N89-24644

SOME COMMENTS ON GLOBAL-LOCAL ANALYSES

Satya N. Atluri
Georgia Institute of Technology
Atlanta, Georgia

ABSTRACT

The main theme of this paper concerns methods that may be classified as global (approximate) and local (exact). Some specific applications of these methods are found in:

(1) Fracture and fatigue analysis of structures with 3-D surface flaws

(2) Large-deformation, post-buckling analysis of large space trusses and space frames, and their control

(3) Stresses around holes in composite laminates

A typical engineering problem is illustrated in figure 1, which shows a corner flaw at the intersection of a nozzle and a pressure vessel. The shape of the surface flaw may often be approximated mathematically as quarter-elliptical or quarter-circular. For the problem shown in figure 1, wherein the crack is located in the longitudinal plane of symmetry of the structure, only the so-called Mode I conditions prevail. In figure 1, the presence of a traction-free crack, in an otherwise unflawed solid, alters the stress-state only locally. From a viewpoint of fracture mechanics, however, the main quantities of interest are only the stress-intensity factors (strengths of asymptotic stress singularities) near the crack front. For analyzing fatigue crack growth and crack instability under thermal shock various flaw sizes and shapes need to be considered. The primary objective of analysis is to determine the variation of the Mode I stress-intensity factor along the border on the surface flaw.

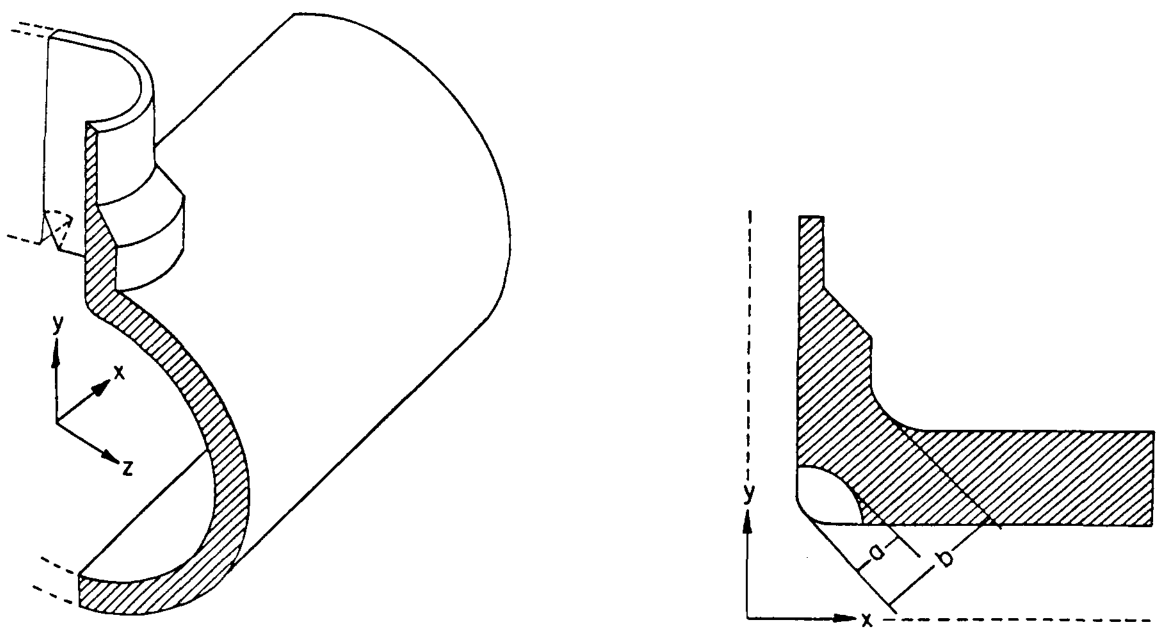


Figure 1. Corner surface-flaw at the pressure-vessel-nozzle intersection.

Figure 2 shows the schematic of a 12-bay space frame. The equations of dynamic motion of the frame, assuming large deformations and plasticity, may be written as:

$$\underset{\sim}{M}\,{}^{(N+1)}\ddot{\underset{\sim}{X}} + \underset{\sim}{C}\,{}^{(N+1)}\dot{\underset{\sim}{X}} + {}^{t}\underset{\sim}{K}\,\Delta\underset{\sim}{X} = \underset{\sim}{f}_C + \underset{\sim}{Q}_E - {}^{(N)}\underset{\sim}{R}$$

where $\underset{\sim}{M}$ is the mass matrix, $\underset{\sim}{C}$ the matrix of passive damping, ${}^{t}\underset{\sim}{K}$ the tangent stiffness matrix (which includes the effect of large deformation and plasticity), $\underset{\sim}{f}_C$ the control-actuator force, $\underset{\sim}{Q}_E$ the external load, ${}^{(N+1)}\ddot{\underset{\sim}{X}}$ the acceleration vector at time t_{N+1}, ${}^{(N+1)}\dot{\underset{\sim}{X}}$ the velocity vector at t_{N+1}, $\Delta\underset{\sim}{X}$ the incremental displacement between t_N and t_{N+1}, and ${}^{(N)}\underset{\sim}{R}$ the internal-force vector at t_N. In order to implement the control algorithms in an efficient manner, the order of the above system of equations must be as small as possible (i.e., each frame member must be modeled by no more than one finite element). Further, the control must be implemented for pulse-type loading of high intensity, such that the above system of equations must be integrated directly rather than using a modal-decomposition. Also requirements of on-line control may necessitate that ${}^{t}\underset{\sim}{K}$, $\underset{\sim}{C}$, and $\underset{\sim}{M}$ be known explicitly (in closed form) for arbitrary values of deformation, without the need for introducing approximate shape functions for deformation of each element and without the need for any numerical integrations over each element. In figure 2, the object of inquiry is what effect does local (member) instability have on global (system) stability? How can we control the dynamic deformations locally to improve global behavior? Each member may be treated as a truss member, or a 3-D beam-type member, depending on joint design. How can local effects be accounted for simply and efficiently, so that algorithms for control of dynamic motion may be implemented, on line, using on-board computers in a large space structure?

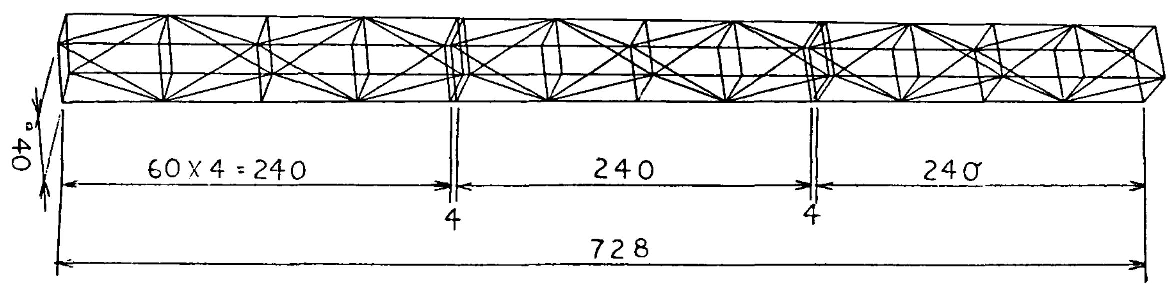


Figure 2. Schematic of a 12-bay space frame.

An appraisal of the computational mechanics methods is given in figure 3. These methods include the finite-element, boundary-element, and edge function methods (fig. 3).

FINITE ELEMENTS:

- TRIAL AND TEST FUNCTIONS ARE BOTH APPROXIMATE
- TRIAL AND TEST FUNCTIONS ARE, IN GENERAL, ALIKE - GALERKIN APPROACH
- IN SOME INSTANCES IT IS BEST TO HAVE TEST FUNCTIONS DIFFERENT FROM TRIAL FUNCTIONS - PETROV-GALERKIN APPROACH
- THE SOLUTION IS BOTH GLOBALLY AND LOCALLY APPROXIMATE
- VERSATILE OR ARBITRARY GEOMETRY, BOUNDARY CONDITIONS, SUITED FOR GLOBALLY APPROXIMATE NONLINEAR SOLUTIONS

BOUNDARY ELEMENTS:

- TEST FUNCTIONS ARE GLOBALLY EXACT FOR THE GIVEN LINEAR PROBLEM, OR AT LEAST FOR THE HIGHEST-ORDER DIFFERENTIAL OPERATOR OF THE PROBLEM
- TRIAL FUNCTIONS ARE APPROXIMATE (AT BOUNDARY ONLY FOR LINEAR PROBLEMS, AND IN INTERIOR ALSO FOR NONLINEAR PROBLEMS)
- THE SOLUTION IS BOTH LOCALLY AND GLOBALLY APPROXIMATE
- NOT AS VERSATILE AS THE FINITE-ELEMENT METHOD, BUT EXCELLENT FOR SOME SPECIFIC PROBLEMS

EDGE FUNCTION METHOD:

- TRIAL FUNCTIONS ARE GLOBALLY EXACT
- TEST FUNCTIONS ASSUMED ONLY AT BOUNDARY
- LIMITED TO LINEAR PROBLEMS POSED BY CLASSICAL DIFFERENTIAL EQUATIONS

Figure 3. Appraisal of computational mechanics methods.

In the most commonly used Galerkin finite-element approach in computational solid mechanics, the trial and test function spaces are identical and consist of simple piecewise continuous algebraic polynomials over each finite element, such that these functions and their appropriate-order derivatives (as dictated by the problem on hand) are continuous at the interelement boundaries. For problems of fourth or higher order, such as those of plates and shells, the development of finite elements has long been, and continues to be, plagued by the need for C^1 (or higher order) continuity at the interelement boundaries. However, the success of the finite-element method in structural mechanics is unparalleled and is mainly due to the intuitive and 'geometric' interpretation of the method. The method is versatile in its ability to deal with complicated structural assemblies, such as of beams, plates, and shells, of the type used in aerospace applications. The solutions obtained through the finite-element method may be classified, in general, as being both globally as well as locally approximate.

On the other hand, in linear and nonlinear solid mechanics, it is often possible to derive certain integral representations for displacements. A key ingredient which makes such derivations possible is the singular solution, in an infinite space, of the corresponding differential equation (in certain linear problems) or of the highest-order differential operator (in the nonlinear case, or even in the linear case when the full linear equation cannot be conveniently solved), for a 'unit' load applied at a generic point in the infinite space. When the problem is linear and the singular solution can be established for the complete linear differential equation of the problem, the aforementioned integral representations for displacements involve only boundary integrals of unknown trial functions and their appropriate derivatives. Such an integral representation, when discretized, leads to the so-called boundary-element method. Such pure boundary-element methods are possible in linear, isotropic, elastostatics, and in problems of static bending of linear elastic isotropic plates. On the other hand, as in such cases as (i) linear problems wherein the singular solutions cannot be established for the entire differential equations, (ii) anisotropic materials, and (iii) problems of large deformation and material inelasticity, the integral representations (if any) for displacements would involve not only boundary integrals but also interior-domain integrals of the trial functions and/or their derivatives. A discretization of such integral equations would lead not only to a simple boundary-element method but also to a sort of hybrid boundary/interior element method.

When asymptotic solutions to the governing differential equations of the problem are used as assumed trial functions, the interior residual error is zero, and only the boundary conditions need to be satisfied in a weighted residual method. Such an approach is called the edge-function method, but is limited mostly to linear problems. For further details, see references 1 through 4.

The surface-flow problems for current methods are noted in figure 4. Problems for the proposed method are also shown.

All Present Methods: Singular stress-state near the flaw border is modeled by locally approximate methods

(I) Finite-Element Methods (singular elements)
Atluri & Kathiresan (refs. 5-10)
(Hybrid crack elements) 3-6,000 d.o.f.
Tracy, Barsoum, Newman & Raju (refs. 11-13)
(Distorted isoparametric elements and singular shape fn.) 5-10,000 d.o.f.
These are very expensive, but accommodate arbitrary geometries of structure and flaw.

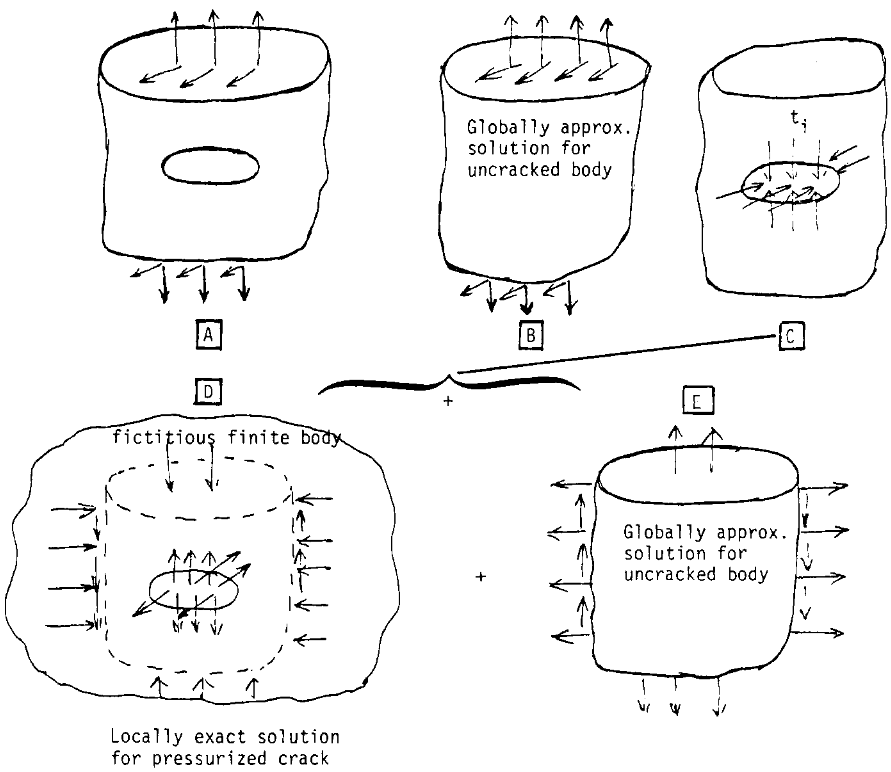
(II) Boundary-Element Methods (for locally approximate stress analysis and K-estimation from stress extrapolation)
Cruse (ref. 14), Heliot et al. (ref. 15).
Not suitable for 'thin' shells with flaws.
Still very expensive.

(III) Line-Spring Method
Limited to simple geometries of structure and flaw.

Proposed Method:

- It is a GLOBALLY APPROXIMATE, but LOCALLY EXACT METHOD
- Singular stress-state near the flaw is NOT MODELED NUMERICALLY
- It is about 30 times cheaper than the singular finite-element method
- Details (Atluri & Nishioka (refs. 16-21) - several papers with varied examples)

Figure 4. Surface-flaw problems.



Remarks: 1. Solution D: A rather complicated analytical solution (Atluri & Vijayakumar, Journal of Applied Mechanics, 1981) (ref. 22)

2. Local solution due to crack-face traction alone is (i.e., the Solution C) the source of singularity. The stresses due to this local solution decay very rapidly. Only one or two iterations are sufficient to obtain K-solutions with 1% accuracy.

About 30 times cheaper than the usual finite-element method for a typical problem such as flawed BWR nozzle.

Figure 5. Global (approximate) and local (exact) analyses of embedded flaws.

Some comments concerning the solution of surface flaws in finite bodies using the present procedure are in order (fig. 4). Since the analytical solution of an elliptical flaw embedded in an infinite solid is used as solution D, it is necessary to define the residual stresses over the entire crack-plane including the fictitious portion of the crack which lies outside of the finite body containing only a surface flaw (fig. 5). Moreover it is well known that the accuracy of the 'least-squares' type function-interpolation inside the interpolated region can be increased with the number of polynomial terms; however, the interpolating curve may change drastically outside of the region of interpolation. The optimum variation of pressure on the crack surface extended into the fictitious region should be as shown in figure 6. For in-depth discussions of a variety of surface problems and their solutions, see refs. 19-21, 23-28.

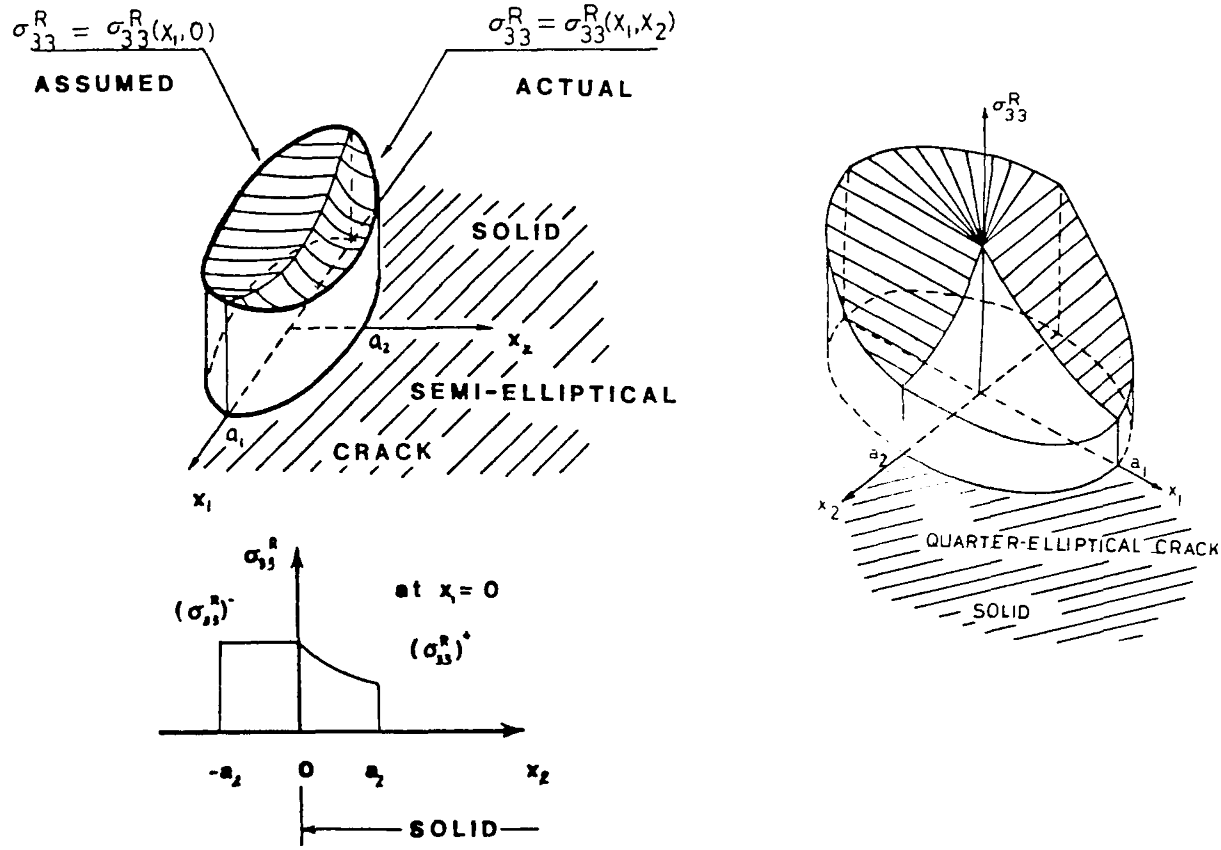


Figure 6. Postulated residual stress distributions on fictitious portions of an elliptical flaw, in the case of semi-elliptical or quarter-elliptical surface flaws.

For both 3-dimensional truss and frame members, explicit (locally exact) tangent stiffness matrices have been derived (fig. 7). Some effects of local (member) buckling on global (structural) behavior are illustrated in figures 8 and 9.

<u>Truss Member:</u>

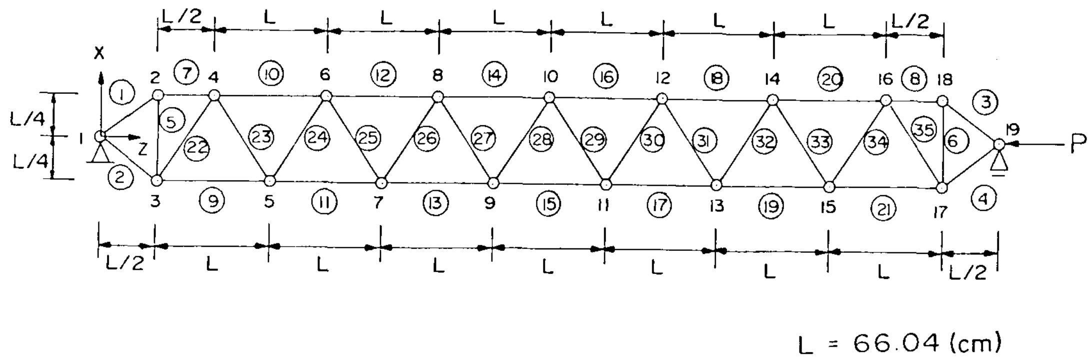
- Each member undergoes large displacement and large rigid rotation
- Member material is nonlinear
- Each member may buckle and become curved (what effect does it have on global stability?)

<u>Frame Member:</u>

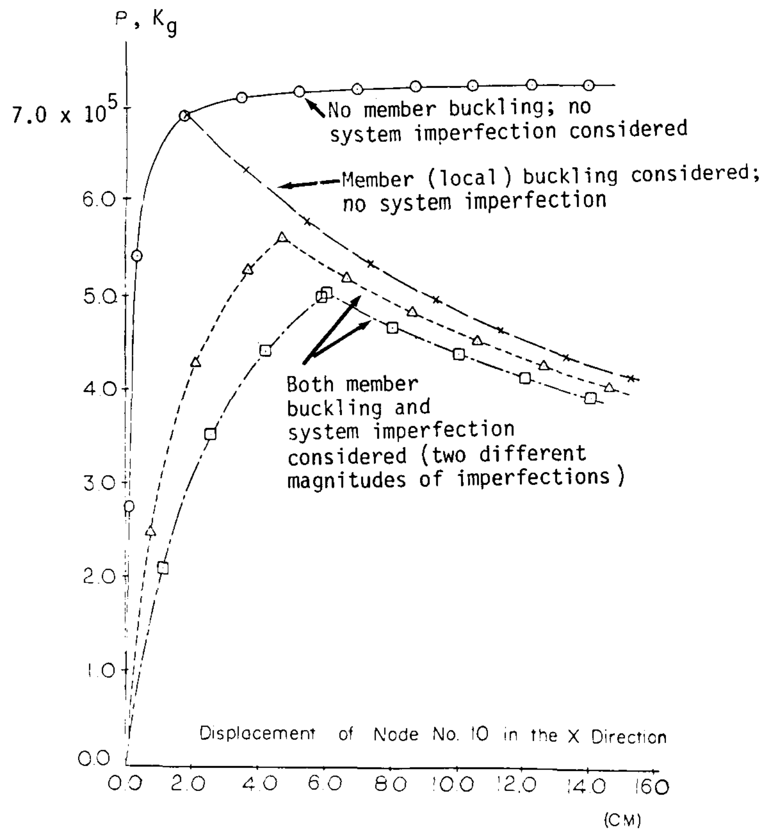
- Concepts of 3-dimensional semi-tangential rotations employed
- Each member undergoes arbitrarily large rigid rotations and rigid displacements
- Bending-stretching coupling incorporated in each member
- Plastic-hinge method used to account for plasticity in each member
- Member forces: axial, shear, and bending-twisting moments

Kondoh and Atluri (refs. 29-30) and Kondoh et al. (refs. 31-32)

Figure 7. Space trusses and space frames.



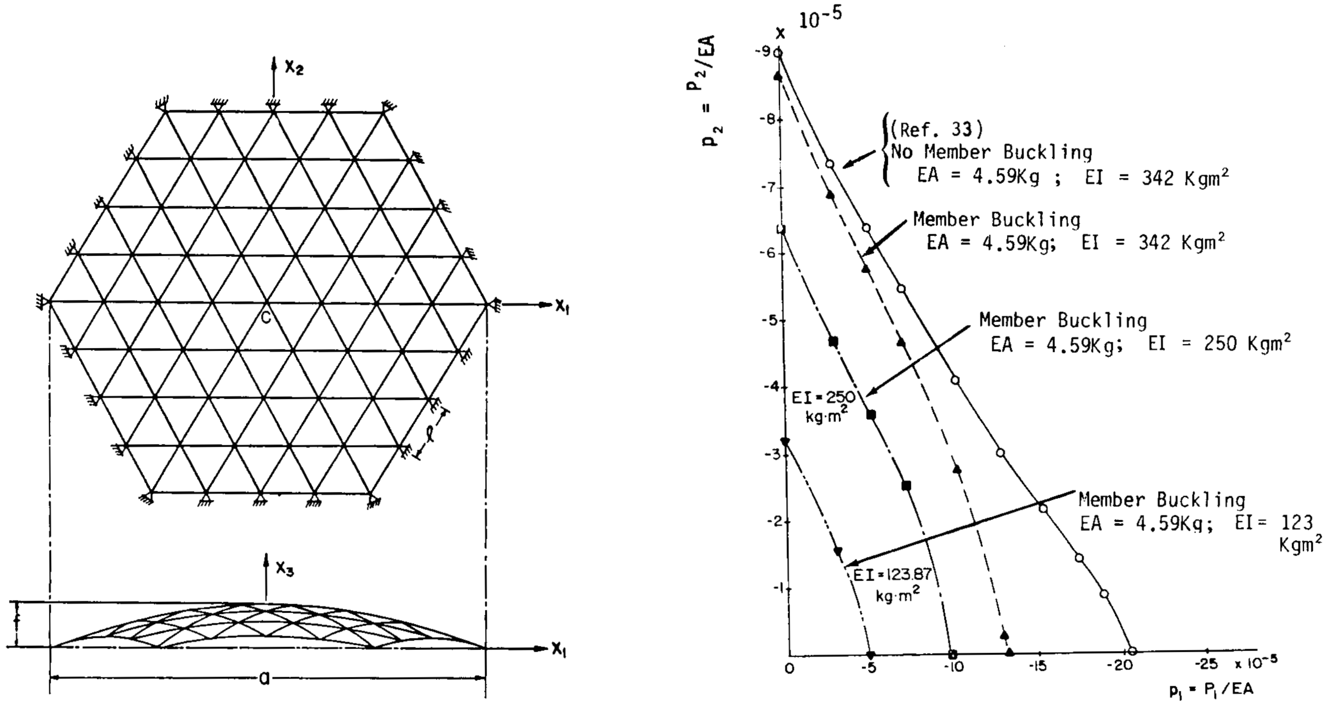
(a) Thompson's Strut



(b) Effect of local (member) buckling on global (structural) behavior.

Tangent stiffness of each member is exact in both the pre-buckled and post-buckled states of member (ref. 29)

Figure 8. Thompson's strut and effect of local buckling on global behavior.



(a) Load System: (i) P_1: Vertical point loads as all nodes; (ii) P_2: Vertical point loads at nodes in quadrants x_1, x_2 > 0

(b) Stability boundary under loads P_1 and P_2

Figure 9. Study of the effect of member buckling on global (system) stability.

Examples of the efficiency of the global/local approach in analyzing frames are illustrated in figures 10 and 11. In figure 10, the classical problem of a two-bar frame is schematically illustrated. In the present approach, the tangent stiffness matrix of each member (respresented by a single finite element) is derived from exact solutions of governing differential equations which account for the bending-stretching coupling. Thus, no "shape functions" are assumed in each element, and no numerical integrations are performed in forming the tangent stiffness matrix. The present numerical integrations are performed in forming the tangent stiffness matrix. The present numerical results are shown to agree excellently with those of Wood and Zienkiewicz (ref. 34), as well as the experimental results of Williams (ref. 35). However, Wood and Zienkiewicz use five finite elements to model each member of the frame.

In figure 11, the problem of plastic collapse of a frame is illustrated. Here again, the tangent stiffness matrix of each member (represented by a single finite element) is derived in closed form, accounting for large deformations, large rotations, and plasticity. A plastic-hinge method is used, and the progressive development of plastic hinges, at various load levels, is indicated in figure 11.

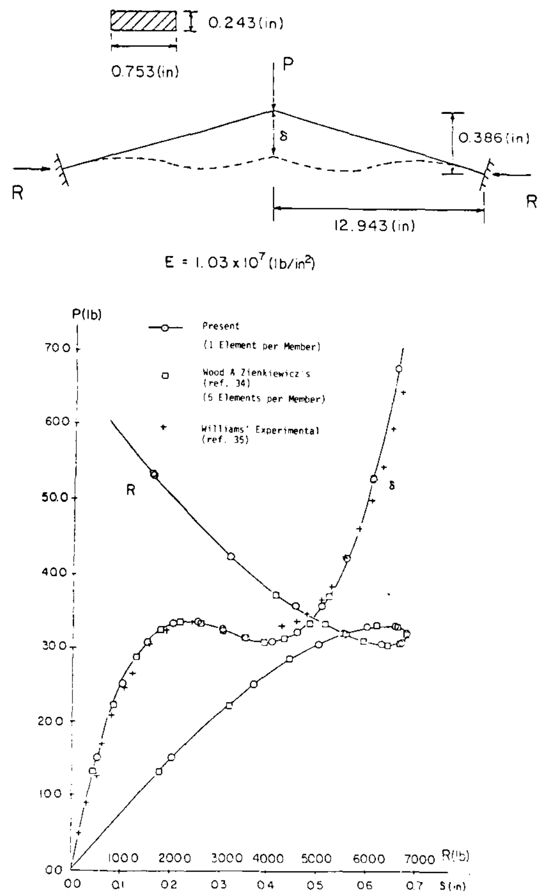
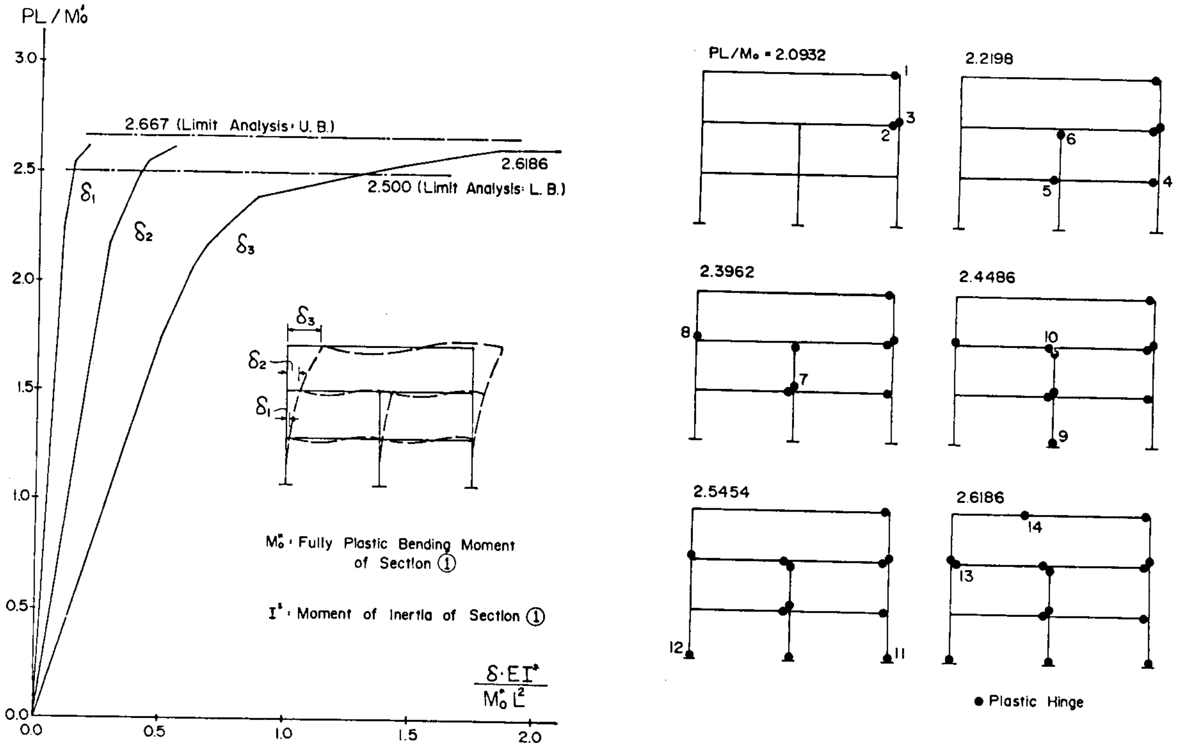


Figure 10. Variation of load-point displacement and support reaction with applied load in a two-bar frame. Tangent stiffness of each 3-D beam member undergoing large deformation, large rotation, and plasticity is exact. Locally exact solution (ref. 30).

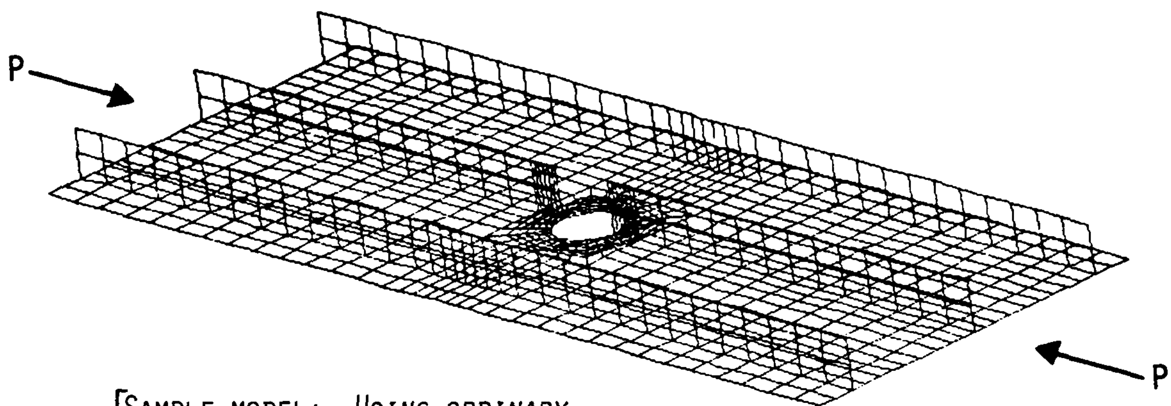


- Tangent stiffness of each member undergoing large deformation, large rotation and plasticity is Explicit and Exact. Plastic-hinge method used.
- Extension to crash analysis of frames being studied.

Figure 11. Plastic collapse of frame.

Figure 12 shows a problem of current interest in the analysis of stiffened composite plates. Issues involve the following: (1) stress concentrations near the hole in a composite laminate, (2) local buckling of stiffeners, (3) effect of geometric imperfections, (4) effect of discontinuities, and (5) three-dimensional effects and delaminations near the hole. An efficient globally approximate and locally exact approach could possibly include: (1) use of locally exact, laminated hole elements with embedded three-dimensional stress state (refs. 36 and 37), (2) use of locally exact stiffener elements as described earlier (ref. 32), (3) techniques for proper interacting of various elements, and (4) hole elements that can be improved by incorporating possible free-edge singularities in σ_{3i}.

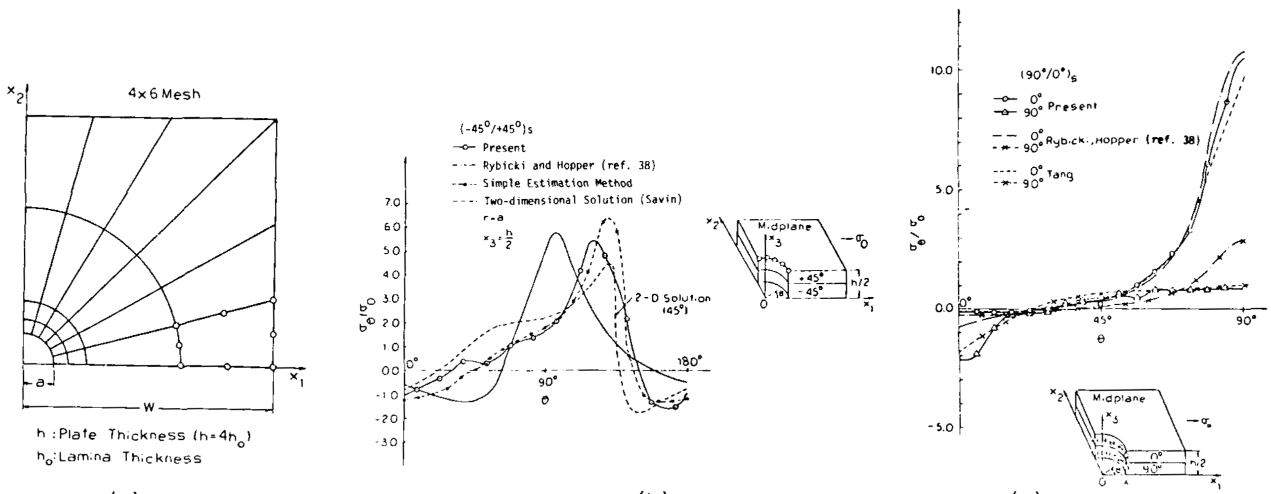
FOCUS PROBLEM



[SAMPLE MODEL: USING ORDINARY (GLOBALLY & LOCALLY APPROXIMATE) FINITE ELEMENTS]

Figure 12. A stiffened laminated-composite panel with a hole.

Another example of the advantages of using a global/local approach is illustrated here in the problem of analysis of stresses near a hole in a laminated composite [two cases of $(-45/+45)_s$ and $(90/0)_s$ laminates are discussed]. Figure 13a shows a typical finite-element model with "special-hole elements" in which a 3-D asymptotic hole solution is embedded. Figures 13b and 13c illustrate the excellent accuracy obtained from the present approach, in comparison with a fully 3-D finite-element solution of Rybicki and Hopper (ref. 38). The present solution is, however, an order of magnitude less expensive. Further details are given in references 36 and 37.



(a) TYPICAL FEM MODEL OF A LAMINATE WITH HOLE, 3-D ASYMPTOTIC "HOLE-SOLUTIONS" EMBEDDED IN ELEMENTS NEAR THE HOLD (Ref. 36)

(b) STRESS AROUND A HOLE IN $(-45/+45)_S$ LAMINATE

(c) STRESS AROUND A HOLE IN $(90/0)_S$ LAMINATE

Figure 13. Analysis of stress state near a hole in laminated composites.

The following conclusions and recommendations are given.

- Hybrid analytical/numerical methodologies should be explored
- Simplified analysis procedures for elasto-plastic should be considered (Dynamic response calculations should be studied (some benchmark problems essential))
- Constitutive models badly need improvement
- Methods of coupling of problem-specific methodologies for use in general purpose programs should be explored
- Trends to treat structural mechanics problems as continuum mechanics problems should be critically reviewed; the knowledge base in structural mechanics should be fruitfully utilized
- Attempts to bridge the gap between micromechanics and macromechanics of heterogeneous (composite) media through computational mechanics should be explored
- Computational stochastic structural analysis methods should be developed
- Algorithms for new computing systems (MIMD) should be explored
- Expert systems, ... (?)

NASA's role should be to provide:

- Increased research support to universities
- Predoctoral NASA fellowships (up to 20K per year, tax-free) that could be awarded to attract the best students
- Long-range funding to properly plan and sustain high-quality research efforts
- Increased access to supercomputers
- Frequent visits to NASA facilities by graduate students to participate in laboratory testing. University facilities in this area are scarce; students in computational mechanics should get some first-hand experience in experimental mechanics

REFERENCES

1. Atluri, S.N. and Kathiresan, K., "Influence of Flaw Shapes on Stress Intensity Factors for Pressure Vessel Surface Flaws and Nozzle Corner Cracks", Journal of Pressure Vessel Technology, Vol. 102, August 1980, pp. 78-286.

2. Atluri, S.N. (Editor and Contributor to Part II, Finite Element Methods), Finite Element Handbook, McGraw-Hill Book Co., 1200 pp. (In Press).

3. Atluri, S.N., "Computational Solid Mechanics (Finite Elements and Boundary Elements): Present Status and Future Directions", (Keynote Paper) in Proc. 4th International Conference on Applied Numerical Modeling (Eds: H.M. Hsia, et al.), pp. 19-37, 1984.

4. O'Donoghue, P.E. and Atluri, S.N., "Control of Dynamic Response of a Continuum Model of a Large Space Structure", AIAA 26th Structures, Structural Dynamics & Materials Conference, Orlando, Florida, April 15-17, 1985.

5. Atluri, S.N., Kathiresan, K., and Kobayashi, A.S., "Three-Dimensional Linear Fracture Mechanics Analysis by a Displacement-Hybrid Finite-Element Model", Trans. of the 3rd International Conference on Structural Mechanics in Reactor Technology, London, Vol. 5, Part L, Paper No. L-7/3, September 1975, 13 pp.

6. Atluri, S.N. and Kathiresan, K., "On a 3-D 'Singularity Element' for Computation of Combined Mode Stress Intensities", Advances in Engineering Science, Vol. 1, November 1976, pp. 267-274.

7. Atluri, S.N., Kathiresan, K., Kobayashi, A.S., and Nakagaki, M., "Inner Surface Cracks in an Internally Pressurized Cylinder Analyzed by a Three-Dimensional Displacement-Hybrid Finite Element Method", Pressure Vessel Technology, Part II - Materials and Fabrication, ASME, 1977, pp. 527-533.

8. Atluri, S.N. and Kathiresan, K., "Outer and Inner Surface Flaws in Thick-Walled Pressure Vessels", Trans. of 4th International Conference on Structural Mechanics in Reactor Technology, Paper G 5/4, 1977, 13 pp.

9. Atluri, S.N. and Kathiresan, K., "Stress Analysis of Typical Flaws in Aerospace Structural Components Using 3-D Hybrid Displacement Finite Element Method", Proceedings of 19th AIAA/ASME/SAE Structures, Structural Dynamics and Materials Conference, Bethesda, Maryland, 1978, pp. 340-351.

10. Atluri, S.N. and Kathiresan, K., "3-D Analysis of Surface Flaws in Thick-Walled Reactor Pressure Vessels Using Displacement-Hybrid Finite Element Method", Nuclear Engineering & Design," Vol. 51, No. 2, January 1979, pp. 163-176.

11. Tracy, D.M., "Finite Elements for Three-Dimensional Elastic Crack Analysis", Nuclear Engineering & Design, Vol. 16, 1977, pp. 282-290.

12. Barsoum, R.S., "On the Use of Isoparametric Finite Elements in Linear Fracture Mechanics", International Journal for Numerical Methods in Engineering, Vol. 10, No. 1, 1976, pp. 25-37.

13. Newman, J.C., Jr. and Raju, I.S., "Stress-Intensity Factor Equations for Cracks in Three-Dimensional Finite Bodies Subjected to Tension and Bending Loads", (S.N. Atluri, Ed.) Computational Method in the Mechanics of Fracture, North-Holland Publishing Co. (In Press).

14. Cruse, T.A., "Boundary-Integral Equation Method for Three-Dimensional Elastic Fracture Mechanics", AFOSR-TR-75-0813, U.S. Air Force, 1975, 52 pp.

15. Heliot, J., Labbens, R.C., and Pellissier-Tannon, A., "Semi-Elliptical Cracks in a Cylinder Subjected to Stress Gradients", in Fracture Mechanics, ASTM STP 677, 1979, pp. 341-364.

16. Atluri, S.N. and Nishioka, T., "Analytical Solution for Embedded Elliptical Cracks, and Finite Alternating Method for Elliptical Surface Cracks, Subjected to Arbitrary Loadings", Engineering Fracture Mechanics, Vol. 17, No. 3, 1982, pp. 247-268.

17. Atluri, S.N. and Nishioka, T., "Integrity Analyses of Surface-Flawed Aircraft Attachment Lugs: A New, Inexpensive, 3-D Alternating Method", Proceedings of 22nd AIAA/ASME/ASCE/AHS Structures and Material Conference, New Orleans, Louisiana, 1982, pp. 287-301.

18. Atluri, S.N. and Nishioka, T., "Analysis of Surface Flaw in Pressure Vessels by a New 3-Dimensional Alternating Method", Aspects of Fracture Mechanics in Pressure Vessels and Piping, PVP, Vol. 58, ASME, New York, 1982, pp. 17-37. Also Journal of Pressure Vessel Technology, ASME Vol. 104, 1982, pp. 299-307.

19. Atluri, S.N. and Nishioka, T., "Stress Intensity Factor Variation with Loading and Crack Depth for Long Flaws of Semi-Elliptical Shape", Advances in Aerospace Structures, AD Vol. 2, ASME, 1982, pp. 47-50.

20. Atluri, S.N. and Nishioka, T., "An Inexpensive 3-D Finite Element-Alternating Method for the Analysis of Surface Flawed Aircraft Structural Components", AIAA Journal, Vol. 21, No. 5, 1983, pp. 749-758.

21. Atluri, S.N. and Nishioka, T., "Multiple Semi-Elliptical Surface Cracks in Pressure Vessels", Journal of Pressure Vessel Technology, Trans. ASME, Vol. 54, 1982, pp. 612-619.

22. Atluri, S.N. and Vijayakumar, K., "An Embedded Elliptical Crack, in an Infinite Solid, Subject to Arbitrary Crack-Face Tractions", Journal of Applied Mechanics, Vol. 103, No. 1, 1981, pp. 88-96.

23. O'Donoghue, P.E., Nishioka, T., and Atluri, S.N., "Multiple, Coplanar, Embedded Elliptical Cracks, in an Infinite Solid Subject to Arbitrary Crack Face Tractions", International Journal of Numerical Methods in Engineering, Vol. 21, 1985, pp. 437-449.

24. Atluri, S.N. and Nishioka, T., "Computational Methods for Three-Dimensional Problems of Fracture", in Computational Methods in the Mechanics of Fracture (S.N. Atluri, Editor), North-Holland (In Press).

25. O'Donoghue, P.E., Nishioka, T., and Atluri, S.N., "Multiplie Surface Cracks in Pressure Vessels", Engineering Fracture Mechanics, Vol. 20, No. 3, 1984, pp. 545-560.

26. Atluri, S.N., "Computational and Theoretical Studies on Dynamic Fracture Mechanics and Three-Dimensional Crack Problems", Keynote Lecture, Proceedings of ICF International Symposium on Fracture Mechanics, Science Press, Beijing, 1983, pp. 1-17.

27. O'Donoghue, P.E., Nishioka, T., and Atluri, S.N., "Analysis of Interaction Behavior of Surface Flaws in Pressure Vessels", Journal of Pressure Vessels Technology, Trans. ASME, 1984.

28. Punch, E.F. and Atluri, S.N., Development and Testing of Stable, Invariant, Isoparametric Curvilinear 2- and 3-D Hybrid Stress Elements", Computer Methods in Applied Mechanics and Engineering, Vol. 47, 1984, pp. 331-356.

29. Kondoh, K. and Atluri, S.N., "Influence of Local Buckling on Global Instability: Simplified, Large Deformation, Post-Buckling Analyses of Plane Trusses", Computers and Structures, Vol. 21, No. 4, 1985, pp. 613-628.

30. Kondoh, K. and Atluri, S.N., "A Simplified Finite Element Method for Large Deformation, Post-Buckling Analyses of Large Frame Structures, Using Explicitly Derived Tangent Stiffness Matrices", International Journal of Numerical Methods in Engineering, Vol. 23, No. 1, 1986, pp. 69-90.

31. Kondoh, K., Tanaka, K., and Atluri, S.N., "LSS-NLAP: A Program for Simplified Nonlinear Analysis of Large Space-Trusses and Frames , Using Explicity Derived Tangent Stiffnesses, and Accounting for Local Buckling", Report to Air Force Wright Aeronautical Laboratories, 1985, 171 pp, USAF, WPAFB.

32. Tanaka, K., Kondoh, K., and Atluri, S.N., "Instability Analysis of Space Trusses Using Exact Tangent-Stiffness Matrices" in Finite Element Analysis & Design, North-Holland, Vol. 1, No. 4, 1985, pp. 291-312.

33. Noor, A.K. and Peters, J.M., "Instability Analysis of Space Trusses", Computer Methods in Applied Mechanical Engineering", Vol. 40, 1983, pp. 199-218.

34. Wood, R.D. and Zienkiewicz, O.C., "Geometrically Nonlinear Finite Element Analysis of Beams, Frames, Arches, and Axisymmetric Shells", Computers and Structures, Vol. 7, 1977, pp. 725-735.

35. Williams, F.W., "An Approach to the Nonlinear Behavior of Members of a Rigid Jointed Framework With Finite Deflections", Quant. J. Mech. Appl. Math., Vol. 17, No. 4, 1964, pp. 451-469.

36. Atluri, S.N. and Nishioka, T., "Stress Analysis of Holes in Angle-Ply Laminates: An Efficient Assumed Stress 'Special-Hole-Element", Approach and a Simple Estimation Method", Computers and Structures, Vol. 15, November 1982, pp. 135-147.

37. Atluri, S.N. and Nishioka, T., "A Simple Estimation Method of Stress Intensity Factors for Through Cracks in Angle-Ply Laminates", Engineering Fracture Mechanics, Vol. 16, No. 4, 1982, pp. 573-583.

38. Rybicki, E.F. and Hopper, A.T., "Analytical Investigation of Stress Concentrations Due to Holes in Fiber Reinforced Plastic Laminated Plates: Three-Dimensional Models", AFML-TR-73-100, Battelle Columbus Labs, 1973.