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Finite Element Simulation of Thunderstorm Electrodynamics in the Proximity of the Storm

Prepared by:	Michael Edward Baginski
Academic Rank:	Assistant Professor
University and Department:	Auburn University Department of Electrical Engineering
NASA/MSFC:	
Laboratory:	Structures and Dynamics
Division:	Earth Science and Application
Branch:	Remote Sensing
MSFC Colleague:	Dr. Hugh Christian Dr. Richard Blakeslee
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by

Michael Edward Baginski
Assistant Professor of Electrical Engineering
Auburn University
Auburn, Alabama

Abstract

At the Eighth International Conference on Atmospheric Electricity observations of electric field, Maxwell current density, and air conductivity over thunderstorms were presented by Blakeslee. The measurements were obtained using electric field mils and conductivity probes installed on a U2 aircraft as the aircraft passed approximately directly over an active thunderstorm at an altitude of 18-20 km. Accurate electrical observations of this type are rare and provide important information to those involved in numerically modeling a thunderstorm. A preliminary set of computer simulations based on this data have been conducted and are described in this paper. The simulations show good agreement with measurements and are used to infer the thundercloud's charging current and amount of charge exchanged per flash.

ACKNOWLEDGEMENTS

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This summer's research endeavors have lead to the submittal of two manuscripts (submitted to The Journal of Geophysical Research-*special issue* and Geophysical Research Letters), neither of which would have ever come about had it not been for my day to day interactions with Dr. Hugh Christian and Dr. Richard Blakeslee. My appreciation for the personal interest both of these scientists have taken in the research can not be stated strongly enough. I am looking forward to future research in the area of Atmospheric Electricity with Dr. Christian and Dr. Blakeslee

Introduction

The focus of this research is to numerically characterize the electrodynamic behavior above a thunderstorm at altitudes of 18-26 km and lateral distances out to 10 km. This range is selected in light of the recently available electric field and Maxwell current data obtained from U2 flights over an active thunderstorm (presented by Blakeslee *et al.*, at the International Conference on Atmospheric Electricity, Uppsala, Sweden). By comparing simulated behavior to measurements a more complete understanding of the thunderstorm's electrical behavior results, especially concerning the storms electrical sources and its possible influence on the global circuit.

This investigation is based on a finite element solution of the governing differential equations (the Maxwell Equations) and includes the thundercloud charging current and the charge rearrangement following intra cloud lightning [Baginski *et al.*, 1988]. A complete description of the computer code used and its operation is given by Baginski (1987). The scope of the present investigation does not include the atmosphere's response to the high current lightning transient (i.e., propagating electromagnetic energy induced by the lightning return stroke's current) but addresses the electrodynamics following the lightning column's cessation.

Historical Perspective

Since the 1950's several electrical models describing the interaction of thunderstorms with the atmosphere have been published. Holzer and Saxon (1952) have assumed concentrated charges in a dipole configuration with spatially varying conductivities to obtain temporally invariant field patterns in the lower atmosphere and ionosphere. The lightning return stroke, however, generates transients in the electric field pattern known as "field changes" [Uman, 1969]. Early workers attributed this temporal recovery to recharging within the thundercloud. Tamura (1955) is credited as the first to note that the surrounding atmosphere is also involved. He defined solutions based on the conservative field assumption (i.e., $\nabla \times E = 0$, $E \sim \exp(-t/T)$, $T = \varepsilon/\sigma$)

that depend on the conductivity at the point of observation. Kasmir (1959) constructed the first dynamic model of the thundercloud system using resistors, a capacitor, and a spark gap. His model connected a current generator, a capacitor, and a resistor in parallel to model the cloud ionospheric connections with the path to earth replaced by a resistor. More dynamic models began to follow. Anderson and Freier (1969) incorporated dynamic changes in the dipole structure with spatially varying conductivities. However, Anderson and Freier omit the total set of Maxwell's equations and a dynamic forcing current in their modeling--only the quasi-static relaxation is included. Additional transient solutions were developed based on the "monopole" model of C. T. R. Wilson (1916) by Illingworth (1972), Park and Dejnakarintra (1973), Greifinger and Greifinger (1976), and Holzworth and Chiu (1982). To date, there have been many more thunderstorm models presented, several of which solve the self consistent set of Maxwell's equations (Faraday's law of induction and Ampere's circuital law) with a high degree a numerical resolution. The errors inherent to numerical solutions of the transient event are now generally reducible to negligible levels; this advance is due mainly to the advent of the super computer (computational intensive codes are no longer a problem).

Since numerical models will (generally) solve the governing set of equations correctly for the specified input parameters, the focus is therefore to determine accurately the input conditions (e.g., "stiff" sources, boundary conditions, and conductivity) pertinent to the thundercloud's description. Thundercloud information of this type is based on experimental data; a complete description of which is unfortunately not presently available. The measurements presented by Blakeslee *et al.* (1988) provide a significant contribution to the existing data pool available and serve as the basis of the modeling described here.

Measurements

Vertical electric field and conductivity measurements were obtained using U2 borne field mils and conductivity probes above active thunderstorms. The plane flew above the thunderstorm at an altitude of approximately 18-20 km at a speed of $\sim$200 m/s. Measurements were

obtained as the plane passed directly over the storm with temporal resolutions of ~ 100 msec. A description of the experiment is given by Blakeslee *et al.* (this issue).

Excerpts of the measured data used for comparison here (Figure 1) represent typical schema of the transient signatures observed. It is the primary intent of this modeling effort to depict the relative magnitudes and temporal signatures observed, with the possibility of predicting the general electrical environment about the storm systems instrumented. The thunderstorms interaction with the Global Electric Circuit is a logical extension of the modeling and will be addressed in future work.

Modeling of the Thunderstorm

The thunderstorms electrical activity is sustained by a constant current generator that exists between upper and lower charge centers (6 and 10 km respectively, upper center positive). Intra cloud lightning, resulting from the accumulation of the generator charge, occurs at time intervals determined primarily by the charging current and amount of charge exchanged per flash. For the purposes of modeling, the effects of the constant current generator will be analyzed separately from that of the charge perturbation associated with intra cloud lightning. The resulting steady state and transient solutions will then be superimposed to determine the total electrical response of interest [Baginski, 1987].

Several implicit assumptions should be noted. The net amount of thundercloud charging caused by effects of charging current and discharging the thundercloud via intra cloud lightning is assumed to be zero (following the initial charge accumulation). This requires that the average amount of charge exchanged via intra cloud lightning equals that of the storm's charging generator. In the modeling a one ampere charging current is used. Charge accumulation at the charge centers continues until a breakdown field strength is obtained [Uman, 1969]. The resulting steady state field mapping at the time breakdown is reached is the steady state component of the relative field signatures of the solutions. Specific details of the spatial and temporal distribution of the thundercloud's charge used in the modeling are given in the following sections.

The computer code used in this modeling is based on an adaptation of a previously developed code that identified middle atmospheric and ionospheric lightning induced signatures [Baginski *et al.*, 1988]. The errors associated with spatial and temporal resolution of the earlier code's results were reduced to minimal levels. Since the simulations of interest to this study are at a significantly closer range and require temporal resolutions $\sim$100 msec (instrumental limit) , the spatial and temporal discretization used previously will be used here as well.

Charging Mechanisms

As previously described, the thunderstorm's electrical activity is sustained by charge separation which induces a net positive upper and negative lower charge center in a dipole configuration. The height of the charge centers is somewhat affected by seasonal and geographic effects. Heights of 10 km for the upper and 6 km for the lower charge center are not unreasonable [Chalmers, 1967] and will be used for the model [Baginski, 1987]. The current resulting from a intra cloud flash is responsible for the charge perturbation [Uman, 1969] simulated. The rate of deposition of lightning current is proportional to the time derivative of the charge perturbation [Uman, 1969]. Therefore, the total charge deposited at time <t> may be expressed as the integral of the lightning current in time:

$$Q_f(t) = \int_0^t I_{ic}(\tau)d\tau \qquad (1)$$

where $I_{ic}(\tau)$ = intra cloud lightning current
$Q_f(t)$ = total displaced charge

Sunde's (1968) lightning return stroke model is selected for this study. Sunde's model consists of two exponential terms and is relatively simple compared to some [Uman, 1969], but for the time frames of interest in this study (t > 100 msec), it includes the necessary temporal information required to predict the late time transient electromagnetic behavior [Sunde, 1968]. The intra cloud charge

perturbation may be expressed in terms of the temporal behavior of this current, as follows:

$$i(t) = I0(\exp(-at) - \exp(-bt)) \quad (2)$$

where i(t) = return stroke current (Sunde's model)

a = 10^4 seconds-1

b = 0.5×10^6

I0 = proportional to amount of charge displaced during return stroke

The temporal structure of the forced charge generator is given as:

$$d\ Q_f(t)/dt = I0(\exp(-at) - \exp(-bt)) \quad (3)$$

A cylindrical coordinate system is used in the model with symmetry assumed about the vertical (z) axis (Figure 2). The spatial structure of the deposited charge (transient and steady state) is given by a modified spherical Gaussian profile:

$$\pm\partial\rho_f(r,z,t)/\partial t = \pm(\partial(Q_f(t)/\partial t \pm \alpha)(f(r,z)) \quad (4)$$

$$f(r,z) = (\exp(-R/(2\lambda)))/(2\pi\lambda)^{1.5}$$

where λ = variance (λ = 4000 m^2 for simulations)

$$R = r^2 + (z - z')^2$$

z' = altitude of charge perturbation (6 or 10 km)

α = steady state charging current
($\pm$1 A used in the modeling)

The spatial distribution of the charge perturbation does not noticeably effect the electric field and Maxwell current density signatures far from its interior [Baginski *et al.*, 1988]; since transient phenomenology exterior to the cloud is of interest here a certain degree of freedom exists in the specification of the distribution.

Geometry of the Region

The region selected (Figure 2) is contained within a perfectly conducting right circular cylinder with a radius of 80 km and height of 110 km. The earth's surface is modeled electrically as a perfect conductor (lower plate). Typical values of 10^{-3} to 10^{-2} mhos/meter [Volland, 1984] are given for the earth's conductivity while 10^{-14} to 10^{-13} mhos/meter is the usual range of the adjacent atmosphere's conductivity. This is a difference of more than 11 orders of magnitude, making the earth's surface appear (electrically) as a perfect conductor with respect to the atmosphere. The simulations of interest were found to be insensitive to increases in either the vertical (110 km) or the radial (80 km) limits. The Hall and Pederson components of the conductivity (present above $\sim$70 km) are neglected in the formulation [Baginski *et al.*, 1988].

The Maxwell Equations

From the Maxwell Equations a single equation is developed in which the electric field is dependent on the source charge and current densities as follows [Holzworth and Chiu, 1982]:

$$\nabla \times \nabla \times E = -\mu_0(\sigma \partial^2 E/\partial t + \partial J/\partial t + \varepsilon_0 \partial^2 E/\partial t^2) \qquad (5)$$

$$\nabla \rho/\varepsilon_0 = \nabla^2 E - \mu_0(\sigma \partial E/\partial t + \partial J_s/\partial t + \varepsilon \partial^2 E/\partial t^2) \qquad (6)$$

where J_s = source current density associated with intra cloud stroke current, is neglected in the simulations

ρ = charge density

The continuity equation is derived by taking the divergence of the Maxwell current density:

$$0 = \nabla\cdot\nabla\times H = \nabla\cdot(\sigma E + \varepsilon_o \partial E/\partial t + J_s) \quad (7)$$

$$0 = \rho\sigma/\varepsilon_o + \nabla\sigma\cdot E + \partial\rho/\partial t + G_s \quad (8)$$

where $\nabla\cdot J_s = G_s$ = source of charge perturbation
(deposition of intra cloud lightning current)

Equations 6 and 8 describe the electrodynamic response of the atmosphere to the assumed thundercloud charge and current configuration. A charge perturbation of 1 C and steady state thundercloud charging current of 1 A are used in the modeling. Intra cloud lightning is simulated by superimposing the transient signatures resulting from total charge perturbations of + 1 C at z'= 6 km and - 1 C at z'= 10 km. The conductivity profile used in the previously referenced high altitude simulations [Baginski *et al.* 1988] is slightly modified here to depict the observed conductivity [Blakeslee *et al.*, 1988].

Results

Transient Signatures

Figure 3 identifies the vertical electric field signatures resulting from a positive charge perturbation at 6 km (modeled cloud to ground lightning) and charge perturbations at 6 km and 10 km with the upper charge sensed negative (intra cloud lightning). The magnitude of the waveforms decrease as radial distance is increased with the temporal structure showing only slight changes. This is expected considering previous numerical studies [Baginski, 1987] and observations [Blakeslee *et al.*, 1988]. The simulated intra cloud lightning waveforms are constructed by superimposing the results of perturbations at 6 and 10 km with the perturbation at 10 km sensed negative.

The corresponding vertical Maxwell current density transients are shown in Figure 4. Following the onset of the transient there is an extremely rapid temporal decay in magnitude out to $\sim$1 msec followed by a much slower decay [Hale and Baginski, 1987]. The temporal

characteristics of these signatures are very similar for the respective perturbations with magnitude decreasing as radial distance is increased.

Simulations of U2 Measurements

In order to accurately simulate the observed electric field and Maxwell current density patterns the assumed thundercloud charging current and intra cloud charge perturbation are scaled appropriately. The measured electric field and Maxwell current density waveforms shown in Figure 1 are assumed to occur as the plane passes approximately above the thunderstorm [Blakeslee *et al.*, 1988]. There is some uncertainty as to the plane's exact location relative to the thunderstorm and therefore reasonable estimates of the scaling parameters must be made.

After a series of simulations were conducted with a wide range of scaling variance the following parameters were selected:

Cloud charging current $\sim$5 Amperes
Amount of charge exchanged during
intra cloud lightning $\sim$10-20 Coulomb
Altitude of observation = 18 km
Time delay prior to 1st intra cloud lightning = 3 seconds
Time delay prior to 2nd intra cloud lightning = 15 seconds

The resulting scaled simulations are shown in Figure 5. There is good overall agreement between the scaled simulations and measurements. The major differences are in the amount of "noise" associated with the measured data. This may be due in part to concurrent thundercloud discharges many orders of magnitude less than the primary discharge. In addition, the simulated Maxwell current density waveforms do not contain large negative transients with temporal durations of $\sim$500 msec of as shown in the measurements in Figure 1. This may partially be a result of an actual flash consisting of multiple strokes and partially an artifact arising from insufficiently resolving $\varepsilon_0 \partial E/\partial t$ during lightning discharges in the derivation of the Maxwell current density from E.

Conclusions

The results of the study add credence to the belief that accurate computer simulations of the atmosphere's response to thunderstorms are possible. A comparison of the (unscaled) simulated and measured Maxwell current densities and electric fields showed good agreement temporally. Agreement in magnitude was attained by incorporating scaling factors in the simulated results. Since the simulation's governing set of differential equations are linear (the Maxwell Equations) , the thunderstorms charging current and net amount of charge exchanged during intra cloud or cloud to ground lightning may be inferred using magnitude scaling.

Future research in electrically modeling a thunderstorm will benefit from a significantly larger data base. Several thunderstorm campaigns that would provide this type of information are being considered for the early 1990's.

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Figure Captions

1. Measurements made during U2 flights over an active thunderstorm presented by Blakeslee at the International Symposium on Atmospheric Electricity. The vertical electric field, Maxwell current density, conduction current density and displacement current density are shown in from top to bottom respectively

2. Geometry of model simulation

3. Simulated vertical electric field transients at an altitude of 18 km for intra cloud lightning and a charge perturbation centered at 6 km. A total charge of 1 coulomb is exchanged.

4. Simulated vertical Maxwell current density transients at an altitude of 18 km for intra cloud lightning and a charge perturbation centered at 6 km. A total charge of 1 coulomb is exchanged.

5. Simulated vertical electric field and Maxwell current density waveforms using scaling factors at an altitude of 18 km for intra cloud lightning. A total charge of 1 coulomb is exchanged with cloud charging current ~ 5 Amperes, amount of charge exchanged during intra cloud lightning ~ 10-20 coulombs.

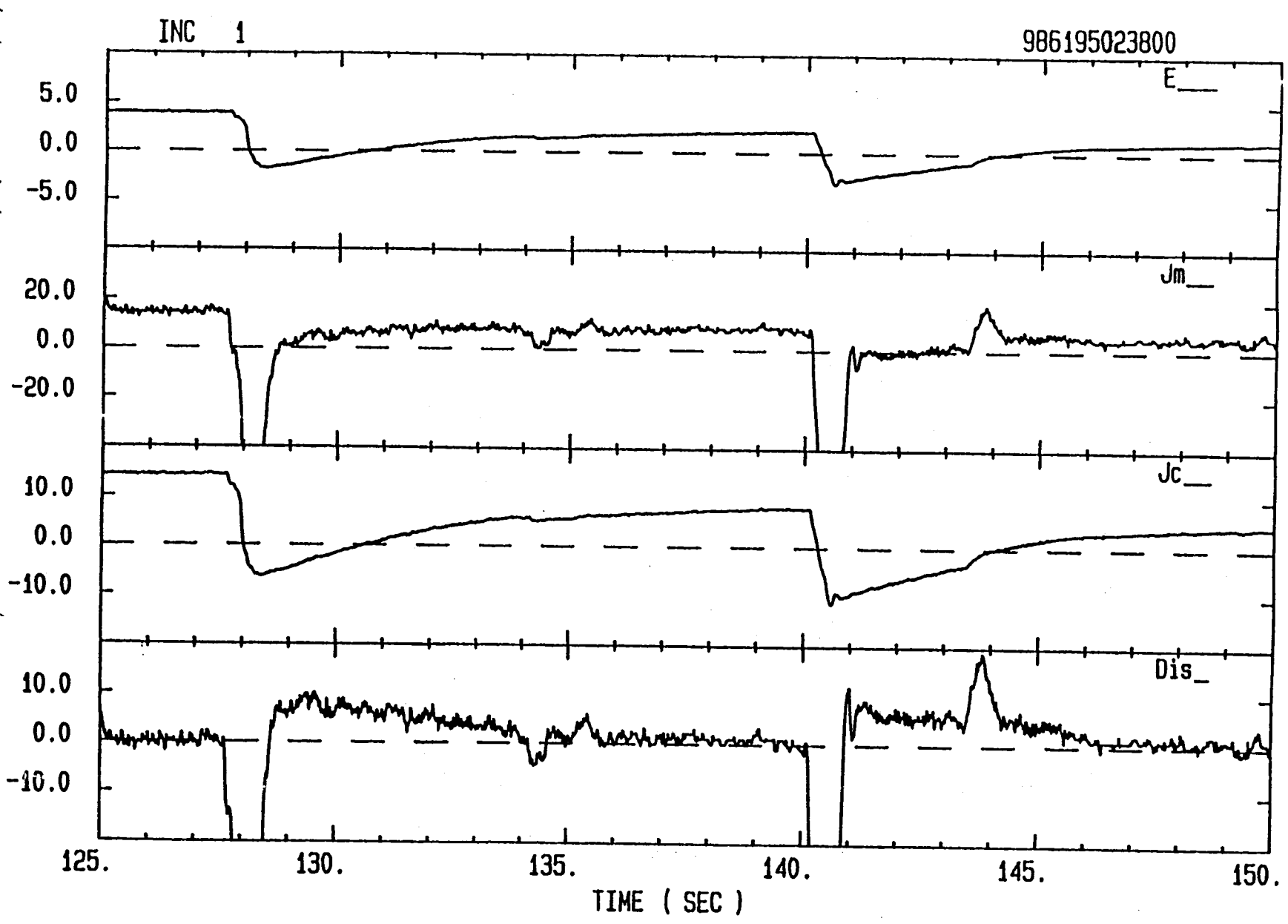
INC 1
986195023800
E
Jm
Jc
Dis
5.0
0.0
-5.0
20.0
0.0
-20.0
10.0
0.0
-10.0
10.0
0.0
-10.0
125.
130.
135.
140.
145.
150.
TIME (SEC)
Electric field (kilovolts/m)
Maxwell (nanoamps/m*2)
Conduction (nanoamps/m*2)
Displacement (nanoamps/m*2)

Figure 1

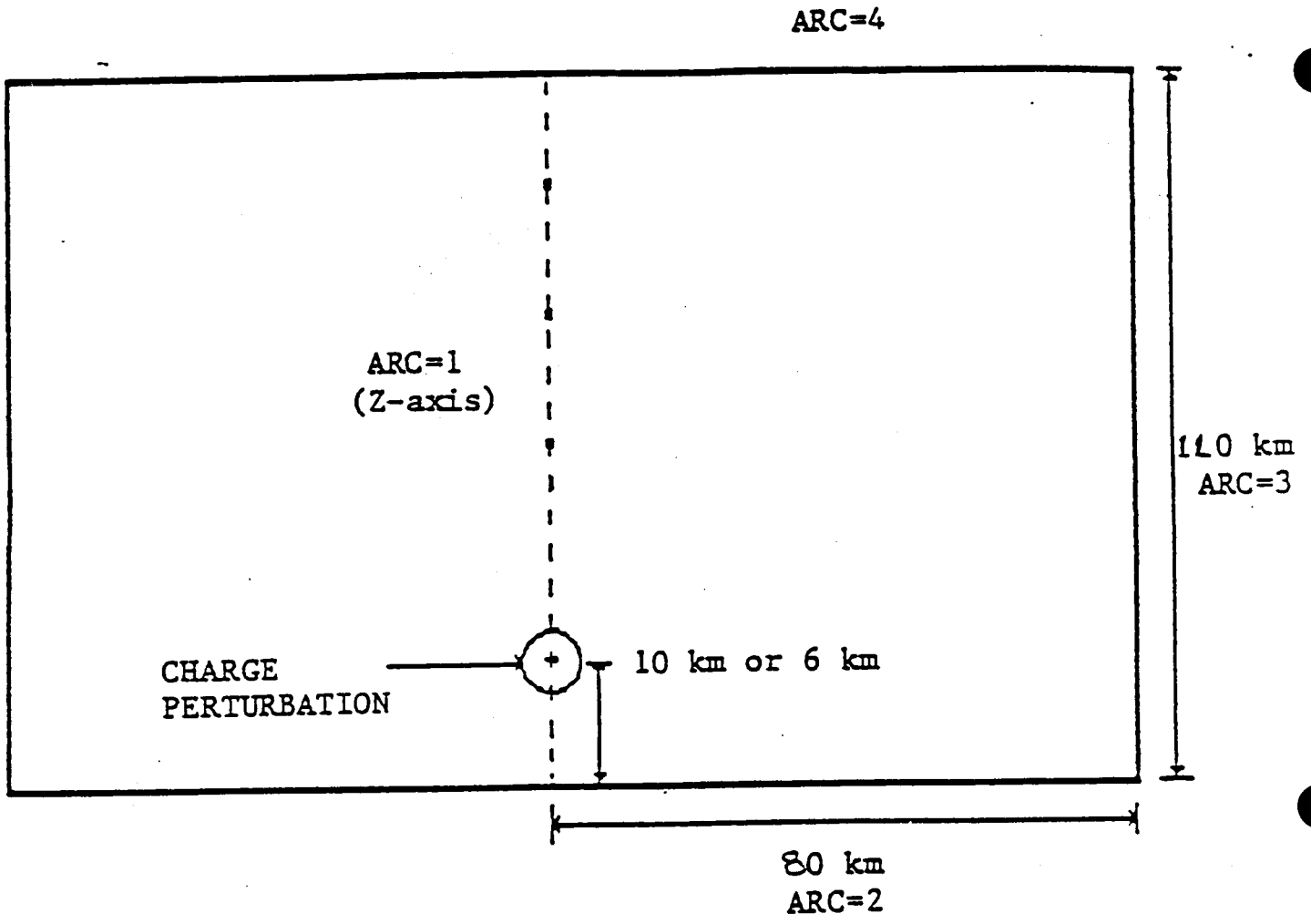
ARC=4
ARC=1
(Z-axis)
110 km
ARC=3
CHARGE
PERTURBATION
10 km or 6 km
80 km
ARC=2

Figure 2.

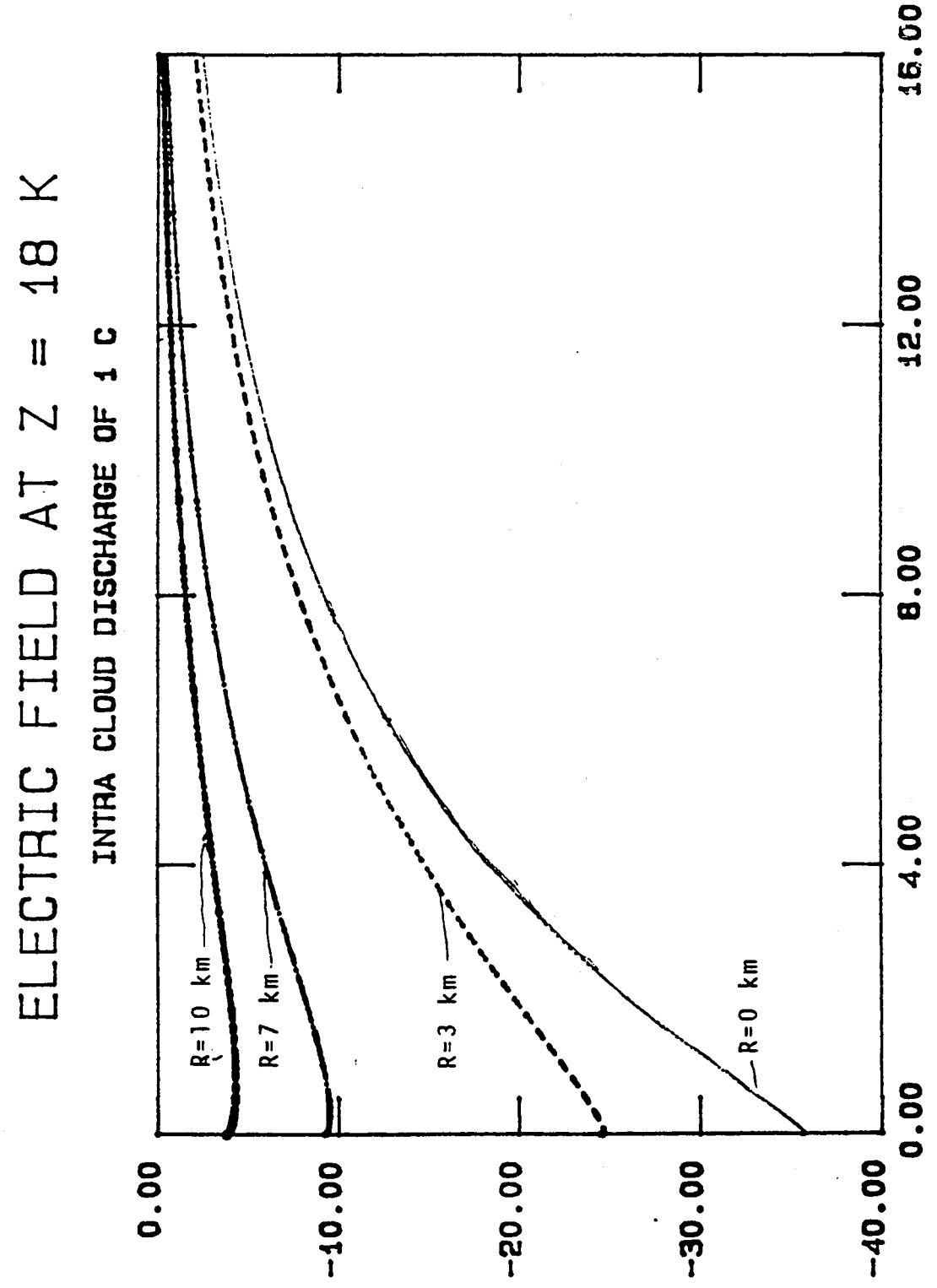
ELECTRIC FIELD AT Z = 18 K
INTRA CLOUD DISCHARGE OF 1 C
R=10 km
R=7 km
R=3 km
R=0 km
0.00
-10.00
-20.00
-30.00
-40.00
0.00
4.00
8.00
12.00
16.00
TIME (SECONDS)
ELECTRIC FIELD (V/m)

Figure 3a

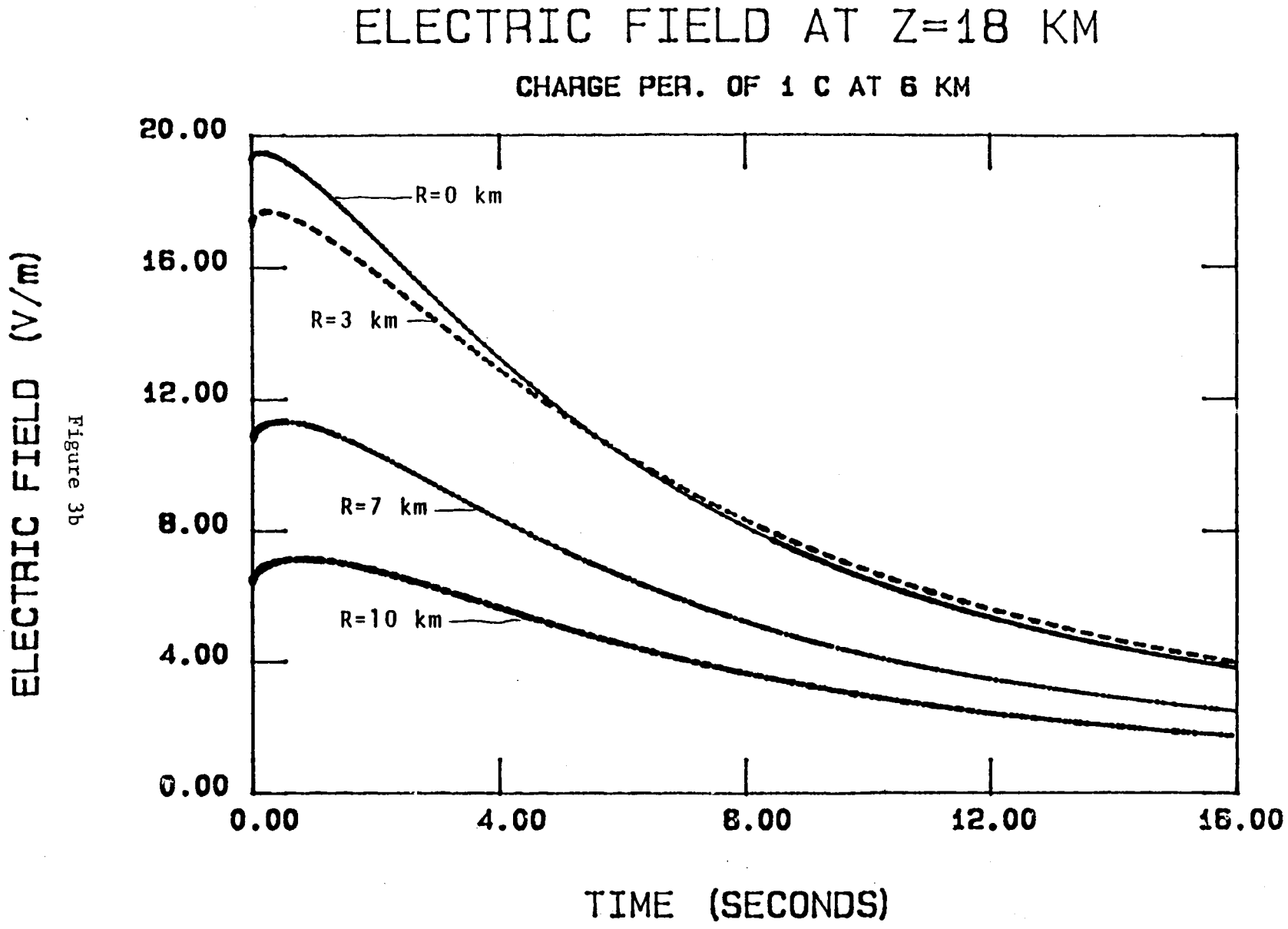
ELECTRIC FIELD AT Z=18 KM
CHARGE PER. OF 1 C AT 6 KM
20.00
16.00
12.00
8.00
4.00
0.00
R=0 km
R=3 km
R=7 km
R=10 km
0.00
4.00
8.00
12.00
16.00
TIME (SECONDS)
ELECTRIC FIELD (V/m)

Figure 3b

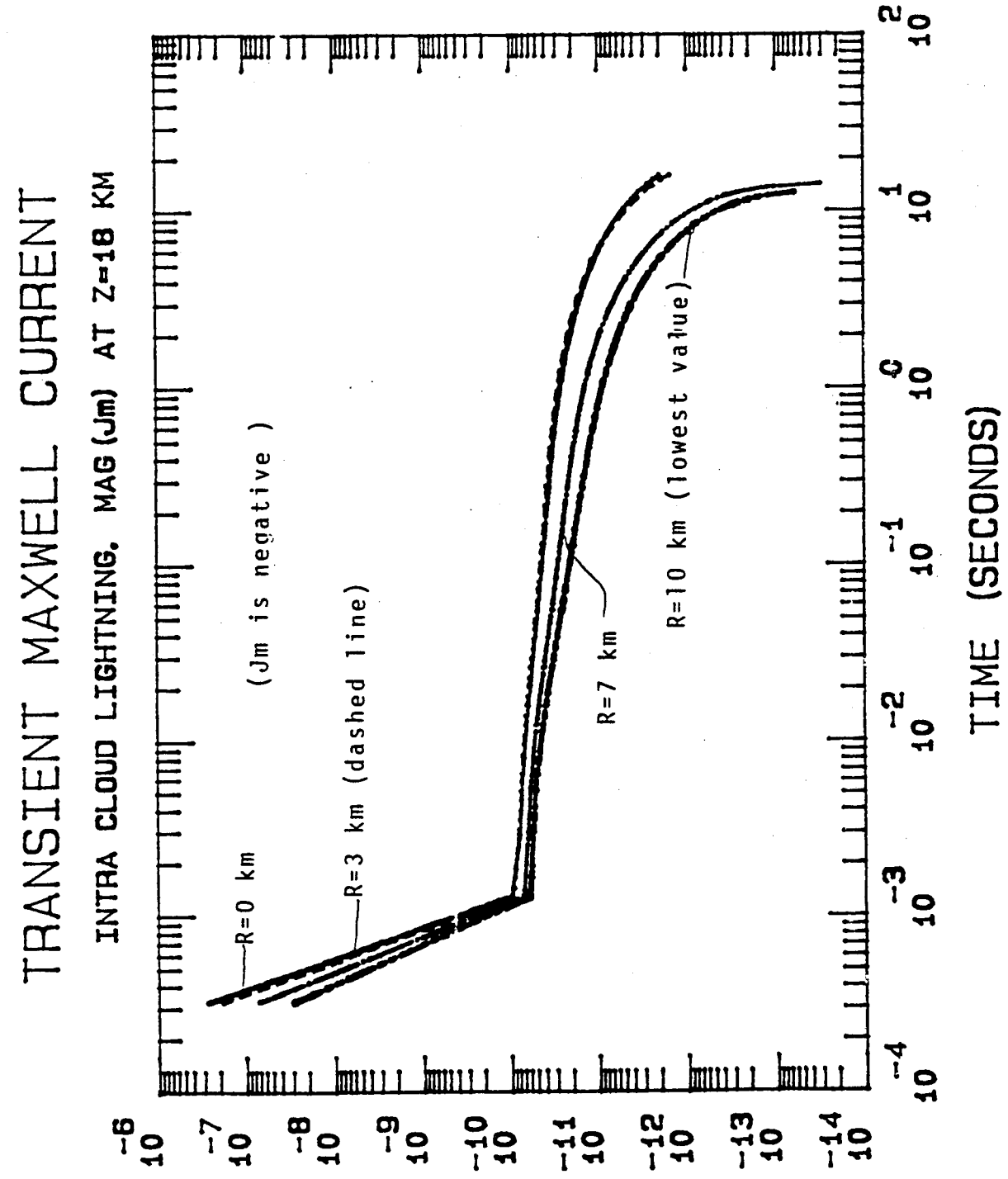
TRANSIENT MAXWELL CURRENT
INTRA CLOUD LIGHTNING, MAG (Jm) AT Z=18 KM
(Jm is negative)
R=0 km
R=3 km (dashed line)
R=7 km
R=10 km (lowest value)
AMPERES/m**2
TIME (SECONDS)
10^-6
10^-7
10^-8
10^-9
10^-10
10^-11
10^-12
10^-13
10^-14
10^-4
10^-3
10^-2
10^-1
10^0
10^1
10^2

Figure 4a

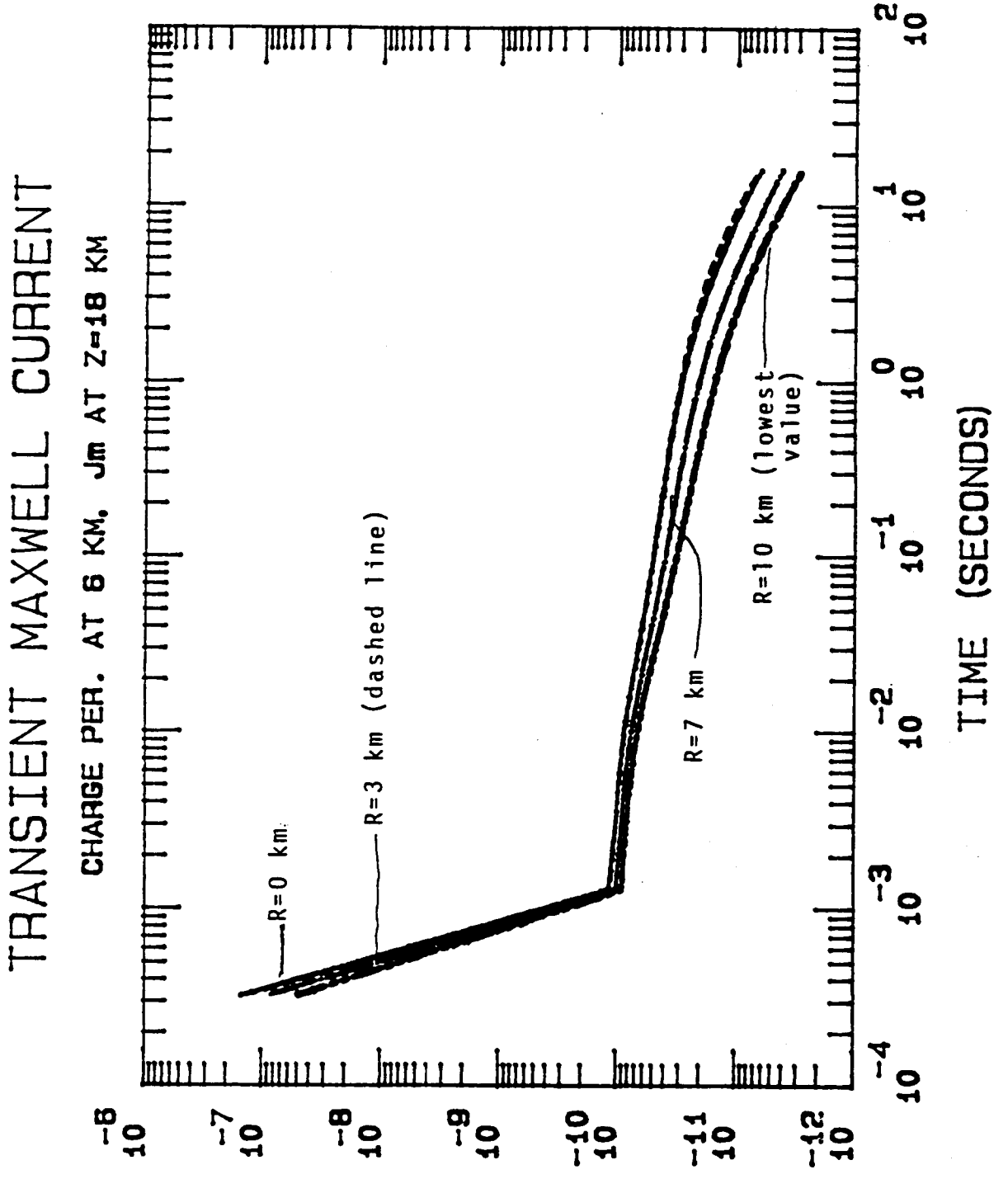
TRANSIENT MAXWELL CURRENT
CHARGE PER. AT 6 KM, Jm AT Z=18 KM
R=0 km
R=3 km (dashed line)
R=7 km
R=10 km (lowest value)
TIME (SECONDS)
AMPERES/m*2

Figure 4B

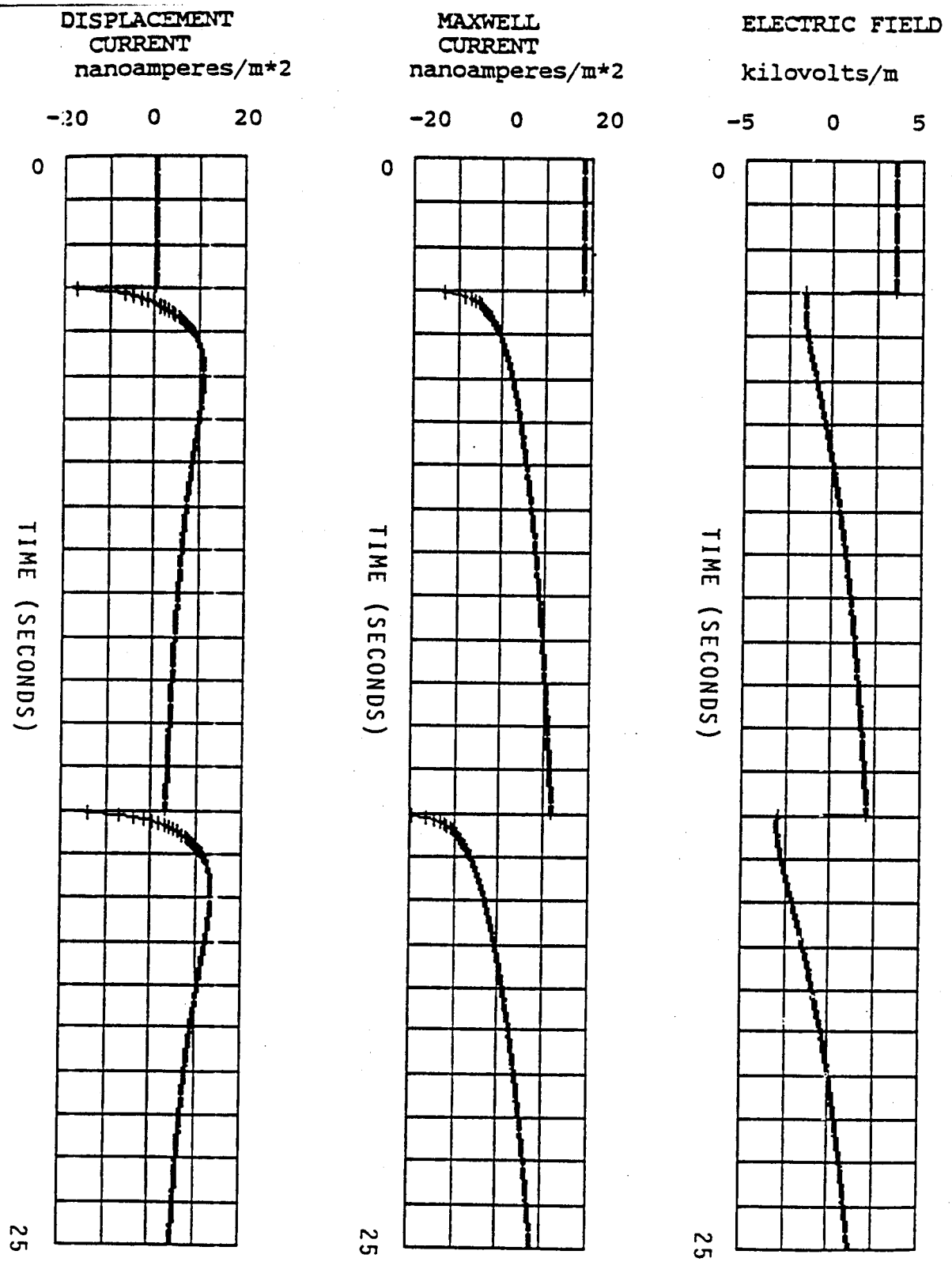
DISPLACEMENT CURRENT
nanoamperes/m*2
-20
0
20
0
25
TIME (SECONDS)
MAXWELL CURRENT
nanoamperes/m*2
-20
0
20
0
25
TIME (SECONDS)
ELECTRIC FIELD
kilovolts/m
-5
0
5
0
25
TIME (SECONDS)

Figure 5