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SSME PROPULSION PERFORMANCE RECONSTRUCTION TECHNIQUES

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ABSTRACT

In view of the complex flight operation of the Space Shuttle propulsion system together with an expected launch rate increase, the flight performance reconstruction process needs to be performed by automated computer programs. These programs must have the capability to quickly and reliably determine the true behavior of the various components of the propulsion system. For the flight reconstruction, measured values from the solid rocket motors, liquid engines, and trajectory are appraised through the Kalman filter technique to identify the most likely flight propulsion performance.

A more detailed data collection program for the single SSME engine captive test firing evaluation is scheduled for startup in September of 1988. Engine performance evaluation for the captive test firing requires a reconstruction process that is similar to the process that is used for the flight reconstruction. This paper describes analytical tools that may be used to reconstruct a propulsion system's true performance under flight and/or test conditions.

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1. INTRODUCTION

During the summer of 1987, Rogers Engineering and Associates (REA) submitted a technical report to Marshall Space Flight Center. This report summarized the results of the Propulsion Estimation Development Verification performed by REA under contract NAS8-36152. During that contract period, REA modified an existing program, which was developed under a previous contract, to include improved models of the Solid Rocket Booster, the Space Shuttle Main Engine (SSME) gain coefficient model, the vehicle trajectory using quaternions, and an improved Kalman filter based on the U-D factorized algorithm. In this report, this modified program will be identified by the symbols PROG1.

Under a current contract, REA has proposed to design, evaluate, and refine a model for a single SSME that does not include the influences of other Space Shuttle components. The refined SSME model is expected to permit the collection of high quality measurements and to provide an improved quality of SSME performance estimation. REA's single SSME model should be available in the Fall of 1988, and this report refers to that program by the symbols PROG2.

Propulsion performance estimation procedures applied in programs PROG1 and PROG2 are basically the same. Each requires the assumption that a correct model of the Space Shuttle's propulsion system exists and the model involves two main equations. They are an equation that models the dynamics of a state vector and a second equation that defines the relationship between an observation vector and the state vector. Also, the engine performance estimation consists of using collected data on the observation vector to obtain a corresponding estimate of the state vector. This state vector estimate is called an estimate or a reconstruction of the engine's performance.

The state vector estimate is based on the satisfaction of a statistical optimization criterion. Furthermore, data quality determination and the quality of the estimated state vector are judged through an application of statistics tools. Hence, in order for the Propulsion Laboratory to properly apply and correctly interpret the results of programs PROG1 and PROG2, an intuitive review of estimation procedures utilized by these programs is needed.

Therefore, the objectives of this paper are to:

1. Examine the assumptions and limitations of the Kalman filtering technique for the intended application to flight performance reconstruction of propulsion systems and single engine performance reconstruction based on static testing data.

2. Select specific topics from the area of statistics and probability for discussion and better understanding with EP55 personnel.

3. Identify some additional applications of contractor's programs.

Throughout this paper, underlined capital letters are used to denote vectors, capital letters denote matrices, and the identity matrix is denoted by the capital letter I. All vectors are of the column type and the transpose of any matrix or vector is denoted by using the letter T at the superscript position. The letter E denotes the expectation operator, $N(\underline{U},\Sigma)$ denotes the multivariate normal probability distribution with mean vector $\underline{U}$ and covariance matrix Σ. The caret symbol "^" written directly above a scalar or vector denotes a statistical estimator of that scalar or vector.

2. NOTATION

For any Space Shuttle flight or single SSME test, let $\underline{Z}_t$ denote the observed values of vector $\underline{Z}$ at time t. Each component of $\underline{Z}_t$ represents a relevant measurable output of the Space Shuttle Propulsion System or a relevant navigation measurement. For example, the components of $\underline{Z}_t$ may be oxygen pressurant flow, fuel volume flow, hydrogen pressurant flow, fuel flow pressure, fuel flow temperature, and chamber pressure whenever $\underline{Z}_t$ represents the output of a single SSME static test. For an actual flight of the Space Shuttle, $\underline{Z}_t$ will contain all of the aforementioned measurements for each of the three main engines plus additional components that represent navigational measurements and one measurement for each of the two Solid Rocket Motors (SRM). For the actual flight, Rogers (1987) lists 35 components for vector $\underline{Z}_t$ and 71 components for state vector $\underline{X}_t$ where $\underline{X}_t$ is defined in the next paragraph. However, for the single engine static test setup, vectors $\underline{Z}_t$ and $\underline{X}_t$ have fewer components because the SRM's and navigational components become inactive.

Vector $\underline{X}_t$ is a state vector of parameters to be estimated at time t. It is assumed that the observation vector $\underline{Z}_t$ is a function of the state vector $\underline{X}_t$. That is,

$$\underline{Z}_t = \underline{h}\,(\underline{X}_t, t) + \underline{V}_t \tag{2.1}$$

where $\underline{h}$ is some function and $\underline{V}_t \sim N(\underline{O}, R_t)$. State vector $\underline{X}_t$ is known to change with respect to time according to the equation

$$\dot{\underline{X}}_t = \underline{f}\,(\underline{X}_t, t) + \underline{W}_t \tag{2.2}$$

where $\underline{f}$ is some function, $\underline{W}_t \sim N(\underline{O},Q_t)$ and $E[\underline{W}_t \underline{V}_t^T] = 0$.

If it is assumed that $\underline{f}$ and $\underline{h}$ in equations (2.1) and (2.2) are linear, then numerical procedures allow us to transform these equations into the form

$$\underline{X}_k = \Phi(k,k-1)\, \underline{X}_{k-1} + \underline{W}_k \tag{2.3}$$

$$\underline{Z}_k = H_k\, \underline{X}_k + \underline{V}_k \tag{2.4}$$

where k represents discrete values of t, $\underline{W}_k \sim N(\underline{O},Q_k)$, $\underline{V}_k \sim N(\underline{O},R_k)$ and

$$E(\underline{W}_k\, \underline{V}_k^T) = E(\underline{W}_i\, \underline{W}_j^T) = E(\underline{V}_i\, \underline{V}_j^T) = 0 \quad ,$$

provided $i \neq j$. The symbol $\Phi(k,k-1)$ denotes the transition matrix that connects vector $\underline{X}_k$ to $\underline{X}_{k-1}$.

After a Space Shuttle flight has taken place or after a static test, the post flight/test reconstruction procedure seeks to use the observed values of $\underline{Z}_k$ and equations (2.3) and (2.4) to reconstruct (estimate) the vector $\underline{X}_k$ so that the estimation error is minimized. The reader should see Gelb (1974) for a review of minimization procedures. The estimated value of $\underline{X}_k$ is denoted by $\hat{\underline{X}}_k$.

3. THE KALMAN FILTER SETUP

Equations (2.3) and (2.4) along with assumed properties of vectors $\underline{W}_k$ and $\underline{V}_k$ allow for the development of the below Kalman filtering process. During the process, two types of estimators of $\underline{X}_k$ are possible for each k. A notation used by Gelb (1974) allows the two estimators to be distinguished. That notation is

$\hat{\underline{X}}_k(-)$ = the estimate of $\underline{X}_k$ using all observations up to and including observation (k-1).

$P_k(-)$ = the estimate of the error covariance matrix for $\underline{X}_k$ using all observations up to and including observation (k-1).

$\hat{\underline{X}}_k^{(+)}$ = the estimate of $\underline{X}_k$ using all observations up to and including observation k.

$P_k^{(+)}$ = the estimate of the error covariance matrix for $\underline{X}_k$ using all observations up to and including observation k.

The values of $\hat{\underline{X}}_k^{(-)}$ and $P_k^{(-)}$ are obtained by using equation (2.3). The $\hat{\underline{X}}_k^{(-)}$ vector is often called the projected ahead value of $\underline{X}_k$ and $P_k^{(-)}$ is the projected ahead variance. Vector $\hat{\underline{X}}_k^{(+)}$ is called the updated estimate of $\underline{X}_k$ and P_k^+ denotes the updated variance. Computation equations (3.1) through (3.5) describe the discrete Kalman filtering process.

$$\hat{\underline{X}}_k^{(-)} = \Phi(k,k-1)\, \hat{\underline{X}}_{k-1}^{(+)} \tag{3.1}$$

$$P_k^{(-)} = \Phi(k,k-1)\, P_{k-1}^{(+)}\, \Phi^T(k,k-1) + Q_{k-1} \tag{3.2}$$

$$K_k = P_k^{(-)} H_k^T (H_k P_k^{(-)} H_k + R_k)^{-1} \tag{3.3}$$

$$\hat{\underline{X}}_k^{(+)} = \hat{X}_k^{(-)} + K_k (\underline{Z}_k - H_k \hat{\underline{X}}_k^{(-)}) \tag{3.4}$$

$$P_k^+ = (I - K_k H_k) P_k^{(-)} \quad . \tag{3.5}$$

To start the computation procedure, initial estimates $\hat{\underline{X}}_0^{(-)}$ and $P_0^{(-)}$ are needed and are usually determined by the users of equations (3.1) through (3.5).

The linear system described in equations (2.3) and (2.4) is essential to the development of the discrete Kalman filtering process given in equations (3.1) through (3.5). Therefore, when functions $\underline{f}$ and $\underline{h}$ are nonlinear, as they are in the case of the flight/test reconstruction model, vector $\underline{X}_k$ has to be estimated through a linearization process. Two different linearization procedures are described in the next paragraph. The first procedure is called the linearized filter and the other is known as the extended Kalman filter.

Let $\underline{X}_k^*$ be some known vector such that $\dot{\underline{X}}_k^* = \underline{f}\,(\underline{X}_k^*,k)$. Often times $\underline{X}_k^*$ is called a reference solution or a reference nominal trajectory. That is, $\underline{X}_k^*$ is a known solution to equation (2.2) where the influence of $\underline{W}_t$ is not considered. If a discrete solution to equation (2.2) is $\underline{X}_k$ where $\underline{X}_k = \underline{X}_k^* + \Delta\underline{X}$, then the linearization process may be used to estimate $\Delta\underline{X}$. Here, we see that for any $\Delta\underline{X}$ and for any k value of t, a first degree Taylor series approximation of functions (2.1) and (2.2) may be determined by

$$\dot{\underline{X}}_k + \Delta\dot{\underline{X}} \approx \underline{f}\,(\underline{X}_k^*,k) + \left[\frac{\partial \underline{f}}{\partial \underline{X}}\right]_{\underline{X}=\underline{X}_k^*} \Delta\underline{X} + \underline{W}_k \tag{3.6}$$

and

$$\underline{Z}_k \approx \underline{h}\,(\underline{X}_k^*,k) + \left[\frac{\partial \underline{h}}{\partial \underline{X}}\right]_{\underline{X}=\underline{X}_k^*} \Delta\underline{X} + \underline{V}_k \quad . \tag{3.7}$$

Since $\dot{\underline{X}}_k = \underline{f}\,(\underline{X}_k,k)$, equations (3.6) and (3.7) reduce to

$$\Delta\dot{\underline{X}} = \left[\frac{\partial \underline{f}}{\partial \underline{X}}\right]_{\underline{X}=\underline{X}_k^*} \Delta\underline{X} + \underline{W}_k \tag{3.8}$$

and

$$Z_k - \underline{h}\,(\underline{X}_k^*,k) = \left[\frac{\partial \underline{h}}{\partial \underline{X}}\right]_{\underline{X}=\underline{X}_k^*} \Delta\underline{X} + V_k \quad . \tag{3.9}$$

Equation (3.8) is called the linearized dynamics equation and (3.9) is the linearized measurement equation. These two equations are linear and are equivalent to the linear equations (2.3) and (2.4). Hence, for each discrete time k, $\Delta\underline{X}_k$ can be estimated and error covariance matrices can be determined. The state vector estimate at time k is then given by

$$\hat{\underline{X}}_k^+ = \underline{X}_k^* + \Delta\hat{\underline{X}}_k^{(+)} \tag{3.10}$$

where $\Delta\hat{\underline{X}}_k^+$ is computed by applying the Kalman filtering process to equations (3.8) and (3.9). When each $\underline{X}_{k+1}^*$ $(k = 0,1,\ldots,T)$ is known prior to the beginning of the application of the Kalman filtering process to equations (3.8) and (3.9), the estimate in equation (3.10) is called the linearized Kalman estimate. When each X_{k+1}^* is determined from the previous estimate of $\underline{X}_k$, i.e., $\underline{X}_{k+1}^* = \hat{\underline{X}}_k^+$, the estimate in equation (3.10) is called the extended Kalman filter estimate.

4. FAILURE DETECTION

The current repalcement cost of a single SSME is about $50 million. This cost, combined with analyses of data from static test failures and shutdowns, suggest that there is a need for a more advanced state test failure detection system for the SSME. A 1985 report by Taniguchi emphasizes the importance of SSME failure detection improvements and identified some possible analytic designs for a failure detection system. Each of Taniguchi's failure detection designs assumed that a correct model of the single SSME exists. Each design also required the application of a statistics test as a decision tool. Therefore, a natural extension of REA's SSME static test model would be into the area of failure detection during static testing. REA's static test model is already undergoing refinements and is expected to be available to Marshall Space Flight Center by the Fall of 1988. Also, according to Taniguchi (1985), data is available on 1200 static test firings of a single SSME. A brief overview of a failure detection setup is provided in the next paragraph.

Assume that equations (2.3) and (2.4) are modified to yield

$$\underline{X}_k = \Phi(k,k-1)\,\underline{X}_{k-1} + A_1\,(k,k-1)\,\underline{b}_k + \underline{W}_k \tag{4.1}$$

$$\underline{Z}_k = H_k\,\underline{X}_k + A_2(k)\,\underline{b}_k + \underline{V}_k \tag{5.2}$$

where vectors $\underline{b}_k$ are called biased vectors. If biases are regarded as additional states with dynamics such that

$$\underline{b}_{k+1} = \underline{b}_k$$

then equations (4.1) and (4.2) may be transformed into the form

$$\underline{Y}_k = \begin{bmatrix} \Phi(k,k-1) & A_1(k,k-1) \\ 0 & I \end{bmatrix} \underline{Y}_{k-1} + \begin{bmatrix} I \\ 0 \end{bmatrix} \underline{W}_k \tag{4.3}$$

and

$$\underline{Z}_k = [H_k \quad A_2(k)]\ Y_k + \underline{V}_k \tag{4.4}$$

where

$$\underline{Y}_k = \begin{bmatrix} \underline{X}_k \\ \underline{b}_k \end{bmatrix} .$$

If Kalman filtering is applied to equations (4.3) and (4.4), the bias vector $\underline{b}_k$ is estimated along with the other components of the state vector.

If no failure has occurred at time k, then it is intuitively reasoned that $\underline{b}_k = \underline{O}$. Of course, the judgement as to whether $\underline{b}_k$ is really zero is determined by a multivariate statistical test.

For additional details on bias vector estimation, the reader should consult Friedland (1983).

5. CONCLUSION

The assumptions and limitations of the Kalman filtering process have been discussed with NASA personnel. These discussions included relevant topics from the area of statistics. In particular, it was pointed out in this paper that equations (2.3) and (2.4) are the essential ingredients for the discrete variable reconstruction. Hence, if functions $\underline{f}$ and $\underline{h}$ of equations (2.1) and (2.2) are nonlinear, $\underline{X}_k$ can be estimated by linearizing $\underline{f}$ and $\underline{h}$ about some nominal vector solution $\underline{X}_k{}^*$.

This report also points out that REA's SSME model may be modified to form a failure detection procedure. The reader should be reminded that the intuitive failure detection approach outlined from equations (4.3) and (4.4) is based on an assumption that $\underline{f}$ and $\underline{h}$ of equations (2.1) and (2.2) are linear functions. Since functions $\underline{f}$ and $\underline{h}$ are generally nonlinear for SSME applications, it is worthwhile to investigate the effectiveness of bias estimation techniques as an SSME static test failure detection device.

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