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Turbulence modeling

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1. Motivation and Objective

Motivation

In recent years codes that use the Navier-Stokes equations to compute aerodynamic flows have evolved from computing two-dimensional flows around simple airfoils to computing flows around full scale aircraft configurations. Most flows of engineering interest are turbulent and turbulence models are needed for their prediction. Yet, we know that present turbulence models are adequate only for simple flows and do poorly in complicated flows such as three-dimensional separation, large-scale unsteadiness, etc. The same progress that allowed the development of these aerodynamic codes, namely the introduction of supercomputers, has allowed us to compute directly turbulent flows, albeit only for simple flows at moderate Reynolds numbers. These direct turbulence simulations provide us with detailed data that experimentalists have not been able to measure. This work is motivated by the fact that data exists for developing better turbulence models and by the need for better models to compute flows of engineering interest.

Objective

The objective of this work is then to develop turbulence models for engineering applications. The model categories that show promise for immediate use are on the two-equation level and the Reynolds-stress level. We will make use of existing methodologies to develop models. The models will be tested using data from direct simulations, experiments and analysis. Specifically, our objectives are as follows:

1. Examine the Reynolds stress budgets using direct simulation flow fields (Mansour et al 1988, Moin et al 1989).

2. Use Rapid Distortion Theory to analytically study the effects of mean deformation on turbulence. In particular, examine the development of the rapid pressure-strain under rapid distortions.

3. Compare existing models with data and theory. Develop models where needed using appropriate expansions and constraints. Test these new models using results from direct simulation, experiment and theory.

4. Use the method of moment generating function to extend second order closures to higher order closures. We know that there exist a close connection between high-order moments and coherent structures. The moment generating

function approach should allow us to include the effect of coherent structures on the models.

5. Extend turbulence modeling to compressible flows.

2. Work Accomplished

1. Numerical simulation of a three-dimensional boundary layer — AIAA paper no. 89-0373 (work supported in part by AFOSR)

The objective of this simulation is to study the mechanics of three-dimensional boundary layers and develop improved models for their predictions. Three-dimensional effects were achieved by direct simulation of a fully developed turbulent channel flow subjected to transverse pressure gradient. To obtain a good statistical sample during the transient period, 14 computer runs were ensemble averaged. Each run started from a different realization of the channel flow (far apart in time). The simulation shows that, in agreement with experimental observations, the Reynolds stresses are reduced and that near the wall a lag develops between the stress and the strain rate. The reduction in the stress is due largely to a drop in the production rate and an increase in the dissipation rate. In the coming year, we will study the performance of existing second order closures in predicting this drop and introduce improvements where needed.

2. k-ϵ modeling

k-ϵ turbulence models are often used in computing engineering flows with moderate success. In general, the flow field is qualitatively well predicted, but quantitative agreement often falls short. In the case of Reynolds stress modeling the question of a length scale for an eddy viscosity does not arise, and the energy dissipation rate, ϵ, is one of the unknowns of the problem. In this work an ϵ equation has been developed following the methodology of Lumley (1978). We used it in connection with both two-equation modeling and Reynolds stress modeling.

A two-equation model for use in low-Reynolds number flows has been tested against the channel data of Kim, Moin and Moser (1987). The modeled equations are given by,

$$k_{,t} + U_k k_{,k} = \left[\left(\frac{\nu_T}{\sigma_k} + \nu\right) k_{,k}\right]_{,k} + \nu_T S_{ij} S_{ij} - \epsilon$$

$$\epsilon_{,t} + U_k \epsilon_{,k} = \left[\left(\frac{\nu_T}{\sigma_\epsilon} + \nu\right) \epsilon_{,k}\right]_{,k} + C_1 \frac{\epsilon}{k} \nu_T S_{ij} S_{ij} - C_2 f_\epsilon \frac{\epsilon \tilde{\epsilon}}{k}$$

$$
\begin{aligned}
\sigma_k &= 1 \\
\sigma_\epsilon &= 1.3 \\
C_1 &= 1.48 \\
C_2 &= 1.8 \\
\tilde{\epsilon} &= \epsilon - \phi \\
\phi &= k_{,j} k_{,j} / (2k) \\
\nu_T &= C_\mu f_\mu k^2 / \tilde{\epsilon} \\
C_\mu &= 0.09 \\
f_\mu &= 1 - \exp(-0.0115 \frac{u_\tau y}{\nu}) \\
f_\epsilon &= 1 - \frac{0.4}{1.8} \exp\left(- \left(\frac{k^2}{6\nu\epsilon} \right)^2 \right)
\end{aligned}
$$

We find that the model adequately predicts the mean velocity profile (see figure 1) and the turbulent kinetic energy (see figure 2).

3. Second order modeling of near-wall low-Reynolds number turbulence

A set of second order closure models for low-Reynolds number turbulence has been developed for the simulation of wall bounded flows without using wall functions. The wall effect is built in the pressure-strain correlation term of the Reynolds stress equation and in the modeled terms of the dissipation rate equation. We find that realizability is particularly important for modeling the near wall turbulence. The proposed models are particularly suitable for surfaces of arbitrary topology since they do not use the wall distance as a parameter. The models are tested by computing the fully developed channel flow. The full set of equations are used to compute the mean velocity, all the Reynolds stresses and the dissipation rate of the turbulent kinetic energy. We find reasonable agreement between the prediction of the mean profile and the data (see Figures 3) which is an indication that the shear stress is well predicted. However, the normal stresses are not as well predicted. In particular the streamwise component is underpredicted (see figure 4) while the transverse component is overpredicted. The cause of this shortcoming is still being investigated.

4. Second order modeling of a passive scalar in turbulent shear flows — AIAA paper no. 89-0607

A model equation for the scalar dissipation rate was proposed using the ansatz that the ratio of mechanical time scale to scalar time scale has an equilibrium value. In addition a model for the pressure related terms in the scalar flux equation was constructed based on consideration of realizability. The models were tested by comparison with experimental data for heated plane and axisymmetric jets.

5. Rapid Distortion Theory and 2-D 2-C turbulence modeling

Rapid distortion theory was used to analyze the development of the Reynolds stress in a 2-D 2-C homogeneous turbulence under mean irrotational strain and mean shear. Here, 2-C refers to two-component turbulence. We found that the development of the Reynolds stress under 2-D 2C conditions is very different from its development under 3-D 3-C conditions. The findings can be summarized as follows:

1. The mean shear or strain have no effects on the isotropy of the Reynolds stress if the initial field is isotropic.
2. Initially anisotropic fields become isotropic under the influence of mean shear or strain.

These findings are opposite to the finding in 3D 3C turbulence where mean shear or strain will drive isotropic turbulence away from isotropy. In 2D 2C turbulence the pressure strain drives the Reynolds stresses back to isotropy.

A general 2D 2C model for the pressure-strain term, T_{ij}, was constructed which contains only one undetermined coefficient C,

$$\frac{T_{ij}}{2k} = S_{ij} + 4Cb_{ik}b_{pj}S_{pk} + 2\Omega_{jk}b_{ik} + 2\Omega_{ik}b_{kj}$$

where,

$$S_{ij} = \frac{1}{2}(U_{i,j} + U_{j,i}), \quad \Omega_{ij} = \frac{1}{2}(U_{i,j} - U_{j,i})$$

$$b_{ij} = \frac{\overline{u_i u_j}}{2k} - \frac{1}{3}\delta_{ij}, \quad k = \frac{1}{2}\overline{u_i u_i}$$

Rapid distortion theory will be used to determine this coefficient. We postulate that the more general 3D 3C model should reduce to this model in the limit of 2D 2C turbulence.

3. Future Plans

1. Examine the performance of existing second order closures in predicting three-dimensional turbulent boundary layers. Introduce improvements where needed.
2. Use Rapid Distortion Theory to study the effects of mean deformation on turbulence. Extend the analysis developed for mean strain and shear to study the effect of rapid rotation on the turbulence.
3. Use the method of moment generating function to extend second order closures to higher order closures.
4. Use non-weighted ensemble averaging method (Shih et al, 1987) to extend the second order models to compressible flows. This averaging technique (as opposed to Favre averaging) will allow us to apply all the incompressible modeling methodologies to modeling compressible flows.

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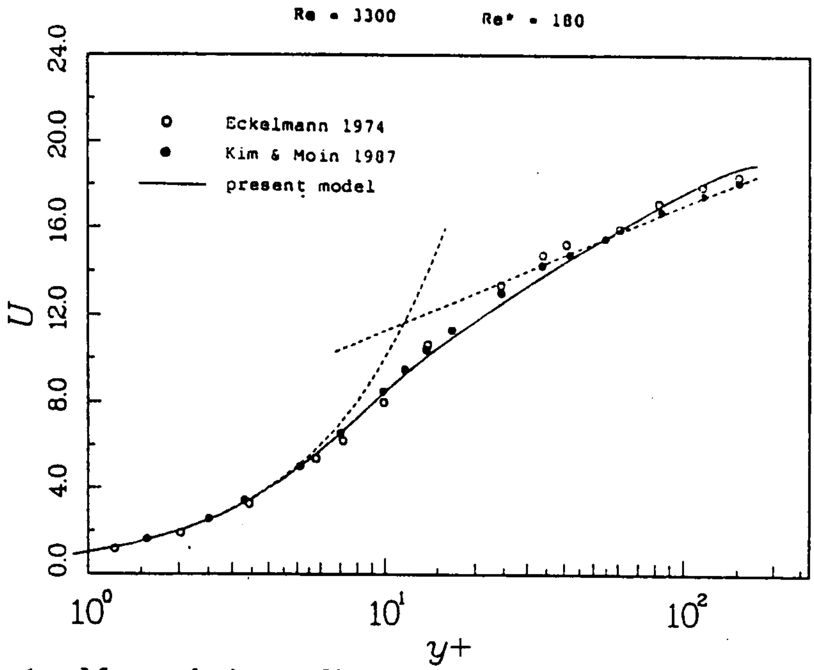

FIGURE 1. Mean velocity profile in a channel as predicted by a two-equation model.

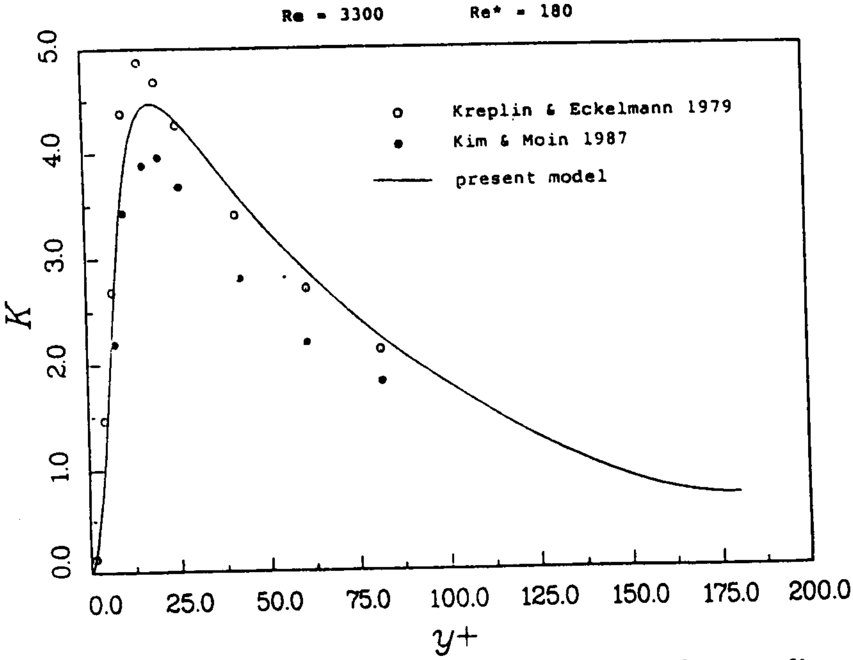

FIGURE 2. Mean turbulent kinetic energy in the channel as predicted by a two-equation model.

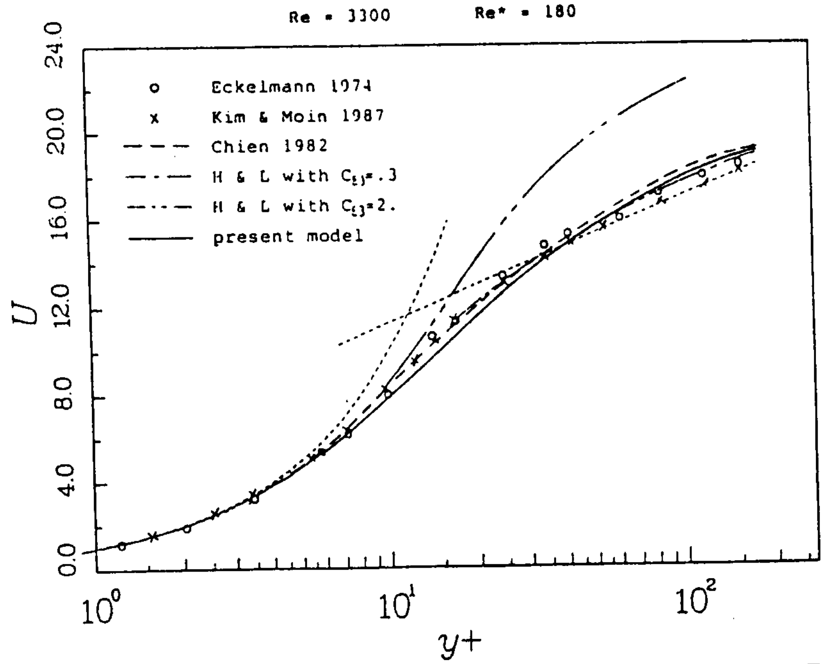

FIGURE 3. Mean velocity profile in a channel as predicted using a Reynolds stress model.

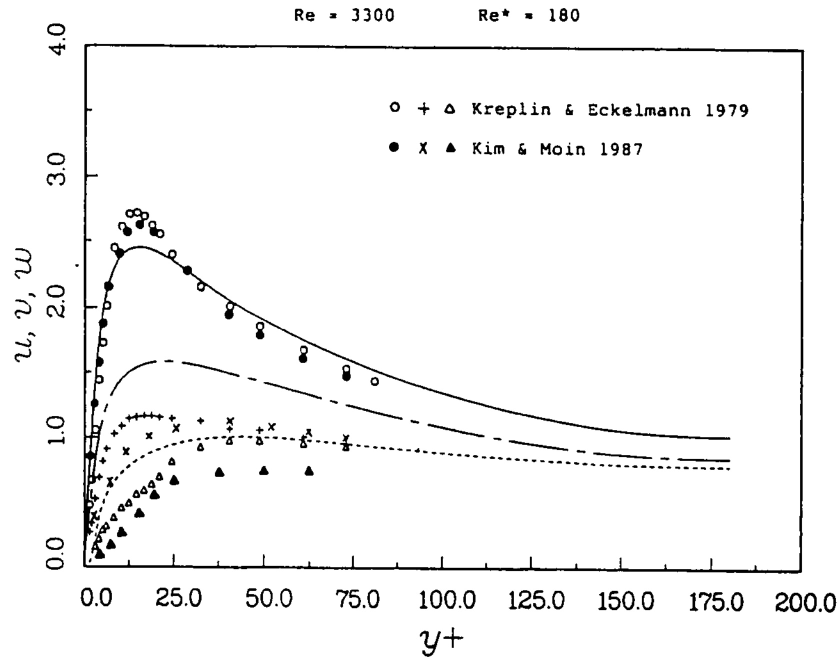

FIGURE 4. Normal stress in a channel as predicted using a Reynolds stress model.