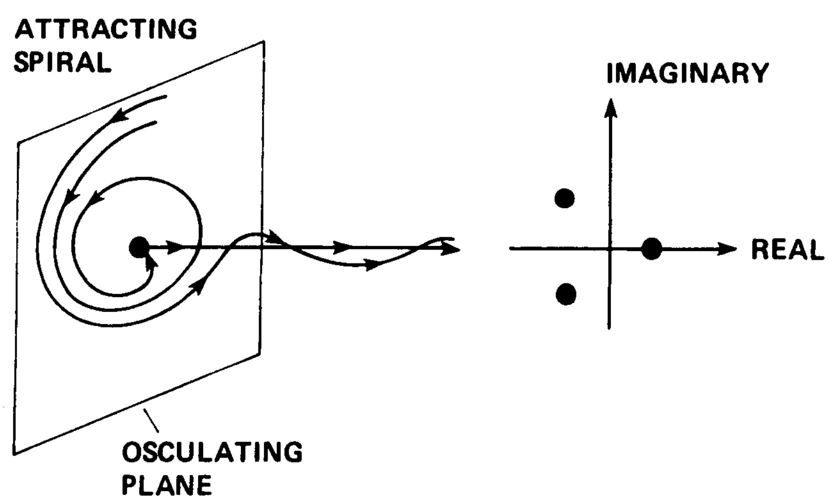


Fig. 1



A SADDLE OF INDEX 2



A SADDLE OF INDEX 1

Fig. 2

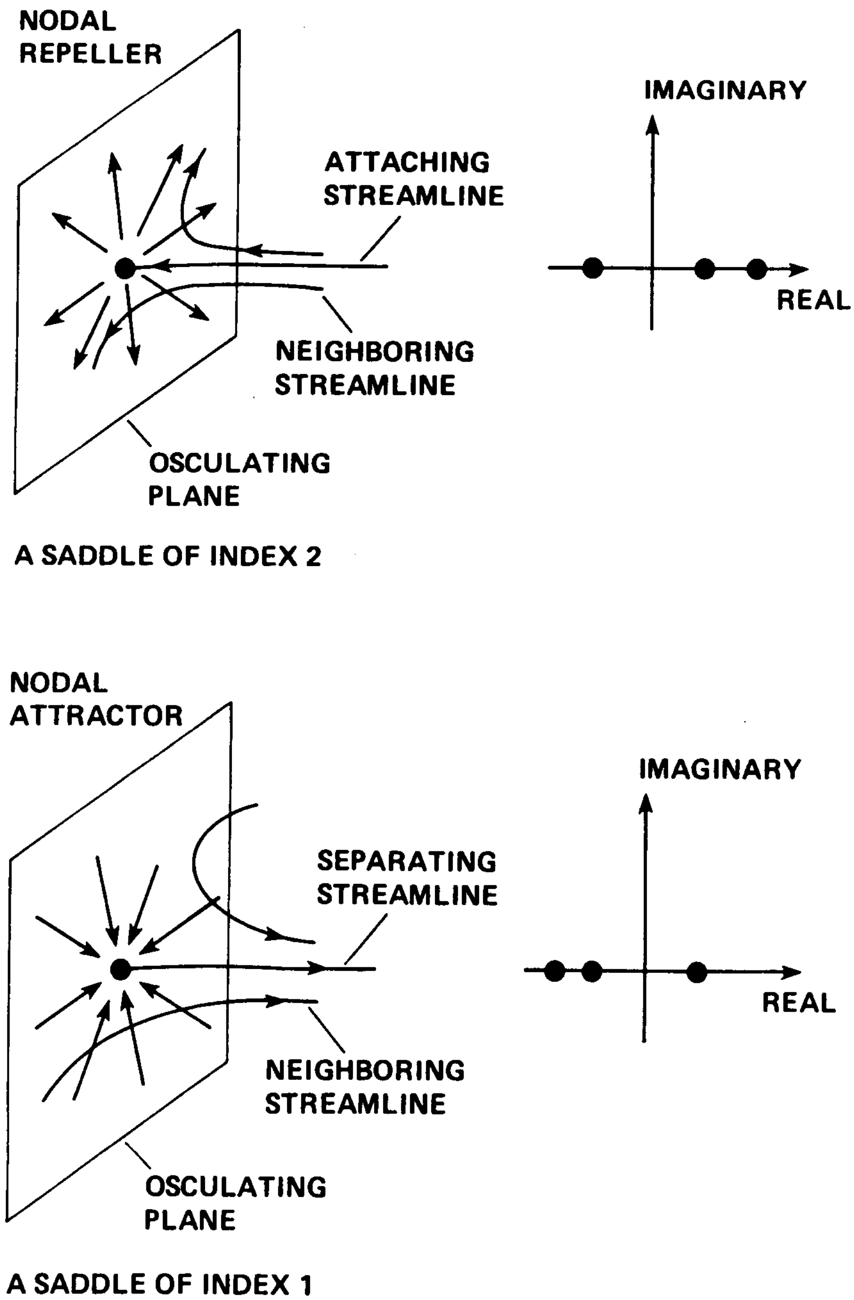

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Fig. 3

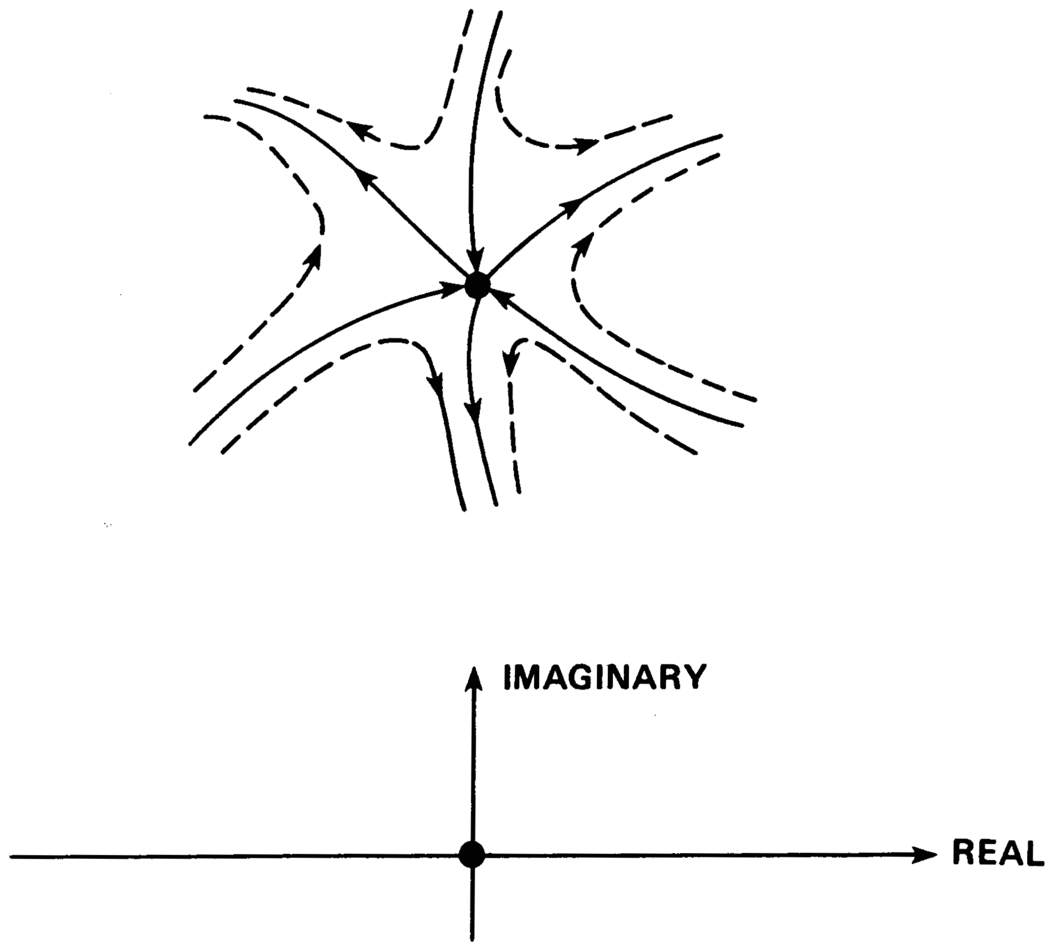


Fig. 4

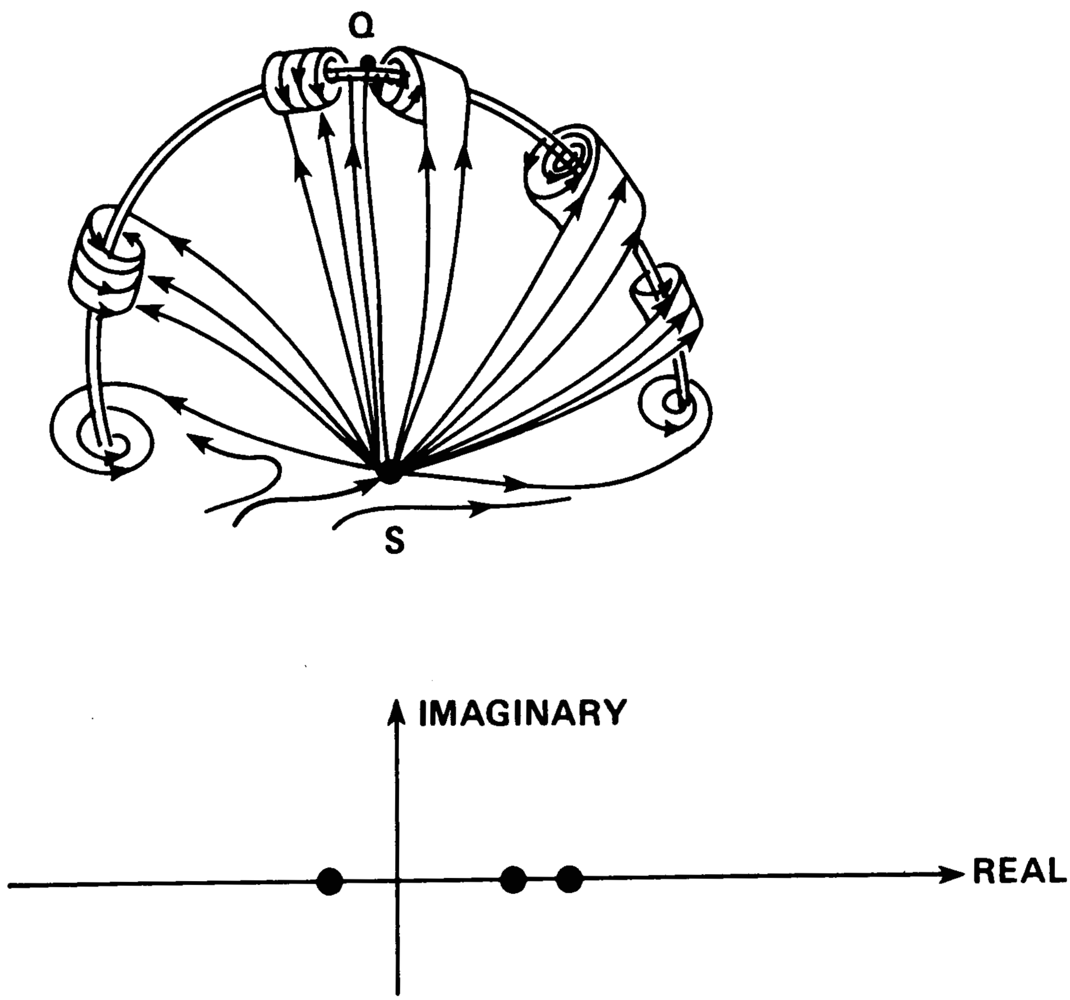
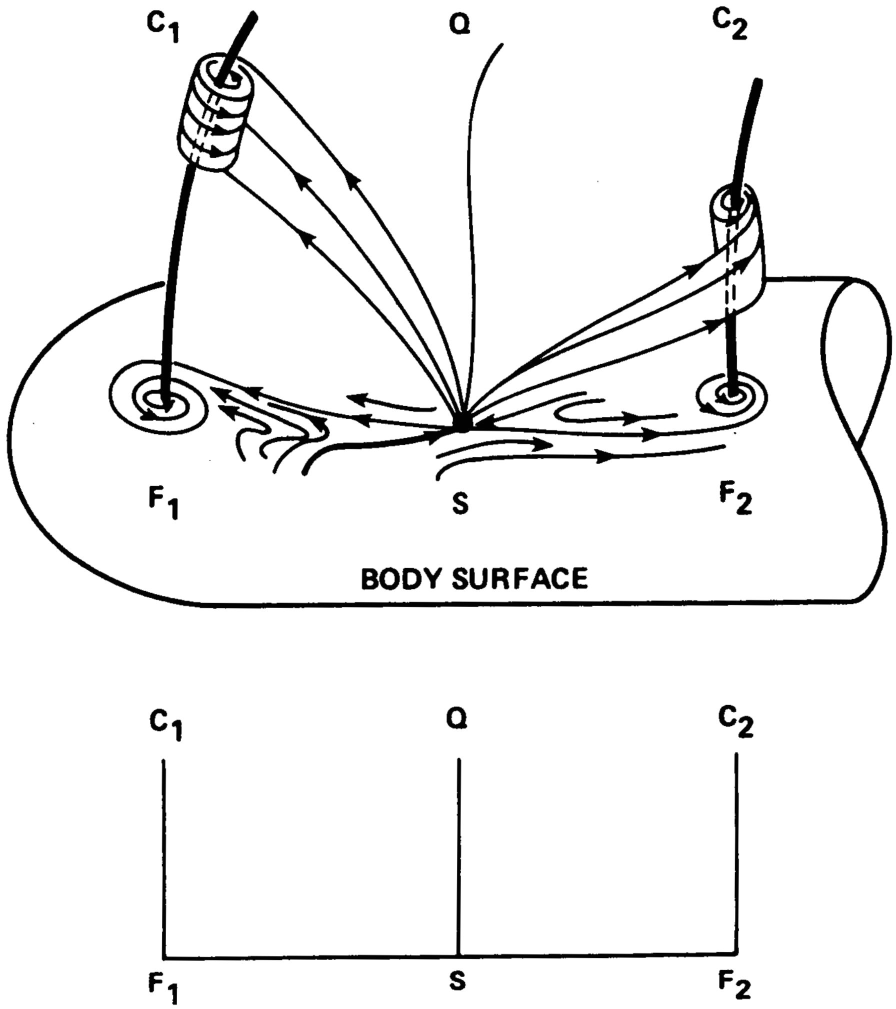


Fig. 5



Q IS A SADDLE OF INDEX ONE.
NOTICE THAT Q CAN BE AT INFINITY

Fig. 6

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