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THE ROLE OF THE TURBULENT PRANDTL NUMBER  
IN TURBINE BLADE HEAT TRANSFER PREDICTION

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ABSTRACT

A study was undertaken to improve the prediction of external (gas-to-blade) heat transfer coefficients in gas turbine engines. The study specifically investigated the effects of improved eddy diffusivity of heat modeling in the turbulence model. A two-dimensional boundary layer code, STAN5, was selected and modified by incorporating several different turbulent Prandtl number models. Results indicated that slight effects were attributable to the modified turbulence model. Boundary layer character appeared to be much more significant.

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## INTRODUCTION

The thermal aspects of blade design is one of the more difficult engineering tasks facing a designer of any modern gas turbine engine. Thermal (and many times aerodynamic) analysis procedures currently available to designers have deficiencies that do not permit achievement of design goals without expensive experimental development programs. For example, the external (gas-to-blade) heat transfer coefficient still eludes satisfactory prediction using computational fluid dynamic codes. Even if consideration is restricted to the nominally two-dimensional midspan region of a turbine blade, prediction is still unsatisfactory. The reasons for the unsatisfactory prediction capability of the codes are complex but ultimately lie in the fundamental concepts and models used to define the fluid dynamic and heat transfer behavior. Without question, the complex gas turbine engine environment pushes current models to their limit. Thus, there exists a need for an improved design approach making use of codes with sufficiently improved turbulence modeling.

The work presented here was undertaken to improve the prediction of gas-to-blade heat transfer coefficients. Specifically, it investigates the effect of modeling the eddy diffusivity of heat via several turbulent Prandtl number models published in the literature.

### OBJECTIVE

The objective of this study was to improve the computational prediction of the external (gas-to-blade) heat transfer coefficient for gas turbine engine applications. Such an improvement would reduce and perhaps eliminate the expensive experimental iterations that current engine designers must endure. The end result would impact engine design in a very positive way.

## PROCEDURE

### CODE SELECTION

Current gas turbine engine design practice is to use a two-dimensional boundary layer analysis to calculate the gas-to-blade heat transfer coefficients. Certainly any computational method which does not solve the full time-dependent Navier-Stokes and energy equations cannot be expected to be universally valid over the entire range of circumstances governed by these equations. However, there are solutions from reduced sets of these equations that are valid for a subset of problems. Such is the case here where it is implied that the flow field immediately adjacent to the surface of an airfoil in typical gas turbine geometries can be analytically modeled using boundary layer equations.

Perhaps the most familiar and widely used boundary layer method is a finite difference technique which relies on algebraic relations for defining turbulence quantities. A very common design tool of this type is STAN5, a code developed by Crawford and Kays [1] and later modified by NASA Lewis Research Center [2]. For boundary layer flow with heat transfer, STAN5 involves the solution of two governing partial differential equations using the numerical scheme of Patankar and Spalding [3]. Turbulence closure is obtained using eddy diffusivity concepts. The STAN5 code has received wide attention because of its careful development, flexibility, and adequate documentation. For those very reasons, STAN5 was selected to be used for this study.

The STAN5 code allows many parameters to be adjusted and it was felt that one set of parameters should be selected and held constant throughout the test so that the influence of the turbulent Prandtl number models could be determined. Of course it was desirable to have the parameters describe a true gas turbine engine flow field as closely as possible.

Reviewing published data for flow over turbine blades, it was decided that a fully turbulent boundary layer on both the suction and pressure surfaces of the blade would be assumed. This is perhaps a point of contention but it was adopted for a couple of reasons. First, many transition models have been tried in the past with limited success [4]. Secondly, a typical gas turbine engine environment flow field has a high free stream turbulence level. Also, any

boundary layer character change (such as relaminarization) that might occur would be modeled through the pressure gradient implicitly contained in the input data.

STAN5 has two eddy diffusivity models, the Prandtl mixing length hypothesis (MLH) and the higher order turbulent kinetic energy (TKE) concept. For this study, the MLH method was selected based on the past attention given to it - especially in gas turbine engine studies. Also the choice of the MLH model can be considered a practical selection. The detailed experimental data required to realistically tune higher order turbulence models for gas turbine engine applications are quite scarce. On the other hand, the global-type boundary layer data normally used to develop lower order turbulence models (such as the MLH) are more common.

Another consideration was whether to assume the blade surface was a flat plate or to include the blade curvature into the analysis. A curvature model was available in STAN5 but previous studies [4] have revealed that using the curvature model did not significantly effect the heat transfer results. Also, as pointed out earlier, current design practice is to assume the flat plate. Therefore a flat plate model of the blade was assumed in this study.

Finally, all specifiable constants in STAN5 were set equal to values suggested by Crawford and Kays.

#### TURBULENT PRANDTL NUMBER MODELS

As part of the eddy diffusivity concept used in the STAN5 turbulence model, a parameter called the turbulent Prandtl number ( $Pr_t$ ) is introduced. This dimensionless parameter links the eddy diffusivity of momentum ( $\epsilon_m$ ) and the eddy diffusivity of heat ( $\epsilon_h$ ). The turbulent Prandtl number concept mirrors the classical laminar approach where the momentum and thermal transport mechanisms are related by the molecular Prandtl number ( $Pr$ ). By definition, the turbulent Prandtl number is:

$$Pr_t = \epsilon_m / \epsilon_h .$$

Typically,  $\epsilon_m$  is solved for using a mixing length hypothesis or the turbulent kinetic energy concept. Then  $\epsilon_h$  is determined assuming a  $Pr_t = 1.0$  (Reynolds Analogy) or some other constant value ( $0.8 \leq Pr_t \leq 0.9$  has received wide acceptance for gas flows). This whole premise of using a constant  $Pr_t$  totally ignores the heat transport mechanism. Direct modeling of  $\epsilon_h$  would provide a much more realistic

picture of the flow physics. The individual  $\epsilon_m$  and  $\epsilon_h$  models could then be combined to yield a  $Pr_t$  model which contains much more information.

A number of attempts have been made to predict turbulent Prandtl numbers containing  $\epsilon_h$  modeling through highly idealized analyses. Many of the resultant models are fundamentally based on an idea first suggested by Jenkins [5]. He hypothesized that a turbulent eddy, while moving transverse to the mean direction of flow, may lose heat at a different rate than it loses momentum. His analysis assumed that the eddy lost heat by simple molecular conduction and lost momentum by the action of viscous shear. He also modeled the eddy as a spherical element of fluid with a radius equal to the size of the mixing length.

Turbulent Prandtl number models for this study were obtained after an extensive review of the literature. To establish a baseline, a constant turbulent Prandtl number model using  $Pr_t = 0.86$  was used. Then four different models were selected in order to test the validity of the various assumptions contained in them. Each of the models is briefly described in the following. For a complete description of each method the reader is referred to the specified reference.

#### 0 Crawford and Kays [1]

Reflecting on Jenkin's hypothesis, Crawford and Kays suggested that the turbulent eddies transfer momentum by the action of impact and pressure forces and that viscous forces are not involved. The success of the mixing-length theory in which viscosity is not a variable would suggest this to be the case. Conversely, there is no mechanism other than molecular conduction whereby heat can be transferred from an eddy. This means that the transport mechanisms must be different. Their final turbulent number model is in terms of the molecular Prandtl number and the eddy diffusivity of momentum.

#### 0 Thomas [6]

Feeling that a fresh approach was needed, Thomas developed a turbulent transport model employing an elementary surface renewal and penetration model. He based his model on the idea of diffusive penetration of eddies through a film, (in this case the viscous sublayer), intermittently renewed by fluid from the region of turbulent flow. He further assumed that the molecular transport is predominate during the time the fluid elements are in the vicinity of the surface. The analogy between heat and



momentum transfer is a result of the renewal mechanism and the resultant model is a function of molecular Prandtl number, eddy diffusivity of momentum and a normalized distance from the surface ( $y^+$ ).

0 Cebeci [7]

Cebeci developed a model for the turbulent Prandtl number based on Stokes flow considerations. Expressions for both  $\epsilon_m$  and  $\epsilon_h$  were obtained by following Van Driest's damped mixing-length representation of the Stokes flow viscous sublayer. It differs from the other models in that his  $\epsilon_h$  expression provides a continuous temperature distribution across the boundary layer and also accounts for any pressure gradient. The model expresses the turbulent Prandtl number in terms of the molecular Prandtl number and a normalized distance from the surface ( $y^+$ ).

0 Tyldesley and Silver [8]

The approach taken by Tyldesley and Silver is quite different from the approaches discussed previously. They abandoned the mixing-length concept and investigated the transport properties of a turbulent fluid by using a simple model to represent the detailed fluid behavior. In the model fluid behavior is attributed to the motions of fluid entities of varying size, shape, and velocity. Their analysis enables them to find expressions for the eddy coefficients of momentum and heat in terms of properties of the turbulence. For example, their model indicates that the turbulent Prandtl number is a function of not only molecular Prandtl number and Reynolds number but also turbulence intensity as well.

#### EXPERIMENTAL DATA

In order to evaluate the predictive capabilities of any computational method, it needs to be compared with experimental data. Many well documented heat transfer studies have been performed and there is a fair amount of reliable data available. This study used the work performed at Detroit Diesel Allison by Hylton et al. [4]. The main reason for selecting this data was that in addition to presenting their experimental results, the authors also provided the necessary STAN5 input data for their experimental configuration. This eliminated the need to develop the required input data thus allowing more time to be devoted to the task at hand.

The experimental program of Hylton et al. studied flow

through a turbine cascade. The cascade contained three blades that were characteristic of a first-stage turbine. The blades were designated as "C3X" airfoils and the profile of one is shown in Figure 1. The center blade in the cascade was instrumented and provided the aerodynamic and heat transfer data. The operating conditions for the data set used for comparison in this study are given below.

Inlet Total Temperature:	1460°F
Inlet Mach Number:	0.16
Inlet Reynolds Number:	640,000
Free-stream Turbulence Level:	6.55%
Blade Surface Temperature:	1182°F

Note: The inlet Reynolds number is based on true chord.

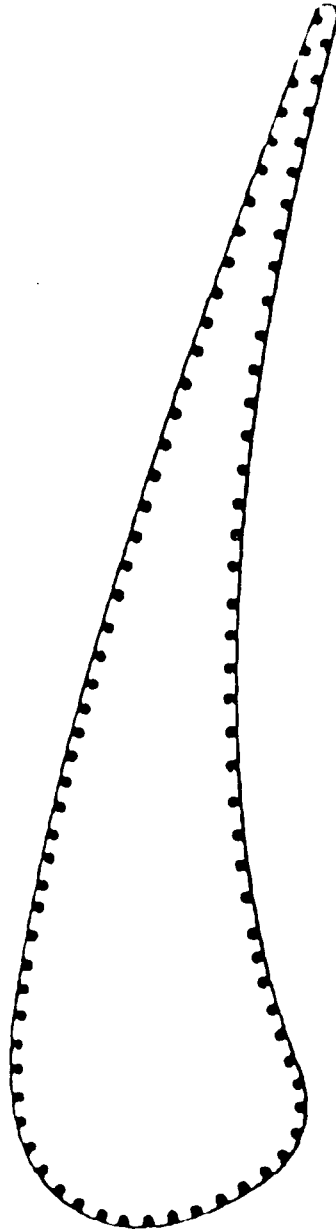


Figure 1. Profile of C3X Turbine Blade

## RESULTS

The heat transfer coefficient predictions produced by STAN5 combined with the various turbulent Prandtl number models can be seen in Figures 2 and 3. For presentation, the heat transfer coefficient ( $H$ ) has been normalized by a reference value ( $H_0$ ) of 200 BTU/Hr/ft<sup>2</sup>/°F and the distance along the blade surface ( $S$ ) is normalized by the total surface arc length (ARC). Also shown with the predictions is the experimental data of Hylton et al.

Figure 2 shows the distribution of the heat transfer coefficient on the blade's suction surface. It can be seen that for the first 20 percent of the surface the models yield identical predictions. A large favorable pressure gradient exists in this region and the predictions are likely representative of a boundary layer forced to relaminarize. This is confirmed by observing the laminar solution which is also shown on the figure. At a surface distance of about 20 percent the flow appears to transition. All the models predict the transition start but then have limited success downstream. The constant  $Pr_t$  assumption and the model of Cebeci overpredict the transition but recover nicely downstream. The model of Thomas predicts transition very well but then immediately underpredicts the heat transfer coefficient along the rest of the blade surface. The Crawford and Kays model yields results similar to Thomas' model but does not underpredict as severely downstream.

Distributions on the blade's pressure surface are shown in Figure 3. It is obvious that none of the turbulent Prandtl number models adequately predict the distribution represented by the experimental data. The laminar solution is also shown for comparison and reveals that the turbulent Prandtl number predictions do not vary from the laminar solution until a surface distance of 60 percent. After that the models predict what appears to be a transition from laminar to turbulent flow with a corresponding increase in heat transfer coefficient.

The turbulent Prandtl number models did not appear to significantly alter the heat transfer predictions and at times, there was little difference between the laminar and turbulent flow assumptions. This suggests that the driving force behind this phenomenon is the character of the boundary layer. One of the assumptions made in this study was that of treating the airfoil as a flat plate and using

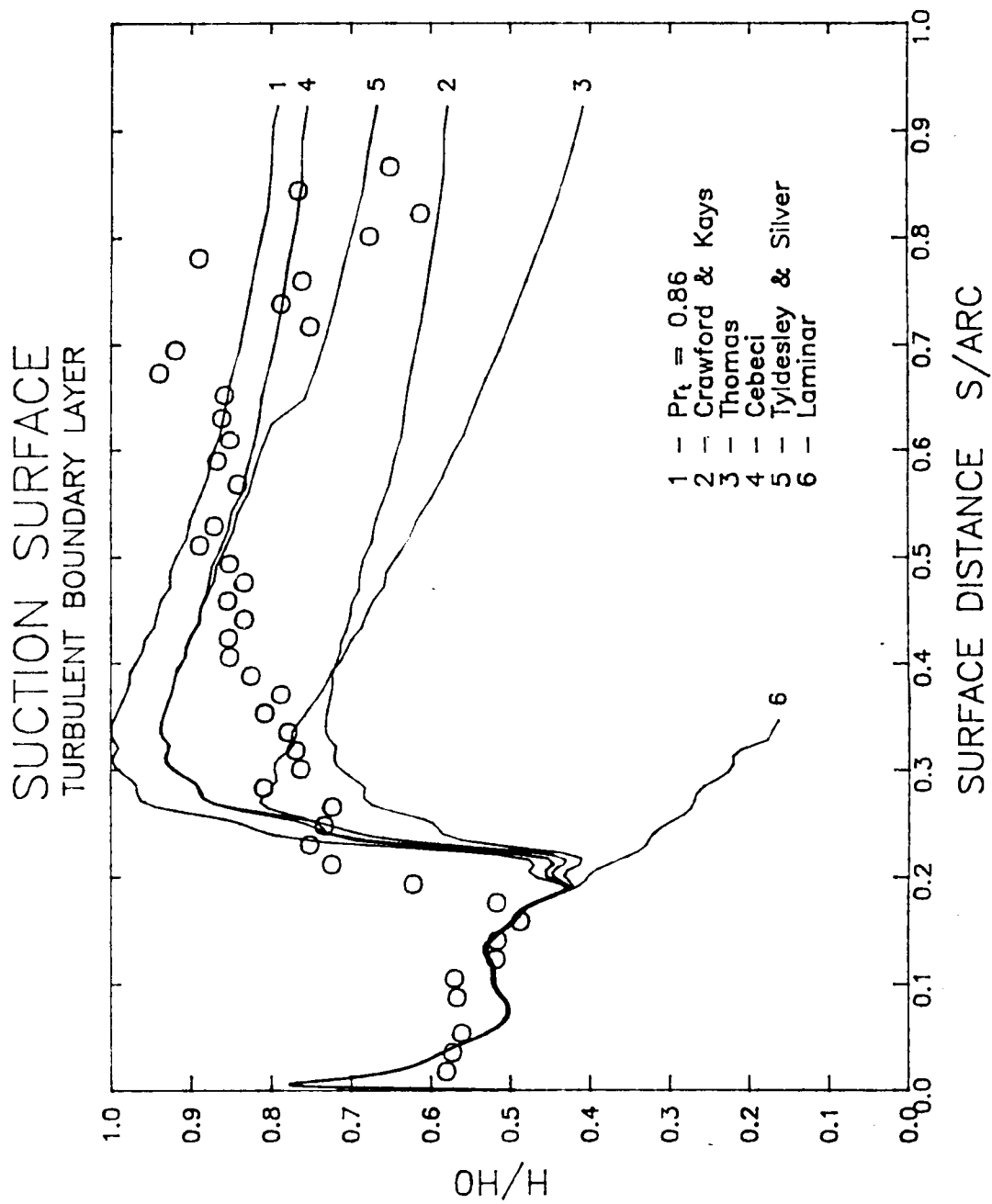


Figure 2. Heat Transfer Coefficient Distribution on the Suction Surface

# PRESSURE SURFACE TURBULENT BOUNDARY LAYER

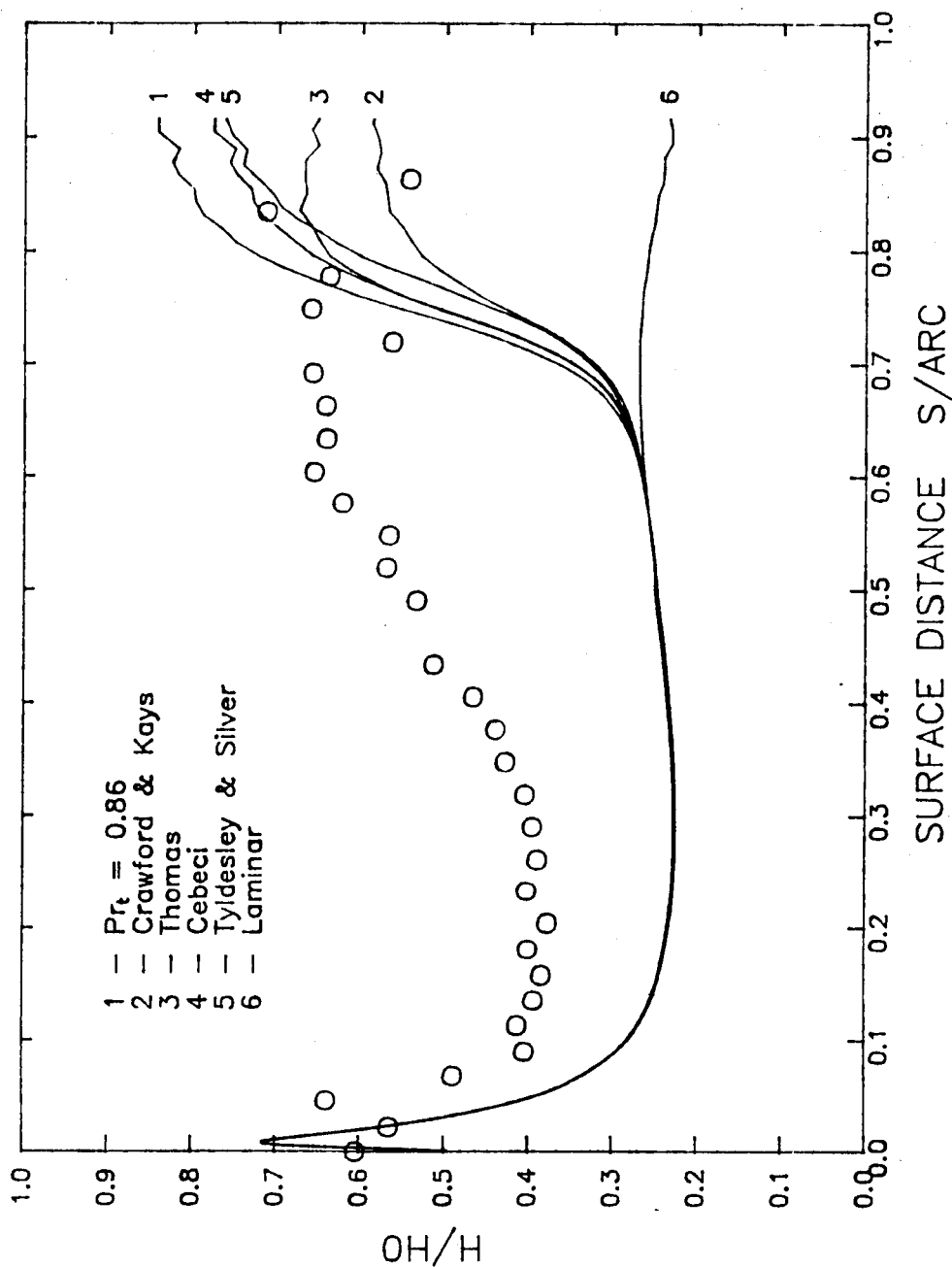


Figure 3. Heat Transfer Coefficient Distribution on the Pressure Surface

all of the correlations accompanying that assumption. Within STAN5 there is an expression to determine the effective viscous sublayer thickness ( $A^+$ ) which is based on flat plate analyses. Also, there is a correction for  $A^+$  depending on the pressure gradient experienced by the flow. An attempt was made to see how well these concepts apply to turbine blades.

After some "trail-and-error", it was found that the  $A^+$  correction factor was significantly effecting the heat transfer coefficient predictions. This can be seen in Figure 4. On the pressure surface very good predictions were obtained with all the models by not using the  $A^+$  correction factor. Thomas' model predicts the trend exhibited by the experimental data very well. The model of Crawford and Kays also agrees well but consistently underpredicts. The other models predict well up to a surface distance of about 60 percent and then overpredict for the remainder of the blade. The suction surface predictions did not respond in a similar manner, however. In fact, not using the  $A^+$  correction factor caused the predictions to become even worse. An example of a typical result is shown in Figure 5.

An attempt was made to further investigate the character of the suction surface boundary layer. Clearly both laminar or fully turbulent flow do not represent what is happening on the suction surface. One set of predictions were obtained assuming that the boundary layer on the suction surface was in transition from leading edge to trailing edge. Representative results are shown in Figure 6. Although the transition seen previously at 20 percent still exists, it is not as abrupt and the downstream levels are not correct. Examples with and without the  $A^+$  correction factor are shown.

# PRESSURE SURFACE TURBULENT BOUNDARY LAYER

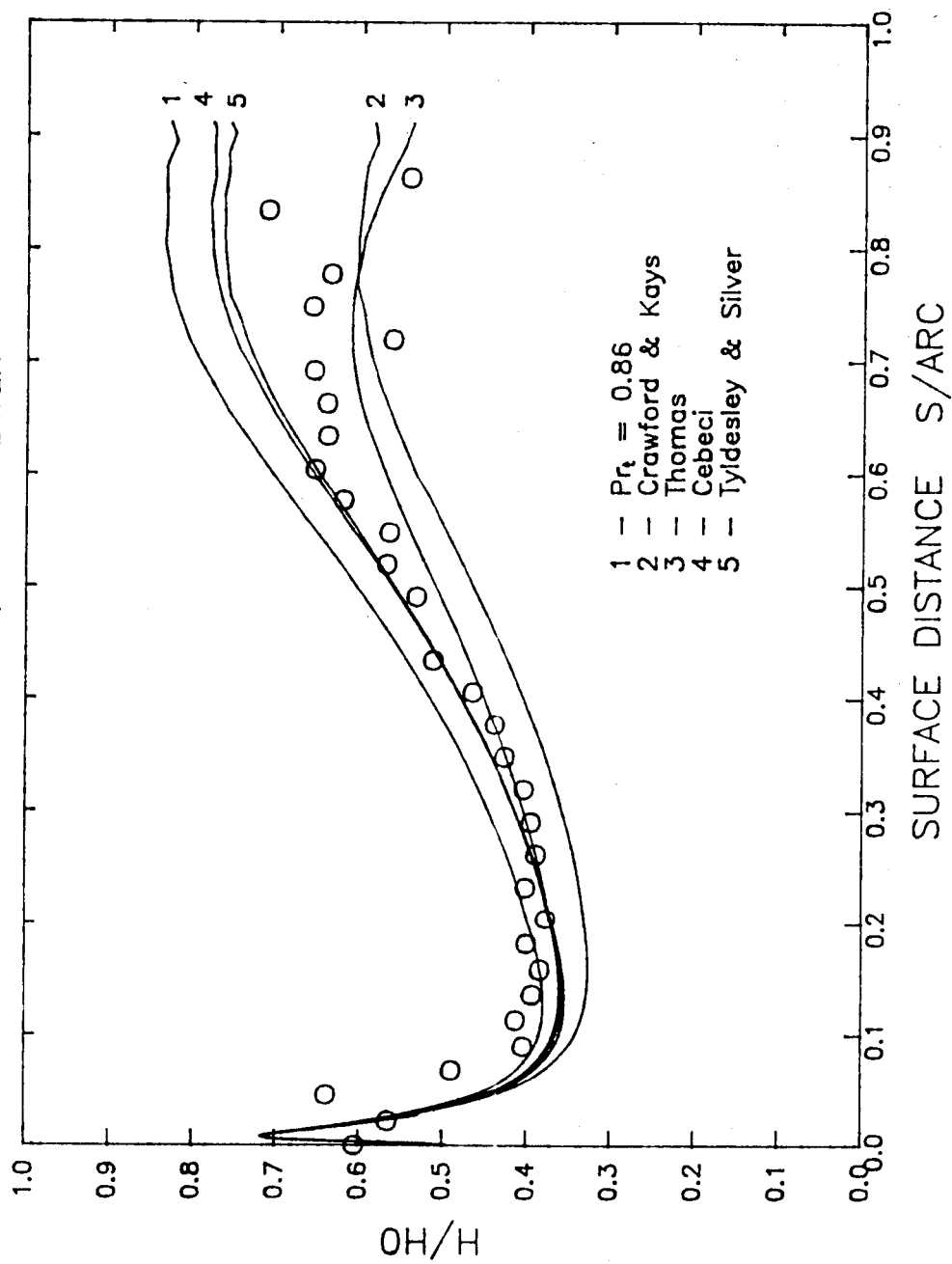


Figure 4. Heat Transfer Coefficient Distribution on the Pressure Surface without using A+ Correction



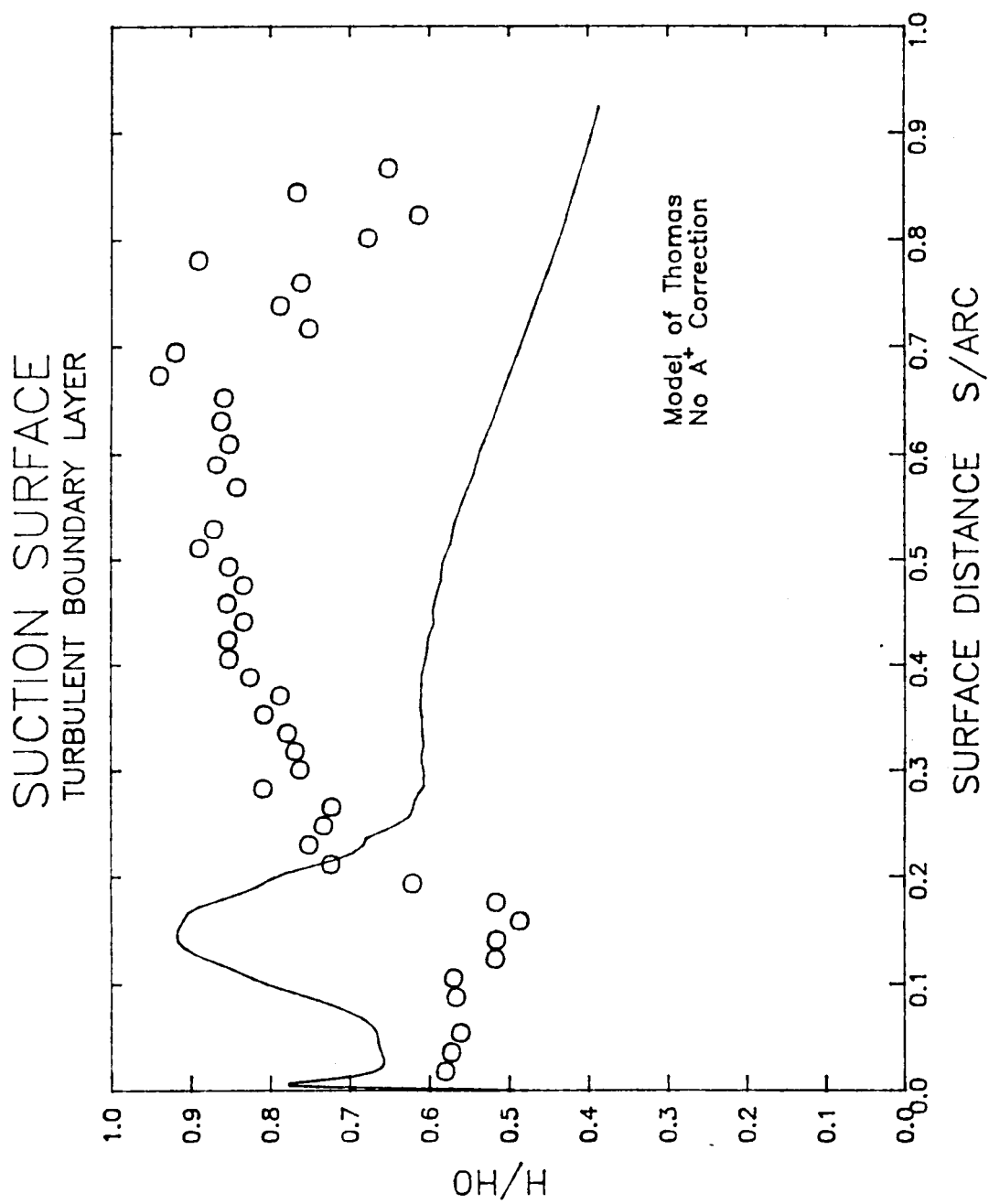


Figure 5. Heat Transfer Coefficient Distribution on the Suction Surface without using A<sup>+</sup> Correction

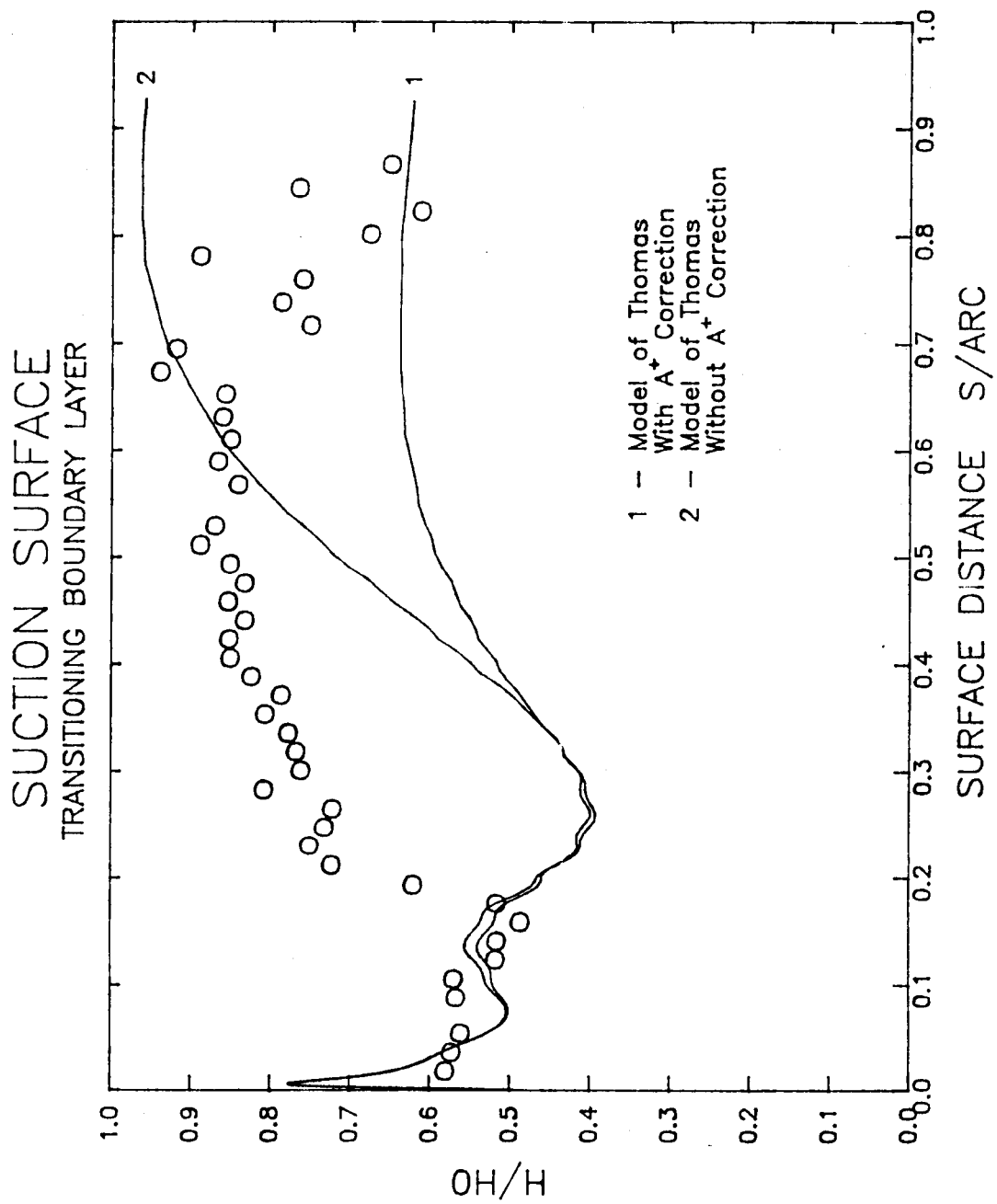


Figure 6. Heat Transfer Coefficient Distribution on the Suction Surface Assuming Transitioning Flow

## CONCLUSIONS AND RECOMMENDATIONS

Based on this preliminary investigation it was concluded that the turbulent Prandtl number is not a significant force in determining the heat transfer coefficient. It appears that the character of the boundary is much more important with the turbulent Prandtl number "fine tuning" the end result. This was seen in the pressure surface results where the predictions went from poor to excellent by altering the boundary layer character via the effective viscous sublayer thickness ( $A^+$ ) correction.

It is also apparent that the suction surface flow is clearly not being modeled correctly. The transition from laminar to turbulent has to date defied description via current transition models. There is certainly a complex interaction between the transition, pressure gradient, curvature, and three dimensional effects not yet understood.

Albeit preliminary, this study has suggested need for the following:

- 0 Boundary layer character on the suction surface must be understood. This suggests a detailed studied of the flow incorporating all of the important parameters such as surface curvature, flow separation, and transition.
- 0 Higher order turbulence models coupled with the turbulent Prandtl number models need to be developed. Also, the models may need to reflect any three-dimensional effects inherent in the flow over turbine blades.
- 0 The experimental data base for gas turbine engine environments must be substantially enlarged. Current prediction methods have a limited number of data by which to compare and some of the data is suspect due to the difficulty in making the measurements. Detailed measurements are necessary to validate the higher order models that appear to be needed.

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