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Observation Model and Parameter Partials for the JPL Geodetic GPS Modeling Software "GPSOMC"

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ABSTRACT

This report describes the physical models employed in GPSOMC, the modeling module of the GIPSY software system developed at JPL for analysis of geodetic Global Positioning Satellite (GPS) measurements. Details of the various contributions to range and phase observables are given, as well as the partial derivatives of the observed quantities with respect to model parameters. A glossary of parameters is provided to enable persons doing data analysis to identify quantities in the current report with their counterparts in the computer programs. The present version is a revision of the original document of the same title, JPL Publication 87-21, dated September 15, 1987, which it supersedes. There are no basic model revisions, with the exceptions of an improved ocean loading model, described in Section 2.2.2, and some new options for handling clock parametrization, mentioned in Section 3 and Appendix B. Such misprints as were discovered have been corrected. The authors hope to publish further revisions of this document in the future, both to include modeling improvements, and to ensure that the model description is always in accord with the current software.

CONTENTS

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SECTION 1

INTRODUCTION

In interpreting measurements of range from satellites of the Global Positioning System (GPS) to ground-based receivers, the observables are passed through a multiparameter estimation routine ("filter") to estimate the parameters of a model. This model describes the spacecraft orbits and the motions of the Earth-fixed receivers, and supplies to the filter *a priori* values of computed observables, as well as their partial derivatives with respect to model parameters. During 1984-5, software was developed at JPL to perform these functions. The purpose of the present report is to describe that portion of the software which concerns the modeling of receiver locations, motions of the whole Earth, and computation of observables and their partial derivatives. Modeling of satellite orbits and parameter estimation form separate units in the software chain and are described in separate documents.

Many aspects of the model required to describe GPS range measurements are identical to the modeling developed during the past decade for Very Long Baseline Interferometry (VLBI). Consequently much of the content of this document, as well as of the associated software package GPSOMC, is borrowed from the VLBI modeling and parameter estimation package MASTERFIT and the document giving the details of its models (Sovers and Fanselow, 1987). There are two major differences. First, in satellite range measurements, the sources (transmitters) are not at infinity, and they contain time standards of their own. Together with the need for orbit determination, this makes the situation considerably more complicated than the VLBI case, where signals are received from fixed sources at infinite distances. The second, minor, model difference is the lack of necessity of describing the passage of the signal through the Earth's ionosphere. It is assumed that ionospheric effects will always be removed by performing measurements at two frequencies.

Three major model components which will be discussed here are: geometry, clocks, and troposphere. Section 2 deals with the coordinate frame for the model and establishes methods for calculating the position of the receiver in that frame, employing the best current models of whole Earth motions and local tidal deformations. This section is nearly identical to the corresponding coordinate frame description in the MASTERFIT document, with slight differences reflecting differences in the implementation of minor effects. The next section (Sec. 3) defines the observables and the intimately associated models of behavior of space vehicle and receiver clocks. Section 4 presents the model employed to describe the passage of the signal through the troposphere. All the partial derivatives of observables with respect to model parameters are given in Sec. 5. Values of the physical constants used in the GPSOMC software, as well as a complete list of the parameters presently available for adjustment, are given in the appendices.

As of August 1988, the model description in this report is intended to correspond to the Fortran code used to generate the executable GPSOMC.EXE;114 on the Section 335 LOGOS and XENOS VAX computers. This correspondence is believed to hold for the dominant components of the observable models. Some of the code implementing seldom-used aspects of modeling (*e.g.*, nutation parameters, time-dependent station positions and zenith tropospheres) has not yet been thoroughly checked; it is intended that complete testing will be performed in the near future.

SECTION 2

COORDINATE FRAME AND GEOMETRY

The geometric range is that portion of the distance between a satellite transmitter and an Earth-fixed receiver which would be measured by perfect instrumentation, perfectly synchronized, if there were a perfect vacuum between the transmitter and receiver. It is by far the largest component of the observed range. The main complexity of this portion of the model arises from the numerous coordinate transformations necessary to relate the inertial reference frame used for locating the spacecraft to the Earth-fixed reference frame in which station locations are represented.

In the following we will assume that "geocentric inertial reference frame" means a geocentric, equatorial frame with the equator and equinox of J2000 as defined by the 1976 IAU conventions with the 1980 nutation series (Seidelmann, 1982, and Kaplan, 1981). On the other hand, we will mean by "Earth-fixed reference frame" some reference frame tied to the mean surface features of the Earth. This is a right-handed version of the CIO reference system with the pole defined by the 1903.0 pole. Implementation is accomplished by defining the position of one of the fiducial stations, and measuring the positions of the other receivers. This section provides the details for the evaluation of the geometric range, including receiver coordinates, tidal effects, and the transformation from Earth-fixed to geocentric inertial coordinate systems.

2.1 RECEIVER LOCATION TIME DEPENDENCE

Normally the receiver position vector $\mathbf{r}_{E_0}$ and its components in the Earth-fixed reference frame r_{sp}, λ, z (radius off spin axis, longitude, and height above the equator, respectively) are considered to be time-invariant. An alternative formulation introduces the time rates of change of the station coordinates as adjustable parameters. The model is linear, with the components of $\mathbf{r}_{E_0}(t)$ at time t expressed as

$$r_{sp} = r_{sp}^0 + \dot{r}_{sp}(t - t_0) \tag{2.1}$$

$$\lambda = \lambda^0 + \dot{\lambda}(t - t_0) \tag{2.2}$$

$$z = z^0 + \dot{z}(t - t_0) \tag{2.3}$$

Here t_0 is a reference epoch, at which the receiver coordinates are $(r_{sp}^0, \lambda^0, z^0)$. The receiver "position vector" may include an antenna phase center offset and an offset to a standard benchmark or monument (see Sec. 2.9). In the model development that follows, $\mathbf{r}_{E_0}$ includes these offsets, but is referred to as the "receiver vector." Besides the linear time variation of Eqs. (2.1-2.3), modeling allows for estimation of a new benchmark position for every observing session.

2.2 TIDAL EFFECTS

In calculating the geometric range, we need to consider the effects of crustal motions on receiver locations. Among these deformations are solid Earth tides, tectonic motions, and alterations of the Earth's surface due to local geological, hydrological, and atmospheric processes. In the standard Earth-fixed coordinate system, tidal effects modify the receiver location $\mathbf{r}_{E_0}$ by an amount

$$\mathbf{\Delta} = \mathbf{\Delta}_{sol} + \mathbf{\Delta}_{ocn} + \mathbf{\Delta}_{pol} \tag{2.4}$$

where the three terms are due to solid Earth tides, ocean loading, and pole tide, respectively. Other Earth-fixed effects would be incorporated by augmenting the definition of $\mathbf{\Delta}$.

2.2.1 Solid Earth Tides

Alteration of the positions of the receivers by solid Earth tides is rather complicated due to their coupling with the ocean tides and to the effects of local geology. We gloss over these complications, and employ the simple quadrupole response model described by J. G. Williams (1970), who used Melchior (1966) as a reference. Let $\mathbf{R}_s$ be the position vector of a perturbing source in the Earth-fixed reference system, and let $\mathbf{r}_{E_0}$ be the receiver position vector in the same coordinates. If h and l are the Love numbers, ψ a phase shift of the tidal effects, and $\mathbf{r}_p$ the phase-shifted receiver location vector in the Earth-fixed reference system, then the vector of tidal displacements in a local Cartesian frame (x axis vertical, y axis eastward, and z axis northward on a spherical Earth) is

$$\boldsymbol{\delta} = \sum_s [hg_{1s},\ lg_{2s},\ lg_{3s}]^T \tag{2.5}$$

where

$$g_{1s} = \frac{3\mu_s r_p^2}{R_s^5}\left[\frac{(\mathbf{r}_p \cdot \mathbf{R}_s)^2}{2} - \frac{r_p^2 R_s^2}{6}\right] \tag{2.6}$$

$$g_{2s} = \frac{3\mu_s r_p^2}{R_s^5}(\mathbf{r}_p \cdot \mathbf{R}_s)(Y_s x_p - X_s y_p)\frac{|\mathbf{r}_p|}{\sqrt{x_p^2 + y_p^2}} \tag{2.7}$$

$$g_{3s} = \frac{3\mu_s r_p^2}{R_s^5}(\mathbf{r}_p \cdot \mathbf{R}_s)\left[\sqrt{x_p^2 + y_p^2}\ Z_s - \frac{z_p}{\sqrt{x_p^2 + y_p^2}}(x_p X_s + y_p Y_s)\right] \tag{2.8}$$

Here μ_s is the ratio of the mass of the disturbing object, s, to the mass of the Earth, and

$$\mathbf{R}_s = [X_s, Y_s, Z_s]^T \tag{2.9}$$

is the vector from the center of the Earth to that body. The summation is over all tide-producing bodies, of which we include only the Sun and the Moon.

The phase-shifted receiver vector is calculated employing a phase lag, or equivalently, employing a right-handed rotation, L, through an angle ψ about the z axis of date, $\mathbf{r}_p = L\mathbf{r}_{E_0}$. This lag matrix, L, is:

$$L = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.10}$$

By a positive value of ψ we mean that the peak response on an Earth meridian occurs at a time $\delta t = \psi/\omega_E$ after that meridian containing $\mathbf{r}_{E_0}$ crosses the tide-producing object, where ω_E is the angular rotation rate of the Earth. In the vertical component, the peak response occurs when the meridian containing $\mathbf{r}_p$ also includes $\mathbf{R}_s$. The difference between geodetic and geocentric latitude can affect this model on the order of the (tidal effect)/(flattening factor) ≈ 0.1 cm.

To convert the locally referenced strain, $\boldsymbol{\delta}$, to the Earth-fixed frame, two additional rotations must be performed. The first, W, rotates by an angle, $-\phi_s$, about the y axis to an equatorial system. The second, V, rotates about the resultant z axis by angle, $-\lambda_s$, to bring the displacements into the standard Earth-fixed coordinate system. The result is

$$\boldsymbol{\Delta}_{sol} = VW\boldsymbol{\delta} \tag{2.11}$$

where

$$W = \begin{pmatrix} \cos\phi_s & 0 & -\sin\phi_s \\ 0 & 1 & 0 \\ \sin\phi_s & 0 & \cos\phi_s \end{pmatrix} \tag{2.12}$$

and

$$V = \begin{pmatrix} \cos\lambda_s & -\sin\lambda_s & 0 \\ \sin\lambda_s & \cos\lambda_s & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.13}$$

Actually, the product of these two matrices is coded:

$$VW = \begin{pmatrix} \cos\lambda_s\cos\phi_s & -\sin\lambda_s & -\cos\lambda_s\sin\phi_s \\ \sin\lambda_s\cos\phi_s & \cos\lambda_s & -\sin\lambda_s\sin\phi_s \\ \sin\phi_s & 0 & \cos\phi_s \end{pmatrix} \tag{2.14}$$

2.2.2 Ocean Loading

This section is concerned with one of the secondary tidal effects, *i.e.*, the elastic response of the Earth's crust to ocean tides, which move the receivers to the extent of a few cm. Such effects are commonly labeled "ocean loading." The most complete recent model appears to be that described by Pagiatakis (1982, 1988) and by Pagiatakis, Langley, and Vanicek (1982), which is implemented as an option in the current GPSOMC. It was recently generalized to include effects of anisotropy and viscoelasticity of the Earth; differences of displacements with those given by the 1982 model are well below 1 cm. The formulation and coding are general enough, however, to permit other inputs to be used in place of the Pagiatakis ocean loading model. Because the receiver motions caused by response to ocean tides appear to be limited to approximately 3 cm for sites well removed from the coast, no estimation capability was deemed necessary at present. This decision is supported by the fact that for locations near the coast, where the effects may be more sizable, and which would thus be expected to produce data useful in parameter estimation, the elastic response modeling is as yet inadequate (Agnew, 1982). The present model entails deriving an expression for the locally referenced displacement $\boldsymbol{\delta}$ due to ocean loading. In the vertical, N-S, E-W local coordinate system at time t,

$$\delta_j = \sum_{i=1}^{N} \xi_i^j \cos(\omega_i t + V_i - \delta_i^j) \tag{2.15}$$

The quantities ω_i (frequency of tidal constituent i) and V_i (astronomical argument of constituent i) depend only on the ephemeris information (positions of the Sun and Moon). On the other hand the amplitude ξ_i^j and Greenwich phase lag δ_i^j of each tidal component j are determined by the particular model assumed for the deformation of the Earth. As of November 1988, software for calculating these deformations at an arbitrary point on the Earth's surface exists at JPL only for the Pagiatakis-Langley model. Six tidal components are included [$N = 6$ in Eq. (2.15)]: the M_2, S_2, K_1, O_1, N_2, and P_1 tides, all of which have periods close to either 12 or 24 hours. The local displacement vector is transformed via Eqs. (2.14) and (2.11) to the displacement Δ_{ocn} in the standard Earth-fixed frame.

Input to GPSOMC provides for specification of an arbitrary number of frequencies and astronomical arguments ω_i and V_i, followed by tables of the local distortions and their phases, ξ_i^j and δ_i^j, calculated from the ocean tidal loading model of choice. In particular, longer-period tidal constituents can be accommodated in this fashion.

There are presently four choices of models. As mentioned above, the six components of the Pagiatakis-Langley model can be calculated by separate software for an arbitrarily located receiver to generate input tables for GPSOMC. This has been done for all stations commonly employed in JPL VLBI experiments. There is no comparable facility to obtain amplitudes and phases for the Agnew (1982), Scherneck (1983), or Goad (1983) models. Consequently for these three models GPSOMC input tables only exist for the limited set of stations considered by these authors. Agnew considers only five components (omitting P_1), while Scherneck includes five components in addition to the Pagiatakis-Langley set: K_2, Q_1, M_f, M_m, and S_{sa}. These all have amplitudes of 1 mm or smaller and, thus, are not expected to be significant at the present level of experimental accuracy. Goad's MERIT standard model only specifies vertical displacements, but includes three components (K_2, Q_1 and M_f) in addition to Pagiatakis-Langley. Due to their bulk, none of the tables of tidal amplitudes are reproduced here, but are available on request in computer-readable form.

2.2.3 Pole Tide

In addition to the effects of ocean tides, another secondary tidal effect is the displacement of a receiver by the elastic response of the Earth's crust to shifts in the spin axis orientation. The spin axis is known to describe a circle of ≈ 20-m diameter at the north pole. Depending on where the spin axis pierces the crust at the instant of a measurement, the "pole tide" displacement will differ from time to time. This effect must be included if centimeter accuracy is desired, especially for measurements spanning an appreciable fraction of a year.

Yoder (1984) derived an expression for the displacement of a point at latitude ϕ, longitude λ due to the pole tide:

$$\begin{aligned}\boldsymbol{\delta} = -\frac{\omega_E^2 R}{g}\big[&\sin\phi\cos\phi(x\cos\lambda + y\sin\lambda)h\hat{\mathbf{r}}\\ &+ \cos 2\phi(x\cos\lambda + y\sin\lambda)l\hat{\boldsymbol{\phi}}\\ &+ \sin\phi(-x\sin\lambda + y\cos\lambda)l\hat{\boldsymbol{\lambda}}\big]\end{aligned} \tag{2.16}$$

Here ω_E is the rotation rate of the Earth, R the radius of the (spherical) Earth, g the acceleration due to gravity at the Earth's surface, and h and l the customary Love numbers. Displacements of the spin axis from the 1903.0 CIO pole position along the x and y axes are given by x and y. Eq. (2.16) shows how these map into receiver displacements along the unit vectors in the radial ($\hat{\mathbf{r}}$), latitude ($\hat{\boldsymbol{\phi}}$), and longitude ($\hat{\boldsymbol{\lambda}}$) directions. With the standard values $\omega_E = 7.292 \times 10^{-5}$ rad/sec, $R = 6378$ km, and $g = 980.665$ g/cm^2, the factor $\omega_E^2 R/g = 3.459 \times 10^{-3}$. Since the maximum values of x and y are of the order of 10 meters, and $h \approx 0.6$, $l \approx 0.08$, the maximum displacement due to the pole tide is 1 to 2 cm, depending on the location of the receiver (ϕ, λ).

As in the case of the previously considered tidal effects, the locally referenced displacement $\boldsymbol{\delta}$ is transformed via the transformation Eq. (2.14) to give the displacement $\boldsymbol{\Delta}_{pol}$ in the standard Earth-fixed coordinate system. After each of the locally referenced tidal displacements has been transformed to these coordinates, the receiver location is

$$\mathbf{r}_E = \mathbf{r}_{E_0} + \boldsymbol{\Delta}_{sol} + \boldsymbol{\Delta}_{ocn} + \boldsymbol{\Delta}_{pol} \tag{2.17}$$

2.3 TRANSFORMATION FROM EARTH-FIXED TO GEOCENTRIC INERTIAL COORDINATE SYSTEMS

The Earth is approximately an oblate spheroid, spinning in the presence of two massive moving objects (the Sun and the Moon) which are positioned such that their time-varying gravitational effects not only produce tides on the Earth, but also subject it to torques. In addition, the Earth is covered by a complicated fluid layer, and also is not perfectly solid internally. As a result, the orientation of the Earth is a very complicated function of time, which to first order can be represented as the composite of a time-varying rotation rate, a wobble, a nutation, and a precession. The exchange of angular momentum between the solid Earth and the fluids on its surface is not readily predictable, and thus must be continually determined experimentally. Nutation and precession are well modeled theoretically. At the centimeter level, however, even these models are not completely adequate.

The rotational transformation, Q, of coordinate frames from the Earth-fixed frame to the geocentric inertial frame is composed of 6 separate rotations (actually 10, since the nutation transformation, N, consists of 3 transformations in itself, as does the "perturbation" transformation, Ω) applied to a vector in the Earth-fixed system:

$$Q = \Omega PNUXY \tag{2.18}$$

Thus, if $\mathbf{r}_E$ is a receiver location expressed in the Earth-fixed system, *e.g.*, the result of Eq. (2.17), that location, $\mathbf{r}_I$, expressed in the geocentric inertial (J2000) system is

$$\mathbf{r}_I = Q\mathbf{r}_E \tag{2.19}$$

Note that since we rotate the Earth rather than the celestial sphere, our rotation matrices, Ω, P, and N, will be the transposes of those used to rotate the inertial system of J2000 to an inertial system of date.

2.4 UT1 AND POLAR MOTION

The first transformation, Y, is a right-handed rotation about the x axis of the Earth-fixed frame by an angle Θ_2. Currently, the Earth-fixed frame is the 1903.0 CIO frame, except that the positive y axis is at 90 degrees east (Moscow). The x axis is coincident with the 1903.0 meridian of Greenwich, and the z axis is the 1903.0 standard pole. The Y rotation matrix is

$$Y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\Theta_2 & \sin\Theta_2 \\ 0 & -\sin\Theta_2 & \cos\Theta_2 \end{pmatrix} \tag{2.20}$$

where Θ_2 is identical to the Θ_y that may be readily obtained from the pole position published by BIH (1987) or IRIS (IAG, 1987).

The next rotation in sequence is the right-hand rotation through an angle Θ_1 about the y axis obtained after the previous rotation has been applied:

$$X = \begin{pmatrix} \cos\Theta_1 & 0 & -\sin\Theta_1 \\ 0 & 1 & 0 \\ \sin\Theta_1 & 0 & \cos\Theta_1 \end{pmatrix} \tag{2.21}$$

In this rotation, Θ_1 is identical to the Θ_x obtainable from the BIH or IRIS value for the pole position. Note that we have incorporated in the matrix definitions the transformation from the left-handed system used by BIH to the right-handed system we use. Note also that any source of polar motion data can be used provided it is represented in a left-handed system. The only effect would be a change in the definition of the Earth-fixed reference system.

The application of "XY" to a vector in the Earth-fixed system of coordinates expresses that vector as it would be observed in a coordinate frame whose z axis was along the Earth's ephemeris pole. The third rotation, U, is about the resultant z axis obtained by applying "XY." It is a rotation through the angle, $-H$, where H is the hour angle of the true equinox of date (*i.e.*, the dihedral angle measured westward between the xz plane defined above and the meridian plane containing the true equinox of date). The equinox of date is the point defined on the celestial equator by the intersection of the mean ecliptic with that equator. It is that intersection where the mean ecliptic rises from below the equator to above it (ascending node).

$$U = \begin{pmatrix} \cos H & -\sin H & 0 \\ \sin H & \cos H & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.22}$$

The angle H is composed of two parts:

$$H = h_\gamma + \alpha_E \tag{2.23}$$

where h_γ is the hour angle of the mean equinox of date, and α_E is the difference in hour angle of the true equinox of date and the mean equinox of date, a difference which is due to the nutation of the Earth. This set of definitions is cumbersome and couples the nutation and precession effects into Earth rotation measurements. However, in order to provide a direct estimate of conventional $UT1$ it is convenient to endure this historical approach, at least for the near future.

$UT1$ (universal time) is defined to be such that the hour angle of the mean equinox of date is given by the following expression (Aoki *et al.*, 1982, and Kaplan, 1981):

$$\begin{aligned} h_\gamma = {} & UT1 + 6^h\ 41^m\ 50^s.54841 + 8640184^s.812866\ T_u \\ & + 0^s.093104\ T_u^2 - 6^s.2 \times 10^{-6}\ T_u^3 \end{aligned} \tag{2.24}$$

where

$$T_u = \frac{(Julian\ UT1\ date)\ -\ 2451545.0}{36525} \tag{2.25}$$

The actual equivalent expression which is coded is:

$$\begin{aligned} h_\gamma =& 2\pi(UT1\ Julian\ day\ fraction) + 67310^s.54841 \\ &+ 8640184^s.812866\ T_u + 0^s.093104\ T_u^2 - 6^s.2 \times 10^{-6}\ T_u^3 \end{aligned} \tag{2.26}$$

This expression produces a time, $UT1$, which tracks the Greenwich hour angle of the real Sun to within 16^m. However, it really is sidereal time, modified to fit our intuitive desire to have the Sun directly overhead at noon on the Greenwich meridian. Historically, differences of $UT1$ from a uniform measure of time, such as atomic time, have been used in specifying the orientation of the Earth. Note that this definition has buried in it the precession constant since it refers to the mean equinox of date.

By the very definition of "mean of date" and "true of date," nutation causes a difference in the hour angles of the mean equinox of date and the true equinox of date. This difference, called the "equation of equinoxes," is denoted by α_E and is obtained accordingly:

$$\alpha_E = \tan^{-1}\left(\frac{y_{\gamma'}}{x_{\gamma'}}\right) = \tan^{-1}\left(\frac{N_{21}^{-1}}{N_{11}^{-1}}\right) = \tan^{-1}\left(\frac{N_{12}}{N_{11}}\right) \tag{2.27}$$

where the vector

$$\begin{pmatrix} x'_\gamma \\ y'_\gamma \\ z'_\gamma \end{pmatrix} = N_{ij}^{-1}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{2.28}$$

is the unit vector, in true equatorial coordinates of date, toward the mean equinox of date. In mean equatorial coordinates of date, this same unit vector is just $(1,0,0)^T$. The matrix ${N^{-1}}_{ij}$ is just the inverse (or equally, the transpose) of the transformation matrix N which will be defined below to effect the transformation from true equatorial coordinates of date to mean equatorial coordinates of date.

Depending on the smoothing used to produce the *a priori* $UT1 - UTC$ series, the short-period (t < 35 days) fluctuations in $UT1$ due to changes in the latitude and size of the mean tidal bulge may or may not be smoothed out. Since we want as accurate an *a priori* as possible, it may be necessary to add this effect to the $UT1$ *a priori* obtained from the series, $UT1_{smoothed}$. If this option is selected, then the desired *a priori* $UT1$ is given by

$$UT1_{a\ priori} = UT1_{smoothed} + \Delta UT1 \tag{2.29}$$

$UT1_{smoothed}$ represents an appropriately smoothed *a priori* measurement of the orientation of the Earth (*e.g.*, BIH circular D smoothed or, even better, $UT1R$), for which the short period (t < 35 days) tidal effects either have been averaged to zero, or, as in the case of $UT1R$, removed before smoothing. This $\Delta UT1$ can be represented as

$$\Delta UT1 = \sum_{i=1}^{N}\left[A_i \sin\left[\sum_{j=1}^{5} k_{ij}\alpha_j\right]\right] \tag{2.30}$$

where N is chosen to include all terms with a period less than 35 days. There are no other contributions until a period of 90 days is reached. However, these long-period terms are included by the measurements of the current Earth-orientation measurement services. The values for k_{ij} and A_i, along with the period involved are given in Table I. The α_i for i = 1, 5 are just the angles defined below in the nutation series as l, l', F, D, and Ω respectively.

It is convenient to apply "UXY" as a group. To parts in 10^{12}, $XY = YX$. However, with the same accuracy $UXY \neq XYU$. Neglecting terms of $O(\Theta^2)$ (which produce receiver location errors of approximately 6×10^{-6} meters):

$$UXY = \begin{pmatrix} \cos H & -\sin H & -\sin\Theta_1\cos H - \sin\Theta_2 \sin H \\ \sin H & \cos H & -\sin\Theta_1 \sin H + \sin\Theta_2\cos H \\ \sin\Theta_1 & -\sin\Theta_2 & 1 \end{pmatrix} \tag{2.31}$$

Table I
Periodic Tidally Induced Variations in UT1
With Periods Less Than 35 Days

Index i	Period (days)	Argument coefficient k_{i1}	k_{i2}	k_{i3}	k_{i4}	k_{i5}	A_i ($0^s.0001$)
1	5.64	1	0	2	2	2	-0.02
2	6.85	2	0	2	0	1	-0.04
3	6.86	2	0	2	0	2	-0.10
4	7.09	0	0	2	2	1	-0.05
5	7.10	0	0	2	2	2	-0.12
6	9.11	1	0	2	0	0	-0.04
7	9.12	1	0	2	0	1	-0.41
8	9.13	1	0	2	0	2	-0.99
9	9.18	3	0	0	0	0	-0.02
10	9.54	-1	0	2	2	1	-0.08
11	9.56	-1	0	2	2	2	-0.20
12	9.61	1	0	0	2	0	-0.08
13	12.81	2	0	2	-2	2	0.02
14	13.17	0	1	2	0	2	0.03
15	13.61	0	0	2	0	0	-0.30
16	13.63	0	0	2	0	1	-3.21
17	13.66	0	0	2	0	2	-7.76
18	13.75	2	0	0	0	-1	0.02
19	13.78	2	0	0	0	0	-0.34
20	13.81	2	0	0	0	1	0.02
21	14.19	0	-1	2	0	2	-0.02
22	14.73	0	0	0	2	-1	0.05
23	14.77	0	0	0	2	0	-0.73
24	14.80	0	0	0	2	1	-0.05
25	15.39	0	-1	0	2	0	-0.05
26	23.86	1	0	2	-2	1	0.05
27	23.94	1	0	2	-2	2	0.10
28	25.62	1	1	0	0	0	0.04
29	26.88	-1	0	2	0	0	0.05
30	26.98	-1	0	2	0	1	0.18
31	27.09	-1	0	2	0	2	0.44
32	27.44	1	0	0	0	-1	0.53
33	27.56	1	0	0	0	0	-8.26
34	27.67	1	0	0	0	1	0.54
35	29.53	0	0	0	1	0	0.05
36	29.80	1	-1	0	0	0	-0.06
37	31.66	-1	0	0	2	-1	0.12
38	31.81	-1	0	0	2	0	-1.82
39	31.96	-1	0	0	2	1	0.13
40	32.61	1	0	-2	2	-1	0.02
41	34.85	-1	-1	0	2	0	-0.09

2.5 NUTATION

With the completion of the $UT1$ and polar motion transformations, we are left with a receiver location vector, $\mathbf{r}_{date}$. This is the receiver location in true equatorial inertial coordinates of date. The last set of transformations are nutation, N, precession, P, and the perturbation rotation, Ω, applied in that order. These transformations give the receiver location, $\mathbf{r}_I$, in geocentric inertial J2000 equatorial coordinates:

$$\mathbf{r}_I = \Omega P N \mathbf{r}_{date} \tag{2.32}$$

Both the nutation and precession rotation angles are defined relative to their values at Julian date 2451545.0 (J2000). The angles are computed from trigonometric and polynomial series as a function of Barycentric Dynamic Time (TDB). This time scale, which is also used to reference the planetary ephemeris, is related to Terrestrial Dynamic Time (TDT) by (Kaplan, 1981)

$$TDB = TDT + 0^s.001658 \sin(g + 0.0167 \sin g) \tag{2.33}$$

$$g = 2\pi(357^\circ.578 + 35999^\circ.050\ TDT)/360^\circ \tag{2.34}$$

The time scale TDT runs at the same rate as International Atomic Time (TAI) and is offset from TAI by a defined constant,

$$TDT = TAI + 32^s.184 \tag{2.35}$$

All time arguments used in nutation and precession computations are measured in centuries of TDB from J2000 [*cf.* Eq. (2.25)].

The transformation matrix N is a composite of three separate rotations (Melbourne *et al.*, 1968):

1. $A(\varepsilon)$: true equatorial coordinates of date to ecliptic coordinates of date.

$$A(\varepsilon) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon & \sin\varepsilon \\ 0 & -\sin\varepsilon & \cos\varepsilon \end{pmatrix} \tag{2.36}$$

where ε is the true obliquity of the ecliptic.

2. $C^T(\delta\psi)$: nutation in longitude from ecliptic coordinates of date to mean ecliptic coordinates of date.

$$C^T(\delta\psi) = \begin{pmatrix} \cos\delta\psi & \sin\delta\psi & 0 \\ -\sin\delta\psi & \cos\delta\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.37}$$

where $\delta\psi$ is the nutation in ecliptic longitude.

3. $A^T(\bar{\varepsilon})$: ecliptic coordinates of date to mean equatorial coordinates.

In ecliptic coordinates of date, the mean equinox is at an angle, $\delta\psi = \tan^{-1}(y_{\overline{\gamma}}/x_{\overline{\gamma}})$. $\delta\varepsilon = \varepsilon - \bar{\varepsilon}$ is the nutation in obliquity, and $\bar{\varepsilon}$ is the mean obliquity (the dihedral angle between the plane of the ecliptic and the mean plane of the equator). "Mean" as used in this section implies that the short-period ($T \leq 18.6$ years) effects of nutation have been removed. Actually, the separation between nutation and precession is rather arbitrary, but historical. The composite rotation is:

$$N = A^T(\bar{\varepsilon})C^T(\delta\psi)A(\varepsilon) \tag{2.38}$$

$$= \begin{pmatrix} \cos\delta\psi & \cos\varepsilon\sin\delta\psi & \sin\varepsilon\sin\delta\psi \\ -\cos\bar{\varepsilon}\sin\delta\psi & \cos\bar{\varepsilon}\cos\varepsilon\cos\delta\psi + \sin\bar{\varepsilon}\sin\varepsilon & \cos\bar{\varepsilon}\sin\varepsilon\cos\delta\psi - \sin\bar{\varepsilon}\cos\varepsilon \\ -\sin\bar{\varepsilon}\sin\delta\psi & \sin\bar{\varepsilon}\cos\varepsilon\cos\delta\psi - \cos\bar{\varepsilon}\sin\varepsilon & \sin\bar{\varepsilon}\sin\varepsilon\cos\delta\psi + \cos\bar{\varepsilon}\cos\varepsilon \end{pmatrix}$$

The 1980 IAU nutation model (Seidelmann, 1982, and Kaplan, 1981) is used for obtaining the values for $\delta\psi$ and $\varepsilon - \bar{\varepsilon}$. The mean obliquity is obtained from Lieske *et al.* (1977) or from Kaplan (1981):

$$\bar{\varepsilon} = 23^\circ\ 26'\ 21.''448\ -\ 46.''8150\ T\ -\ 5.''9 \times 10^{-4}T^2\ +\ 1.''813 \times 10^{-3}T^3 \tag{2.39}$$

This nutation in longitude ($\delta\psi$) and in obliquity ($\delta\varepsilon = \varepsilon - \bar{\varepsilon}$) can be represented by a series expansion of the sines and cosines of linear combinations of five fundamental arguments. These are (Kaplan, 1981, Cannon, 1981):

1. The mean anomaly of the Moon:

$$\alpha_1 = l = 485866''.733 + (1325^r + 715922''.633)\,T + 31''.310\,T^2 + 0''.064\,T^3 \tag{2.40}$$

2. The mean anomaly of the Sun:

$$\alpha_2 = l' = 1287099''.804 + (99^r + 1292581''.224)\,T - 0''.577T^2 - 0.''012\,T^3 \tag{2.41}$$

3. The mean argument of latitude of the Moon:

$$\alpha_3 = F = 335778''.877 + (1342^r + 295263''.137)\,T - 13''.257\,T^2 + 0''.011\,T^3 \tag{2.42}$$

4. The mean elongation of the Moon from the Sun:

$$\alpha_4 = D = 1072261''.307 + (1236^r + 1105601''.328)\,T - 6''.891\,T^2 + 0''.019\,T^3 \tag{2.43}$$

5. The mean longitude of the ascending lunar node:

$$\alpha_5 = \Omega = 450160''.280 - (5^r + 482890''.539)\,T + 7''.455\,T^2 + 0''.008\,T^3 \tag{2.44}$$

where $1^r = 360° = 1296000''$.

With these fundamental arguments, the nutation quantities then can be represented by

$$\delta\psi = \sum_{j=1}^{N}\left[(A_{0j} + A_{1j}T)\sin\left[\sum_{i=1}^{5} k_{ji}\alpha_i(T)\right]\right] \tag{2.45}$$

and

$$\delta\varepsilon = \sum_{j=1}^{N}\left[(B_{0j} + B_{1j}T)\cos\left[\sum_{i=1}^{5} k_{ji}\alpha_i(T)\right]\right] \tag{2.46}$$

where the various values of α_i, k_{ji}, A_j, and B_j are listed in Table II.

An additional set of terms can be optionally added to the nutations $\delta\psi$ and $\delta\varepsilon$ in Eqs. (2.45) and (2.46). These include the out-of-phase nutations, and the free-core nutations (Yoder, 1983) with period ω_f (nominally 460 days). The "nutation tweaks" $\Delta\psi$ and $\Delta\varepsilon$ are arbitrary constant increments of the nutation angles $\delta\psi$ and $\delta\varepsilon$. Unlike the usual nutation expressions, they have no time dependence. The out-of-phase nutations, which are not included in the IAU 1980 nutation series, are identical to Eqs. (2.45) and (2.46), with the replacements sin $\leftrightarrow$ cos:

$$\delta\psi = \sum_{j=1}^{N}\Big[(A_{2j} + A_{3j}T)\cos\Big[\sum_{i=1}^{5} k_{ji}\alpha_i(T)\Big]\Big] \tag{2.47}$$

and

$$\delta\varepsilon = \sum_{j=1}^{N}\Big[(B_{2j} + B_{3j}T)\sin\Big[\sum_{i=1}^{5} k_{ji}\alpha_i(T)\Big]\Big] \tag{2.48}$$

Table II

1980 IAU Theory of Nutation

Index j	Period (days)	k_{j1}	k_{j2}	k_{j3}	k_{j4}	k_{j5}	A_{0j} (0″.0001)	A_{1j} (0″.0001)	B_{0j} (0″.0001)	B_{1j} (0″.0001)
1	6798.4	0	0	0	0	1	-171996	-174.2	92025	8.9
2	3399.2	0	0	0	0	2	2062	0.2	-895	0.5
3	1305.5	-2	0	2	0	1	46	0.0	-24	0.0
4	1095.2	2	0	-2	0	0	11	0.0	0	0.0
5	1615.7	-2	0	2	0	2	-3	0.0	1	0.0
6	3232.9	1	-1	0	-1	0	-3	0.0	0	0.0
7	6786.3	0	-2	2	-2	1	-2	0.0	1	0.0
8	943.2	2	0	-2	0	1	1	0.0	0	0.0
9	182.6	0	0	2	-2	2	-13187	-1.6	5736	-3.1
10	365.3	0	1	0	0	0	1426	-3.4	54	-0.1
11	121.7	0	1	2	-2	2	-517	1.2	224	-0.6
12	365.2	0	-1	2	-2	2	217	-0.5	-95	0.3
13	177.8	0	0	2	-2	1	129	0.1	-70	0.0
14	205.9	2	0	0	-2	0	48	0.0	1	0.0
15	173.3	0	0	2	-2	0	-22	0.0	0	0.0
16	182.6	0	2	0	0	0	17	-0.1	0	0.0
17	386.0	0	1	0	0	1	-15	0.0	9	0.0
18	91.3	0	2	2	-2	2	-16	0.1	7	0.0
19	346.6	0	-1	0	0	1	-12	0.0	6	0.0
20	199.8	-2	0	0	2	1	-6	0.0	3	0.0
21	346.6	0	-1	2	-2	1	-5	0.0	3	0.0
22	212.3	2	0	0	-2	1	4	0.0	-2	0.0
23	119.6	0	1	2	-2	1	4	0.0	-2	0.0
24	411.8	1	0	0	-1	0	-4	0.0	0	0.0
25	131.7	2	1	0	-2	0	1	0.0	0	0.0
26	169.0	0	0	-2	2	1	1	0.0	0	0.0
27	329.8	0	1	-2	2	0	-1	0.0	0	0.0
28	409.2	0	1	0	0	2	1	0.0	0	0.0
29	388.3	-1	0	0	1	1	1	0.0	0	0.0
30	117.5	0	1	2	-2	0	-1	0.0	0	0.0
31	13.7	0	0	2	0	2	-2274	-0.2	977	-0.5
32	27.6	1	0	0	0	0	712	0.1	-7	0.0
33	13.6	0	0	2	0	1	-386	-0.4	200	0.0
34	9.1	1	0	2	0	2	-301	0.0	129	-0.1
35	31.8	1	0	0	-2	0	-158	0.0	-1	0.0
36	27.1	-1	0	2	0	2	123	0.0	-53	0.0
37	14.8	0	0	0	2	0	63	0.0	-2	0.0
38	27.7	1	0	0	0	1	63	0.1	-33	0.0
39	27.4	-1	0	0	0	1	-58	-0.1	32	0.0
40	9.6	-1	0	2	2	2	-59	0.0	26	0.0

Table II cont.

1980 IAU Theory of Nutation

Index j	Period (days)	k_{j1}	k_{j2}	k_{j3}	k_{j4}	k_{j5}	A_{0j} (0''.0001)	A_{1j}	B_{0j} (0''.0001)	B_{1j}
41	9.1	1	0	2	0	1	-51	0.0	27	0.0
42	7.1	0	0	2	2	2	-38	0.0	16	0.0
43	13.8	2	0	0	0	0	29	0.0	-1	0.0
44	23.9	1	0	2	-2	2	29	0.0	-12	0.0
45	6.9	2	0	2	0	2	-31	0.0	13	0.0
46	13.6	0	0	2	0	0	26	0.0	-1	0.0
47	27.0	-1	0	2	0	1	21	0.0	-10	0.0
48	32.0	-1	0	0	2	1	16	0.0	-8	0.0
49	31.7	1	0	0	-2	1	-13	0.0	7	0.0
50	9.5	-1	0	2	2	1	-10	0.0	5	0.0
51	34.8	1	1	0	-2	0	-7	0.0	0	0.0
52	13.2	0	1	2	0	2	7	0.0	-3	0.0
53	14.2	0	-1	2	0	2	-7	0.0	3	0.0
54	5.6	1	0	2	2	2	-8	0.0	3	0.0
55	9.6	1	0	0	2	0	6	0.0	0	0.0
56	12.8	2	0	2	-2	2	6	0.0	-3	0.0
57	14.8	0	0	0	2	1	-6	0.0	3	0.0
58	7.1	0	0	2	2	1	-7	0.0	3	0.0
59	23.9	1	0	2	-2	1	6	0.0	-3	0.0
60	14.7	0	0	0	-2	1	-5	0.0	3	0.0
61	29.8	1	-1	0	0	0	5	0.0	0	0.0
62	6.9	2	0	2	0	1	-5	0.0	3	0.0
63	15.4	0	1	0	-2	0	-4	0.0	0	0.0
64	26.9	1	0	-2	0	0	4	0.0	0	0.0
65	29.5	0	0	0	1	0	-4	0.0	0	0.0
66	25.6	1	1	0	0	0	-3	0.0	0	0.0
67	9.1	1	0	2	0	0	3	0.0	0	0.0
68	9.4	1	-1	2	0	2	-3	0.0	1	0.0
69	9.8	-1	-1	2	2	2	-3	0.0	1	0.0
70	13.7	-2	0	0	0	1	-2	0.0	1	0.0
71	5.5	3	0	2	0	2	-3	0.0	1	0.0
72	7.2	0	-1	2	2	2	-3	0.0	1	0.0
73	8.9	1	1	2	0	2	2	0.0	-1	0.0
74	32.6	-1	0	2	-2	1	-2	0.0	1	0.0
75	13.8	2	0	0	0	1	2	0.0	-1	0.0
76	27.8	1	0	0	0	2	-2	0.0	1	0.0
77	9.2	3	0	0	0	0	2	0.0	0	0.0
78	9.3	0	0	2	1	2	2	0.0	-1	0.0
79	27.3	-1	0	0	0	2	1	0.0	-1	0.0
80	10.1	1	0	0	-4	0	-1	0.0	0	0.0

Table II cont.

1980 IAU Theory of Nutation

Index j	Period (days)	k_{j1}	k_{j2}	k_{j3}	k_{j4}	k_{j5}	A_{0j} (0″.0001)	A_{1j} (0″.0001)	B_{0j} (0″.0001)	B_{1j} (0″.0001)
81	14.6	-2	0	2	2	2	1	0.0	-1	0.0
82	5.8	-1	0	2	4	2	-2	0.0	1	0.0
83	15.9	2	0	0	-4	0	-1	0.0	0	0.0
84	22.5	1	1	2	-2	2	1	0.0	-1	0.0
85	5.6	1	0	2	2	1	-1	0.0	1	0.0
86	7.3	-2	0	2	4	2	-1	0.0	1	0.0
87	9.1	-1	0	4	0	2	1	0.0	0	0.0
88	29.3	1	-1	0	-2	0	1	0.0	0	0.0
89	12.8	2	0	2	-2	1	1	0.0	-1	0.0
90	4.7	2	0	2	2	2	-1	0.0	0	0.0
91	9.6	1	0	0	2	1	-1	0.0	0	0.0
92	12.7	0	0	4	-2	2	1	0.0	0	0.0
93	8.7	3	0	2	-2	2	1	0.0	0	0.0
94	23.8	1	0	2	-2	0	-1	0.0	0	0.0
95	13.1	0	1	2	0	1	1	0.0	0	0.0
96	35.0	-1	-1	0	2	1	1	0.0	0	0.0
97	13.6	0	0	-2	0	1	-1	0.0	0	0.0
98	25.4	0	0	2	-1	2	-1	0.0	0	0.0
99	14.2	0	1	0	2	0	-1	0.0	0	0.0
100	9.5	1	0	-2	-2	0	-1	0.0	0	0.0
101	14.2	0	-1	2	0	1	-1	0.0	0	0.0
102	34.7	1	1	0	-2	1	-1	0.0	0	0.0
103	32.8	1	0	-2	2	0	-1	0.0	0	0.0
104	7.1	2	0	0	2	0	1	0.0	0	0.0
105	4.8	0	0	2	4	2	-1	0.0	0	0.0
106	27.3	0	1	0	1	0	1	0.0	0	0.0

Expressions similar to these are adopted for the free-core nutations:

$$\delta\psi = (A_{00} + A_{10}T)\sin(\omega_f T) + (A_{20} + A_{30}T)\cos(\omega_f T) \tag{2.49}$$

and

$$\delta\varepsilon = (B_{00} + B_{10}T)\cos(\omega_f T) + (B_{20} + B_{30}T)\sin(\omega_f T) \tag{2.50}$$

The nutation model thus contains a total of 856 parameters: A_{ij} (i=0,3; j=1,106) and B_{ij} (i=0,3; j=1,106) plus the free-nutation amplitudes A_{i0} (i=0,3), B_{i0} (i=0,3). The only nonzero *a priori* amplitudes are the A_{0j}, A_{1j}, B_{0j}, B_{1j} (j=1,106) of the 1980 IAU nutation series.

The nutation tweaks are just constant additive factors to the angles $\delta\psi$ and $\delta\varepsilon$:

$$\delta\psi \rightarrow \delta\psi + \Delta\psi \tag{2.51}$$

and

$$\delta\varepsilon \rightarrow \delta\varepsilon + \Delta\varepsilon \tag{2.52}$$

It is emphasized that, for the present, the default nutation model in GPSOMC is just the 1980 IAU nutation model.

2.6 PRECESSION

The next transformation in going from the Earth-fixed frame to the geocentric inertial frame is the rotation P. This is the precession transformation from mean equatorial coordinates of date to the equatorial coordinates of the reference epoch (*e.g.*, J2000). It is the transpose of the matrix given on page 7 of Melbourne *et al.* (1968):

$$P_{11} = \cos\varsigma_A \cos\Theta_A \cos Z_A - \sin\varsigma_A \sin Z_A \tag{2.53}$$
$$P_{12} = \cos\varsigma_A \cos\Theta_A \sin Z_A + \sin\varsigma_A \cos Z_A \tag{2.54}$$
$$P_{13} = \cos\varsigma_A \sin\Theta_A \tag{2.55}$$
$$P_{21} = -\sin\varsigma_A \cos\Theta_A \cos Z_A - \cos\varsigma_A \sin Z_A \tag{2.56}$$
$$P_{22} = -\sin\varsigma_A \cos\Theta_A \sin Z_A + \cos\varsigma_A \cos Z_A \tag{2.57}$$
$$P_{23} = -\sin\varsigma_A \sin\Theta_A \tag{2.58}$$
$$P_{31} = -\sin\Theta_A \cos Z_A \tag{2.59}$$
$$P_{32} = -\sin\Theta_A \sin Z_A \tag{2.60}$$
$$P_{33} = \cos\Theta_A \tag{2.61}$$

With the angular units in arc seconds, the arguments are:

$$\varsigma_A = 2306.2181\,T + 0.30188\,T^2 + 0.017998\,T^3 \tag{2.62}$$
$$Z_A = 2306.2181\,T + 1.09468\,T^2 + 0.018203\,T^3 \tag{2.63}$$
$$\Theta_A = 2004.3109\,T - 0.42665\,T^2 - 0.041833\,T^3 \tag{2.64}$$

These expressions are given by Lieske *et al.* (1977) and by Kaplan (1981). This completes the standard model for the orientation of the Earth.

2.7 PERTURBATION ROTATION

The standard model for the rotation of the Earth as a whole may need a small incremental rotation about any one of the resulting axes, for example, in compensating for defects in the *a priori* precession model. Define this perturbation rotation matrix as

$$\Omega = \Delta_x \Delta_y \Delta_z \tag{2.65}$$

where

$$\Delta_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\delta\Theta_x \\ 0 & \delta\Theta_x & 1 \end{pmatrix} \tag{2.66}$$

with $\delta\Theta_x$ being a small angle rotation about the x axis, in the sense of carrying y into z;

$$\Delta_y = \begin{pmatrix} 1 & 0 & \delta\Theta_y \\ 0 & 1 & 0 \\ -\delta\Theta_y & 0 & 1 \end{pmatrix} \tag{2.67}$$

with $\delta\Theta_y$ being a small angle rotation about the y axis, in the sense of carrying z into x; and

$$\Delta_z = \begin{pmatrix} 1 & -\delta\Theta_z & 0 \\ \delta\Theta_z & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.68}$$

with $\delta\Theta_z$ being a small angle rotation about the z axis, in the sense of carrying x into y. For angles of the order of 1 arc second we can neglect terms of order $\delta\Theta^2 R_E$ as they give effects on the order 0.015 cm. Thus, in that approximation

$$\Omega = \begin{pmatrix} 1 & -\delta\Theta_z & \delta\Theta_y \\ \delta\Theta_z & 1 & -\delta\Theta_x \\ -\delta\Theta_y & \delta\Theta_x & 1 \end{pmatrix} \tag{2.69}$$

In general,

$$\delta\Theta_i = \delta\Theta_i(t) = \delta\Theta_{i_0} + \delta\dot{\Theta}_i T + f_i(T) \tag{2.70}$$

which is the sum of an offset, a time-linear rate, and some higher-order or oscillatory terms. Currently, only the offset and linear rate are implemented. In particular, $\delta\dot{\Theta}_y$ is equivalent to a change in the precession constant. Setting

$$\delta\Theta_x = \delta\Theta_y = \delta\Theta_z = 0 \tag{2.71}$$

gives the effect of applying only the standard rotation matrices.

Starting with the Earth-fixed vector, $\mathbf{r}_{E_0}$, we have in Sections 2.1–2.6 above shown how we obtain the same vector, $\mathbf{r}_I$, expressed in the geocentric inertial frame:

$$\mathbf{r}_I = \Omega PNUXY(\mathbf{r}_{E_0} + \mathbf{\Delta}) \tag{2.72}$$

2.8 GEOCENTER OFFSET AND COORDINATE SCALING

The Earth-fixed reference frame is essentially derived from VLBI measurements, which are completely insensitive to the location of the Earth's center of mass. Practically, receiver coordinates are based on the location of a reference station, which is derived either from spacecraft tracking data or satellite laser ranging (SLR) measurements. Consequently, there might be sizable errors in fiducial station locations ($\sim$ 10 m or $\sim$ 1 m for the two techniques, respectively). GPS tracking data is capable of determining the position of the geocenter to an accuracy dependent on the quality and quantity of the range observations. In order to allow for the possibility of solving for the location of the geocenter, Cartesian offsets were introduced, such that the receiver coordinates (including those of the reference station(s)) are

$$\begin{aligned} x_{E_0} &\rightarrow x_{E_0} + x_{GC} \\ y_{E_0} &\rightarrow y_{E_0} + y_{GC} \\ z_{E_0} &\rightarrow z_{E_0} + z_{GC} \end{aligned} \tag{2.73}$$

or in vector notation,

$$\mathbf{r}_{E_0} \rightarrow \mathbf{r}_{E_0} + \mathbf{r}_{GC} \tag{2.74}$$

Another problem with coordinate systems at the present level of understanding is the uncertainty that the coordinate transformations for *a priori* satellite orbits are being carried out correctly. Introduction of a scale factor α, which multiplies Earth-fixed coordinates relative to the geocentric inertial coordinates of the satellites, permits empirical detection of such problems. It gives rise to the transformations

$$\begin{aligned} x_{E_0} &\rightarrow \alpha x_{E_0} \\ y_{E_0} &\rightarrow \alpha y_{E_0} \\ z_{E_0} &\rightarrow \alpha z_{E_0} \end{aligned} \tag{2.75}$$

or

$$\mathbf{r}_{E_0} \rightarrow \alpha \mathbf{r}_{E_0} \tag{2.76}$$

in vector notation, for the coordinates of the receivers.

Upon inclusion of both of these modifications in the Earth-fixed station locations $\mathbf{r}_{E_0}$ of Eq. (2.72), the final expression for the station location vector in geocentric inertial coordinates becomes

$$\mathbf{r}_I = \Omega PNUXY(\alpha \mathbf{r}_{E_0} + \mathbf{r}_{GC} + \mathbf{\Delta}) = Q(\alpha \mathbf{r}_{E_0} + \mathbf{r}_{GC} + \mathbf{\Delta}) \tag{2.77}$$

2.9 PHASE CENTER OFFSETS

Geometric offsets may exist between transmitter and receiver phase centers and the standard reference points. In the case of the ground-based receiver, this offset takes the form of a "site vector" from a surveyor's benchmark to the receiver phase center. Normally, the position of the benchmark, $\mathbf{r}_{BM}$, is desired, rather than the position of the phase center, $\mathbf{r}_{PC}$. These two vectors are related through the site vector $\mathbf{r}_{SURV}$:

$$\mathbf{r}_{E_0} = \mathbf{r}_{BM} = \mathbf{r}_{PC} + \mathbf{r}_{SURV} \tag{2.78}$$

For GPS satellites, the ephemeris reference point is the spacecraft center of mass (CM). The vector offset between the phase center and CM is given by $0.211\,\mathbf{i} + 0.886\,\mathbf{k}$ meters in a coordinate system in which the unit vector $\mathbf{k}$ points from the spacecraft CM to the Earth center, $\mathbf{j}$ is the normalized cross product of $\mathbf{k}$ with the unit vector from the spacecraft to the Sun, and $\mathbf{i}$ completes a right-handed system (Winn, 1984). Both of these geometric offsets are incorporated into GPSOMC modeling.

SECTION 3

OBSERVABLES AND CLOCK PARAMETERS

Range measurements from GPS satellites are based on the detection of the phase of transmitted electromagnetic signals. The physical observable is the difference between the reference phase of the transmitter at the time of signal emission and the reference phase of the receiver at the time of signal reception. Such measurements of necessity involve detailed and careful consideration of a number of time scales. These are intimately related to the definitions of the observables, and for that reason this section contains detailed descriptions of both the observables and clock models.

The GPS satellite transmissions are one-way transmissions originating at the spacecraft. A carrier signal modulated by a pseudo-random noise code is broadcast (Spilker, 1978). Both a "carrier phase" observable φ and a "pseudo-range" observable $\mathcal{R}$ are modeled in `GPSOMC`. The *observed* observables are related in a prescribed way to the physically detected phase. The *computed* observables are defined in terms of clock differences since the reference phase at the receiver is derived from the station clock and the reference phase at the transmitter is derived from the space vehicle clock. For both carrier phase and pseudo-range the computed observable is given by the theoretical difference between the space vehicle and station clocks plus a bias term. The clock difference is accumulated as the sum of five components – the first is the difference between station clock time and proper time at the station; the second is the difference between proper time and coordinate time at the station; the third is the difference between the coordinate time of signal emission and the coordinate time of signal reception; the fourth is the difference between coordinate time and proper time at the spacecraft; the fifth is the difference between proper time at the spacecraft and spacecraft clock time.

These terms are readily interpreted. The first term is referred to as a station clock error and the fifth term is referred to as a space vehicle clock error. The second and fourth terms are general relativistic time transformations. The third term is the solution to the light time equation connecting the events of signal emission and signal reception. Of these five contributions the third term is normally dominant. This term is also referred to as the geometric range between spacecraft and receiver, whence the use of "range" in referring to GPS observables. Tropospheric and ionospheric delays are not considered here; they are discussed in Sections 1 and 4 of this report. The observables are assumed to have been calibrated to remove the effects of instrumental delays. A detailed model for instrumental phase delay is not provided, but a bias term is included in the definition of both data types. This bias might account for

(i) an unknown integer number of cycles of phase,
(ii) an uncalibrated offset in the absolute phase of an oscillator,
(iii) uncalibrated delays within transmitter or receiver electronics, or
(iv) an uncalibrated offset between the phase reference used for signal detection and the phase reference used for time-tag generation within a receiver.

The geometric portion of the observable is calculated in the geocentric reference frame, moving with the Earth, with inertial J2000 oriented axes. This reference frame is the generalization of the local inertial reference frame containing the Earth (Ashby and Bertotti, 1984) and provides a simple setting for applying all necessary relativistic corrections. Terrestrial Dynamic Time (TDT) is used as coordinate time (Thomas, 1975 and Ashby and Allan, 1979). The spacecraft equations of motion are integrated in this reference frame using TDT as the independent variable. As described in Section 2 of this report, station coordinates are transformed into this reference system. Care must be taken when selecting *a priori* values for model parameters since particle mass and physical distance in the geocentric system differ by a scale factor from current determinations of same in the celestial coordinate system (Hellings, 1986).

We use t_2 to denote the time of signal emission and t_3 to denote the time of signal reception. Let $\tilde{t}_3$, $\bar{t}_3$, and t_3 refer to, respectively, station clock time, proper time, and coordinate time at the receiver. Let $\tilde{t}_2$, $\bar{t}_2$, and t_2 refer to, respectively, space vehicle clock time, proper time, and coordinate time at the transmitter.

3.1 PSEUDO-RANGE

Observed pseudo-range is given by

$$\mathcal{R}^O = c\ (\ \tilde{t}_3 - \tilde{t}_2\ ) \tag{3.1}$$

with time tag $\tilde{t}_3$, where the times $\tilde{t}_3$ and $\tilde{t}_2$ are the actual times kept by the station and spacecraft clocks, respectively. Correlation of the received pseudo-random noise code generated and transmitted by the space vehicle with a local copy of the code within the receiver allows for direct conversion between the detected received phase and clock readings.

Computed pseudo-range is given by

$$\mathcal{R}^C = c\ (\ \tilde{t}_3 - \tilde{t}_2 + B_{STN}\ ) \tag{3.2}$$

with time tag $\tilde{t}_3$, where now the times $\tilde{t}_3$ and $\tilde{t}_2$ are the modeled times of, respectively, the station and space vehicle clocks. To facilitate computation, this expression is rewritten as a sum of six terms,

$$\begin{aligned}\mathcal{R}^C = \ & c\ (\ \tilde{t}_3 - \bar{t}_3 \\ & + \bar{t}_3 - t_3 \\ & + t_3 - t_2 \\ & + t_2 - \bar{t}_2 \\ & + \bar{t}_2 - \tilde{t}_2 \\ & + B_{STN}\ )\end{aligned} \tag{3.3}$$

The first five terms are the five clock differences discussed above, and the last term is the bias discussed above.

The station clock error is modeled as a quadratic function,

$$\tilde{t}_3 - \bar{t}_3 \ = \ a_{STN,\mathcal{R}} \ + \ b_{STN,\mathcal{R}}(\tilde{t}_3 - \tilde{t}_{3_0}) \ + \ c_{STN,\mathcal{R}}(\tilde{t}_3 - \tilde{t}_{3_0})^2 \tag{3.4}$$

On the right hand side $\tilde{t}_{3_0}$ is a specified epoch. The coefficients $a_{STN,\mathcal{R}}$, $b_{STN,\mathcal{R}}$, and $c_{STN,\mathcal{R}}$ are generally unknown but presumed to be small. Ideally, in geocentric coordinates, all clocks fixed to the surface of the Earth run at approximately the same rate, with the precise rate depending weakly on the geodetic coordinates of the clock. We assume that all clocks are adjusted as necessary so that their average rate is the same as that of coordinate time. The rate of coordinate time has been defined to agree with the SI second (Kaplan, 1981). Thus, in our model, station proper time, ideal UTC time, and coordinate time all run at the same rate. This is not a limitation since the deviation of the rate of any clock from the assumed rate can be modeled as a station clock error.

The difference between proper time and coordinate time at the station is given by

$$\bar{t}_3 - t_3 = -\ [\ (TAI - UTC) + (TDT - TAI)\ ] \tag{3.5}$$

where $TAI - UTC$ is an integer number of leap seconds which changes approximately once a year and $TDT - TAI$ is defined to be 32.184 sec.

The geometric range is given by the solution to the light time equation,

$$t_3 - t_2 = \frac{|\ \mathbf{r}_{STN}(t_3) - \mathbf{r}_{SV}(t_2)|}{c} + \Delta t_{rel_{geom}} \tag{3.6}$$

where $\mathbf{r}_{STN}(t_3)$ is the inertial receiver position at the time of signal reception, $\mathbf{r}_{SV}(t_2)$ is the space vehicle position at the time of signal transmission, and $\Delta t_{rel_{geom}}$ is the general relativistic correction to the geometric range. To begin the computation of the range, we start from the measurement epoch $\tilde{t}_3$. This is converted to $\bar{t}_3$ using Eq. (3.4) and to t_3 using Eq. (3.5). The station position in geocentric inertial coordinates is then calculated at the epoch t_3 using Eq. (2.77). Thus station clock errors affect the computation of the geometric range. Given the station position and a PV file containing the geocentric inertial spacecraft position as a function of coordinate time, the light time equation is iteratively solved to obtain t_2:

$$t_2^{(i+1)} = t_2^{(i)} + \frac{t_3 - t_2^{(i)} - r_{12}^{(i)}/c - \Delta t_{rel_{geom}}^{(i)}}{1 - \mathbf{r}_{12}^{(i)} \cdot \dot{\mathbf{r}}_2^{(i)}/(c r_{12}^{(i)})} \tag{3.7}$$

The space vehicle position used in this computation is that of the antenna phase center, as discussed in Section 2.9 of this report. The gravitational delay $\Delta t_{rel_{geom}}$ is given by (Tausner, 1966 and Holdridge, 1967)

$$\Delta t_{rel_{geom}} = \frac{(1 + \gamma_{PPN})\mu_E}{c^3} \ln \frac{r_1 + r_2 + r_{12}}{r_1 + r_2 - r_{12}} \tag{3.8}$$

where γ_{PPN} is 1 for general relativity, μ_E is the Earth's gravitational constant, and, as in Eq. (3.7),

$$\begin{aligned} \mathbf{r}_1 &= \mathbf{r}_{STN}(t_3) & r_1 &= |\mathbf{r}_1| \\ \mathbf{r}_2 &= \mathbf{r}_{SV}(t_2) & r_2 &= |\mathbf{r}_2| \\ \mathbf{r}_{12} &= \mathbf{r}_1 - \mathbf{r}_2 & r_{12} &= |\mathbf{r}_{12}| \end{aligned} \tag{3.9}$$

We use $\mathbf{r}_{SV}(t_3)$ as the initial estimate for $\mathbf{r}_{SV}(t_2)$; the light time iteration converges very fast.

The rate of an ideal space vehicle clock will differ from the rate of coordinate time due to the difference in gravitational potential and speed between the space vehicle clock and the reference clock. This rate difference may be decomposed into two components: a bias component which depends only on the nominal semi-major orbit axis of the space vehicle, and a periodic component which depends mainly on the orbit eccentricity. The frequencies of the GPS satellite clocks were purposely set low before launch, relative to the nominal published values, to offset the nominal bias rate difference between space vehicle time and coordinate time (Spilker, 1978). Accordingly, our model rate difference between proper time and coordinate time at the spacecraft contains only a periodic component. This component depends only on orbit eccentricity. A bias component due to off-nominal semi-major orbit axis and a small periodic component due to Earth oblateness and solar perturbations are absorbed in the space vehicle clock error model. The difference between proper time and coordinate time at the space vehicle is given by

$$t_2 - \bar{t}_2 = (TAI - UTC) + (TDT - TAI) + \Delta t_{rel_{clk}} \tag{3.10}$$

where the clock correction term is

$$\Delta t_{rel_{clk}} = 2\, \mathbf{r}_{SV}(t_2) \cdot \dot{\mathbf{r}}_{SV}(t_2) \tag{3.11}$$

This result may be derived from integration of the usual differential equation relating coordinate and proper time [*e.g.*, Eq. (1) of Thomas (1975)] in the geocentric reference system, modeling the space vehicle orbit about a point-mass Earth as elliptical while ignoring the differential gravitational potential due to perturbing bodies. The resulting rate expression is accurate to about one part in 10^{12} for the GPS orbits, which is comparable to the instability of the GPS satellite clocks. Note that the large ($\approx$ 50 sec) defined offset between coordinate and proper time appears in Eqs. (3.5) and (3.10) with opposite sign, and hence does not affect the computation. It is necessary, though, to include this offset due to our convention of basing observable time tags on UTC while indexing the PV file by TDT.

Analogously to the station clock error, the space vehicle clock error is also modeled as a quadratic function,

$$\bar{t}_2 - \tilde{t}_2 \;=\; a_{SV,\mathcal{R}} \;+\; b_{SV,\mathcal{R}}(\bar{t}_2 - \bar{t}_{2_0}) \;+\; c_{SV,\mathcal{R}}(\bar{t}_2 - \bar{t}_{2_0})^2 \tag{3.12}$$

On the right hand side $\bar{t}_{2_0}$ is a specified epoch. The space vehicle clock error does not affect geometric range. The last term in the model is the pseudo-range bias B_{STN}, which depends on the station but is independent of the spacecraft. It also does not affect the value of the computed geometric range.

3.2 CARRIER PHASE

Observed carrier phase is given by

$$\varphi^O = (-c/\omega_n)\ (\phi_{SV} - \phi_{STN}) \tag{3.13}$$

with time tag $\tilde{t}_3$, where ϕ_{SV} is the received RF (radio frequency) phase of the space vehicle signal at time $\tilde{t}_3$, ϕ_{STN} is the receiver reference phase at time $\tilde{t}_3$, and ω_n is the nominal value for both the transmitted frequency of the space vehicle signal and the receiver mixing frequency.

The receiver reference phase is modeled as

$$\phi_{STN}(\tilde{t}_3) = \omega_n\ (\ \tilde{t}_3 - \tilde{t}'_{3_0}) \tag{3.14}$$

Since phase is a physical invariant, the received phase of the space vehicle signal at the time of signal reception is the same as the space vehicle reference phase at the time of signal transmission,

$$\phi_{SV}(\tilde{t}_3) = \phi_{ref}(\tilde{t}_2) \tag{3.15}$$

The space vehicle reference phase is modeled as

$$\phi_{ref}(\tilde{t}_2) = \omega_n\ (\ \tilde{t}_2 - \tilde{t}'_{2_0}) \tag{3.16}$$

Thus the computed carrier phase is given by

$$\varphi^C = c\ (\ \tilde{t}_3 - \tilde{t}_2 + B_{SV,STN}) \tag{3.17}$$

with time tag $\tilde{t}_3$. The integration constants $\tilde{t}'_{3_0}$ and $\tilde{t}'_{2_0}$ have been absorbed in the carrier phase bias $B_{SV,STN}$. Deviation of the station mixing frequency from nominal is modeled as a station clock error, and deviation of the space vehicle transmitter frequency from nominal is modeled as a space vehicle clock error. Note that the modeled value for carrier phase appears to be the same as the modeled value for pseudo-range, except for the substitution of the carrier phase bias for the pseudo-range bias. A further distinction is that we allow the modeled values of station time and spacecraft time to be different for the two data types.

Just as for pseudo-range the expression for computed carrier phase is rewritten as a sum of six terms,

$$\begin{aligned}\varphi^C = \ & c\ (\ \tilde{t}_3 - \bar{t}_3 \\ & + \bar{t}_3 - t_3 \\ & + t_3 - t_2 \\ & + t_2 - \bar{t}_2 \\ & + \bar{t}_2 - \tilde{t}_2 \\ & + B_{SV,STN}\ )\end{aligned} \tag{3.18}$$

The station clock error is modeled as

$$\tilde{t}_3 - \bar{t}_3\ =\ a_{STN,\varphi}\ +\ b_{STN,\varphi}(\bar{t}_3 - \bar{t}_{3_0})\ +\ c_{STN,\varphi}(\bar{t}_3 - \bar{t}_{3_0})^2 \tag{3.19}$$

and the space vehicle clock error is modeled as

$$\bar{t}_2 - \tilde{t}_2 = a_{SV,\varphi} + b_{SV,\varphi}(\bar{t}_2 - \bar{t}_{2_0}) + c_{SV,\varphi}(\bar{t}_2 - \bar{t}_{2_0})^2 \tag{3.20}$$

The carrier phase bias $B_{SV,STN}$ depends on both the station and the space vehicle. The other terms in Eq. (3.18) are evaluated just as for pseudo-range. They will, however, have different numerical values unless the station clock error coefficients are identical for the two data types. The value of the station clock error affects the geometric range, while the values of the carrier phase bias and space vehicle clock error do not.

In cases where the behavior of transmitter and receiver clocks permits, both the range and phase observables may be modeled with a common set of parameters. By analogy with Eqs. (3.4), (3.12), (3.19), and (3.20), the common clock errors may be written as

$$\tilde{t}_3 - \bar{t}_3 = a_{STN} + b_{STN}(\tilde{t}_3 - \tilde{t}_{3_0}) + c_{STN}(\tilde{t}_3 - \tilde{t}_{3_0})^2 \tag{3.21}$$

and

$$\bar{t}_2 - \tilde{t}_2 = a_{SV} + b_{SV}(\bar{t}_2 - \bar{t}_{2_0}) + c_{SV}(\bar{t}_2 - \bar{t}_{2_0})^2 \tag{3.22}$$

SECTION 4

TROPOSPHERE

The delay experienced by the incoming signal due to the Earth's atmosphere can be modeled using a spherical-shell troposphere consisting of a wet component and a dry component. If E is the apparent geodetic elevation angle of the spacecraft, the tropospheric contribution to the range is

$$\rho_{trop} = \rho_{Z_{dry}} R_{dry}(E) + \rho_{Z_{wet}} R_{wet}(E) \tag{4.1}$$

where ρ_Z is the additional delay at local zenith due to the presence of the troposphere, and R is an elevation angle mapping function. For recent VLBI data, it was found that modeling the zenith delay as a linear function of time improves troposphere modeling considerably. The dry and wet zenith parameters are therefore written as

$$\rho_{Z_{d,w}} = \rho^0_{Z_{d,w}} + \dot{\rho}_{Z_{d,w}}(t - t_0) \tag{4.2}$$

where t_0 is a reference time.

The analytic mapping function developed by Lanyi (1984) is used for mapping zenith values to the observed elevation angles. In its simplest form, this mapping function employs average values of atmospheric constants. Provision is made for specifying surface meteorological data acquired at the time of the experiments, which may override the average values. An approximate partial derivative is obtained with respect to one parameter in the Lanyi mapping function; this permits adjustment even in the absence of surface data.

Here we attempt to give a minimal summary of the final formulas. The tropospheric delay is written as:

$$\rho_{trop} = F(E)/\sin E \tag{4.3}$$

where

$$\begin{aligned} F(E) &= \rho_{Z_{dry}} F_{dry}(E) + \rho_{Z_{wet}} F_{wet}(E) \\ &+ [\rho^2_{Z_{dry}} F_{b1}(E) + 2\rho_{Z_{dry}}\rho_{Z_{wet}} F_{b2}(E) + \rho^2_{Z_{wet}} F_{b3}(E)]/\Delta + \rho^3_{Z_{dry}} F_{b4}(E)/\Delta^2 \end{aligned} \tag{4.4}$$

The quantities $\rho_{Z_{dry}}$ and $\rho_{Z_{wet}}$ have the usual meaning: zenith dry and wet tropospheric delays. Δ is the atmospheric scale height, $\Delta = kT_0/mg_c$, with k = Boltzmann's constant, T_0 = average surface temperature, m = mean molecular mass of dry air, and g_c = gravitational acceleration at the center of gravity of the air column. With the standard values $k = 1.38066 \times 10^{-16}$ erg/K, $m = 4.8097 \times 10^{-23}$ g, $g_c = 978.37$ erg/g·cm, and the average temperature for DSN stations $T_0 = 292$ K, the scale height $\Delta = 8567$ m.

The dry, wet, and bending contributions to the delay, $F_{dry}(E)$, $F_{wet}(E)$, and $F_{b1,b2,b3,b4}(E)$, are expressed in terms of moments of the refractivity as

$$F_{dry}(E) = A_{10}(E)G(\lambda M_{110}, u) + 3\sigma u M_{210} G^3(M_{110}, u)/4 \tag{4.5}$$

$$F_{wet}(E) = A_{01}(E)G(\lambda M_{101}/M_{001}, u)/M_{001} \tag{4.6}$$

$$F_{b1}(E) = [\sigma G^3(M_{110}, u)/\sin^2 E - M_{020}G^3(M_{120}/M_{020}, u)]/2\tan^2 E \tag{4.7}$$

$$F_{b2}(E) = -M_{011}G^3(M_{111}/M_{011}, u)/2M_{001}\tan^2 E \tag{4.8}$$

$$F_{b3}(E) = -M_{002}G^3(M_{102}/M_{002}, u)/2M^2_{001}\tan^2 E \tag{4.9}$$

$$F_{b4}(E) = M_{030}G^3(M_{130}/M_{030}, u)/2\tan^4 E \tag{4.10}$$

A misprinted sign in the last of Eqs. (5) of Appendix B of Lanyi (1984) has been corrected in Eq. (4.10). Here $G(q,u)$ is a geometric factor given by

$$G(q, u) = (1 + qu)^{-1/2} \tag{4.11}$$

with

$$u = 2\sigma / \tan^2 E \tag{4.12}$$

where σ is a measure of the curvature of the Earth's surface Δ/R with standard value 0.001345.

The quantities $A_{lm}(E)$ and M_{ilm} are related to moments of the atmospheric refractivity, and are defined below. $A_{10}(E)$ involves the dry refractivity, while $A_{01}(E)$ is the corresponding wet quantity. The $A_{lm}(E)$ are given by

$$A_{lm}(E) = M_{0lm} + \sum_{n=1}^{10} \sum_{k=0}^{n} \frac{(-1)^{n+k}(2n-1)!!M_{n-k,l,m}}{2^n k!(n-k)!} \left[\frac{u}{1+\lambda u M_{1lm}/M_{0lm}}\right]^n \left[\frac{\lambda M_{1lm}}{M_{0lm}}\right]^k \tag{4.13}$$

with the scale factor $\lambda = 3$ for $E < 10°$ and $\lambda = 1$ for $E > 10°$. Only the two combinations $(l, m) =$ (0,1) and (1,0) are needed for the $A_{lm}(E)$. The moments of the dry and wet refractivities are defined as

$$M_{nij} = \int_0^\infty q^n f^i_{dry}(q) f^j_{wet}(q) dq \tag{4.14}$$

where $f_{dry,\, wet}(q)$ are the surface-normalized refractivities. Here, n ranges from 0 to 1, i from 0 to 3, and j from 0 to 2; not all combinations are needed. Carrying out the integration in Eq. (4.14) for a three-section temperature profile gives an expression for the general moment M_{nij}:

$$\begin{aligned} M_{nij}/n! &= (1-e^{-aq_1})/a^{n+1} + e^{-aq_1}\left[1 - T_2^{b+n+1}(q_1,q_2)\right] \prod_{i=0}^{n} \frac{\alpha}{b+i+1} \\ &\quad + e^{-aq_1} T_2^{b+n+1}(q_1,q_2)/a^{n+1} \end{aligned} \tag{4.15}$$

Here,

$$T_2(q_1,q_2) = 1 - (q_2 - q_1)/\alpha \tag{4.16}$$

The quantities q_1 and q_2 are the scale-height normalized inversion and tropopause altitudes, respectively. For the standard atmospheric model, $q_1 = 0.1459$ and $q_2 = 1.424$. The constants a and b are functions of the dry ($\alpha = 5.0$) and wet ($\beta = 3.5$) model parameters, as well as of the powers of the refractivities (i and j) in the moment definitions. Table III gives the necessary a's and b's.

Table III

Dependence of the Constants a and b on Tropospheric Model Parameters

i	j	a	b
1	0	1	α-1
0	1	β	$\alpha\beta - 2$
2	0	2	$2(\alpha - 1)$
1	1	$\beta + 1$	$\beta(\alpha + 1) - 3$
0	2	2β	$2(\alpha\beta - 2)$
3	0	3	$3(\alpha - 1)$

Note that the normalization is such that $M_{010} = 1$; this moment has therefore not been explicitly written in Eqs. (4.5)–(4.10).

At present, provision is made for input of four meteorological parameters to override the standard (average) values of the Lanyi model. These are: (1) the surface temperature T_0, which determines the atmosphere scale height (default value 292 K); (2) the temperature lapse rate W, which determines the dry model parameter α (default values $W = 6.8165$ K/km, $\alpha = 5.0$); (3) the inversion altitude h_1, which determines $q_1 = h_1/\Delta$ (default value $h_1 = 1.25$ km); and (4) the tropopause altitude h_2, which determines $q_2 = h_2/\Delta$ (default value $h_2 = 12.2$ km). A fifth parameter, the surface pressure p_0, is not used at present. Approximate sensitivity of the tropospheric delay (at 15° elevation) to the meteorological parameters is -0.3 cm/K for surface temperature, 1 cm/(K/km) for lapse rate, and -1 cm/km for inversion and 0.2 cm/km for tropopause altitude, respectively.

SECTION 5

DERIVATIVES OF OBSERVABLES WITH RESPECT TO MODEL PARAMETERS

This section presents expressions for the partial derivatives of the computed observables with respect to the three classes of model parameters described in this document, as well as with respect to dynamic parameters related to the spacecraft state. The relativistic corrections, the tropospheric correction, and the space vehicle clock error term in the computed observable depend weakly on geometric parameters, on the station clock error, and on dynamic parameters. These functional dependences are neglected when computing partial derivatives.

We begin with some notation. From Eqs. (3.2) through (3.17) the computed observable can be written as

$$C = c\ (\tilde{t}_3 - \bar{t}_3)\ +\ \rho\ +\ c\ (\bar{t}_2 - \tilde{t}_2)\ +\ c\ B_\nu \tag{5.1}$$

where C is $\mathcal{R}^C$ for pseudo-range and φ^C for carrier phase, and ν is STN for pseudo-range and SV, STN for carrier phase.

The clock error terms are given by

$$\tilde{t}_3 - \bar{t}_3\ =\ a_{STN,\beta}\ +\ b_{STN,\beta}(\tilde{t}_3 - \tilde{t}_{3_0})\ +\ c_{STN,\beta}(\tilde{t}_3 - \tilde{t}_{3_0})^2 \tag{5.2}$$

and

$$\bar{t}_2 - \tilde{t}_2\ =\ a_{SV,\beta}\ +\ b_{SV,\beta}(\bar{t}_2 - \bar{t}_{2_0})\ +\ c_{SV,\beta}(\bar{t}_2 - \bar{t}_{2_0})^2 \tag{5.3}$$

where β is $\mathcal{R}$ for pseudo-range and φ for carrier phase. The geometric range is given by

$$\rho = |\ \mathbf{r}_{STN} - \mathbf{r}_{SV}| \tag{5.4}$$

where $\mathbf{r}_{STN}$ is the geocentric inertial station position at the time t_3 of signal reception and $\mathbf{r}_{SV}$ is the space vehicle position, in the same system, at the time t_2 of signal emission. The station position comes from Eq. (2.77),

$$\mathbf{r}_{STN}\ =\ \Omega PNUXY\ (\ \alpha \mathbf{r}_{STN_0} + \mathbf{r}_{GC} + \boldsymbol{\Delta}) \tag{5.5}$$

The space vehicle position and the times of signal emission and reception are related through the light time equation, from which we may write the geometric range as

$$\rho = c\ (\ t_3 - t_2\ ) \tag{5.6}$$

and, equivalently,

$$c^2(t_3 - t_2)^2 = (\ \mathbf{r}_{STN} - \mathbf{r}_{SV}) \cdot (\ \mathbf{r}_{STN} - \mathbf{r}_{SV}) \tag{5.7}$$

5.1 GEOMETRIC PARAMETERS

The class of "geometric" parameters includes receiver coordinates and their rates, Love numbers and tide phase, relativistic γ, geocenter offsets, coordinate scale factor, UTPM parameters, nutation amplitudes, and the coefficients of the perturbation rotation. Computed observables for both the carrier phase and pseudo-range have identical functional dependence on these geometric parameters through the geometric range. The partials will therefore be written for the geometric range given by Eq. (5.4). Symbolizing one of the geometric parameters by η, we have

$$\begin{aligned}\frac{\partial\rho}{\partial\eta} &= \frac{1}{\rho}(\mathbf{r}_{STN} - \mathbf{r}_{SV})^T\ \frac{\partial}{\partial\eta}(\mathbf{r}_{STN} - \mathbf{r}_{SV})\\ &= \frac{1}{\rho}(\mathbf{r}_{STN} - \mathbf{r}_{SV})^T\ \left[\frac{\partial\mathbf{r}_{STN}}{\partial\eta} - \frac{\partial\mathbf{r}_{SV}}{\partial\mathbf{r}_{STN}}\ \frac{\partial\mathbf{r}_{STN}}{\partial\eta}\right]\\ &= \frac{1}{\rho}(\mathbf{r}_{STN} - \mathbf{r}_{SV})^T\ \left[I_3 - \frac{\partial\mathbf{r}_{SV}}{\partial\mathbf{r}_{STN}}\right]\frac{\partial\mathbf{r}_{STN}}{\partial\eta}\end{aligned} \tag{5.8}$$

The light time correction term is computed first:

$$\frac{\partial \mathbf{r}_{SV}}{\partial \mathbf{r}_{STN}} = \dot{\mathbf{r}}_{SV} \frac{\partial t_2}{\partial \mathbf{r}_{STN}} \tag{5.9}$$

The partial on the right hand side is obtained from Eq. (5.7) by implicit differentiation:

$$2c^2(t_3 - t_2)\Big(-\frac{\partial t_2}{\partial \mathbf{r}_{STN}}\Big) = 2(\mathbf{r}_{STN} - \mathbf{r}_{SV})^T \Big[I_3 - \dot{\mathbf{r}}_{SV} \frac{\partial t_2}{\partial \mathbf{r}_{STN}}\Big] \tag{5.10}$$

Solving for $\frac{\partial t_2}{\partial \mathbf{r}_{STN}}$ gives

$$\frac{\partial t_2}{\partial \mathbf{r}_{STN}} = \frac{-(\mathbf{r}_{STN} - \mathbf{r}_{SV})^T}{c\rho\Big(1 - \frac{\mathbf{r}_{STN} - \mathbf{r}_{SV}}{\rho} \cdot \frac{\dot{\mathbf{r}}_{SV}}{c}\Big)} \tag{5.11}$$

The computation of the geometric partials is complete except for the parameter-dependent terms $\frac{\partial \mathbf{r}_{STN}}{\partial \eta}$.

Taking the parameters in order of their appearance on the right hand side of Eq. (5.5), the partial derivatives are

$$\frac{\partial \mathbf{r}_{STN}}{\partial\ \delta\Theta_{x,y,z_0}} = \frac{\partial \Omega}{\partial\ \delta\Theta_{x,y,z_0}} PNUXY(\alpha \mathbf{r}_{STN_0} + \mathbf{r}_{GC} + \mathbf{\Delta}) \tag{5.12}$$

$$\frac{\partial \mathbf{r}_{STN}}{\partial\ \delta\dot{\Theta}_{x,y,z}} = \frac{\partial \Omega}{\partial\ \delta\dot{\Theta}_{x,y,z}} PNUXY(\alpha \mathbf{r}_{STN_0} + \mathbf{r}_{GC} + \mathbf{\Delta}) \tag{5.13}$$

for the perturbation rotation parameters;

$$\frac{\partial \mathbf{r}_{STN}}{\partial A_{ij}} = \Omega P\Big(\frac{\partial NU}{\partial A_{ij}}\Big)XY(\alpha \mathbf{r}_{STN_0} + \mathbf{r}_{GC} + \mathbf{\Delta}) \tag{5.14}$$

$$\frac{\partial \mathbf{r}_{STN}}{\partial B_{ij}} = \Omega P\Big(\frac{\partial NU}{\partial B_{ij}}\Big)XY(\alpha \mathbf{r}_{STN_0} + \mathbf{r}_{GC} + \mathbf{\Delta}) \tag{5.15}$$

for the nutation series amplitudes;

$$\frac{\partial \mathbf{r}_{STN}}{\partial \Delta\psi} = \Omega P\Big(\frac{\partial NU}{\partial \Delta\psi}\Big)XY(\alpha \mathbf{r}_{STN_0} + \mathbf{r}_{GC} + \mathbf{\Delta}) \tag{5.16}$$

$$\frac{\partial \mathbf{r}_{STN}}{\partial \Delta\varepsilon} = \Omega P\Big(\frac{\partial NU}{\partial \Delta\varepsilon}\Big)XY(\alpha \mathbf{r}_{STN_0} + \mathbf{r}_{GC} + \mathbf{\Delta}) \tag{5.17}$$

for the nutation offsets;

$$\frac{\partial \mathbf{r}_{STN}}{\partial (UT1 - UTC)} = \Omega PN \frac{\partial U}{\partial (UT1 - UTC)} XY(\alpha \mathbf{r}_{STN_0} + \mathbf{r}_{GC} + \mathbf{\Delta}) \tag{5.18}$$

for universal time, and

$$\frac{\partial \mathbf{r}_{STN}}{\partial \Theta_{1,2}} = \Omega PNU\Big(\frac{\partial XY}{\partial \Theta_{1,2}}\Big)(\alpha \mathbf{r}_{STN_0} + \mathbf{r}_{GC} + \mathbf{\Delta}) \tag{5.19}$$

for polar motion. The only quantities remaining to be evaluated in Eqs. (5.12– 5.19) are the partial derivatives of the various rotation matrices. These are exactly the same as the partials entering modeling of VLBI observations, and are set out in some detail in the publication of Sovers and Fanselow (1987), Sec. 2.12. For example, the first partial in Eq. (5.12) is

$$\frac{\partial \Omega}{\partial\ \delta\Theta_{x_0}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \tag{5.20}$$

Continuing the evaluation of partial derivatives with respect to geometric parameters, and using $Q = \Omega PNUXY$ from Eq. (2.18) to simplify notation, we have for the station coordinates and their time rates of change,

$$\mathbf{r}_{STN_0} = \mathbf{r}^0_{STN} + \dot{\mathbf{r}}^0_{STN}(t - t_0) \tag{5.21}$$

$$\mathbf{r}_{STN} = Q(\alpha \mathbf{r}_{STN_0} + \mathbf{r}_{GC} + \mathbf{\Delta}) \tag{5.22}$$

$$\frac{\partial \mathbf{r}_{STN}}{\partial \mathbf{r}^0_{STN}} = \alpha Q \tag{5.23}$$

$$\frac{\partial \mathbf{r}_{STN}}{\partial \dot{\mathbf{r}}^0_{STN}} = \alpha(t - t_0)Q \tag{5.24}$$

For the geocenter offset components,

$$\frac{\partial \mathbf{r}_{STN}}{\partial \mathbf{r}_{GC}} = Q \tag{5.25}$$

For the coordinate scale factor,

$$\frac{\partial \mathbf{r}_{STN}}{\partial \alpha} = Q\mathbf{r}_{STN_0} \tag{5.26}$$

The only parameters in the tidal displacement Δ are the two solid-Earth-tide Love numbers h and l, and the tide phase angle ψ. The corresponding partial derivatives of the range are somewhat complicated:

$$\frac{\partial \mathbf{r}_{STN}}{\partial h} = QVW \begin{pmatrix} g_{1s} \\ 0 \\ 0 \end{pmatrix} \tag{5.27}$$

$$\frac{\partial \mathbf{r}_{STN}}{\partial l} = QVW \begin{pmatrix} 0 \\ g_{2s} \\ g_{3s} \end{pmatrix} \tag{5.28}$$

$$\frac{\partial \mathbf{r}_{STN}}{\partial \psi} = QVW \begin{pmatrix} \dfrac{\partial g_1(i)}{\partial \psi_i} \\ \dfrac{\partial g_2(i)}{\partial \psi_i} \\ \dfrac{\partial g_3(i)}{\partial \psi_i} \end{pmatrix} \tag{5.29}$$

where the tide phase partials of the g_is of Eqs. (2.6–2.8) are

$$\frac{\partial g_{1s}}{\partial \psi} = \frac{3\mu_s r_p^2}{R_s^5} h\mathbf{r}_p \cdot \mathbf{R}_s (X_s y_p - Y_s x_p) \tag{5.30}$$

$$\frac{\partial g_{2s}}{\partial \psi} = \frac{3\mu_s r_p^2}{R_s^5}\ l\ \frac{|\mathbf{r}_p|}{\sqrt{x_p^2 + y_p^2}} \left[(\mathbf{r}_p \cdot \mathbf{R}_s)(x_p X_s + y_p Y_s) - (Y_s x_p - X_s y_p)^2\right] \tag{5.31}$$

$$\frac{\partial g_{3s}}{\partial \psi} = \frac{3\mu_s r_p^2}{R_s^5}\, l\,(X_s y_p - Y_s x_p)\left[\sqrt{x_p^2 + y_p^2}\,Z_s - \frac{z_p}{\sqrt{x_p^2+y_p^2}}(2\mathbf{r}_p \cdot \mathbf{R}_s - z_p Z_s)\right] \tag{5.32}$$

Finally, the partial derivative of range with respect to the post-Newtonian parameter γ_{PPN} [see Eq. (3.8)] is

$$\frac{\partial \rho}{\partial \gamma_{PPN}} = \frac{\mu_E}{c^2} \ln \frac{r_1 + r_2 + r_{12}}{r_1 + r_2 - r_{12}} \tag{5.33}$$

5.2 CLOCK PARAMETERS

Clock parameters include the coefficients of the station and space vehicle clock error models and also the pseudo-range and carrier phase biases. From Eq. (5.1), the partial of pseudo-range with respect to the first station clock error parameter is

$$\begin{aligned} \frac{\partial \mathcal{R}^C}{\partial a_{STN,\mathcal{R}}} &= c\,\frac{\partial}{\partial a_{STN,\mathcal{R}}}(\tilde{t}_3 - \bar{t}_3) + \frac{\partial \rho}{\partial t_3} \cdot \frac{\partial t_3}{\partial a_{STN,\mathcal{R}}} \\ &= c - \dot{\rho} \end{aligned} \tag{5.34}$$

where we have used

$$\frac{\partial}{\partial a_{STN,\mathcal{R}}}(\tilde{t}_3 - \bar{t}_3) = 1 \tag{5.35}$$

$$\frac{\partial t_3}{\partial a_{STN,\mathcal{R}}} = \frac{\partial}{\partial a_{STN,\mathcal{R}}}[\tilde{t}_3 - (\tilde{t}_3 - t_3)] = -1 \tag{5.36}$$

and

$$\dot{\rho} = \frac{\partial \rho}{\partial t_3} \tag{5.37}$$

The range rate $\dot{\rho}$ is, from Eq. (5.6), given by

$$\dot{\rho} = c\left(1 - \frac{\partial t_2}{\partial t_3}\right) \tag{5.38}$$

The partial on the right hand side is obtained from implicit differentiation of Eq. (5.7):

$$2c^2(t_3 - t_2)\left(1 - \frac{\partial t_2}{\partial t_3}\right) = 2\,(\mathbf{r}_{STN} - \mathbf{r}_{SV}) \cdot \left(\dot{\mathbf{r}}_{STN} - \dot{\mathbf{r}}_{SV}\frac{\partial t_2}{\partial t_3}\right) \tag{5.39}$$

Solving for $\frac{\partial t_2}{\partial t_3}$ gives

$$\frac{\partial t_2}{\partial t_3} = \frac{c\rho - (\mathbf{r}_{STN} - \mathbf{r}_{SV}) \cdot \dot{\mathbf{r}}_{STN}}{c\rho - (\mathbf{r}_{STN} - \mathbf{r}_{SV}) \cdot \dot{\mathbf{r}}_{SV}} \tag{5.40}$$

where we have used Eq. (5.6). Substituting into Eq. (5.38) gives

$$\dot{\rho} = \frac{(\mathbf{r}_{STN} - \mathbf{r}_{SV}) \cdot (\dot{\mathbf{r}}_{STN} - \dot{\mathbf{r}}_{SV})}{\rho\left(1 - \frac{\mathbf{r}_{STN} - \mathbf{r}_{SV}}{\rho} \cdot \frac{\dot{\mathbf{r}}_{SV}}{c}\right)} \tag{5.41}$$

The station velocity is obtained from the time derivative of Eq. (5.5), and the space vehicle velocity is obtained from interpolation of the PV file.

The partial with respect to the first space vehicle clock error parameter is

$$\frac{\partial \mathcal{R}^C}{\partial a_{SV,\mathcal{R}}} = c\,\frac{\partial}{\partial a_{SV,\mathcal{R}}}(\bar{t}_2 - \tilde{t}_2) = c \tag{5.42}$$

The pseudo-range bias partial is

$$\frac{\partial \mathcal{R}^C}{\partial B_{STN}} = c \tag{5.43}$$

The pseudo-range partials with respect to the other clock error parameters are similarly computed, as are the carrier phase partials; here we summarize the results.

$$\begin{aligned}
\frac{\partial \mathcal{R}^C}{\partial a_{STN,\mathcal{R}}} &= c - \dot{\rho} & \qquad \frac{\partial \mathcal{R}^C}{\partial a_{SV,\mathcal{R}}} &= c \\
\frac{\partial \mathcal{R}^C}{\partial b_{STN,\mathcal{R}}} &= (c - \dot{\rho})(\tilde{t}_3 - \tilde{t}_{3_0}) & \qquad \frac{\partial \mathcal{R}^C}{\partial b_{SV,\mathcal{R}}} &= c(\bar{t}_2 - \bar{t}_{2_0}) \\
\frac{\partial \mathcal{R}^C}{\partial c_{STN,\mathcal{R}}} &= (c - \dot{\rho})(\tilde{t}_3 - \tilde{t}_{3_0})^2 & \qquad \frac{\partial \mathcal{R}^C}{\partial c_{SV,\mathcal{R}}} &= c(\bar{t}_2 - \bar{t}_{2_0})^2
\end{aligned} \tag{5.44}$$

$$\frac{\partial \mathcal{R}^C}{\partial B_{STN}} = c \tag{5.45}$$

$$\begin{aligned}
\frac{\partial \varphi^C}{\partial a_{STN,\varphi}} &= (c - \dot{\rho}) & \qquad \frac{\partial \varphi^C}{\partial a_{SV,\varphi}} &= c \\
\frac{\partial \varphi^C}{\partial b_{STN,\varphi}} &= (c - \dot{\rho})(\tilde{t}_3 - \tilde{t}_{3_0}) & \qquad \frac{\partial \varphi^C}{\partial b_{SV,\varphi}} &= c(\bar{t}_2 - \bar{t}_{2_0}) \\
\frac{\partial \varphi^C}{\partial c_{STN,\varphi}} &= (c - \dot{\rho})(\tilde{t}_3 - \tilde{t}_{3_0})^2 & \qquad \frac{\partial \varphi^C}{\partial c_{SV,\varphi}} &= c(\bar{t}_2 - \bar{t}_{2_0})^2
\end{aligned} \tag{5.46}$$

$$\frac{\partial \varphi^C}{\partial B_{SV,STN}} = c \tag{5.47}$$

5.3 TROPOSPHERE PARAMETERS

Partials of the range with respect to the dry and wet zenith delays are obtained from Eqs. (4.3) and (4.4):

$$\begin{aligned}\frac{\partial \rho}{\partial \rho_{Z_{dry}}} &= \left[F_{dry}(E) + 2\rho_{Z_{dry}} F_{b1}(E)/\Delta\right]/\sin E \\ &\quad + \left[2\rho_{Z_{wet}} F_{b2}(E)/\Delta + 3\rho^2_{Z_{dry}} F_{b4}(E)/\Delta^2\right]/\sin E \end{aligned} \tag{5.48}$$

$$\frac{\partial \rho}{\partial \rho_{Z_{wet}}} = \left[F_{wet}(E) + 2\rho_{Z_{dry}} F_{b2}(E)/\Delta + 2\rho_{Z_{wet}} F_{b3}(E)/\Delta\right]/\sin E \tag{5.49}$$

For the zenith rate parameters:

$$\frac{\partial \rho}{\partial \dot{\rho}_{Z_{d,w}}} = (t - t_0) R_{d,w} \tag{5.50}$$

In the analysis of data for which meteorological parameters are not available, it is convenient to introduce an approximation to the mapping function [Eqs. (4.3) and (4.4)] which involves a one-parameter estimate. This parameter p accounts for deviations from standard meteorological conditions. The tropospheric range is expressed as

$$\rho = \left(\rho_{Z_{dry}} + \rho_{Z_{wet}}\right)/\sin E + p\frac{\partial \rho}{\partial p} \tag{5.51}$$

where the partial derivative is

$$\begin{aligned}\frac{\partial \rho}{\partial p} &= -\frac{\left(\rho_{Z_{dry}} + \rho_{Z_{wet}}\right) u M_{110}}{G(M_{110}, u)\left[1 + G(M_{110}, u)\right]\sin E} \\ &\quad + \frac{\rho_{Z_{wet}} u \left(M_{110} - M_{101}/M_{001}\right)}{G(M_{110}, u) G(M_{101}/M_{001}, u)\left[G(M_{110}, u) + G(M_{101}/M_{001}, u)\right]\sin E}\end{aligned} \tag{5.52}$$

5.4 SATELLITE PARAMETERS

The final class of parameters in modeling GPS range measurements includes the dynamic parameters characterizing the forces on the satellites, and the six "epoch state" position and velocity parameters, $\mathbf{r}_{SV_0}$, $\dot{\mathbf{r}}_{SV_0}$ for each space vehicle. Carrier phase and pseudo-range are affected in the same way, through the geometric range ρ, by these parameters. Here we use ξ to represent any parameter of this class. The partial derivative of geometric range with respect to ξ is calculated from Eq. (5.6):

$$\frac{\partial \rho}{\partial \xi} = -c\frac{\partial t_2}{\partial \xi} \tag{5.53}$$

The partial on the right hand side is obtained from implicit differentiation of Eq. (5.7):

$$\begin{aligned} 2c^2(t_3 - t_2)\left(-\frac{\partial t_2}{\partial \xi}\right) &= 2(\mathbf{r}_{STN} - \mathbf{r}_{SV})^T\left(-\frac{\partial}{\partial \xi}\mathbf{r}_{SV}\right) \\ &= -2(\mathbf{r}_{STN} - \mathbf{r}_{SV})^T\left(\left.\frac{\partial \mathbf{r}_{SV}}{\partial \xi}\right|_{t_2} + \dot{\mathbf{r}}_{SV}\cdot\frac{\partial t_2}{\partial \xi}\right) \end{aligned} \tag{5.54}$$

Solving for $\frac{\partial t_2}{\partial \xi}$ gives

$$\frac{\partial t_2}{\partial \xi} = \frac{(\mathbf{r}_{STN} - \mathbf{r}_{SV})^T \left.\frac{\partial \mathbf{r}_{SV}}{\partial \xi}\right|_{t_2}}{c\rho\left(1 - \frac{\mathbf{r}_{STN} - \mathbf{r}_{SV}}{\rho}\cdot\frac{\dot{\mathbf{r}}_{SV}}{c}\right)} \tag{5.55}$$

Substituting this result into Eq. (5.53) gives

$$\frac{\partial \rho}{\partial \xi} = \frac{-(\mathbf{r}_{STN} - \mathbf{r}_{SV})^T \left.\frac{\partial \mathbf{r}_{SV}}{\partial \xi}\right|_{t_2}}{\rho\left(1 - \frac{\mathbf{r}_{STN} - \mathbf{r}_{SV}}{\rho}\cdot\frac{\dot{\mathbf{r}}_{SV}}{c}\right)} \tag{5.56}$$

The space vehicle velocity $\dot{\mathbf{r}}_{SV}$ and the variational partial $\left.\frac{\partial \mathbf{r}_{SV}}{\partial \xi}\right|_{t_2}$ are read from the PV file at time t_2. Detailed descriptions of the spacecraft force models and variational partials are found in the OASIS Mathematical Description document (Wu *et al.*, 1986).

SECTION 6

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APPENDIX A

Parameter Units and Physical Constants

Early during the development of software for the GIPSY project, it was decided that the units of all parameters and partial derivatives shall be kks (kilometers, kilograms, seconds), instead of the more customary mks. With the exception of clock parameters and range biases, we adhere to the kks convention. To prevent numerical instabilities in parameter estimation arising from the disparity of magnitudes of the clock parameters and the observables, clock offsets and range biases are in units of μsec (10^{-6} sec), rates in 10^{-12} sec/sec, and rate rates in 10^{-18} sec/sec^2. All lengths were measured in units of km, velocities in km/sec, and accelerations in km/sec^2. Angular quantities, such as tide lag, UTPM, the perturbation tweaks, and nutation angles are in radians (or rad/sec for their rates). Finally, the zenith troposphere quantities are measured in km, and their rates in km/sec. These conventions do not necessarily apply to the formatted GPSOMC input file, where units may be used for some parameters that make them more easily recognizable.

We have tried to use the constants recommended by the IAU project MERIT (Melbourne *et al.*, 1983). Those that have not been defined in the text above, but which affect the results that are obtained using GPSOMC, are given below:

Symbol	Value	Quantity
c	299792.458	Velocity of light (km/sec)
R_E	6378.140	Equatorial radius of the Earth (km)
ω_E	$7.2921151467 \times 10^{-5}$	Rotation rate of the Earth (rad/sec)
f	298.257	Flattening factor of the geoid
h	0.609	Vertical Love number
l	0.0852	Horizontal Love number

APPENDIX B

GPSOMC Parameters

Table B.I

Glossary of GPSOMC Parameters

Parameter	GPSOMC name	Definition	Reference
$a_{SV,\varphi}$	SAT PEPOvvvvvv	Coefficients in	(3.20)
$b_{SV,\varphi}$	SAT PRATvvvvvv	space vehicle clock	(3.20)
$c_{SV,\varphi}$	SAT PRRTvvvvvv	model (phase)	(3.20)
$a_{STN,\varphi}$	STA PEPO ss	Coefficients in	(3.19)
$b_{STN,\varphi}$	STA PRAT ss	station clock	(3.19)
$c_{STN,\varphi}$	STA PRRT ss	model (phase)	(3.19)
$B_{SV,STN}$	BIAS ^vv_ss;nn	Carrier phase bias	(3.18)
$a_{SV,\mathcal{R}}$	SAT REPOvvvvvv	Coefficients in	(3.12)
$b_{SV,\mathcal{R}}$	SAT RRATvvvvvv	space vehicle clock	(3.12)
$c_{SV,\mathcal{R}}$	SAT RRRTvvvvvv	model (range)	(3.12)
$a_{STN,\mathcal{R}}$	STA REPO ss	Coefficients in	(3.4)
$b_{STN,\mathcal{R}}$	STA RRAT ss	station clock	(3.4)
$c_{STN,\mathcal{R}}$	STA RRRT ss	model (range)	(3.4)
B_{STN}	BIAS PSR ss	Pseudo-range bias	(3.3)
a_{SV}	SAT CEPOvvvvvv	Coefficients in	(3.22)
b_{SV}	SAT CRATvvvvvv	space vehicle	(3.22)
c_{SV}	SAT CRRTvvvvvv	clock model	(3.22)
a_{STN}	STA CEPO ss	Coefficients in	(3.21)
b_{STN}	STA CRAT ss	station clock	(3.21)
c_{STN}	STA CRRT ss	model	(3.21)

vvvvvv satellite name
vv satellite number
ss station number
nn sequence number

Table B.I cont.

Glossary of GPSOMC Parameters

Parameter	GPSOMC name	Definition	Reference
r_{sp}	RSPINAX ss	Cylindrical	(2.1)
λ	LONGTUD ss	station	(2.2)
z	POLPROJ ss	coordinates	(2.3)
$\dot{r}_{sp}$	DRSP/DT ss	Time rates of	(2.1)
$\dot{\lambda}$	DLON/DT ss	change of	(2.2)
$\dot{z}$	DPOL/DT ss	station coordinates	(2.3)
h	VLOVE ss	Vertical Love number	(2.5)
l	HLOVE ss	Horizontal Love number	(2.5)
ψ	TIDPHAS ss	Tide lag	(2.10)
γ_{PPN}	GEN REL GAMMA	Post-Newtonian gamma	(3.8)
x_{GC}	GEOCENTER X	Coordinate frame offset	(2.73)
y_{GC}	GEOCENTER Y	Cartesian	(2.73)
z_{GC}	GEOCENTER Z	components	(2.73)
α	COORD SCALE	Scale factor	(2.75)
x_{SV_0}	X vvvvvv	Space vehicle epoch	
y_{SV_0}	Y vvvvvv	position Cartesian	
z_{SV_0}	Z vvvvvv	components	Sec. 5.4
$\dot{x}_{SV_0}$	DX vvvvvv	Space vehicle epoch	
$\dot{y}_{SV_0}$	DY vvvvvv	velocity Cartesian	
$\dot{z}_{SV_0}$	DZ vvvvvv	components	

ss station number

vvvvvv satellite name

Table B.I cont.

Glossary of GPSOMC Parameters

Parameter	GPSOMC name	Definition	Reference
Θ_1	X POLE MOTION	Pole position	(2.21)
Θ_2	Y POLE MOTION	components	(2.20)
$UT1 - UTC$	UT1 MINUS UTC	UT1 - UTC	(2.24)
$\delta\Theta_{x,y,z_0}$	† ROT TWEAK OFFS	Perturbation rotation	(2.70)
$\delta\dot{\Theta}_{x,y,z}$	† ROT TWEAK RATE	coefficients	(2.70)
A_{0j}, A_{2j}	NUT AMPLPSI ynnn	Nutation amplitudes	(2.45, 2.47)
A_{1j}, A_{3j}	NUT AMPLPSITynnn	in longitude	(2.45, 2.47)
$\Delta\psi$	NUT AMPLPSIA	Longitude nutation tweak	(2.51)
B_{0j}, B_{2j}	NUT AMPLEPS ynnn	Nutation amplitudes	(2.46, 2.48)
B_{1j}, B_{3j}	NUT AMPLEPSTynnn	in obliquity	(2.46, 2.48)
$\Delta\varepsilon$	NUT AMPLEPSA	Obliquity nutation tweak	(2.52)
ρz_{dry}	DRYZTROP ss	Dry zenith delay	(4.1)
ρz_{wet}	WETZTROP ss	Wet zenith delay	(4.1)
$\dot{\rho} z_{dry}$	DDTRP/DT ss	Zenith delay	(4.2)
$\dot{\rho} z_{wet}$	DWTRP/DT ss	time rates	(4.2)
p	DRYMAPSG ss	Lanyi map parameter	(5.51)

† X, Y, or Z

y S or C for sine or cosine

nnn number of term in Wahr series

ss station number