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Energy Transfer in Isotropic Turbulence at Low Reynolds Numbers

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Detailed measurements were made of energy transfer among the scales of motion in incompressible turbulent fields at low Reynolds numbers generated by direct numerical simulation. It was observed that although the transfer resulted from triad interactions that were non-local in k space, the energy always transferred locally. The results are consistent with the notion of non-uniform advection of small weak eddies by larger and stronger ones, similar to transfer processes in the far dissipation range at high Reynolds numbers.

1. Introduction

Our goal was to analyze velocity fields generated by direct numerical simulations of homogeneous, isotropic turbulence to better understand how energy is transferred among different scales of motion in such flows. At the present time there is some controversy concerning the importance of local versus nonlocal energy transfer processes. A number of theoretical works (Deissler 1978, Kraichnan 1971, 1976, and Dannevik 1987) and one experimental work (Lii et. al. 1976) predict relatively large energy transfer between eddies of disparate sizes (nonlocal energy transfer). On the other hand the classical argument of Kolmogoroff stresses a local energy cascade as a leading cause of the universal subrange. The numerical work of Domaradzki et al. (1987), Domaradzki (1988), Brasseur and Corrsin (1987), and Kerr (1988) indicates that at low Reynolds number very little energy is transfered between distant wavenumbers; the energy transfer occurs between similar wavenumbers. The resolution of these contradictions is needed since essentially all turbulence theories and models rely on assumptions about the energy transfer and those assumptions are tested only indirectly by comparing predictions of the models with available experimental data.

2. The computed velocity fields

We have used velocity fields generated by numerical simulations initialized with three different energy spectra. All simulations were run for sufficiently long times to establish nonlinear interactions and the energy spectra decreased by at least three orders of magnitude between the energy peak and the maximum resolved

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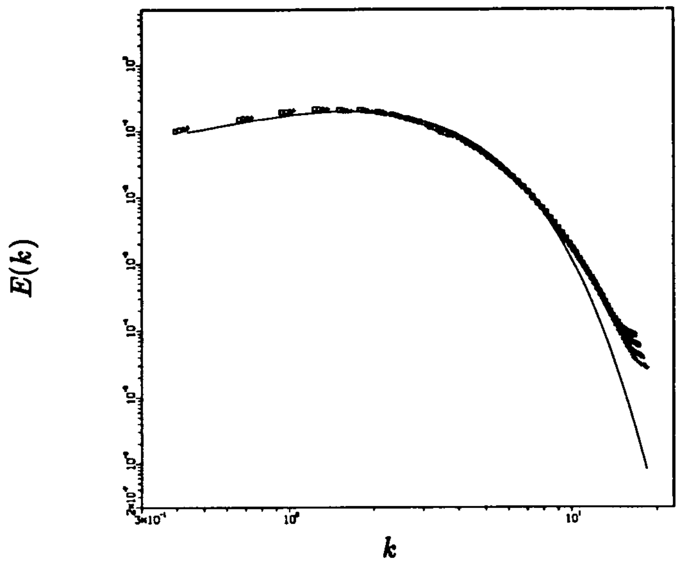


FIGURE 1. Three-dimensional energy spectra, normalized by total energy and Taylor microscale. —— data of Ling & Huang (1970), □ ○ △ + simulation I128D2 at several times.

wavenumber. The transfer spectra are well resolved and approach zero at the largest wavenumber. More detailed information about these fields is given in Table I.

The computed case I128D2D attempts to match the experimental results of Ling and Huang (1970) who found turbulence decay between microscale Reynolds numbers 30 and 3 to be self-similar. At this low Reynolds number the energy and dissipation ranges coincide, and there is only a single length scale. Use of the Taylor microscale collapses the results of the numerical simulations at different times (see figure 1) as it does the experimental data, and for this reason we consider the simulated velocity fields to be a fair model of laboratory, isotropic turbulence at low Reynolds numbers.

The case HIE24 was obtained from simulations of Lee and Reynolds (1985) that were initialized with an energy spectrum decreasing as $exp(-\alpha k^2)$ for large wavenumbers k. Such spectra are commonly used as initial conditions in direct numerical simulations of turbulence even though they decrease much faster than experimental spectra which behave as $exp(-\alpha k)$ for large wavenumbers.

The case F64DR is a result of simulations of turbulence forced in a manner suggested by Yakhot and Orszag (1986). Such forcing is used in the Renormalization Group (RNG) theory to generate a velocity field with a Kolmogoroff $k^{-5/3}$ spectrum that is stationary in time. Forcing of this type was used recently in simulations by Yakhot et al. (1988) to test numerically some predictions of the RNG theory. We were unable to obtain a significant inertial subrange in our forced simulations, the spectrum had a k^{-1} range at low wavenumber and decayed exponentially at higher wavenumbers. Nevertheless this velocity field is an important example of simulated

Table 1. Summary of computed cases

Case	N	k_{max}	k_{peak}	ν	R_λ	S
I128D2D	128^3	60.34	5	0.065	13.6	-0.44
HIE24	128^3	60.34	2	0.01377	20.4	-0.48
F64DR	64^3	30.16	1	0.1	46.0	-0.45

N = computational grid size
k_{max} = maximum resolved wavenumber
k_{peak} = location of maximum of energy spectrum
ν = molecular kinematic viscosity
R_λ = Reynolds number based on v_{rms} and Taylor microscale
S = velocity derivative skewness

turbulence that contains a range of wavenumbers k in which the energy spectrum decays algebraically. Such dependence is characteristic of high Reynolds number turbulence.

3. Basic measured quantities

The quantity of principal interest here is the energy exchange between a given mode $\mathbf{k}$ and all pairs of modes $\mathbf{p}$ and $\mathbf{q} = \mathbf{k} - \mathbf{p}$ that form a triangle with $\mathbf{k}$ as one of the legs and where $\mathbf{p}$ and $\mathbf{q}$ lie in some prescribed regions, $\mathcal{P}$ and $\mathcal{Q}$ respectively of the spectral space. For isotropic fields it is natural to choose $\mathcal{P}$ and $\mathcal{Q}$ as spherical shells $k - \frac{1}{2}\Delta k < |\mathbf{k}| < k + \frac{1}{2}\Delta k$ in the wave space with a shell thickness Δk. In addition to the velocity field $u_n(\mathbf{k})$ given on the entire Fourier mesh we define a truncated velocity field

$$u_n^{(\mathcal{P}\mathcal{Q})}(\mathbf{k}) = \begin{cases} u_n(\mathbf{k}), & \text{if } \mathbf{k} \in \mathcal{P} \text{ or } \mathbf{k} \in \mathcal{Q}; \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

The transfer for the truncated field is then

$$T_{\mathcal{P}\mathcal{Q}}(\mathbf{k}) = Im\{u_n^*(\mathbf{k}) P_{new}(\mathbf{k}) \int u_e^{(\mathcal{P}\mathcal{Q})}(\mathbf{p}) u_w^{(\mathcal{P}\mathcal{Q})}(\mathbf{k}-\mathbf{p}) d^3p\}, \tag{2}$$

where

$$P_{new}(\mathbf{k}) = k_w(\delta_{ne} - k_n k_e/k^2) + k_e(\delta_{nw} - k_n k_w/k^2), \tag{3}$$

the asterisk denotes complex conjugate, and the summation convention is assumed. We take regions $\mathcal{P}$ and $\mathcal{Q}$ to be spherical shells of radius p and q, and average over a spherical shell of radius k giving

$$T(k|p,q) = \begin{cases} 4\pi k^2 < T_{\mathcal{P}\mathcal{Q}}(\mathbf{k}) >, & \mathcal{P} \equiv \mathcal{Q} \\ 4\pi k^2 < T_{\mathcal{P}\mathcal{Q}}(\mathbf{k}) - T_{\mathcal{P}\mathcal{P}}(\mathbf{k}) - T_{\mathcal{Q}\mathcal{Q}}(\mathbf{k}) >, & \mathcal{P} \not\equiv \mathcal{Q} \end{cases}$$

Here $< \ldots >$ denotes the spherical averaging over $\mathbf{k}$. Note that $T(k|p,q)$ is the transfer into band k resulting from all triads having one leg in p, one in q, and one

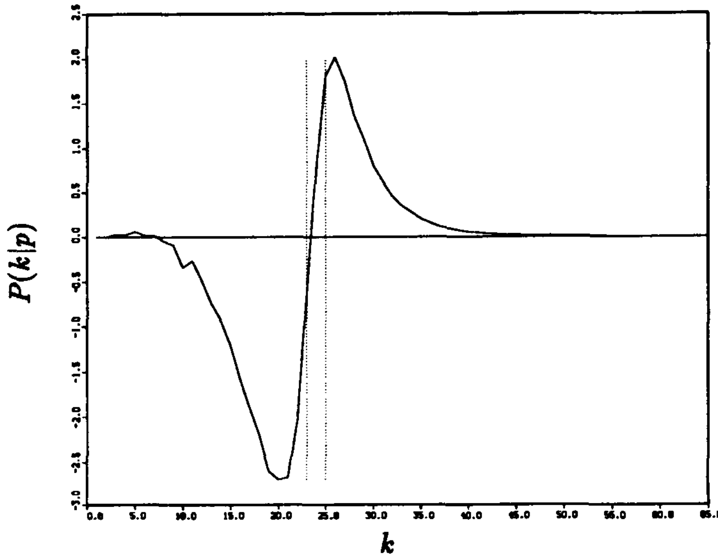


FIGURE 2. Energy transfer contributed by a single band. The total energy transferred by all triads having at least one leg in $23 < p < 25$ is shown.

in k. If the wave space is divided into distinct shells and $T(k|p,q)$ is summed over q for fixed p one obtains

$$P(k|p) = \sum_q T(k|p,q). \tag{4}$$

The function $P(k|p)$ calculated by Domaradzki (1988) by a different method was used to validate the calculations in this work. Finally, summing $P(k|p)$ over all shells p gives the total energy transfer to wavenumber k

$$T(k) = \sum_p P(k|p). \tag{5}$$

Functions $T(k), P(k|p)$, and $T(k|p,q)$ give progressively more detailed information about the energy transfer between different scales of motion.

4. Analysis of the function $P(k|p)$

Deissler (1978) estimated the energy transfer function $P(k|p)$ from the experimental data of Ling and Huang (1970) and concluded that their experimental data supports the notion of nonlocal energy transfer. Specifically, the calculated function $P(k|p)$ indicated that a wavenumber band k loses most of its energy to a band at wavenumber p that is about an order of magnitude greater than k.

In view of our results we believe that Deissler's analysis is in error. As Deissler points out, it is not possible to solve (5) uniquely for $P(k|p)$ because any function having zero sum may be added to the solution. Our measured function $P(k|p)$, for the same conditions as in figure 2a of Deissler, is shown in our figure 2 with wavenumber band p indicated by the two dotted vertical lines. The peaks in the transfer are close to this band $23 < p < 25$ indicating that the energy is transferred among wavenumbers k that are comparable to p, and moreover, energy is transfered from smaller to larger wavenumbers i.e. from larger to smaller scales of motion. In figure 2 the function $P(k|p)$ is negligible for k outside the interval $\frac{1}{2}p < k < 2p$, and we conclude that most of the energy is transfered between modes with wavenumber ratios not exceeding two.

The shape of the function $P(k|p)$ for a fixed p shown in figure 2 is typical for all values of p beyond the energy peak and for all velocity fields considered in our work. When the wavenumber band p is near the peak of the energy spectrum the function $P(k|p)$ still has two peaks for k close to the band p but they are broader, and values of $P(k|p)$ for $k > 2p$ are significant.

The function $P(k|p)$ does not provide information about the third leg of the wavevector triad, two legs of which have lengths k and p respectively. That information is provided by the function $T(k|p,q)$.

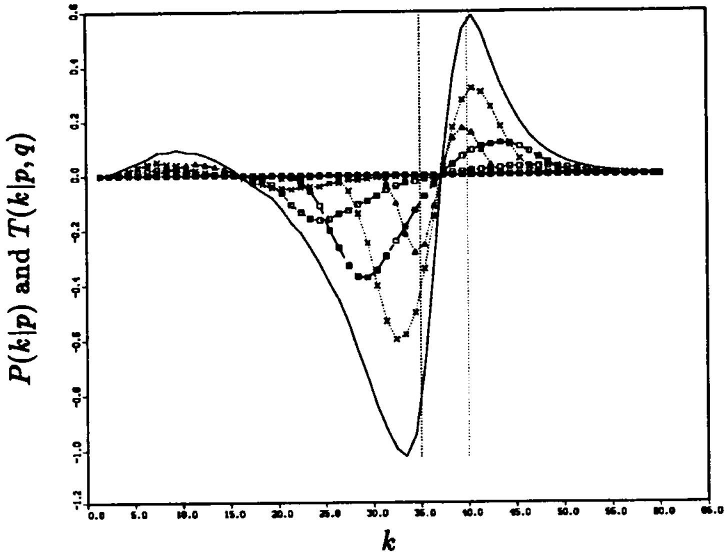
5. Analysis of the function $T(k|p,q)$

For the velocity field I128D2D we have divided Fourier space into 13 spherical shells of thickness $\Delta k = 5$. In figure 3a we show the decomposition of the function $P(k|p)$ into functions $T(k|p,q)$ for p fixed in the wavenumber band $35 < p < 40$. The solid line represents $P(k|p)$ and the lines with symbols represent the individual contributions $T(k|p,q)$ of wavenumber bands q to the sum (4). Even though there is a total of thirteen bands q in the sum (4), only three or four contribute significantly. Those with the largest contributions are wavenumbers $q < 15$.

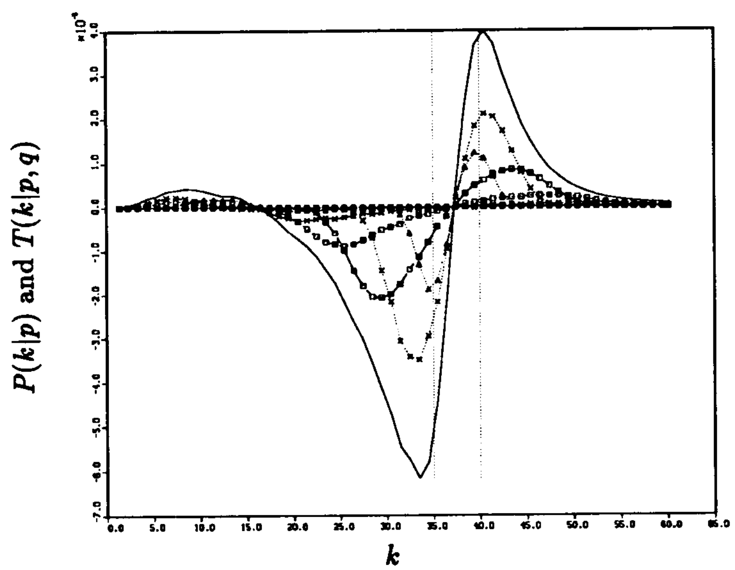
The total transfer $P(k|p)$ is mostly local since the peaks in the curve $P(k|p)$ are in the vicinity of the prescribed band p. Therefore, as explained in the previous section, the energy is transfered between two wavenumbers k of comparable magnitude to p, in this case $30 < k < 45$ and $35 < p < 40$. However, the decomposition (4) indicates that of all the triangles satisfying this condition only triangles with a *significantly smaller* third leg $q < 15$ contribute to the transfer. Similar behaviour was observed for other velocity fields considered in this work. The transfer curves for case HIE24 shown in figure 3b and for case F64DR shown in figure 4 exhibit the same qualitative behaviour as those in figure 3a. In all cases the transferring triads had a leg near the peak of the energy spectrum.

We conclude from this analysis that at low Reynolds numbers the *local energy transfer* between similar wavenumbers located beyond the energy containing range is due to *nonlocal wavevector triads* that have one leg much shorter than the other two.

It is interesting to note in the figures 3 and 4 that the effect of such nonlocal interactions on the smallest leg of the triad (large scale) is a small increase in its energy as represented by the positive values for $k < 15$ (figure 3), and for $k < 7$ (figure 4). This means that small (and less energetic) scales of motion transfer a small amount of energy to large (and more energetic) scales. This is a surprising result since the generally accepted models of energy transfer in turbulence assume



(a)



(b)

FIGURE 3. Detailed triad contributions to energy transfer: (a) case I128D2D, (b) case HIE24. The transfer spectra $T(k|p,q)$ are shown for band $35 < p < 40$, and all bands q that make a significant contribution to $P(k|p)$. △ ········ $0 < q < 5$, × ········ $5 < q < 10$, □ ——— $10 < q < 15$, ■ ---- $15 < q < 20$, × —·— $20 < q < 25$, ▲ —·— $25 < q < 30$, × ---- $30 < q < 35$, ——— $P(k|p)$.

that energy is transferred from large to small scales by nonlocal interactions i.e. one small wavenumber transfering large amounts of energy to two large wavenumbers,

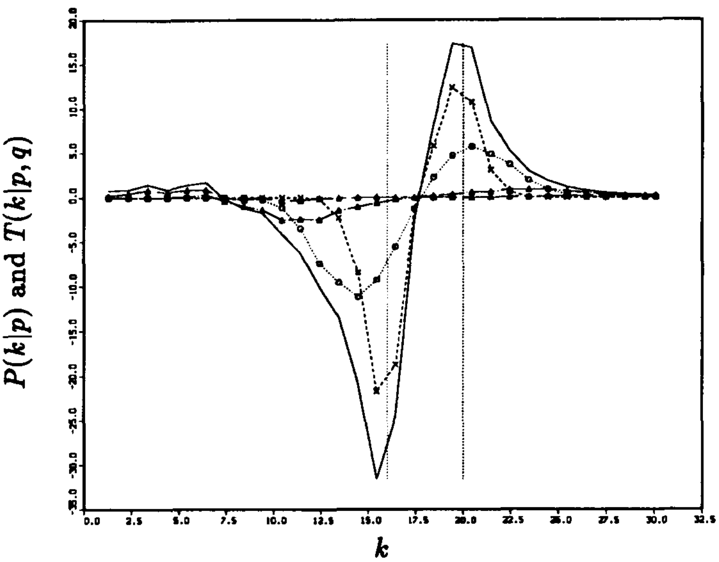


FIGURE 4. Detailed triad contributions to energy transfer for case F64DR. The transfer spectra $T(k|p,q)$ are shown for band $16 < p < 20$, and all bands q that make a significant contribution to $P(k|p)$. ×---- $0 < q < 4$, ○········ $4 < q < 8$, △ —·— $8 < q < 12$, —— $P(k,p)$.

and that eddy viscosity concepts provide an appropriate model.

6. Linear theory

The behavior, at high Reynolds numbers, of the turbulent energy spectrum in the far dissipation range, and the scalar spectrum in the viscous-convection range at high Prandtl number, have been predicted using assumptions about the structure of the velocity field at scales smaller than the Kolmogorov length. See Monin & Yaglom (1975), and the works cited there. The velocity field is assumed to contain small material regions, intermittent in space, in which the strain rate is high and spatially uniform. In addition to the assumption of disparate space scales, the time scales must also be assumed disparate to linearize the equation for vorticity. With these assumptions, the problem becomes a simple convection and decay problem in wave space with a spatially linear velocity field, and the general solution can be written explicitly (Saffman 1963). The interaction between scales is explicitly non-local with the larger scales, given by the Kolmogorov length, straining the smaller scales of the far dissipation range. Batchelor (1958) assumes the strain rates are constant in time, but Kraichnan (1968) points out that a rapidly varying strain rate would alter the solution significantly at high wavenumber.

We propose a similar situation at low Reynolds number, where the straining scales are again the dissipation scales, but now the energy and dissipation ranges are one and the same. It still seems reasonable to consider the interaction of disparate length and time scales and to linearize the vorticity equation, and we are led to the same convection-decay equation in wave space encountered by Batchelor and Kraichnan at high Reynolds numbers. However, we can no longer assume that dissipation (straining) occurs in distinct material regions, intermittent in space and constant in time, and we must account for the energy decay and the increasing length and time scales of the straining scales. Therefore it seems natural to consider the unsteady initial-value problem of Kraichnan rather than the steady boundary-value problem of Batchelor. Kraichnan's approach leads to an energy spectrum with a $exp(-\alpha k)$ form at high wavenumber that is closer to the experimental measurements of Ling & Huang (1970) than is the $exp(-\alpha k^2)$ form predicted by Batchelor.

7. Conclusions

The major conclusion from this work is that the energy transfer in low Reynolds number turbulence is due to triad interactions that involve one short and two long legs of comparable lengths.

The energy is transfered mainly between the pair of large wavenumbers i.e. between comparable small scales, with one small eddy losing energy to a somewhat smaller one. In that sense the energy transfer is predominantly *local* since the eddies exchanging energy are of similar size. However, the triads responsible for such transfer are *nonlocal* since in addition to two small scales, one large scale of motion (small wavenumber) is involved.

Interactions of this type seem to be inconsistent with the older as well as modern (RNG) concepts of eddy viscosity that postulate nonlocal interactions moving energy from one large scale to two small ones. In that context it is important to distinguish between local (nonlocal) *energy transfer* and local (nonlocal) *triad interactions*. Energy transfer is often implicitly assumed to involve only two scales of motion: if they are similar,the transfer is local, if they are disparate the transfer is nonlocal. Triad interactions are local if all three legs of the triad are of comparable lengths and nonlocal if one of the legs is much shorter than the other two. With this distinction the eddy viscosity notion is based on the assumption that both the energy transfer and the triad interactions are nonlocal. The results of our work point toward *local energy transfer* that is caused by *nonlocal triad interactions*.

It seems possible that this transfer process can be described by a linear theory similiar to that of the far dissipation range at high Reynolds numbers, but first it is necessary to inquire in more detail about the space-time structure of the dissipation scales.

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