Numerical simulation of viscous vortex rings

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1. Motivation & Objectives

This work is directed toward understanding vortex interactions and their role in turbulent flow. The objectives are twofold. First, to use the existing axisymmetric code to study the annihilation process of colliding vortex rings and determine the relevance of this problem to similar 3D phenomena. The second objective is to extend the code to three dimensions. The code under development is unique in that it can compute flows in a truly infinite domain (i.e. without periodic boundary conditions or approximations from truncating the domain). Because of this, we are able to compute the far field sound, and therefore, contribute to improved models of turbulence generated noise for this class of flows. Issues which can be addressed by the code include: effects of viscosity on mode selection in azimuthal breakdown of vortex rings (i.e. the Widnall instability); reconnection, the associated production of small scales, and the time scale of the process.

2. Accomplishments

Objective 1: Study head-on collision of viscous vortex rings

During the CTR Summer Program, a collaboration was undertaken between the author, K. Shariff and F. Hussain to study the head-on collision of two identical axisymmetric viscous vortex rings through direct simulations of the incompressible Navier-Stokes equations. The result of this effort is described in the proceedings for the 1988 Summer Program of the CTR (Stanaway *et al.* 1988).

Initial conditions of varying core shapes (thin rings, and Hill's spherical vortex rings) and Reynolds numbers (350 to 1000) were considered and the subsequent annihilation process was studied through time history of circulation, vorticity contours and local contributions to annihilation. The results provide a database to model such a problem and also information about the level of detail required to accurately model this phenomenon. A typical interaction proceeds as follows. Two vortices of opposite sense and the same strength approach each other by self-induction, the radii increase by mutual induction, and vorticity cancels through viscous diffusion across the collision plane. Following contact, we observed (for the cases considered here) that the vorticity distribution in the core forms a head-tail structure, a behavior which has also been seen in inviscid calculations (Shariff *et al.* 1988), 3D viscous calculations with periodic boundary conditions

(Melander & Hussain 1988, MH), and experiments (Oshima 1978). In examining the local contributions to the total vorticity annihilation in both the head and tail regions, we conclude that the tail does contain a significant part of the vorticity and should be included in an accurate model of the annihilation process.

The characteristic time of vorticity annihilation is compared with that of a 3D collision experiment (Schatzle 1987) and 3D numerical simulations (MH). It is found that the annihilation timescale for the axisymmetric collision is faster than the viscous timescale, a_o^2/ν, and slower than the timescale set by the circulation, $a_o^2/(\Gamma_o \nu)^{1/2}$, where a_o is the initial core radius, Γ_o is the initial circulation, and ν is the viscosity. This indicates that the local effects are important in enhancing annihilation, however, nonlocal effects such as vorticity realignment are also important in 3D. One might expect that during the initial stages of the collision, local effects are dominant, and as the circulation in the symmetry plane weakens, the bridges strengthen and the out-of-plane strain becomes the more important effect.

The flow is also computed to the large time Stokes flow limit where the circulation decays as $t^{-3/2}$ and the vorticity distribution agrees with the quadrupole solution of the Stokes equations. In this limit, the self-annihilation is exactly twice the mutual annihilation. For one of the cases computed, the far-field quadrupole sound is compared with the experimental results of Kambe & Minota (1983). The agreement is quite good even though the Reynolds numbers are very different.

Objective 2: Extend axisymmetric spectral code to three dimensions

An operational axisymmetric code which solves the Navier-Stokes equations in an unbounded domain is being modified to compute 3D flows. The method (Stanaway, Cantwell & Spalart 1988) uses divergence free basis functions, having the advantages of reducing the number of unknowns from four to two, and eliminating the need to solve a Poisson equation for the pressure at each time step. The solution is expanded in polar coordinates with the associated Legendre functions in the polar direction and Jacobi polynomials matched to an algebraic mapping of the radial coordinate. The third dimension, the azimuthal direction, is expanded in Fourier series. The above functions are used in order that the resulting matrix equations are numerically attractive; they are completely orthogonal in two directions and have a constant and relatively narrow bandwidth in the third direction (bandwidth ≤ 11). In addition, in the present method the matrices are symmetric and positive definite.

The unknown coefficients in wave space, referred to as the $+$ and $-$ modes (in keeping with the notation of similar approaches), each have an ordinary differential equation describing their evolution. The axisymmetric basis functions are a subset of the three dimensional functions, meaning that in extending the axisymmetric code to 3D, much of the code remains intact. Specifically, the evolution equation for the $+$ modes is unchanged. In the 3D case, an additional evolution equation is required for the − modes. In the development of the axisymmetric code, it was necessary to compute the matrix elements analytically rather than numerically in order to minimize errors leading to ill conditioned matrices. This process, which was rather involved due to the complex coordinate system and mapping, was made possible with the aid of MACSYMA, an algebraic manipulation program. It is worth noting that the effort required to compute the elements of the matrices for the − modes was considerably less than it was for the + modes. The major effort, therefore, in extending the code to three dimensions has been in developing the transforms to and from wave space for the 3D basis functions. This step is nearly finished and will be tested in a straightforward manner by starting with an initial vorticity field, transforming to wave space, and then back to real space.

So the problem of extending the code involves four phases:

(i) Deriving the basis functions and evolution equations.

(ii) Forming the matrices for the − mode evolution equation.

(iii) Developing and testing three dimensional transforms going to and from wave space.

(iv) Running test cases.

Considerable progress has been made in extending the code to three dimensions and it is expected that the 3D code will soon be operational.

3. Future Plans

This new method gives us the unique capability to study many important and often controversial phenomena accurately, notably

- the Widnall instability of an almost axisymmetric ring, in particular, the mode selection process;
- viscous connection and splitting;
- redistribution of vorticity by colliding or pairing vortex rings; and
- generation, intensification and annihilation of vorticity through vortex ring interactions.

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